A multiquark description of the $D_{sJ}(2860)$ and $D_{sJ}(2700)$

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Abstract

Within a theoretical framework that accounts for all open-charm mesons, including the $D_0^*(2308)$, the $D_{sJ}^*(2317)$ and the $D_{sJ}(2460)$, we analyze the structure and explore possible quantum number assignments for the $D_{sJ}(2860)$ and the $D_{sJ}(2700)$ mesons reported by BABAR and Belle Collaborations. The open-charm sector is properly described if considered as a combination of conventional quark-antiquark states and four-quark components. All negative parity and 2^+ states can be understood in terms only of $q\bar{q}$ components, however the description of the 0^+ and 1^+ mesons is improved whenever the mixing between two- and four-quarks configurations is included. We analyze all possible quantum number assignments for the $D_{sJ}(2860)$ in terms of both $c\bar{s}$ and $cn\bar{s}\bar{n}$ configurations. We discuss the role played by the electromagnetic and strong decay widths as basic tools to distinguish among them. The broad structure reported by BABAR near 2.7 GeV is also analyzed.

Keywords: Charm mesons, Charm-strange mesons, quark models.

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The open-charm sector does not cease to amaze both theorists and experimentalists with new states that defy our understanding of the heavy-meson spectra. Two new mesons have recently joined the open-charm zoo. BABAR Collaboration reported the observation of a new D_s state, the $D_{sJ}(2860)$, with a mass of $2856.6\pm1.5\pm5.0$ MeV and a width of $48\pm7\pm10$ MeV in the analysis of the DK spectra. No structures seem to appear in the D^*K invariant mass distribution in the same range of masses. This state was reported together with a broad enhancement near 2.7 GeV with a tentative mass of $2688\pm4\pm2$ MeV and a width $112\pm7\pm36$ MeV [1]. Since the only reported decay channels observed by BABAR correspond to two pseudoscalar mesons, the assignment of natural parity quantum numbers, $J^P = 0^+, 1^-, 2^+, 3^-, \dots$, is strongly favored. The second state, the $D_{sJ}(2700)$, was observed by Belle Collaboration in the decay $B^+ \to \overline{D^0}D^0K^+$ with a mass of $2708\pm9^{+11}_{-10}$ MeV, a width of $108\pm23^{+30}_{-31}$ MeV, and quantum numbers $J^P = 1^-$ [2]. The $D_{sJ}(2860)$ is not observed in the Belle data.

The $D_{s,l}(2860)$ and $D_{s,l}(2700)$ mesons are new members of a long list of charm resonances reported during the last few years. In 2003 BABAR reported the observation of a charmstrange state, the $D_{s,I}^*(2317)$, with a mass of 2316.8±0.4±3 MeV and a width of less than 4.6 MeV [3]. This state was soon after confirmed by CLEO [4] and Belle Collaborations [5]. Besides, BABAR also pointed out the existence of another charm-strange meson, the $D_{sJ}(2460)$ [3]. This resonance was measured by CLEO [4] and confirmed by Belle [5] with a mass of $2457.2\pm1.6\pm1.3$ MeV and a width less than 5.5 MeV. Belle results are consistent with the spin-parity assignments of $J^P = 0^+$ for the $D_{sJ}^*(2317)$ and $J^P = 1^+$ for the $D_{sJ}(2460)$. Thus, these two states are definitively well established and confirmed independently by different experiments. In the nonstrange sector Belle reported the observation of a nonstrange broad scalar resonance named D_0^* with a mass of $2308 \pm 17 \pm 15 \pm 28$ MeV and a width $276 \pm 21 \pm 18 \pm 60$ MeV [6]. A state with similar properties has been suggested by FOCUS Collaboration [7] during the measurement of excited charm mesons D_2^* . SELEX Collaboration at Fermilab [8] has reported an state with a mass of 2632.5 ± 1.7 MeV and a width smaller than 17 MeV. However, up to now no other experiment has been able to confirm the existence of this resonance [1, 9].

The positive parity open-charm mesons present unexpected properties quite different from those predicted by quark potential models if a pure $c\bar{q}$ configuration is considered. If they would correspond to standard P-wave mesons made of a charm quark and a light antiquark their masses would be larger, around 2.48 GeV for the $D_{sJ}^*(2317)$, 2.55 GeV for the $D_{sJ}(2460)$, and 2.46 GeV for the $D_0^*(2308)$. They would be therefore above the DK, D^*K , and $D\pi$ thresholds respectively, being broad resonances. However, as stated above, the $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ are very narrow. In the case of the $D_0^*(2308)$ the large width observed would be expected but not its low mass. Although there are several theoretical interpretations for the masses and widths of some of the positive parity states $D_0^*(2308)$, $D_{sJ}^*(2317)$, and $D_{sJ}(2460)$ [10], in Ref. [11] it was shown that the difficulties to identify the three of them with conventional $c\overline{q}$ mesons are rather similar to those appearing in the lightscalar meson sector and may be indicating that other configurations, as for example fourquark components, may be playing a role. $q\bar{q}$ states are more easily identified with physical hadrons when virtual quark loops are not important. This is the case of the pseudoscalar and vector mesons, mainly due to the P-wave nature of this hadronic dressing. On the contrary, in the scalar sector it is the $q\bar{q}$ pair the one in a P-wave state, whereas quark loops may be in a S-wave. In this case the intermediate hadronic states that are created may play a crucial role in the composition of the resonance, in other words unquenching may be important. The vicinity of these components to the lightest $q\bar{q}$ state implies that they have to be considered. This has been shown as a possible interpretation of the low-lying light-scalar mesons, where the coupling of the scalar $q\bar{q}$ nonet to the lightest $qq\bar{q}\bar{q}$ configurations allows for an almost one-to-one correspondence between theoretical states and experiment [12]. The possible role played by $\text{non}-q\bar{q}$ components in the description of the $D_{sJ}(2860)$ was illustrated in Ref. [13]. In this work it was proposed that this state could be understood within a unitarized meson model as a quasi-bound $c\bar{s}$ state coupled with the nearby S—wave DK threshold, therefore being the first radial excitation of the $D_{sJ}^*(2317)$.

In non-relativistic quark models gluon degrees of freedom are frozen and therefore the wave function of a zero baryon number (B=0) hadron may be written as

$$|B = 0\rangle = \Omega_1 |q\bar{q}\rangle + \Omega_2 |qq\bar{q}\bar{q}\rangle + \dots$$
 (1)

where q stands for quark degrees of freedom and the coefficients Ω_i take into account the mixing of two– and four–quark states. $|\mathbf{B}=0\rangle$ systems could then be described in terms of a hamiltonian

$$H = H_0 + H_1 \quad \text{being} \quad H_0 = \begin{pmatrix} H_{q\bar{q}} & 0\\ 0 & H_{qq\bar{q}\bar{q}} \end{pmatrix}$$

and
$$H_1 = \begin{pmatrix} 0 & V_{q\bar{q} \leftrightarrow qq\bar{q}\bar{q}}\\ V_{q\bar{q} \leftrightarrow qq\bar{q}\bar{q}} & 0 \end{pmatrix}, \tag{2}$$

where H_0 is a constituent quark model hamiltonian described below and H_1 , that takes into account the mixing between $q\overline{q}$ and $qq\bar{q}\bar{q}$ configurations, includes the annihilation operator of a quark-antiquark pair into the vacuum. This operator could be described using the 3P_0 model, however, since this model depends on the vertex parameter, we prefer in a first approximation to parametrize this coefficient by looking to the quark pair that is annihilated and not to the spectator quarks that will form the final $q\overline{q}$ state. Therefore we have taken $V_{q\overline{q} \leftrightarrow qq\overline{q}\overline{q}} = \gamma$. If this coupling is weak enough one can solve independently the eigenproblem for the hamiltonians $H_{q\overline{q}}$ and $H_{qq\overline{q}\overline{q}}$, treating H_1 perturbatively. The two-body problem has been solved exactly by means of the Numerov algorithm [14]. The four-body problem has been solved by means of a variational method using the most general combination of gaussians as trial wave functions [15]. In particular, the so-called mixed terms (mixing the various Jacobi coordinates) that are known to have a great influence in the light quark case have been considered.

It is our purpose in this work to use a standard constituent quark model that provides with a good description of the meson and baryon spectra and also the baryon-baryon phenomenology for the description of the open-charm mesons. For this purpose, we will address the study of hadrons with zero baryon number described as clusters of quarks confined by a realistic interaction. The model is based on the assumption that the constituent quark mass appears because of the spontaneous breaking of the original $SU(3)_L \otimes SU(3)_R$ chiral symmetry at some momentum scale. As a consequence of such a symmetry breaking, quarks acquire a constituent mass and Goldstone bosons are exchanged between the quarks. Beyond the chiral symmetry breaking scale, one expects the dynamics to be governed by QCD perturbative effects, that are taken into account through the one-gluon-exchange potential. Finally, any model imitating QCD should incorporate another nonperturbative effect, confinement. It remains an unsolved problem to derive confinement from QCD in an analytic manner. The only indication we have on the nature of confinement is through lattice studies, showing that $q\bar{q}$ systems are well reproduced at short distances by a linear potential. Such

TABLE I: $c\overline{s}$ masses (QM), in MeV, below 3 GeV. Experimental data (Exp.) are taken from Ref. [17].

$nL J^P$	State	QM	Exp.
$1S \ 0^{-}$	D_s	1981	1968.49 ± 0.34
$2S \ 0^-$	_	2699	_
$1S \ 1^{-}$	D_s^*	2112	$2112.3 {\pm} 0.5$
$2S 1^-$	_	2764	_
$1P \ 0^{+}$	$D_{sJ}^*(2317)$	2489	$2317.8 {\pm} 0.6$
$2P \ 0^{+}$	_	2966	_
$1P \ 1^{+}$	$D_{sJ}(2460)$	2578	$2459.6 {\pm} 0.6$
$1P \ 1^{+}$	$D_{s1}(2536)$	2543	$2535.35 \pm 0.34 \pm 0.5$
$1P \ 2^{+}$	$D_{s2}(2573)$	2582	$2572.6 {\pm} 0.9$
$1D \ 1^{-}$	_	2873	_
$1D \ 2^{-}$	_	2883	_
$1D \ 3^{-}$	_	2882	_

potential can be physically interpreted in a picture in which the quark and the antiquark are linked with a one-dimensional color flux tube. The spontaneous creation of light-quark pairs may give rise to a breakup of the color flux tube, what has been proposed that translates into a screened potential [16], in such a way that the potential saturates at some interquark distance. Explicit expressions of the interacting qq and $q\bar{q}$ potentials and a more detailed description of the model can be found in Refs. [12, 14] where the various parameters are given.

A thoroughly study of the full meson spectra has been presented in Ref. [14], with special attention in Ref. [11] to the open-charm sector. Using this model we have calculated the $c\bar{s}$ masses up to 3 GeV listed in Table I. It can be seen how the open-charm states are easily identified with standard $c\overline{q}$ mesons except for the cases of the $D_{sJ}^*(2317)$ and the $D_{sJ}(2460)$. This behavior is shared by almost all quark potential model calculations [10]. Although the situation from lattice QCD is far from being definitively established, similar difficulties have been observed both in quenched and unquenched approaches [18]. The same conclusion may also be drawn from heavy quark symmetry arguments. Within this approach the scalar $c\bar{s}$ state belongs to the j=1/2 doublet, but since the j=3/2 doublet is identified with the narrow $D_{s2}(2573)$ and $D_{s1}(2536)$ (with total widths of 15^{+5}_{-4} MeV and < 2.3 MeV, respectively) the scalar state is expected to have a much larger width than the one measured for the $D_{s,I}^*(2317)$ [19]. Thus, one possibility for these states beyond the naive $q\bar{q}$ assignment is to interpret them as four-quark resonances within the quark model. The results obtained for the $cn\bar{s}\bar{n}$ and $cn\bar{n}\bar{n}$ configurations in Ref. [11] using the constituent quark model outlined above are shown in Table II. It can be seen that the I=1 and I=0states obtained are far above the corresponding strong decay threshold and therefore should be broad, what rules out a pure four-quark interpretation of the positive-parity open-charm mesons.

As discussed above, for P-wave mesons the hadronic dressing is in a S-wave, thus physical states may correspond to a mixing of two- and four-body configurations, Eq. (1).

TABLE II: $cn\bar{s}\bar{n}$ and $cn\bar{n}\bar{n}$ masses, in MeV.

cn	$\bar{s}\bar{n}$	$cn\bar{n}\bar{n}$
$J^P = 0^+$	$J^P = 1^+$	$J^P = 0^+$
I=0 $I=1$	$I = 0 \ I = 1$	I = 1/2
2731 2699	2841 2793	2505

TABLE III: Probabilities (P), in %, of the wave function components and masses (QM), in MeV, of the open-charm mesons with I=0 (left) and I=1/2 (right) once the mixing between $q\bar{q}$ and $qq\bar{q}\bar{q}$ configurations is considered. Experimental data (Exp.) are taken from Ref. [17] for I=0 and from Ref. [6] for I=1/2.

I = 0					I = 1/2			
	$J^P = 0^+ J^P = 1^+$			$J^P = 0^+$				
QM	2339	2847	QM	2421	2555	QM	2241	2713
Exp.	2317.8 ± 0.6	_	Exp.	2459.6 ± 0.6	$2535.35 \pm 0.34 \pm 0.5$	Exp.	$2308 \pm 17 \pm 32$	_
$P(cn\bar{s}\bar{n})$	28	55	$P(cn\bar{s}\bar{n})$	25	~ 1	$\mathrm{P}(cn\bar{n}\bar{n})$	46	49
$P(c\bar{s}_{1^3P})$	71	25	$P(c\bar{s}_{1^1P})$	74	~ 1	$P(c\bar{n}_{1P})$	53	46
$P(c\bar{s}_{2^3P})$	~ 1	20	$P(c\bar{s}_{1^3P})$	~ 1	98	$P(c\bar{n}_{2P})$	~ 1	5

In the isoscalar sector, the $cn\bar{s}\bar{n}$ and $c\bar{s}$ states get mixed, as it happens with $cn\bar{n}\bar{n}$ and $c\bar{n}$ for the I=1/2 case. The parameter γ was fixed in Ref. [11] to reproduce the mass of the $D_{sJ}^*(2317)$ meson, being $\gamma=240$ MeV. The results obtained are shown in Table III. From these results one can appreciate that the description of the positive parity open-charm mesons improves when four–quark components are considered.

With respect to the new resonance reported by BABAR, it can be see from Tables I and III that among all possibilities only three states are close to its experimental mass, $2856.6\pm1.5\pm5.0$ MeV. They correspond to the 0^+ $c\bar{s}+cn\bar{s}\bar{n}$ (45% and 55% probability respectively) excitation and the 1⁻ and 3⁻ $c\bar{s}$ D—waves, being their energies 2847, 2873, and 2882 MeV. All other possibilities, like for instance the 2S 1⁻ or the 2P 2⁺, are more than 100 MeV above or below the experimental energy. The 2S 1⁻ excitation obtained within our model with an energy of 2764 MeV is a good candidate to be identified with the $D_{sJ}(2700)$ reported by Belle. Concerning the broad bump reported by BABAR at 2.7 GeV, if different from the $D_{sJ}(2700)$, two states appear as possible candidates, the 2S 0⁻ radial $c\bar{s}$ excitations and the isovector 0⁺ $cn\bar{s}\bar{n}$ ground state, both of them with a mass of 2699 MeV.

From the analysis of the masses alone it is not possible to distinguish among all candidates for the new resonances. However, the structure of the D_{sJ}^* mesons could be scrutinizedy apart from their masses, also through the study of their decay widths. The strong decay width of a hypothetical $J^P = 0^+ D_{sJ}(2860)$, either $c\bar{s}$ or $cn\bar{s}\bar{n} + c\bar{s}$, into D^*K or DK^* is forbidden due to quantum number conservation. This is consistent with the absence of DK^* or D^*K

TABLE IV: $\Gamma[D_{sJ}(2860) \to D^*K]/\Gamma[D_{sJ}(2860) \to DK]$ ratio and $\Gamma[D_{sJ}(2860) \to DK]$ strong decay width in MeV for different approaches.

J^P	$\frac{\Gamma[D_s]}{\Gamma[D_s]}$	$J(2860) \to D^*K$ $SJ(2860) \to DK$	$\Gamma[D_{sJ}(2860) \to DK]$		
	Exp:	Not observed	Exp:	$48{\pm}7\pm10~\mathrm{MeV}$	
	[20]	[21]	[20]	[21]	
1-	0.17	0.06	84	>1000	
3-	0.59	0.37	22	Narrow	

signals in the experimental data [1]. The ratio $\Gamma[D_{sJ}(2860) \to D^*K]/\Gamma[D_{sJ}(2860) \to DK]$ has been studied for several different quantum numbers using the ${}^{3}P_{0}$ model [20] and arguments based on heavy quark expansions [21]. Some of their results are quoted in Table IV. A possible assignment L=2 $J^P=1^-$ would result in a too large $\Gamma[D_{sJ}(2860) \to DK]$ decay width, although very different values are obtained depending on the approach considered, while the L=2 $J^P=3^ \Gamma[D_{sJ}(2860)\to D^*K]/\Gamma[D_{sJ}(2860)\to DK]$ ratio seems to indicate a sizable $\Gamma[D_{sJ}(2860) \to D^*K]$ decay width that has not yet been observed. Concerning the electromagnetic decay widths of the $J^P = 0^+$ candidate, the formalism necessary to study the $\Gamma[c\bar{s} \to c\bar{s} + \gamma]$, $\Gamma[cn\bar{s}\bar{n} \to cn\bar{s}\bar{n} + \gamma]$, and $\Gamma[cn\bar{s}\bar{n} \to c\bar{s} + \gamma]$ processes has been described in detail in Ref. [11], where it was proposed that the ratio R = $D_{sJ}(2460) \to D_s^+ \gamma/D_{sJ}(2460) \to D_s^{*+} \gamma$ could be an important tool to distinguish between a $q\bar{q}$ structure $(R\approx 1)$ or a $qq\bar{q}q+q\bar{q}$ one $(R\approx 100)$ for the open-charm mesons. Following this formalism one obtains for the electromagnetic decay $\Gamma[D_{sJ}(2860) [0^+] \to D_s^* \gamma] = 13.67$ keV. At the same time, if a pure 1P $c\bar{s}$ structure is assumed for the $D_{sJ}(2317)$ then the 0^+ $D_{sJ}(2860)$ should correspond to a 2P excitation. One can also evaluate this decay obtaining a much smaller value, $\Gamma[D_{sJ}(2860) \, 0^+ \to D_s^* \, \gamma] = 1.8$ eV, due to the presence of a node in the 2P wave function. Therefore, the mixed scenario would produce a sizable value for the electromagnetic decay width $\Gamma[D_{sJ}(2860) \to D_s^* \gamma]$ only if an scalar state with a dominant four-quark component is present.

If the $D_{sJ}(2860)$ is definitively confirmed as a scalar meson, this will point to the existence of a non-strange partner with an energy of 2713 MeV and an important four–quark component (49%), being this result in the same line as the one reported in Ref.[13]. Furthemore, an isovector 1^+ $cn\bar{s}\bar{n}$ state with a mass of 2793 MeV is also predicted.

The interpretation we have just presented for the new resonances measured by BABAR and Belle within a formalism that includes the mixing of two— and four—quark states has also been used to account for the other experimentally observed open-charmed states and also for the light-scalar mesons within the same constituent quark model [11, 12]. It is therefore the first time that a coherent analysis of all known states within the meson spectra in terms of two— and four—quark states is performed, what gives us confidence on the mechanism proposed. Nonetheless, one should not forget that in the literature there is a wide variety of interpretations for the open-charm mesons. Therefore, the final answer could only be obtained from precise experimental data that would allow to discriminate between the predictions of different theoretical models [22].

As a summary, we have obtained a rather satisfactory description of the open-charm mesons in terms of two— and four—quark configurations, including the new states recently reported by BABAR and Belle. The mixing between these two components is responsible for

the unexpected low mass and widths of the $D_{sJ}^*(2317)$, $D_{sJ}(2460)$, and $D_0^*(2308)$ and also offers a possible interpretation for the $D_{sJ}(2860)$ as a scalar meson. The electromagnetic and strong decay widths give hints that would help in distinguishing the nature of these states. In particular, the study of the decays $\Gamma[D_{sJ}(2860) \to D_s^* \gamma]$ and $\Gamma[D_{sJ}(2860) \to D^* K]$ are ideally suited for this task. A clear signal for this electromagnetic decay mode together with the absence of the strong one would point to an scalar state with an involved structure in terms of two– and four–quark components. The 1⁻ state at 2708 MeV observed by Belle can be interpreted as a $c\bar{s}$ 2S excitation whereas for the broad bump around 2.7 GeV reported by BABAR two candidates can be found, although more experimental data is needed before drawing any conclusion. We encourage experimentalists on the confirmation of the results reported by BABAR and Belle and on the measurement of the electromagnetic and strong decay widths of the open-charm positive parity states. Such a study would help to distinguish not only among the possible quantum numbers allowed for the new BABAR resonance, but also to clarify the exciting situation of the open-charm mesons and the role played by multiquark configurations in the meson spectra.

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