# An explanation of the $\Delta_{5/2^-}(1930)$ as a $\rho\Delta$ bound state.

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## Abstract

We use the $\rho\Delta$ interaction in the hidden gauge formalism to dynamically generate $N^*$ and $\Delta^*$ resonances. We show, through a comparison of the results from this analysis and from a quark model study with data, that the $\Delta_{5/2^-}(1930)$, $\Delta_{3/2^-}(1940)$ and $\Delta_{1/2^-}(1900)$ resonances can be assigned to $\rho\Delta$ bound states. More precisely the $\Delta_{5/2^-}(1930)$ can be interpreted as a $\rho\Delta$ bound state whereas the $\Delta_{3/2^-}(1940)$ and $\Delta_{1/2^-}(1900)$ may contain an important $\rho\Delta$ component. This interpretation allows for a solution of a long-standing puzzle concerning the description of these resonances in constituent quark models. In addition we also obtain degenerate $J^P = 1/2^-, 3/2^-, 5/2^-$ $N^*$ states but their assignment to experimental resonances is more uncertain.

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## I. INTRODUCTION

The interpretation of spectra of baryons is still a thriving field that is attracting much attention. The traditional view of baryons as made of three constituent quarks [1] is being substituted by a more extended view of baryonic states involving three quark ($3q$) as well as four quark-one antiquark ($4q1\overline{q}$) components. In particular some baryonic resonances (a paradigmatic case is the $\Lambda(1405)$) may be better interpreted as molecular states of mesons and baryons. Though such ideas have been advocated in the past [2], it has only been in recent years that detailed quantitative studies have been done based on the combination of chiral dynamics with unitary nonperturbative techniques in coupled channels of mesons and baryons. Thus the low lying $J^P = 1/2^-$ baryonic resonances are relatively well interpreted in terms of meson–baryon molecules [3, 4], indicating the relevance of these $4q1\overline{q}$ components for their description. More technically, they are dynamically generated from the interaction of the octet of mesons containing the $\pi$ and the octet of baryons including the proton. Similarly, the interaction of the octet of mesons of the $\pi$ with the baryon decuplet of the $\Delta$ leads to dynamically generated states of $J^P = 3/2^-$, which can be associated to existing resonances [5, 6]. A further step in this molecular engineering is done with the study of $J^P = 1/2^+$ states stemming from the interaction of a pair of pseudoscalar mesons with a baryon of the octet of the nucleon [7], corresponding indeed to $5q2\overline{q}$ components. A common denominator of these generated baryon states is the use of pseudoscalar mesons as building blocks. Here we undertake the task of extending the study of the dynamical generation of resonances to the vector meson-baryon sector. Besides having its own interest as an extension of the theoretical formalism, this study is particularly relevant in this moment from a phenomenological point of view since there are clear indications that $\rho N$ components [8] as well as $\rho\Delta$ ($\omega\Delta$) ones [9] may play an essential role in the precise description of the negative parity $\Delta$ spectrum below 2.0 GeV.

A framework that makes the study of vector mesons interacting with baryons accurate and manageable is the hidden gauge formalism [10]. There, pseudoscalars and vectors are introduced with an interaction which respects chiral symmetry. The consideration of the interaction of vector mesons with baryons allows then to reinterpret the pseudoscalar meson-baryon chiral Lagrangians as the result of the exchange of vector mesons in the t-channel. The novelty of such a framework is that it also contains the coupling of vector mesons among themselves and therefore one can construct their interaction with baryons. The combination of such an interaction with chiral unitary techniques has been rather successful. In particular, the $\rho\rho$ interaction has been shown to lead to the dynamical generation of the $f_2(1270)$ and $f_0(1370)$ resonances [11], with a branching ratio for the sensitive $\gamma\gamma$ decay channel in good agreement with experimental data [12].

In this work we present the formalism and results for the $\rho\Delta \to \rho\Delta$ interaction which leads to baryonic states in fair agreement with known resonances, within experimental and theoretical uncertainties. The approximate degeneracy of the experimental $J^P = 1/2^-, 3/2^-, 5/2^-$ $\Delta^*$ states around 1920 MeV appears as a dynamical feature of the theory. In the case of the predicted $J^P = 1/2^-, 3/2^-, 5/2^-$ $N^*$ states an assignment to known $N^*$ resonances around 1700 MeV is also feasible although the presence of corresponding $3q$ states close below in mass points out to the need of incorporating both components ($3q$ and $\rho\Delta$) in their description. These contents are distributed as follows. In section II the formalism for the dynamical generation of resonances from the $\rho\Delta \to \rho\Delta$ interaction is derived. In Sections III, IV and V the results obtained in different approximations are presented. Section VI is devoted to an analysis of the possible contribution from anomalous terms involving $\rho\omega\pi$ vertexes. The comparison of our results with experimental data is done in section VII. Finally in section VIII we establish our main conclusions.

## II. FORMALISM FOR THE $VV$ AND $VB$ INTERACTION

We follow the formalism of the hidden gauge interaction for vector mesons [10] (see also [13] for a practical set of Feynman rules). The Lagrangian involving the interaction of vector mesons among themselves is given by

$$\mathcal{L}_{III} = -\frac{1}{4}\langle V_{\mu\nu} V^{\mu\nu}\rangle \ , \tag{1}$$

where the symbol $\langle\rangle$ stands for the trace in the $SU(3)$ space and $V_{\mu\nu}$ is expressed as

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu] \ , \tag{2}$$

with $g$ given by

$$g = \frac{M_V}{2f} \ , \tag{3}$$

with $f = 93$ MeV being the pion decay constant. The value of $g$ of Eq. (3) is one of the ways to account for the $KSFR$ rule [14] which is tied to vector meson dominance [15]. The magnitude $V_\mu$ is the $SU(3)$ matrix of the vectors of the octet of the $\rho$

$$V_\mu = \left( \begin{array}{ccc} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \\ \end{array} \right)_\mu . \tag{4}$$

The interaction of $\mathcal{L}_{III}$ gives rise to a contact term coming for $[V_\mu, V_\nu][V^\mu, V^\nu]$ of the form

$$\mathcal{L}_{III}^{(c)} = \frac{g^2}{2} \langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu \rangle \ , \tag{5}$$

and also to a three vector vertex,

$$\mathcal{L}_{III}^{(3V)} = ig \langle (\partial_\mu V_\nu - \partial_\nu V_\mu) V^\mu V^\nu \rangle \ . \tag{6}$$

It is useful to rewrite this last term as:

$$\begin{aligned} \mathcal{L}_{III}^{(3V)} &= ig \langle V^\nu \partial_\mu V_\nu V^\mu - \partial_\nu V_\mu V^\mu V^\nu \rangle = ig \langle V^\mu \partial_\nu V_\mu V^\nu - \partial_\nu V_\mu V^\mu V^\nu \rangle \\ &= ig \langle (V^\mu \partial_\nu V_\mu - \partial_\nu V_\mu V^\mu) \, V^\nu \rangle \end{aligned} \tag{7}$$

in complete analogy with the coupling of a vector to pseudoscalar mesons in the same theory, which is given in Ref. [10] as

$$\mathcal{L}^{VPP} = -ig \langle [\Phi, \partial_\nu \Phi] \, V^\nu \rangle \tag{8}$$

with $\Phi$, the analogous matrix to Eq. (4), containing the pseudoscalar fields ($P$). This analogy allows us to obtain the interaction of vector mesons with the decuplet of baryons in a straightforward way by realizing that the chiral Lagrangian of Ref. [16] for the interaction of the octect of pseudoscalar mesons with the decuplet of baryons is obtained from the exchange of a vector meson between the pseudoscalar mesons and the baryon, as depicted in Fig.1[a], in the limit of $q^2/M_V^2 \to 0$, being $q$ the momentum transfer.

Then by substituting the vertex of Eq. (8) by that of Eq. (7) the physical picture goes from diagram [a] to [b] of Fig.1. Notice that diagram [b] has a more complicated structure since one has three vector fields to destroy or create either vector, whereas in diagram [a] there is only one vector field and hence no choice. Yet, in the implicit approximations leading to the

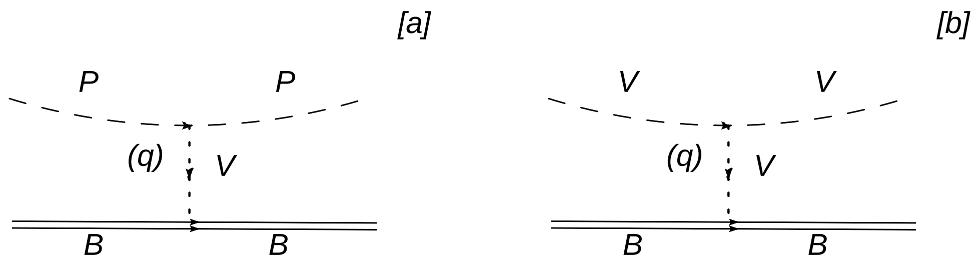


FIG. 1: Diagrams obtained in the effective chiral Lagrangians for interaction of pseudoscalar [a] or vector [b] mesons with the decuplet of baryons.

effective chiral Lagrangian one can go a step further, in line with neglecting $q^2/M_V^2$. Thus, we will neglect the three momentum of the external vectors relative to their mass. Since the vector polarizations have $\epsilon^0$ either zero for the transverse polarizations, or $|\vec{k}|/M_V$ , with $\vec{k}$ the vector meson trimomentum, for the longitudinal component, all the external vectors will have zero $\epsilon^0$ component. Following the same argument, $V^\nu$, appearing in the term $\partial_\nu V^\nu$ in Eq. (7), cannot be an external vector since then $\partial_\nu$ would lead to a three-momentum, which is neglected in the approach. Then $V^\nu$ corresponds to the exchanged vector, $V^\mu V_\mu$ gives rise to $-\vec{\epsilon}\vec{\epsilon}\,'$ for the external vectors and then the PB and VB interactions become formally identical (including the sign), with $\vec{\epsilon}\vec{\epsilon}\,'$ factorized in the VB interaction. For practical reasons one can replace the matrix $\Phi$ by the matrix $V$ in the interaction Lagrangian of Ref. [16], and this is equivalent to substituting $\pi^+ \to \rho^+$, $K^+ \to K^{*+}$, etc... in the matrix elements of PB→ PB to obtain those of VB→VB. There is only a small amendment to be done concerning the $\phi$ and $\omega$ since the ideal mixing implies:

$$\begin{aligned} \omega &= \frac{2}{\sqrt{6}}\omega_1 + \frac{1}{\sqrt{3}}\omega_8 \\ \phi &= \frac{1}{\sqrt{3}}\omega_1 - \frac{2}{\sqrt{6}}\omega_8 \end{aligned} \qquad (9)$$

and the structure of Eq. (7) does not give any contribution for the $\omega_1$, which appears diagonal as diag$(\frac{1}{\sqrt{3}}\omega_1, \frac{1}{\sqrt{3}}\omega_1, \frac{1}{\sqrt{3}}\omega_1)$ in the $V$ matrix. As a consequence of that, the matrix elements of the potential involving $\omega(\phi)$ are obtained from those of $\eta_8$ of $PB \to PB$ multiplying by $\frac{1}{\sqrt{3}}(-\sqrt{\frac{2}{3}})$.

After this discussion we can use directly the results of Ref. [6] to get the $\rho\Delta \to \rho\Delta$

potential with different charges. We find

$$V_{ij} = -\frac{1}{4f^2} C_{ij} (k^0 + k^{'0}) \vec{\epsilon} \vec{\epsilon}\,' \tag{10}$$

where $k^0$ and $k^{'0}$ are the energies of the incoming/outgoing vectors and the $C_{ij}$ coefficients are given in Ref. [6] substituting $\pi$ by $\rho$. One can do the corresponding isospin projections and the results are equally given in Ref. [6] (for $I = 5/2$, corresponding to $\Delta^{++}\pi^+ \to \Delta^{++}\pi^+$, the result can be found in the appendix of this reference):

$$\begin{aligned} \rho\Delta\,, I = \frac{1}{2} &: C = 5 \\ \rho\Delta\,, I = \frac{3}{2} &: C = 2 \\ \rho\Delta\,, I = \frac{5}{2} &: C = -3 \end{aligned} \tag{11}$$

As in Ref. [6] we shall solve the Bethe-Salpeter equation with the on-shell factorized potential [4, 17] and, thus, the T–matrix will be given by

$$T = \frac{V}{1 - VG} \vec{\epsilon} \vec{\epsilon}\,' \,, \tag{12}$$

being $V$ the potential of Eq. (10) in the isospin basis removing the factor $\vec{\epsilon}\vec{\epsilon}\,'$, and $G$ the loop function for intermediate $\rho\Delta$ states, also given in Ref. [6], regularized both with a cuttoff prescription or with dimensional regularization.

In the present work we ignore the coupling with the $\Sigma^* K^*$ channel. The reason being that having a mass 275 MeV above the $\rho\Delta$ threshold it is expected not to play a relevant role in the description of the $\rho\Delta$ states.

The prescription after Eq.(10) to obtain couplings to $\rho\Delta \to \omega\Delta$ or $\omega\Delta \to \omega\Delta$, together with the tables of Ref. [6], gives zero for these transitions. Actually, this is easy to visualize in the vector exchange model since the $\rho\rho\omega$ and $\omega\omega\omega$ vertexes violate $G-$parity while the $\rho\omega\omega$ vertex violates isospin. Thus, for the problem of the lightest states of the vector-baryon decuplet one can consider single $\rho\Delta$ channels.

For the interactions implicit in the Bethe–Salpeter equation, see Fig. 2, one has at second order

$$-\,it = -iV\epsilon_i\epsilon_i iG(-i)V\epsilon_j\epsilon_j \tag{13}$$

with $i, j$ spatial components, and summing over polarizations

$$\sum_\lambda \epsilon_i\epsilon_j = \delta_{ij} + \frac{q_i q_j}{m_V^2}\,. \tag{14}$$

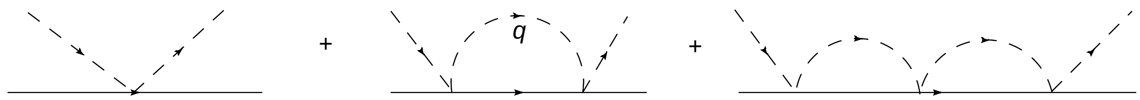


FIG. 2: Terms appearing in the Bethe–Salpeter equation of Eq.(12).

In Ref. [18] following the on-shell factorization, the $\frac{q_i q_j}{m_V^2}$ term was included in the loop function, giving rise to a correction term $\frac{\vec{q}^{\,2}}{3m_V^2}$ which was very small. Consistently with the approximations done here , $\frac{q^2}{m_V^2} = 0$, we also neglect this term. The factor $\vec{\epsilon}\vec{\epsilon}\,'$ appears in all iterations and, thus, factorizes in the T–matrix.

## III. RESULTS WITH NO WIDTH FOR $\rho$ AND $\Delta$.

From Eqs. (10), (11) and (12) we find two attractive, $I = 1/2$ and $I = 3/2$, and one repulsive, $I = 5/2$, channels. Hence, no bound states are obtained in the exotic $I = 5/2$ channel. On the contrary, bound states ($|T|^2$ goes to infinity in Eq. (12)) can be clearly observed in the $I = 1/2$, and $I = 3/2$ channels. The strength of the interaction indicates that the $I = 1/2$ state is more bound than the $I = 3/2$.

Another issue worth mentioning is that the only spin dependence comes from the $\vec{\epsilon}\vec{\epsilon}\,'$ factor of the vector mesons. The spin of the $\Delta$ does not appear in our formalism due to the approximations done. Only the $\gamma^0$ term of the $VBB$ vertex, which has not spin structure to leading order, has been kept [6]. The $\vec{\epsilon}\vec{\epsilon}\,'$ scalar structure tells us that all spin states of the $\rho\Delta$ system behave according to the same interactions and therefore, one has degeneracy for the $J^P = 1/2^-, 3/2^-, 5/2^-$ states, both for $I = 1/2$ and $I = 3/2$.

In Figs. 3 and 4 we show $|T|^2$ as a function of $\sqrt{s}$ for $\rho\Delta \to \rho\Delta$ both in $I = 1/2$ and $I = 3/2$. In order to check the dependence of our results on the choice made for the value of the cutoff, a 10% variation over the value taken in Ref. [6] ($q_{max} = 700$ MeV) has been considered. We shall take the corresponding dispersion of results as an indication of their uncertainty. By comparing Figs. 3 and 4 it is clear that the results for $I = 3/2$, with bound state masses ranging from 1940 MeV to 1980 MeV, show much less dispersion than the results for $I = 1/2$, with bound state masses ranging from approximately 1700 MeV to 1800 MeV. This has to do with the more attractive interaction for $I = 1/2$, so that the more

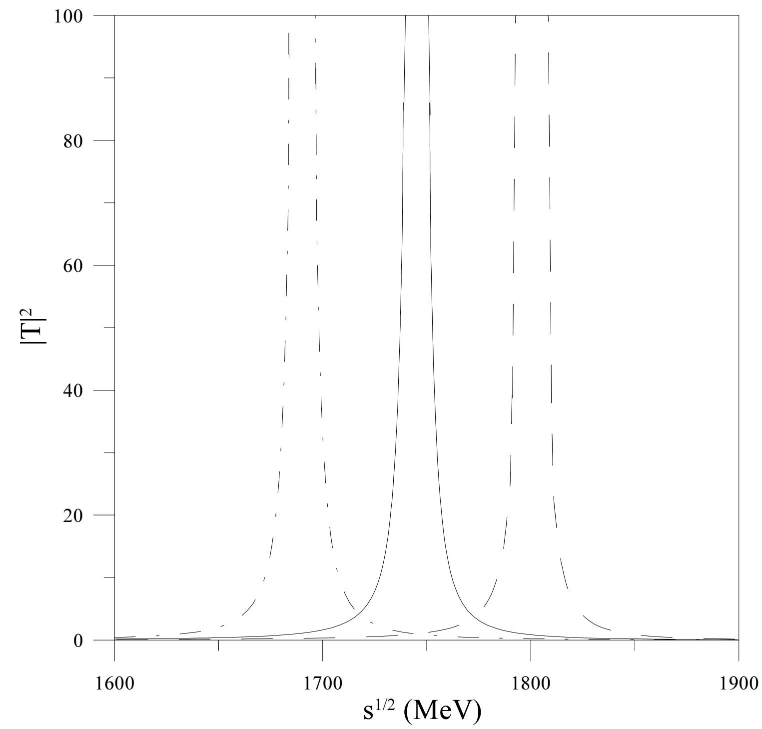


FIG. 3: $|T|^2$ for $\rho\Delta \to \rho\Delta$ in the $I = 1/2$ channel for several values of the cutoff $q_{max}$: solid line $q_{max} = 770$ MeV, dashed line $q_{max} = 700$ MeV, dashed-dotted line $q_{max} = 630$ MeV.

bound the sate the more uncertain the prediction of its mass.

We postpone the comparison with experimental $N^*$ and $\Delta^*$ resonances until a discussion of possible corrections to these results coming from the inclusion of $\rho$ and $\Delta$ widths or from anomalous terms is carried out.

## IV. CONVOLUTION DUE TO THE $\rho$ AND $\Delta$ MASS DISTRIBUTIONS

The strong attraction in the $I = 1/2, 3/2$ channels produces $\rho\Delta$ bound states and thus with no width within the model. However, this is not strictly true since the $\rho$ and $\Delta$ have a large width, or equivalently a mass distribution that allows the states obtained to decay in $\rho\Delta$ for the low mass components of the $\rho$ and $\Delta$ mass distributions. To take this into account we follow the usual procedure consisting in convoluting the $G$ function with the

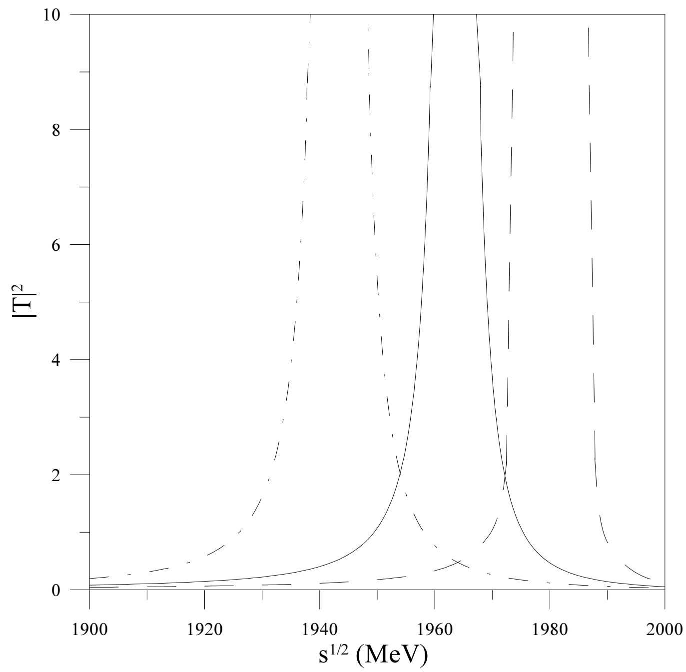


FIG. 4: Same as Fig. 3 for $I = 3/2$.

mass distributions of the $\rho$ and $\Delta$ [19] so that the $G$ function is replaced by $\tilde{G}$ as follows

$$\tilde{G}(s) = \frac{1}{N_\rho N_\Delta} \int_{m_\Delta - 2\Gamma_\Delta}^{m_\Delta + 2\Gamma_\Delta} d\tilde{M} (-\frac{1}{\pi}) \mathcal{I}m \frac{1}{\tilde{M} - M_\Delta + i\frac{\Gamma_1(\tilde{M})}{2}}$$
$$\times \int_{(m_\rho - 2\Gamma_\rho)^2}^{(m_\rho + 2\Gamma_\rho)^2} d\tilde{m}^2 (-\frac{1}{\pi}) \mathcal{I}m \frac{1}{\tilde{m}^2 - m_\rho^2 + i\tilde{m}\Gamma_2(\tilde{m})} G(s, \tilde{M}, \tilde{m}^2) \ , \tag{15}$$

with

$$N_\rho = \int_{(m_\rho - 2\Gamma_\rho)^2}^{(m_\rho + 2\Gamma_\rho)^2} d\tilde{m}^2 (-\frac{1}{\pi}) \mathcal{I}m \frac{1}{\tilde{m}^2 - m_\rho^2 + i\tilde{m}\Gamma_2(\tilde{m})} \tag{16}$$
$$N_\Delta = \int_{m_\Delta - 2\Gamma_\Delta}^{m_\Delta + 2\Gamma_\Delta} d\tilde{M} (-\frac{1}{\pi}) \mathcal{I}m \frac{1}{\tilde{M} - M_\Delta + i\frac{\Gamma_1(\tilde{M})}{2}} \ ,$$

where

$$\Gamma_1(\tilde{M}) = \Gamma_\Delta \left( \frac{\lambda^{1/2}(\tilde{M}^2, M_N^2, m_\pi^2) 2M_\Delta}{\lambda^{1/2}(M_\Delta^2, M_N^2, m_\pi^2) 2\tilde{M}} \right)^3 \theta(\tilde{M} - M_N - m_\pi) \tag{17}$$
$$\Gamma_2(\tilde{m}) = \Gamma_\rho (\frac{\tilde{m}^2 - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2})^{3/2} \theta(\tilde{m} - 2m_\pi)$$

with $\lambda(x,y,z) = x^2+y^2+z^2-2xy-2xz-2yz$, $\Gamma_\Delta = 120$ MeV and $\Gamma_\rho = 150$ MeV (for $\Gamma_2(\tilde{m})$ we have taken the $\rho$ width for the decay into two pions in $P$-wave). The use of $\tilde{G}$ in Eq. (12) provides a width to the states obtained.

## V. RESULTS WITH $\rho$ AND $\Delta$ WIDTHS

In Figs. 5 and 6 we show the results taking into account the widths of the $\rho$ and $\Delta$ as discussed in the previous section. We can see that the states develop a width leading to more realistic results. Yet, this is not the whole width of the states since the $\pi N$ decay channels have not been included in the approach. As discussed in Ref. [6] these channels are supposed to play a minor role as building blocks of the resonance since they are far apart in energy from the masses of the states obtained. However, given the fact that there is more phase space for the decay into these channels, they can have some contribution to the total width. One should note though that the inclusion of the width hardly changes the position of the peaks.

## VI. CONTRIBUTION FROM ANOMALOUS TERMS

In the previous sections we mentioned that we do not have contributions from either $\rho\Delta \to \omega\Delta$ or $\omega\Delta \to \omega\Delta$ transitions within the vector exchange approach of our model. Yet it is possible to have $\omega\Delta$ intermediate states through anomalous terms involving the $\rho \to \omega\pi$ transition. The contribution to $\rho\Delta \to \rho\Delta$ from intermediate $\omega\Delta$ states is studied through the diagram depicted in Fig. 7. The anomalous coupling $\rho \to \omega\pi$ is considered using the same normalization of Ref. [19],

$$\mathcal{L}_{VVP} = \frac{G'}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} \langle \partial_\mu V_\nu \partial_\alpha V_\beta P \rangle \tag{18}$$

where $G' = 3g'^2/4\pi^2 f$, $g' = -G_V M_\rho/\sqrt{2} f^2$, $G_V \approx 55$ MeV. For $\rho^+(k) \to \omega(q)\pi^+$ we get the coupling

$$-it = iG' \epsilon^{\mu\nu\alpha\beta} q_\mu k_\alpha \epsilon_\nu(\omega) \epsilon_\beta(\rho^+) \tag{19}$$

and the same contribution for $\rho^0 \to \omega\pi^0$.

However, it must be taken into account that the three momenta of the external vector mesons has been neglected and, thus, $k_\alpha \to k_0$, forcing $\mu, \nu$, and $\beta$ to be spatial components.

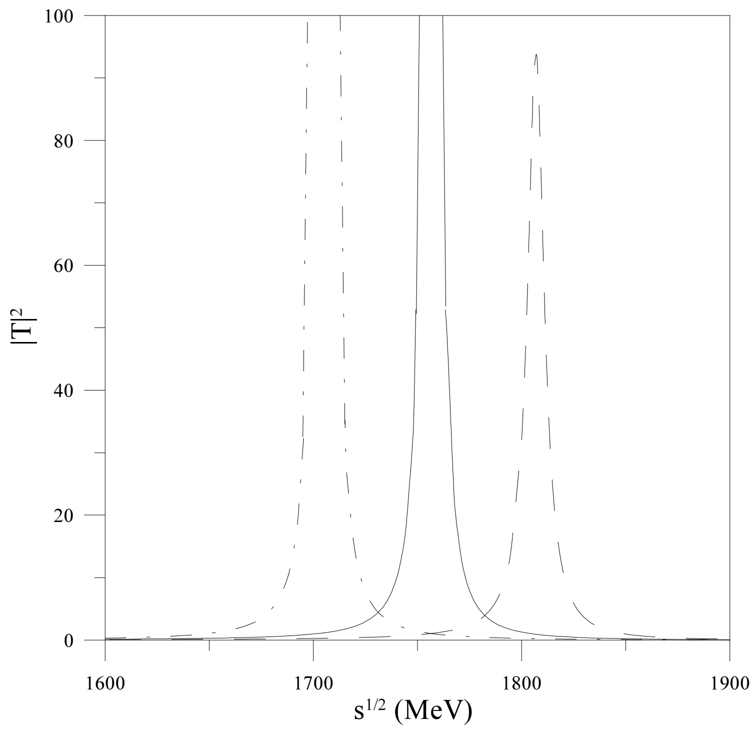


FIG. 5: $|T|^2$ for $\rho\Delta \to \rho\Delta$ in the $I = 1/2$ channel for several values of the cutoff $q_{max}$ including $\rho$ and $\Delta$ mass distributions: solid line $q_{max} = 770$ MeV, dashed line $q_{max} = 700$ MeV, dashed-dotted line $q_{max} = 630$ MeV.

Hence,

$$-it = -iM_\rho G' \epsilon_{ijl} q^i \epsilon^j(\omega)\epsilon^l(\rho^+)\,. \tag{20}$$

Note that in the diagram of Fig. 7 one could also have a $N$ in the intermediate state. We show next that the contribution of the diagram of Fig. 7 is pretty small. Therefore we shall neglect it as well as the one with an intermediate nucleon.

To evaluate the diagram of Fig. 7 one needs the $\pi\Delta\Delta$ coupling, which have been taken from [20],

$$\mathcal{L}_{\Delta\Delta\pi} = -\frac{f_\Delta}{m_\pi}\Psi^+_\Delta S_{\Delta,i}(\partial_i\phi^\lambda)T^\lambda_\Delta\Psi_\Delta \tag{21}$$

with $f_\Delta = 0.802$, where $\vec{S}_\Delta(\vec{T}_\Delta)$ is the spin(isospin) operator for the $\Delta$, $\vec{S}^2_\Delta = S(S+1)$ and $\vec{T}^2_\Delta = T(T+1)$).

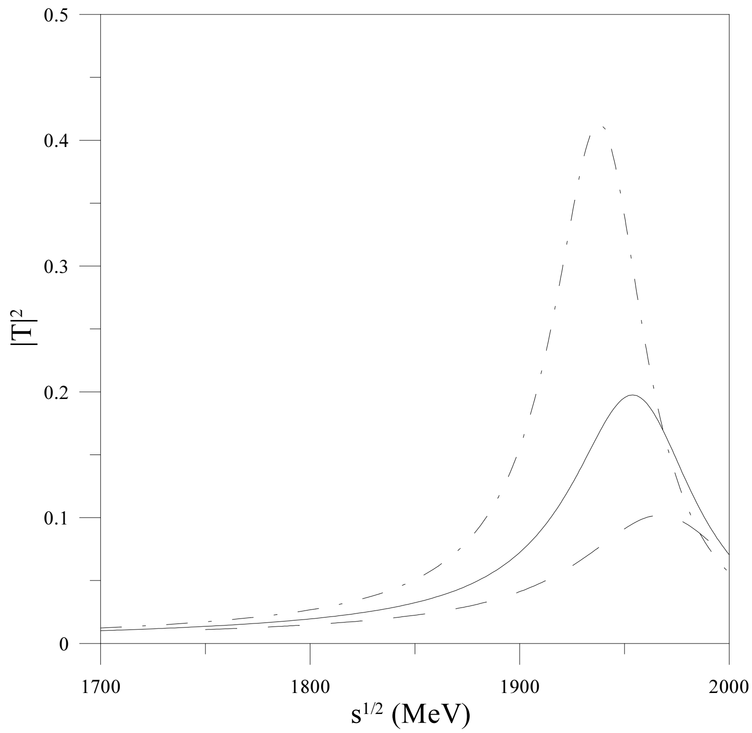


FIG. 6: Same as Fig. 5 for $I = 3/2$.

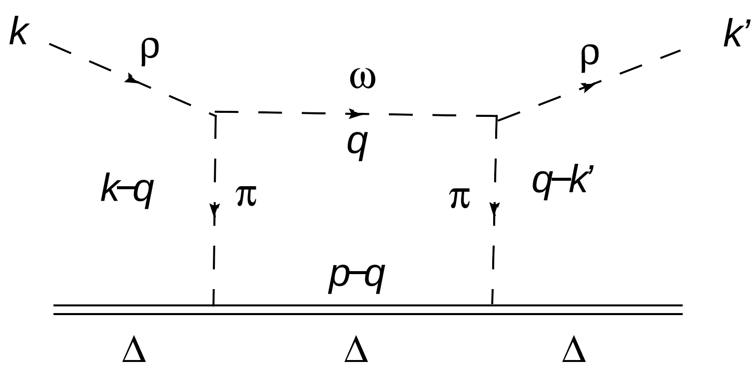


FIG. 7: $\rho\Delta \to \rho\Delta$ term involving $\omega\Delta$ intermediate states through anomalous $\rho \to \pi\omega$ couplings.

By taking the usual isospin convention $|\rho^+\rangle = -|1, +1\rangle$, one has

$$|\Delta\rho; 3/2, +3/2\rangle = \sqrt{\frac{3}{5}}\Delta^{++}\rho^0 + \sqrt{\frac{2}{5}}\Delta^+\rho^+ \tag{22}$$

$$|\Delta\rho; 1/2, +1/2\rangle = \sqrt{\frac{1}{2}}\Delta^{++}\rho^- - \sqrt{\frac{1}{3}}\Delta^+\rho^0 - \sqrt{\frac{1}{6}}\Delta^0\rho^+$$

The only contribution obtained corresponds to isospin $I = 3/2$ $\rho\Delta$ states, since $\omega\Delta$ only couples to this isospin. In doing so one obtains

$$\langle \Delta\rho, I=3/2 | (\vec{T}_\Delta \vec{\phi})^2 | \Delta\rho, I=3/2\rangle = \frac{15}{4} \tag{23}$$

by means of which the contribution of the diagram of Fig. 7 to the $\rho\Delta \to \rho\Delta$ transition is given by

$$\begin{aligned} -it = & \frac{15}{4} M_\rho^2 (G')^2 \left(\frac{f_\Delta}{m_\pi}\right)^2 \epsilon_{ijk}\epsilon_{i'j'k'}(-i)(-i) \int \frac{d^4q}{(2\pi)^4} q^i \epsilon^j(\omega)\epsilon^k(\rho) q^{i'} \epsilon^{j'}(\omega)\epsilon^{k'}(\rho) \times \\ & \times \frac{i}{(k-q)^2 - m_\pi^2 + i\epsilon} \, \frac{i}{(q-k')^2 - m_\pi^2 + i\epsilon} \left(\vec{S}_\Delta \cdot \vec{q}\right) \left(\vec{S}_\Delta \dot{(} -\vec{q})\right) \times \\ & \times \frac{i}{q^2 - M_\omega^2 + i\epsilon} \, \frac{i}{P - q - E_\Delta(q) + i\epsilon} \end{aligned} \tag{24}$$

By summing over the polarizations, setting $\vec{k}$, $\vec{k}'$ equal to zero, taking into account that

$$\int \frac{d^3q}{(2\pi)^3} f(\vec{q}^{\,2}) q_i q_j q_l q_m = \frac{1}{15} \int \frac{d^3q}{(2\pi)^3} f(\vec{q}^{\,2}) \vec{q}^{\,4} (\delta_{ij}\delta_{lm} + \delta_{il}\delta_{jm} + \delta_{im}\delta_{jl}) \,, \tag{25}$$

and ignoring the tensor part $(S_{\Delta_i} S_{\Delta_j} - \frac{1}{3}\vec{S}_\Delta^2 \delta_{ij})\epsilon_i \epsilon'_j$ versus the dominant scalar part $\frac{5}{3}\vec{\epsilon}\vec{\epsilon}\,'\vec{S}_\Delta^2 = \frac{25}{4}\vec{\epsilon}\vec{\epsilon}\,'$ (we estimate the contribution of the tensor part to be of the order of 10% of the scalar one), one arrives to

$$\begin{aligned} -it = & \frac{25}{8} \vec{\epsilon}\vec{\epsilon}\,' \left(M_\rho G' \frac{f_\Delta}{m_\pi}\right)^2 \int \frac{d^4q}{(2\pi)^4} \vec{q}^{\,4} \frac{1}{(k'-q)^2 - m_\pi^2 + i\epsilon} \times \\ & \times \frac{1}{(q-k)^2 - m_\pi^2 + i\epsilon} \, \frac{1}{q^2 - M_\omega^2 + i\epsilon} \, \frac{1}{(P^0 - q^0) - E_\Delta(q) + i\epsilon} \end{aligned} \tag{26}$$

The $q^0$ integration is now performed summing the residues of the poles of the propagator and one finally finds

$$\begin{aligned} t^{(anomalous)}_{\rho\Delta\to\rho\Delta} = & \frac{25}{8} \vec{\epsilon}\vec{\epsilon}\,' \left(M_\rho G' \frac{f_\Delta}{m_\pi}\right)^2 \int \frac{d^3q}{(2\pi)^3} \vec{q}^{\,4} \frac{1}{2\omega} \frac{1}{(P^0 - \omega) - E_\Delta + i\epsilon} \times \\ & \times \left(\frac{1}{\omega - k^0 + \omega_\pi}\right)^2 \left(\frac{1}{\omega + k^0 + \omega_\pi}\right)^2 \left(\frac{1}{P^0 - k^0 - \omega_\pi - E_\Delta}\right)^2 \frac{1}{(2\omega_\pi)^3} 4 \times \\ & \times \{ 2\omega_\pi^5 + 4(E_\Delta + k^0 + 2\omega - P^0)\omega_\pi^4 + 2[E_\Delta^2 + 2k^0 E_\Delta + 6\omega E_\Delta \\ & + (k^0)^2 + 6\omega^2 + (P^0)^2 + 4k^0\omega - 2(E_\Delta + k^0 + 3\omega)P^0]\omega_\pi^3 + \\ & + \omega(5E_\Delta + 7k^0 + 8\omega - 5P^0)(E_\Delta + \omega - P^0)\omega_\pi^2 + 2\omega(E_\Delta + \omega - P^0)\omega_\pi \times \\ & \times [-(k^0)^2 + 2\omega k^0 + \omega(2E_\Delta + \omega - 2P^0)] + \omega(E_\Delta + \omega - P^0) \times \\ & \times (E_\Delta + k^0 - P^0)(\omega^2 - (k^0)^2) \} \, FF(\vec{q})^2 \end{aligned} \tag{27}$$

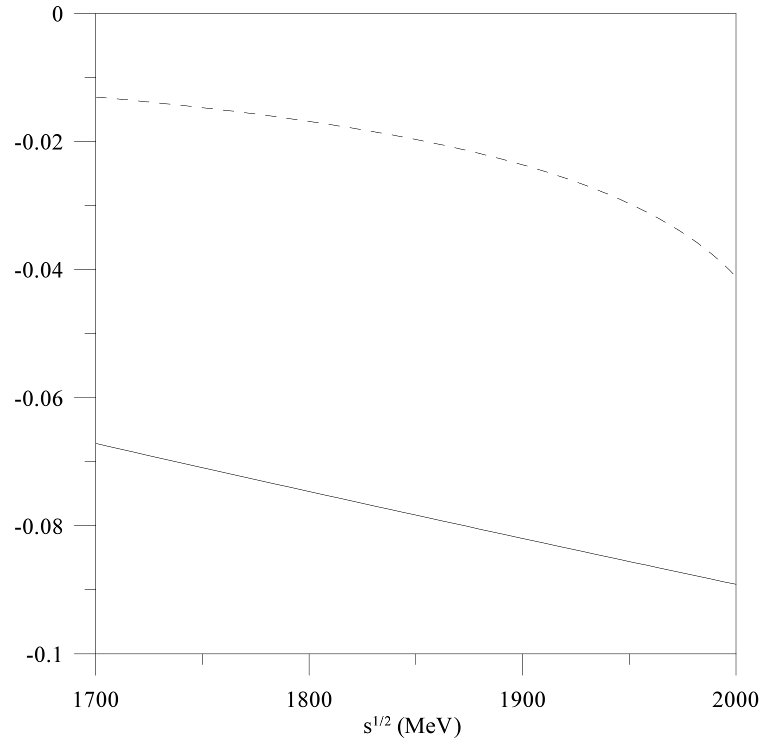


FIG. 8: $t^{(anomalous)}_{\rho\Delta\to\rho\Delta}$ (dashed lines) compared to $V$ (solid lines) as a function of $\sqrt{s}$ for $q_{max} = 770$ MeV/c.

In Eq. (27) a form factor

$$FF(\vec{q}) = \frac{\Lambda^2}{\Lambda^2 - \vec{q}^{\,2}} \tag{28}$$

has been included for each $\Delta\Delta\pi$ vertex, with $\Lambda = 1$ GeV, as customary in the Yukawa coupling of pions and baryons.

In Fig. 8 we show the results for $t^{(anomalous)}_{\rho\Delta\to\rho\Delta}$ compared to $V$ for the same channel for the exchange of vector mesons. As can be seen, the contribution of the anomalous term is reasonably smaller than the dominant one of vector meson exchange and, hence, neglecting it, as we have done in the former section, is a good approximation. Note that this is not the only source of smaller contributions to the potential. We have also ignored terms with $\pi N$ and $\pi\Delta$ in the intermediate states. Actually, in Ref. [11], the equivalent terms with $\pi\pi$ intermediate states in the $\rho\rho$ scattering are found to have a relatively small real part compared to the contribution from the dominant $\rho$ exchange terms. Furthermore, they have opposite sign to the anomalous contributions, leading to additional cancellations of these small terms that we also expect here.

## VII. COMPARISON TO EXPERIMENTAL STATES

To compare our dynamically generated meson-baryon bound states (DGBS) with experimental resonances we should keep in mind that a calculated DGBS mass will only fit precisely an experimental mass if the corresponding resonance has either i) a very dominant meson-baryon ($4q1\overline{q}$) component or ii) weakly coupled $3q$ and meson-baryon components giving rise separately to bound states of the same mass.

Option i) seems to be approximately at work at least for some of our degenerate $I = 3/2$, $J^P = 1/2^-, 3/2^-, 5/2^-$ states, with a mass between 1940 MeV ($q_{max} = 770$ MeV) and 1980 MeV ($q_{max} = 630$ MeV), which can be respectively assigned to $\Delta(1900)S_{31}(**)$, $\Delta(1940)D_{33}(*)$ and $\Delta(1930)D_{35}(***)$ from the Particle Data Group (PDG) Review [21]. Indeed the mass of these three resonances can not be reproduced by $3q$ models based on two-quark interactions which predict significant higher values [9]. Specifically for $\Delta(1930)D_{35}$, due to quark Pauli blocking, there are not allowed $3q$ configurations in the first energy band of negative parity and consequently any $3q$ mass prediction is about 300 MeV higher than the PDG average mass. Then we can interpret $\Delta(1930)D_{35}$ as a $\rho\Delta$ bound state whereas the corresponding $3q$ bound state will lye 300 MeV above. For $J^P = 1/2^-, 3/2^-$ the predicted $3q$ radial excitations of the lowest states in the first energy band of negative parity are located at $\sim 2050$ MeV (see for instance [22]). If they were overpredicted, as it is known to happen for radial excitations in the positive parity sector, there could be forming part of $\Delta(1900)S_{31}$ and $\Delta(1940)D_{33}$ altogether with the $\rho\Delta$ bound state and option ii) would be preferred. These considerations get additional support from some experimental analyses [23, 24] where the inclusion of $\rho\Delta$ as an effective inelastic channel becomes essential for the extraction of the $\Delta(1930)D_{35}$.

For $I = 1/2$, $J^P = 1/2^-, 3/2^-, 5/2^-$ and $q_{max} = 770$ MeV we obtain a DGBS mass of $\sim 1700$ MeV. A look to the PDG data suggests the identification with the almost degenerate $N(1650)S_{11}(****)$, $N(1700)D_{13}(***)$ and $N(1675)D_{15}(****)$ respectively. However, although for these resonances $3q$ models systematically underpredict their masses, the predicted $3q$ values differ less than 100 MeV from the PDG averages. This makes option ii) more reliable as we show next. Let us centre first on $D_{15}$ for which the $3q$ mass prediction is usually closer to data. If the $(3q - \rho\Delta)_{D_{15}}$ coupling were weak there would be two bound states close in mass, differing very little from the $3q$ and $\rho\Delta$ bound states. As there is only one experimental candidate in the energy region under consideration (the closest PDG $D_{15}$ state is $N(2200)D_{15}(**)$) we would be forced to conclude that both states actually form the $N(1675)D_{15}$. On the other hand if the $(3q-\rho\Delta)_{D_{15}}$ coupling were strong, but not enough to make the $\rho\Delta$ unbound, there would be two bound states much more separated in mass than the previous $3q$ and $\rho\Delta$ ones (see Ref. [9]). The two new bound states should appear as two distinctive resonances what is not confirmed experimentally. Therefore we conclude that the $N(1675)D_{15}$ may be the result of the overlapping of the $3q$ and $\rho\Delta$ bound states.

For $N(1650)S_{11}$ and $N(1700)D_{13}$ the analysis is more difficult since there can be $3q$ configuration mixing with the low-lying $N(1535)S_{11}$ and $N(1520)D_{13}$ and also a possible coupling to the S-wave $\rho N$ channel that we have not taken into account. The study of the $\rho N$ channel and its effect on the $N$ and $\Delta$ spectrum deserves special attention and will be the subject of a future analysis.

Regarding the dependence of our results on the cutoff it should be remarked that for any value in the considered interval $q_{max} \in [$ 630 MeV, 770 MeV$]$ the only available assignment of our $I=3/2$ $\rho\Delta$ bound states to $\Delta^*$ resonances is the one performed above. This is easy to understand by realizing that even enlarging the $q_{max}$ interval the mass of the closest $\Delta(J^-)$ PDG resonances would be far above or below our predicted masses. Moreover as the change of the prediction over the whole interval is very modest, going from 1940 MeV to 1980 MeV, there can only have a little effect on the probability of the $\rho\Delta$ component in the assigned resonance.

For $I=1/2$ the greater sensitivity of the predicted masses to changes in the cutoff values (from 1700 MeV to 1800 MeV in the considered $q_{max}$ interval) might leave room for an alternative assignment although a quick look to the PDG catalog also locates the next $N(J^P=5/2^-)$ resonance, $N(2200)D_{15}(**)$, very far above our predicted masses. However, a careful look to the data values used to obtain its average mass indicates two sets of them: one giving a mass about 1900 MeV and another one reporting values around 2200 MeV. The same circumstance is repeated for $N(2090)S_{11}(*)$ and $N(2080)D_{13}(**)$ so that the existence of two distinctive resonances may be considered. Actually in Ref. [23] two $D_{13}$ resonances with masses 2080 MeV and 1880 MeV were reported. Then, by enlarging the cutoff interval, our $\rho\Delta$ bound state prediction (for $q_{max}=530$ MeV we predict 1880 MeV) could be pretty close in mass to the $N(1900)(J^P=1/2^-,3/2^-,5/2^-)$ and their assignment would be feasible. Again the presence of $3q$ models configurations close in mass could imply a $3q-\rho\Delta$ structure.

In such a case the $N(1650)S_{11}$, $N(1700)D_{13}$ and $N(1675)D_{15}$ should consistently not contain a $\rho\Delta$ component.

## VIII. CONCLUSIONS

We have performed a study of the $\rho\Delta$ interaction within the framework of the hidden gauge formalism for vector mesons and a unitary approach via the use of the Bethe Salpeter equation. We find that the interaction potential for the $\rho$ and $\Delta$ is attractive in $I = 1/2$ and $I = 3/2$ and repulsive in $I = 5/2$ channels. Then, we found bound states of $\rho\Delta$ in the $I = 1/2, 3/2$ channels and no bound states in the $I = 5/2$ channel. It is interesting to observe that, even if the $\rho\Delta$ structure allows for $I = 5/2$, the dynamics of the problem precludes the formation of bound states. Note that an $I = 5/2$ bound state would be exotic since a three quark structure does not allow such an isospin to appear.

We also find the $\rho\Delta$ interaction with $I = 1/2$ to be stronger than with $I = 3/2$, leading to $N^*$ states more bound that the $\Delta^*$ ones. The other dynamical feature of the model is the spin degeneracy in $J^P = 1/2^-, 3/2^-, 5/2^-$ of both the $N^*$ and $\Delta^*$ states. This is a consequence of the approximations done, corresponding to the $\rho\Delta$ interaction in $S-$ wave, neglecting the three momentum of the vector mesons. As much as the approximations are sensible, we expect the predictions on the degeneracy to be realistic.

When it comes to compare our results with existing resonances, we find for $\Delta^*$ a good quantitative agreement with $\Delta(1900)S_{31}(**)$, $\Delta(1940)D_{33}(*)$ and $\Delta(1930)D_{35}(***)$. The small sensitivity of the predicted masses to changes in the cutoff parameter used in our calculation added to the lack of alternative identifications with existing data takes us to unambiguously assign our $\rho\Delta$ bound states to these resonances. For $\Delta(1930)D_{35}$ the much larger mass predicted by $3q$ models makes clear that it contains almost exclusively a $\rho\Delta$ component. For $\Delta(1900)S_{31}$ and $\Delta(1940)D_{33}$ the presence of a $3q$ radial excitation not far above in energy could leave room for a significant probability of this component.

Concerning $N^*$ we find again a good quantitative agreement of our predicted masses with $N(1650)S_{11}(****)$, $N(1700)D_{13}(***)$ and $N(1675)D_{15}(****)$. The analysis of $3q$ models predictions makes clear that with this assignment each of these experimental $N^*$ resonances would come out from the overlapping of two different states containing both $3q$ as well as $\rho\Delta$ components. However the close mass of these two states and the possible existence of additional components in them would make difficult their experimental disentanglement. Alternatively another assignment is possible. The larger sensitivity of the predicted masses to changes in the cutoff could make feasible in this case the assignment to non-cataloged $N(1900)(J^P = 1/2^-, 3/2^-, 5/2^-)$ resonances which would be hidden in the cataloged $N(2200)D_{15}(**)$, $N(2090)S_{11}(*)$ and $N(2080)D_{13}(**)$.

Therefore we conclude that the $\Delta(1930)D_{35}(***)$ can be interpreted as $\rho\Delta$ bound state and that the $\Delta(1900)S_{31}(**)$ and $\Delta(1940)D_{33}(*)$ contain a significant probability of the $\rho\Delta$ component providing an explanation to the elusive description of these resonances by $3q$ models. In the $I = 1/2$ sector although an assignment of our $\rho\Delta$ bound states to $N(1650)S_{11}(****)$, $N(1700)D_{13}(***)$ and $N(1675)D_{15}(****)$ can be done it would be also possible to associate them to non cataloged $N^*$ states around 1900 MeV. Certainly a refinement of our results, from a more complete calculation incorporating $3q$ altogether with meson-baryon components in a consistent manner, could be more predictive in this case. Nevertheless the detailed analysis we have performed makes clear that an additional experimental input is needed anyhow to definitely settle the point. We encourage an experimental effort along this line.

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