Few-Body Systems 0, 1–6 (2014)

Few-Body Systems

Quark-model hadron structure

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Abstract. We review some selected recent results on hadron spectroscopy and related theoretical studies based on constituent quark models.

1 Introduction

Hadron spectroscopy has undergone a great renaissance in recent years [1]. The new findings include: low-lying excitations of D and B mesons, long-awaited missing states and new states near 4 GeV/c^2 in the charmonium spectrum, charmed and bottom baryons, and evidence for doubly charmed baryons. The light hadron sector remains also restless reporting new scalar mesons or showing a deep theoretical interest in the high energy part of both the meson and baryon spectra.

The hadron spectra should be described in terms of QCD. Tested to very high accuracy in the perturbative regime, its low energy sector (*strong QCD*) comprising hadron physics, remains challenging because neither lattice nor perturbative methods are accurate. As in most other areas of physics, the keys to a qualitative understanding of strong QCD are to identify the appropriate degrees of freedom and the effective forces between them [2]. All roads lead to valence constituent quarks as the appropriate degrees of freedom [3]. The effective forces, summarizing the basic properties of QCD, must at least contain: a confining mechanism, a spin-spin force and a long-range term. This framework is what we know as the constituent quark model. Although quark models differ in their details, the qualitative aspects of their spectra are determined by features that they share in common. These common ingredients can be used to project expectations for new sectors [4].

The limitations of the quark model are as obvious as its successes. Nevertheless almost all hadrons can be classified as relatively simple configurations of a few confined quarks. Nowadays, we have the tools to deepen our understanding of strong QCD. On one side we have at our disposal powerful numerical techniques imported from few-body physics: Faddeev calculations in momentum space [5], hyperspherical harmonic expansions [6] and stochastic variational methods [7]. On the other we have an increasing number of experimental data. In this work

Table 1. Experimentally known heavy baryons.

State	J^P	$Q=s$	$Q=c$	$Q=b$	J^P	$Q=s$	$Q=c$	$Q=b$
$\Lambda(udQ)$	$1/2^+$	1116,1600	2286,2765*	5625	$1/2^-$	1405,1670	2595	
	$3/2^+$	1890	2940*		$3/2^-$	1520,1690	2628,2880*	
$\Sigma(uuQ)$	$1/2^+$	1193,1660	2454,2980*	5811*	$1/2^-$	1480,1620	2800*	
	$3/2^+$	1385,1840	2518,3077*	5833*	$3/2^-$	1560,1670		
$\Xi(usQ)$	$1/2^+$	1318	2469,2577	5792*	$1/2^-$		2790	
	$3/2^+$	1530	2645		$3/2^-$	1820	2815	
$\Omega(ssQ)$	$1/2^+$		2698		$1/2^-$			
	$3/2^+$	1672	2770*		$3/2^-$			

we review some recent results and selected theoretical analysis on heavy baryons, heavy mesons, and light baryons.

2 Heavy baryons

It was already in 1985 when Bjorken wrote [8]: ”We should strive to study triply charmed baryons because their excitation spectrum should be close to the perturbative QCD regime”. The larger the number of heavy quarks the simpler the system. In particular, doubly and triply heavy baryons are driven only by a perturbative one-gluon exchange (OGE), while single heavy baryons include the dynamics of light and heavy-light quark pairs. Baryons with one, two or three heavy quarks are the ideal laboratory to test the assumed flavor independence of confinement [9]. Table 1 resumes the experimental situation of strange, charmed and bottom baryons. The number of experimental data in the charm and bottom sectors is increasing rapidly, states denoted by a star have been reported within the last two years with quantum numbers still not determined.

The dynamics of the light-quark pair plays a relevant role in the strange sector, being mainly responsible for the spin splitting. Thus, it is expected a similar contribution for charmed and bottom baryons. The experimental spin splitting has been measured in the charm and bottom sectors obtaining: $M[\Sigma_c(3/2^+)] - M[\Lambda_c(1/2^+)] = 232$ MeV, $M[\Sigma_c(3/2^+)] - M[\Sigma_c(1/2^+)] = 64$ MeV, $M[\Sigma_b(3/2^+)] - M[\Lambda_b(1/2^+)] = 209$ MeV, and $M[\Sigma_b(3/2^+)] - M[\Sigma_b(1/2^+)] = 22$ MeV. These results are rather well reproduced with quark models containing only gluons or gluons and pions in the light quark dynamics.

Table 2. Contribution of the one-pion exchange (OPE) to the baryon mass [9].

M(MeV)	Full	No OPE	ΔE	M(MeV)	Full	No OPE	ΔE
$\Sigma_b(1/2^+)$	5807	5820	−13	$\Sigma(1/2^+)$	1408	1417	−9
$\Sigma_b(3/2^+)$	5829	5839	−10	$\Sigma(3/2^+)$	1454	1462	−8
$\Lambda_b(1/2^+)$	5624	5804	−180	$\Lambda(1/2^+)$	1225	1405	−180
$\Lambda_b(3/2^+)$	6388	6388	<1				

In Table 2 we show the contribution of pions separately, noting how pions give the same contribution for strange and bottom baryons. Therefore, the contribution of gluons is diminished for models considering pions. If we now go to doubly charmed baryons, where pions do not contribute, the predictions are parameter free and experiment will confirm or defeat these results giving hints on the underlying dynamics of the system:

$$\begin{aligned} M[\Xi_{cc}(3/2^+)] - M[\Xi_{cc}(1/2^+)] &= 66 \text{ MeV} \\ M[\Omega_{cc}(3/2^+)] - M[\Omega_{cc}(1/2^+)] &= 54 \text{ MeV}\,. \end{aligned} \tag{1}$$

3 Heavy mesons

More than thirty years after the so-called November revolution [8], heavy meson spectroscopy is being severely tested by new experiments [1]. This challenging situation arose in the open-charm sector with the discovery of the $D_{sJ}^*(2317)$, the $D_{sJ}(2460)$ and the $D_0^*(2308)$ mesons, positive parity states with masses smaller than expectations from quark potential models. One could say in general that the area phenomenologically understood in the open-charm meson spectrum extends to states where the $q\bar{q}$ pair is in relative $S-$wave. In the positive parity sector, $P-$wave states, is where the problems arise. This has been said as an example where naive quark models are probably too naive [3]. Out of the many explanations suggested for these states, the unquenching of the naive quark model has been successful [10]. When a $q\bar{q}$ pair occurs in a $P-$wave but can couple to hadron pairs in $S-$wave the latter will distort the $q\bar{q}$ picture. In the examples mentioned above, the 0^+ and 1^+ $c\bar{s}$ states predicted above the $DK(D^*K)$ thresholds couple to the continuum. This mixes $DK(D^*K)$ components in the wave function. This idea can be easily formulated in terms of a meson wave-function described by

$$|\psi\rangle = \sum_i \alpha_i \, |q\bar{q}\rangle_i + \sum_j \beta_j \, |qq\bar{q}\bar{q}\rangle_j \tag{2}$$

where q stands for quark degrees of freedom and the coefficients α_i and β_j take into account the admixture of four-quark components in the $q\bar{q}$ picture.

Results for the open-charm mesons [10] show that they are easily identified with standard $c\bar{q}$ states except for the $D_{sJ}^*(2317)$, the $D_{sJ}(2460)$, and the $D_0^*(2308)$. Thus, one could be tempted to interpret them as four-quark resonances within the quark model. Other results obtained with the same interacting potential [10] are: for $cn\bar{s}\bar{n}$, $(J^P, I) = (0^+, 0)$ 2731 MeV, $(0^+, 1)$ 2699 MeV, $(1^+, 0)$ 2841 MeV, $(1^+, 1)$ 2793 MeV, and for $cn\bar{n}\bar{n}$, $(0^+, 1/2)$ 2505 MeV. All of them are far above the corresponding strong decay threshold and therefore broad in contrast to experiment, what rules out a pure four-quark interpretation.

Thus, physical states may correspond to a mixing of two- and four-body configurations, Eq. (2). The results obtained are shown in Table 3 In the non-strange sector once the mixing is considered one obtains a state at 2241 MeV with 46% of four-quark component and 53% of $c\bar{n}$ pair. The lowest state, representing the $D_0^*(2308)$, is above the isospin preserving threshold $D\pi$, being broad as observed experimentally. The mixed configuration compares much better with

Table 3. Probabilities, in %, of the wave function components and masses (QM), in MeV, of the open-charm mesons once the mixing is considered [10].

$J^P=0^+(I=0)$			$J^P=1^+(I=0)$			$J^P=0^+(I=1/2)$		
QM	2339	2847	QM	2421	2555	QM	2241	2713
Exp.	2317.4	–	Exp.	2459.3	2535.3	Exp.	2308±36	–
P($cn\bar{s}\bar{n}$)	28	55	P($cn\bar{s}\bar{n}$)	25	~ 1	P($cn\bar{n}\bar{n}$)	46	49
P($c\bar{s}_{1^3P}$)	71	25	P($c\bar{s}_{1^1P}$)	74	~ 1	P($c\bar{n}_{1P}$)	53	46
P($c\bar{s}_{2^3P}$)	~ 1	20	P($c\bar{s}_{1^3P}$)	~ 1	98	P($c\bar{n}_{2P}$)	~ 1	5

the experimental data than the pure $c\bar{n}$ state. The orthogonal state appears at 2713 MeV, with and important four-quark component. In the strange sector, the $D^*_{sJ}(2317)$ and the $D_{sJ}(2460)$ are dominantly $c\bar{s}$ $J=0^+$ and $J=1^+$ states, respectively, with almost 30% of four-quark component. Without being dominant, this percentage is fundamental to shift the mass of the unmixed states to the experimental values below the DK and D^*K thresholds and, therefore, they are expected to have small widths.

The above arguments have also open the discussion about the presence of compact four-quark states in the charmonium spectrum, with special emphasis on the nature of the $X(3872)$. It is a member of an heterogeneous group, including the $Y(2460)$ and the recently reported $Z(4430)$, whose properties make their identification as traditional $q\bar{q}$ states unlikely. Although some caution is still required an isoscalar $J^{PC}=1^{++}$ state seems to be the best candidate to describe the $X(3872)$ properties.

Charmonium four-quark states have been studied solving the four-body Schrödinger equation using the hyperspherical harmonic (HH) formalism [6] with two standard quark-quark interaction models: one based only on the one-gluon exchange (BCN), and the other containing also boson exchanges (CQC). As can be seen in Table 4 there appear no bound states for any set of quantum numbers, including the suggested assignments of the $X(3872)$, 1^{++}. Independently of the quark-quark interaction and the quantum numbers considered, the system evolves to a well separated two-meson state. Thus, in any manner one can claim for the existence of a bound state for the $c\bar{c}n\bar{n}$ system unless additional

Table 4. Mass, in MeV, of the different J^{PC} $c\bar{c}n\bar{n}$ states, E_{4q}, calculated including up to $K_{\max}$ HH, and difference with the lowest two-meson threshold, Δ_E [6].

	CQC		BCN			CQC		BCN	
$J^{PC}(K_{\max})$	E_{4q}	Δ_E	E_{4q}	Δ_E	$J^{PC}(K_{\max})$	E_{4q}	Δ_E	E_{4q}	Δ_E
0^{++} (24)	3779	+34	3249	+75	0^{--} (17)	3791	+108	3405	+172
0^{+-} (22)	4224	+64	3778	+140	0^{-+} (17)	3839	+94	3760	+105
1^{++} (20)	3786	+41	3808	+153	1^{--} (19)	3969	+97	3732	+94
1^{+-} (22)	3728	+45	3319	+86	1^{-+} (19)	3829	+84	3331	+157
2^{++} (26)	3774	+29	3897	+23	2^{--} (21)	4054	+52	4092	+52
2^{+-} (28)	4214	+54	4328	+32	2^{-+} (21)	3820	+75	3929	+55

ingredients either in the interaction or in the wave function are considered.

4 Light baryons

The high energy part of the baryonic spectrum has been a subject of interest in the last decade, the aim being to get a better understanding of the confinement mechanism and the hadronization process. In particular the idea of a parity multiplet classification scheme at high excitation energies as due to chiral symmetry was suggested some years ago [11] and put in question later on [12].

The use of a quark-quark screened confining potential supplemented by a minimal OGE (coulomb+hyperfine) [13], allows to obviate the missing state problem. A correct prediction of the number and ordering of the known N and Δ resonances, up to 2.4 GeV mass is obtained. The unambiguous assignment of quantum numbers to the dominant configuration of any J^P ground and first non-radial states up to $J=11/2$ translates into a well defined symmetry pattern.

In Table 5 we group experimental resonances according to their dominant configuration. States denoted by a question mark have not an assigned mass in the Particle Data Review. To express the spatial part we use the quantum numbers $(K, L, Symmetry)$. K, defines the parity of the state, $P=(-)^K$, and its centrifugal barrier energy. L is the total orbital angular momentum. *Symmetry* specifies the spatial symmetry. A look at the table makes manifest the underlying $SU(4)\otimes O(3)$ symmetry providing a $(20, L^P)$ classification scheme, the 20plet structure coming out naturally from the product of irreducible quark representations: $4\otimes 4\otimes 4 = 20_S \oplus 20_M \oplus 20_M \oplus \overline{4}$.

From the spectral pattern represented by Table 5 experimental regularities and degeneracies for $J \geq 5/2$ ground states come out: 1.- $E_{N,\Delta}(J+2) - E_{N,\Delta}(J) \approx 400-500$ MeV; 2.- $N(J^\pm) \approx \Delta(J^\pm)$ for $J=\frac{4n+3}{2}$, $n=1,2...$; 3.- $N(J^+) \approx N(J^-)$ for $J=\frac{4n+1}{2}$, $n=1,2.$; and 4.- $(N(J), \Delta(J))^\bullet \approx$

Table 5. Dominant spatial-spin configurations for $N's$ and $\Delta's$.

$(K, L, Symmetry)$	$S=1/2$	$S=3/2$
$(0,0,[3])$	$N(1/2^+)(940)$	$\Delta(3/2^+)(1232)$
$(2,2,[3])$	$N(5/2^+)(1680), N(3/2^+)(1720)$	$\Delta(7/2^+)(1950)$
$(4,4,[3])$	$N(9/2^+)(2220)$	$\Delta(11/2^+)(2420)$
$(6,6,[3])$	$N(13/2^+)(**)(2700)$	$\Delta(15/2^+)(**)(2950)$
$(2,0,[21])$	$N(1/2^+)(***)(1710),\ \Delta(1/2^+)(1750)$	
$(2,2,[21])$	$N(5/2^+)(**)(2000),\ \Delta(5/2^+)(1905)$	$N(7/2^+)(**)(1990)$
$(4,4,[21])$	$N(9/2^+)(2220),\ \Delta(9/2^+)(**)(2300)$	$N(11/2^+)(?)$
$(6,6,[21])$	$N(13/2^+)(2700),\ \Delta(13/2^+)(?)$	$N(15/2^+)(?)$
$(1,1,[21])$	$N(3/2^-)(1520), N(1/2^-)(1535)$ $\Delta(3/2^-)(1700), \Delta(1/2^-)(1620)$	$N(5/2^-)(1675)$
$(3,3,[21])$	$N(7/2^-)(2190),\ \Delta(7/2^-)(*)(2200)$	$N(9/2^-)(2250)$
$(5,5,[21])$	$N(11/2^-)(***)(2600),\ \Delta(11/2^-)(?)$	$N(13/2^-)(?)$
$(3,3,[3])$	$N(7/2^-)(?)$	$\Delta(9/2^-)(**)(2400)$
$(5,5,[3])$	$N(11/2^-)(?)$	$\Delta(13/2^-)(**)(2750)$

Table 6. Predicted $N's$ and $\Delta's$ [13].

$J=7/2$	$N(7/2^+)^\bullet(2220)$	$N(7/2^-)^\bullet(2250)$		$\Delta(7/2^-)^\bullet(2400)$
$J=9/2$	$N(9/2^+)^\bullet(2450)$	$N(9/2^-)^\bullet(2600)$	$\Delta(9/2^+)^\bullet(2420)$	$\Delta(9/2^-)^\bullet(2650)$
$J=11/2$	$N(11/2^+)(2450)$			$\Delta(11/2^-)(2650)$
	$N(11/2^+)^\bullet(2700)$	$N(11/2^-)^\bullet(2650)$	$\Delta(11/2^+)^\bullet(2850)$	$\Delta(11/2^-)^\bullet(2750)$
$J=13/2$		$N(13/2^-)(2650)$	$\Delta(13/2^+)(2850)$	
	$N(13/2^+)^\bullet(2900)$		$\Delta(13/2^+)^\bullet(2950)$	
$J=15/2$	$N(15/2^+)(2900)$			

$(N(J+1), \Delta(J+1))$. The black dot denotes the first non-radial excitation. Taking into account these rules and the symmetry pattern one can make predictions for, until now, unknown states from 2 to 3 GeV, Table 6. They may serve of some help to guide future experimental searches.

Acknowledgement. This work has been partially funded by MCyT under Contract No. FPA2007-65748 and by JCyL under Contract No. SA016A07.

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First received November 30, 2007.