$B \to \rho$ semileptonic decays and $|V_{ub}|$

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We reevaluate the $B \to \rho\, l^+ \nu_l$ decay width as a full $B \to \pi\pi\, l^+ \nu_l$ four-particle decay, in which the two final pions are produced via an intermediate ρ meson. The decay width can be written as a convolution of the $B \to \rho\, l^+ \nu_l$ decay width, for an off-shell ρ, with the $\rho \to \pi\pi$ line shape. This allows to fully incorporate the effects of the finite ρ meson width and a better comparison with actual experiments. We use an Omnès representation to provide the dependence of the $B \to \rho$ semileptonic form factors on q^2. The Omnés subtraction constants and the overall normalization parameter $|V_{ub}|$ are fitted to light cone sum rules and lattice QCD theoretical form-factor calculations, in the low and high q^2 regions respectively, together to the CLEO, BaBar and Belle experimental partial branching fraction distributions. The extracted value from this global fit is $|V_{ub}| = (3.40 \pm 0.15) \times 10^{-3}$, in agreement with $|V_{ub}|$ extracted using all other inputs in CKM fits and the exclusive semileptonic $B \to \pi$ channel, but showing a clear disagreement with $|V_{ub}|$ extracted from inclusive semileptonic $b \to u$ decays. As estimated by Ulf-G. Meißner and Wei Wang in JHEP **1401**, 107 (2014), taking into account the ρ meson width effects and the actual acceptance of the experiments are essential to render the $|V_{ub}|$ determinations from exclusive $B \to \pi$ and $B \to \rho$ decays totally compatible

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I. INTRODUCTION

A precise determination of V_{ub} is essential to check the consistency of the Standard Model, especially the description of CP violations. However, V_{ub} is still the least well known element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. At present, there is a clear tension between the $|V_{ub}|$ values extracted from the analysis of inclusive and exclusive decays. Determinations based on inclusive semileptonic decays have their largest uncertainties coming from the error on the $b-$ quark mass, but their values tend to be consistent. From these analyses, the average value quoted by the Particle Data Group (PDG) in its 2013 update [1] is $|V_{ub}| = (4.41 \pm 0.15^{+0.15}_{-0.17}) \times 10^{-3}$. The corresponding average value extracted from exclusive determinations is dominated by the $B \to \pi$ semileptonic decay value $|V_{ub}| = (3.23 \pm 0.31) \times 10^{-3}$ [1]. In this case the error is dominated by form factor normalizations. Another problem which will be addressed here, is the existing tension between the exclusive determinations using the $B \to \rho$ and $B \to \pi$ semileptonic decays. From $B \to \rho$ decays lower values have been traditionally reported, thus for instance, BaBar presented a value of $|V_{ub}| = (2.75 \pm 0.24) \times 10^{-3}$ in [2], while in the approach of Ref. [3], similar to the one followed here and based on the Omnès representation of the form factors, was obtained $|V_{ub}| = (2.76 \pm 0.21) \times 10^{-3}$. Very recent analyses, using light cone sum rules (LCSR), also find central values, $|V_{ub}| = (2.91 \pm 0.19) \times 10^{-3}$ and $|V_{ub}| = (3.11 \pm 0.19) \times 10^{-3}$ [4], below those found from $B \to \pi$ decays ($|V_{ub}| = (3.47 \pm 0.29 \pm 0.03) \times 10^{-3}$ [5] $|V_{ub}| = (3.6 \pm 0.4_{\text{stat}} \pm 0.2^{+0.6}_{\text{syst}-0.4\text{thy}}) \times 10^{-3}$ [6], $|V_{ub}| = (3.41^{+0.37}_{-0.32}|_{\text{th}} \pm 0.06|_{\text{exp}}) \times 10^{-3}$ [7], $|V_{ub}| = (3.52 \pm 0.29) \times 10^{-3}$ [8]). As pointed out in Ref. [9], part of this systematic discrepancy could be due to the fact that the $B \to \rho$ analyses do not take into account the effect of the broad $\rho-$width. This is relevant since only a limited range of the $\pi\pi$ invariant masses is considered in the different experiments [2, 8, 10, 11].

In Ref. [12] the authors propose to extract $|V_{ub}|$ from the analysis of the four-body semileptonic decay $B \to \pi\pi l^+\nu_l$ taking into account $\pi\pi$ rescattering effects and the effect of the rho meson. Their approach is based on dispersion theory and does not rely on specific resonant contributions. In our calculation we do a simpler study of the four-body decay in which the two pions are produced via an intermediate ρ meson $B \to \pi\pi(\rho)l^+\nu_l$. The decay width can then be expressed as an integration over the ρ meson invariant mass available in the $B \to \rho l^+\nu_l$ decay for an off-shell ρ, weighted by the $\rho \to \pi\pi$ line shape distribution that fully takes into account ρ meson width effects. In this way one can easily select the ρ meson invariant mass range covered by the different experiments. In fact, this type of analysis has been recently done by the Belle collaboration in Ref. [8] with the result that a larger $|V_{ub}|$ value, in better agreement with the determination from $B \to \pi$ semileptonic decay, is obtained.

In this work we perform a combined fit to the latest partial branching fraction distributions by the different experimental collaborations, while at the same time we substantially improve on the treatment of the form factors over previous works. In this respect we shall follow Ref. [3], where the $B \to \rho$ form factors are described using a multiply subtracted Omnès dispersion relation. The Omnès functional form depends on the form factor values at the subtraction points and those values are treated as free parameters. These, together with $|V_{ub}|$, are fitted both to $B \to \rho l^+\nu_l$ recent partial branching fraction measurements from Belle [8], BaBar [2] and CLEO [10] collaborations, as well as to theoretical results for the $B \to \rho$ form factors obtained using LCSR [13] and lattice calculations by the SPQcdR [14] and UKQCD [15] collaborations. For the $\rho \to \pi\pi$ decay we use a phenomenological vertex where the coupling constant has been fixed to the on-shell ρ meson decay width.

The paper is organized as follows. In Sec.II we present all the expressions needed to evaluate the decay width. We shall give a triple differential decay width distribution with respect to p_ρ^2, q^2 and x_l, with p_ρ^2 the ρ meson invariant mass square, q the total four-momentum of the final lepton system, and x_l the cosinus of the angle formed by the momentum of the charged lepton, measured in the lepton center of mass system, and the momentum of the virtual ρ in the B meson rest frame. These are the variables used by the experiments, and in order to obtain the fractional branching fractions (see below) we just have to integrate over their corresponding ranges. Sec. III describes the fitting procedure that follows closely Ref. [3], and finally, in Sec. IV we present and discuss the main results of this work. In Appendix A, we give details on the helicity amplitude formalism used to evaluate the product of the leptonic and hadronic tensors, while in Appendix B we provide the correlation matrix resulting from our global fit.

II. $\Gamma[B \to \pi\pi(\rho)\, l^+\nu_l]$ DECAY WIDTH

Working in the exact isospin limit, the $B \to \pi\pi(\rho)\, l^+\nu_l$ decay width is given by

$$\Gamma = \left(\frac{G_F}{\sqrt{2}}\right)^2 |V_{ub}|^2 C_\rho^2 \frac{1}{2m_B} \frac{1}{(2\pi)^8} \int \frac{d^3p_l}{2E_l} \int \frac{d^3p_\nu}{2E_\nu} \int \frac{d^3p_{\pi_1}}{2E_{\pi_1}} \int \frac{d^3p_{\pi_2}}{2E_{\pi_2}} \, \delta^{(4)}(p_B - p_l - p_\nu - p_{\pi_1} - p_{\pi_2})$$

$$\times \sum_{s_l} \sum_{s_\nu} \left| h_{\alpha\sigma}(p_B, p_\rho) \frac{-g^{\sigma\beta} + \frac{p_\rho^\sigma p_\rho^\beta}{p_\rho^2}}{p_\rho^2 - m_\rho^2 + i\sqrt{p_\rho^2}\,\Gamma_\rho(p_\rho^2)} (p_{\pi_1} - p_{\pi_2})_\beta \, \bar{u}_{s_\nu}(p_\nu)\gamma^\alpha(1 - \gamma_5)v_{s_l}(p_l) \right|^2, \qquad (1)$$

where only the transverse part of the ρ meson propagator contributes in that limit [16]. $G_F = 1.166378 \times 10^{-5}\,\mathrm{GeV}^{-2}$ [1] is the Fermi decay constant and $C_\rho = 6$ is the effective $\rho \to \pi\pi$ coupling constant with

$$\Gamma_\rho(p_\rho^2) = \frac{C_\rho^2}{6\pi\,p_\rho^2}\Big(\frac{p_\rho^2}{4} - m_\pi^2\Big)^{3/2} \tag{2}$$

being the ρ meson width for $\sqrt{p_\rho^2}$ invariant mass. Besides, $p_B = (m_B, \vec{0})$, $p_\rho = p_B - p_l - p_\nu$ and

$$h_{\alpha\sigma}(p_B, p_\rho) = \frac{2V(q^2)}{m_B + \sqrt{p_\rho^2}}\,\epsilon_{\alpha\gamma\delta\sigma}p_B^\gamma p_\rho^\delta - i(m_B + \sqrt{p_\rho^2})A_1(q^2)\,g_{\alpha\sigma}$$
$$+i\frac{A_2(q^2)}{m_b + \sqrt{p_\rho^2}}\,q_\sigma(p_B + p_\rho)_\alpha - i\frac{2A(q^2)}{q^2}\sqrt{p_\rho^2}\,q_\alpha(p_B + p_\rho)_\sigma, \tag{3}$$

where V and A_1, A_2, A are respectively the vector and axial form factors for the $B \to \rho$ weak transition. Here we use $\epsilon_{0123} = +1$ and we have defined $q = p_B - p_\rho = p_l + p_\nu$, which is the total four-momentum carried by the leptons. In the above expression for $h_{\alpha\sigma}$ we have substituted m_ρ by $\sqrt{p_\rho^2}$ with respect to the corresponding expression in Ref. [3] The above expression for Γ can be rewritten as

$$\Gamma = \Big(\frac{G_F}{\sqrt{2}}\Big)^2 |V_{ub}|^2 \frac{1}{m_B}\frac{1}{(2\pi)^6} \int \frac{d^3p_l}{E_l} \int \frac{d^3p_\nu}{E_\nu}\,\mathcal{L}^{\alpha\alpha'}(p_l, p_\nu)$$
$$\times \sum_{r=\pm 1,0}\sum_{s=\pm 1,0} \frac{h_{\alpha\sigma}(p_B, p_\rho)\epsilon_r^{\sigma\,*}(p_\rho)h_{\alpha'\sigma'}^*(p_B, p_\rho)\epsilon_s^{\sigma'}(p_\rho)}{|p_\rho^2 - m_\rho^2 + i\sqrt{p_\rho^2}\,\Gamma_\rho(p_\rho^2)|^2}\,2\sqrt{p_\rho^2}\,\Gamma_\rho^{rs}(p_\rho^2) \tag{4}$$

where we have used that

$$\Big(-g^{\sigma\beta} + \frac{p_\rho^\sigma p_\rho^\beta}{p_\rho^2}\Big) = \sum_{r=\pm 1,0}\epsilon_r^{\sigma\,*}(p_\rho)\epsilon_r^\beta(p_\rho), \tag{5}$$

with $\epsilon_r(p_\rho)$, $r = \pm 1, 0$ the three polarization vectors of a ρ meson with invariant mass given by $\sqrt{p_\rho^2}$. $\mathcal{L}^{\alpha\alpha'}(p_l, p_\nu)$ is the lepton tensor given by

$$\mathcal{L}^{\alpha\alpha'}(p_l, p_\nu) = p_l^\alpha p_\nu^{\alpha'} - g^{\alpha\alpha'}p_l \cdot p_\nu + p_l^{\alpha'}p_\nu^\alpha \pm i\epsilon^{\gamma\alpha\delta\alpha'}p_{l\gamma}p_{\nu\,\delta}, \tag{6}$$

where the $\pm$ sign corresponds to $l^+\nu_l$ or $l\,\bar{\nu}_l$ decays respectively and

$$\Gamma_\rho^{rs}(p_\rho^2) = C_\rho^2\frac{\epsilon_r^\beta(p_\rho)\epsilon_s^{\beta'\,*}(p_\rho)}{2\sqrt{p_\rho^2}\,(2\pi)^2} \int \frac{d^3p_{\pi_1}}{2E_{\pi_1}} \int \frac{d^3p_{\pi_2}}{2E_{\pi_2}}\delta^{(4)}(p_\rho - p_{\pi_1} - p_{\pi_2})(p_{\pi_1} - p_{\pi_2})_\beta\,(p_{\pi_1} - p_{\pi_2})_{\beta'}. \tag{7}$$

The integrals in $\Gamma_\rho^{rs}(p_\rho^2)$ can be readily evaluated using Lorentz covariance and one gets that

$$\Gamma_\rho^{rs}(p_\rho^2) = -\delta_{rs}\,\Gamma_\rho(p_\rho^2), \tag{8}$$

Then,

$$\Gamma = \Big(\frac{G_F}{\sqrt{2}}\Big)^2 |V_{ub}|^2 \frac{1}{m_B}\frac{1}{(2\pi)^5} \int \frac{d^3p_l}{E_l} \int \frac{d^3p_\nu}{E_\nu}\mathcal{L}^{\alpha\alpha'}(p_l, p_\nu)\,\mathcal{H}_{\alpha\alpha'}(p_B, p_\rho)\,\delta_\rho(p_\rho^2), \tag{9}$$

where we have defined the hadronic tensor

$$\mathcal{H}_{\alpha\alpha'}(p_B, p_\rho) = \sum_{r=\pm 1,0} h_{\alpha\sigma}(p_B, p_\rho)\epsilon_r^{\sigma\,*}(p_\rho)h_{\alpha'\sigma'}^*(p_B, p_\rho)\epsilon_r^{\sigma'}(p_\rho), \tag{10}$$

and the ρ meson line shape function

$$\delta_\rho(p_\rho^2) = \frac{1}{\pi}\frac{\sqrt{p_\rho^2}\,\Gamma_\rho(p_\rho^2)}{|p_\rho^2 - m_\rho^2 + i\sqrt{p_\rho^2}\,\Gamma_\rho(p_\rho^2)|^2}. \tag{11}$$
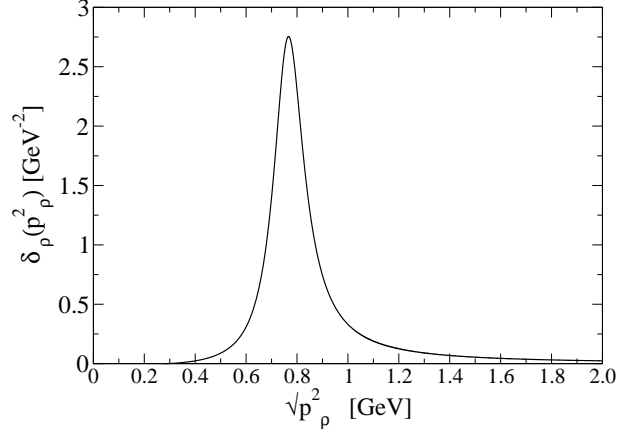
FIG. 1. $\delta_\rho(p_\rho^2)$ representation as a function of $\sqrt{p_\rho^2}$.

A representation of the latter as a function of the ρ invariant mass is given in Fig. 1. In the $\Gamma_\rho(p_\rho^2) \to 0$ limit, one would have

$$\delta_\rho(p_\rho^2) \approx \delta(p_\rho^2 - m_\rho^2), \tag{12}$$

and in that case Γ would be given by

$$
\begin{aligned}
\Gamma &\approx \left(\frac{G_F}{\sqrt{2}}\right)^2 |V_{ub}|^2 \frac{1}{m_B} \frac{1}{(2\pi)^5} \int \frac{d^3 p_l}{E_l} \int \frac{d^3 p_\nu}{E_\nu} \mathcal{L}^{\alpha\alpha'}(p_l, p_\nu) \mathcal{H}_{\alpha\alpha'}(p_B, p_\rho)\, \delta(p_\rho^2 - m_\rho^2) \\
&= \left(\frac{G_F}{\sqrt{2}}\right)^2 |V_{ub}|^2 \frac{1}{m_B} \frac{1}{(2\pi)^5} \int \frac{d^3 p_l}{E_l} \int \frac{d^3 p_\nu}{E_\nu} \int d^4 p_\rho\, \delta(p_\rho^2 - m_\rho^2)\delta^{(4)}(p_B - p_l - p_\nu - p_\rho)\, \mathcal{L}^{\alpha\alpha'}(p_l, p_\nu) \mathcal{H}_{\alpha\alpha'}(p_B, p_\rho) \\
&= \left(\frac{G_F}{\sqrt{2}}\right)^2 |V_{ub}|^2 \frac{1}{m_B} \frac{1}{(2\pi)^5} \int \frac{d^3 p_l}{E_l} \int \frac{d^3 p_\nu}{E_\nu} \int \frac{d^3 p_\rho}{2 E_\rho} \delta^{(4)}(p_B - p_l - p_\nu - p_\rho)\, \mathcal{L}^{\alpha\alpha'}(p_l, p_\nu) \mathcal{H}_{\alpha\alpha'}(p_B, p_\rho) \\
&= \Gamma(B \to \rho\, l^+ \nu_l) \tag{13}
\end{aligned}
$$

recovering the expression for the $B \to \rho\, l^+ \nu_l$ decay width. The reasoning in Ref. [9] is that because experimental data are collected for a limited range of ρ meson invariant masses, one would have

$$\mathcal{N} = \int_{p_{\rho\,\text{inf}}^2}^{p_{\rho\,\text{sup}}^2} dp_\rho^2\, \delta_\rho(p_\rho^2) < 1 \tag{14}$$

and thus one would expect

$$\Gamma \approx \mathcal{N} \times \Gamma(B \to \rho l^+ \nu_l), \tag{15}$$

from where a larger $|V_{ub}|$ value, by an approximate factor $\frac{1}{\sqrt{\mathcal{N}}}$, would be needed to actually describe the data.

Going back to the full expression, it can be rewritten as

$$
\begin{aligned}
\Gamma &= \left(\frac{G_F}{\sqrt{2}}\right)^2 \frac{|V_{ub}|^2}{m_B (2\pi)^5} \int \frac{d^3 p_\nu}{E_\nu} \int \frac{d^3 p_l}{E_l} \mathcal{L}^{\alpha\alpha'}(p_l, p_\nu)\, \mathcal{H}_{\alpha\alpha'}(p_B, p_B - p_l - p_\nu)\, \delta_\rho(p_\rho^2) \\
&= \left(\frac{G_F}{\sqrt{2}}\right)^2 \frac{|V_{ub}|^2}{m_B (2\pi)^5} \int dp_\rho^2\, \delta_\rho(p_\rho^2) \int \frac{d^3 p_\rho}{2 E_\rho} \int \frac{d^3 p_\nu}{E_\nu} \int \frac{d^3 p_l}{E_l} \mathcal{L}^{\alpha\alpha'}(p_l, p_\nu)\, \mathcal{H}_{\alpha\alpha'}(p_B, p_\rho)\delta^{(4)}(p_B - p_\rho - p_l - p_\nu)
\end{aligned}
$$

with $E_\rho = p_\rho^0 = \sqrt{p_\rho^2 + \vec{p}_\rho^{\,2}}$. Then,

$$\Gamma = \int dp_\rho^2\, \delta_\rho(p_\rho^2) \left\{ \left(\frac{G_F}{\sqrt{2}}\right)^2 \frac{|V_{ub}|^2}{m_B (2\pi)^5} \int \frac{d^3 p_\rho}{2 E_\rho} \int \frac{d^3 p_\nu}{E_\nu} \int \frac{d^3 p_l}{E_l} \mathcal{L}^{\alpha\alpha'}(p_l, p_\nu)\, \mathcal{H}_{\alpha\alpha'}(p_B, p_\rho)\delta^{(4)}(p_B - p_\rho - p_l - p_\nu) \right\}, \tag{16}$$

where the term in curly brackets represents the $B \to \rho\, l^+ \nu_l$ decay width for the case of a final ρ meson with invariant mass $\sqrt{p_\rho^2}$. The integrals on neutrino variables can be evaluated using Lorentz covariance

$$\int \frac{d^3 p_\nu}{E_\nu} \int \frac{d^3 p_l}{E_l} \mathcal{L}^{\alpha\alpha'}(p_l, p_\nu)\, \mathcal{H}_{\alpha\alpha'}(p_B, p_\rho)\delta^{(4)}(q - p_l - p_\nu) = 2\pi \frac{q^2 - m_l^2}{2q^2} \int_{-1}^{1} dx_l\, \mathcal{L}^{\alpha\alpha'}(\tilde{p}_l, \tilde{p}_\nu)\mathcal{H}_{\alpha\alpha'}(\Lambda p_B, \Lambda p_\rho), \quad (17)$$

where Λ is a rotation that takes $\vec{p}_\rho$ to the negative Z axis followed by a boost to the center of mass of the two final leptons. In that case

$$\Lambda p_B = \frac{1}{2\sqrt{q^2}} \left(m_B^2 + q^2 - p_\rho^2, 0, 0, -\lambda^{1/2}(m_B^2, q^2, p_\rho^2) \right),$$

$$\Lambda p_\rho = \frac{1}{2\sqrt{q^2}} \left(m_B^2 - q^2 - p_\rho^2, 0, 0, -\lambda^{1/2}(m_B^2, q^2, p_\rho^2) \right). \quad (18)$$

It is clear now that the product of tensors $\mathcal{L}^{\alpha\alpha'}(\tilde{p}_l, \tilde{p}_\nu)\mathcal{H}_{\alpha\alpha'}(\Lambda p_B, \Lambda p_\rho)$ does not depend on the lepton φ_l azimuthal angle that can then be integrated out to give a factor 2π. The lepton $\tilde{p}_l$ and $\tilde{p}_\nu$ momenta can been chosen for simplicity as

$$\tilde{p}_l = (\frac{q^2 + m_l^2}{2\sqrt{q^2}}, \frac{q^2 - m_l^2}{2\sqrt{q^2}}\sqrt{1 - x_l^2}, 0, -\frac{q^2 - m_l^2}{2\sqrt{q^2}}x_l), \quad (19)$$

$$\tilde{p}_\nu = (\frac{q^2 - m_l^2}{2\sqrt{q^2}}, -\frac{q^2 - m_l^2}{2\sqrt{q^2}}\sqrt{1 - x_l^2}, 0, \frac{q^2 - m_l^2}{2\sqrt{q^2}}x_l). \quad (20)$$

With this definition, x_l is the cosinus of the angle formed by the momentum of the charged lepton measured in the center of mass of the two leptons, with the direction of the momentum of the virtual ρ meson measured in the reference frame in which the B meson is at rest. Since there is no dependence on the $\vec{p}_\rho$ angular variables we find

$$\Gamma = \int dp_\rho^2\, \delta_\rho(p_\rho^2) \left\{ \left(\frac{G_F}{\sqrt{2}} \right)^2 \frac{|V_{ub}|^2}{2m_B(2\pi)^3} \int \frac{\lambda^{1/2}(m_B^2, q^2, p_\rho^2)}{2m_B} dE_\rho \frac{q^2 - m_l^2}{2q^2} \int_{-1}^{1} dx_l\, \mathcal{L}^{\alpha\alpha'}(\tilde{p}_l, \tilde{p}_\nu) \mathcal{H}_{\alpha\alpha'}(\Lambda p_B, \Lambda p_\rho) \right\}, \quad (21)$$

from where one can write the following differential decay width

$$\frac{d\Gamma}{dp_\rho^2\, dq^2\, dx_l} = \delta_\rho(p_\rho^2) \frac{G_F^2 |V_{ub}|^2}{128\pi^3 m_B^3 q^2} \lambda^{1/2}(m_B^2, q^2, p_\rho^2)(q^2 - m_l^2)\, \mathcal{L}^{\alpha\alpha'}(\tilde{p}_l, \tilde{p}_\nu)\, \mathcal{H}_{\alpha\alpha'}(\Lambda p_B, \Lambda p_\rho). \quad (22)$$

The product $\mathcal{L}^{\alpha\alpha'}(\tilde{p}_l, \tilde{p}_\nu)\, \mathcal{H}_{\alpha\alpha'}(\Lambda p_B, \Lambda p_\rho)$ can be evaluated using the formalism of helicity amplitudes (see for instance Ref. [17]) that we discuss in Appendix A. The final expression for the triple differential decay width is

$$\frac{d\Gamma}{dp_\rho^2\, dq^2\, dx_l} = \delta_\rho(p_\rho^2) \frac{G_F^2 |V_{ub}|^2}{512\pi^3 m_B^3 q^2} \lambda^{1/2}(m_B^2, q^2, p_\rho^2)(q^2 - m_l^2)^2$$

$$\times \left\{ 2(1 - x_l^2)H_{00} + (1 \mp x_l)^2 H_{+1+1} + (1 \pm x_l)^2 H_{-1-1} \right.$$

$$\left. + \frac{m_l^2}{q^2}[(1 - x_l^2)(H_{+1+1} + H_{-1-1}) + 2x_l^2 H_{00} + 2H_{tt} + 4x_l H_{t0}] \right\}. \quad (23)$$

where the different H_{rs} hadronic helicity amplitudes are defined and given in Appendix A. Besides de upper sign corresponds to $l^+ \nu_l$ decays, like experiments in Refs. [2, 10, 11], while the lower sign corresponds to $l^- \bar{\nu}_l$ ones, like in the latest Belle [8] analysis. This difference is only relevant if the integration over x_l does not cover its full range $[-1, 1]$ as in the case of CLEO data [10]. Neglecting lepton masses, a good approximation for light $l = e, \mu$ final leptons, one arrives at the expression

$$\frac{d\Gamma}{dp_\rho^2\, dq^2\, dx_l} \approx \delta_\rho(p_\rho^2) \frac{G_F^2 |V_{ub}|^2}{512\pi^3 m_B^3} q^2 \lambda^{1/2}(q^2, m_B^2, p_\rho^2)\Big[2(1 - x_l)^2\, \mathcal{H}_{00} + (1 \mp x_l)^2\, \mathcal{H}_{+1+1} + (1 \pm x_l)^2\, \mathcal{H}_{-1-1}\Big]. \quad (24)$$

where the corresponding helicity amplitudes depend only on the $V(q^2)$, $A_1(q^2)$ and $A_2(q^2)$ form factors.

III. FITTING PROCEDURE

The fitting procedure that we shall use is, with minor modifications, the one followed in Ref. [3]. We describe our $B \to \rho$ form factors using a multiply subtracted Omnès dispersion relation [18, 19], the latter being based in unitarity and analyticity. We will have

$$F(q^2) = \frac{1}{s_0 - q^2} \prod_{j=0}^{n} [F(q_j^2)(s_0 - q_j^2)]^{\alpha_j(q^2)} \quad , \quad \alpha_j(q^2) = \prod_{\substack{k=0 \\ k \neq j}}^{n} \frac{q^2 - q_k^2}{q_j^2 - q_k^2}, \quad F = V, A_1, A_2. \tag{25}$$

s_0 corresponds to the pole of the form factor and we shall use $s_0 = m_{B^*} = 5.3252\,\text{GeV}$ [1] for the vector form factor and $s_0 = 5.7235\,\text{GeV}$ [1] (the mass of the 1^+ B meson) for the two axial form factors. As in Ref. [3], we use three subtraction points at $q^2 = 0, 2q_{\max}^2/3, q_{\max}^2$ where we take $q_{max}^2 = (m_B - m_\rho)^2 = 20.3\,\text{GeV}^2$ as used in Refs. [2, 10, 11]. Note however the latest Belle analysis in Ref. [8] works with q^2 values up to 22-24 GeV^2. The values of $F(0), F(2q_{\max}^2/3)$ and $F(q_{\max}^2)$, for $F = V, A_1, A_2$, are treated as free parameters as will be $|V_{ub}|$. The values of these ten parameters are then fitted to reproduce form factor theoretical results obtained in LCSR [13] and lattice calculations [14, 15], and experimental measurements of partial branching fractions obtained by the CLEO [10], BaBar [2] and Belle [8] collaborations. The partial branching fractions are defined as

$$B = \frac{1}{\Gamma(B^0)} \int_{p_{\rho\,\text{inf}}^2}^{p_{\rho\,\text{sup}}^2} dp_\rho^2 \int_{q_{\text{inf}}^2}^{q_{\text{sup}}^2} dq^2 \int_{x_{l\,\text{inf}}}^{x_{l\,\text{sup}}} dx_l \, \frac{d\Gamma}{dp_\rho^2 \, dq^2 \, dx_l}, \tag{26}$$

where for the B^0 lifetime we use $\tau_0 = 1/\Gamma(B^0) = (1.519 \pm 0.007) \times 10^{-12}\,\text{s}$ [1], while for the lower (inf) and upper (sup) limits in each of the integration variables we use the values provided by the experiments (see Table I). For the B^+ lifetime to be used below we take $\tau_+ = (1.641 \pm 0.008) \times 10^{-12}\,\text{s}$ [1].

A. Experimental and theoretical input

Experimental data by the CLEO [10], BaBar [2] and Belle [8] collaborations consist of partial branching fractions as defined in Eq.(26). Their values together with statistical and systematic errors are collected in Table I. CLEO has made used of isospin symmetry to combine results for neutral and charged B meson decays. For BaBar data we have combined their $B^0 \to \rho^- l^+ \nu_l$ 4-mode and $B^+ \to \rho^0 l^+ \nu_l$ data in the following way: Denoting as σ and ϵ the statistical and systematic errors respectively we have evaluated

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_{\rho^-}^2} + \frac{1}{(2\frac{\tau_0}{\tau_+}\sigma_{\rho^0})^2},$$

$$\frac{1}{\epsilon^2} = \frac{1}{\epsilon_{\rho^-}^2} + \frac{1}{(2\frac{\tau_0}{\tau_+}\epsilon_{\rho^0})^2},$$

$$\frac{B}{\sigma^2 + \epsilon^2} = \frac{B_{\rho^-}}{\sigma_{\rho^-}^2 + \epsilon_{\rho^-}^2} + \frac{2\frac{\tau_0}{\tau_+}B_{\rho^0}}{(2\frac{\tau_0}{\tau_+}\sigma_{\rho^0})^2 + (2\frac{\tau_0}{\tau_+}\epsilon_{\rho^0})^2} \tag{27}$$

In the case o the newest Belle's data [8] we treat separately the neutral and charge meson decays since they have been evaluated for different q^2 bins. However in order to perform the fit we multiply the ρ^0 data by $2\tau_0/\tau_+$.

The theoretical input consists of form factors values. For q^2 in the $[0, 10]\,\text{GeV}^2$ range we will use the LCSR form factor values obtained from the parameterizations given in Ref. [13]. For higher q^2 we will use the lattice results by the SPQcdR [14] and UKQCD [15] collaborations. All of them are collected in Table II. For the LCSR form factors, and following Ref. [2], we have assumed a 10% error at $q^2 = 0$ that increases linearly to 13% at $q^2 = 14\,\text{GeV}^2$. SPQcdR errors include both systematic and statistical uncertainties while in the case of UKQCD data both statistical and systematic errors are shown. The latter are highly asymmetric. Following Ref. [3], and in order to perform the fit, we put the UKQCD form factors values in the center of their systematic range and we use half that range as the systematic error.

	$q^2\,[\text{GeV}^2]$	x_l	$\sqrt{p_\rho^2}$	$10^4 B$
CLEO [10]	$0-2$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	$0.45 \pm 0.20 \pm 0.15$
	$2-8$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	$0.96 \pm 0.20 \pm 0.29$
	$8-16$	$[0,1]$	$m_\rho \pm 2\Gamma_\rho$	$0.75 \pm 0.16 \pm 0.14$
	$16-20.3$	$[0,1]$	$m_\rho \pm 2\Gamma_\rho$	$0.35 \pm 0.07 \pm 0.05$
	$8-20.3$	$[-1,0]$	$m_\rho \pm 2\Gamma_\rho$	$0.42 \pm 0.18 \pm 0.31$
BaBar [2]	$0-8$	$[-1,1]$	$[0.65,0.85]\,\text{GeV}$	$0.587 \pm 0.084 \pm 0.097$
	$8-16$	$[-1,1]$	$[0.65,0.85]\,\text{GeV}$	$0.928 \pm 0.047 \pm 0.103$
	$16-20.3$	$[-1,1]$	$[0.65,0.85]\,\text{GeV}$	$0.263 \pm 0.017 \pm 0.042$
Belle [8]				
ρ^+ data	$0-4$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.373 ± 0.106
	$4-8$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.718 ± 0.116
	$8-12$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.806 ± 0.123
	$12-16$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.723 ± 0.125
	$16-20$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.626 ± 0.115
	$20-24$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.017 ± 0.079
ρ^0 data $\times\, 2\tau_0/\tau_+$	$0-2$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.2296 ± 0.0629
	$2-4$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.2851 ± 0.0574
	$4-6$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.3314 ± 0.0629
	$6-8$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.4017 ± 0.0629
	$8-10$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.2647 ± 0.0537
	$10-12$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.3684 ± 0.0629
	$12-14$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.4147 ± 0.0629
	$14-16$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.4017 ± 0.0611
	$16-18$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.3240 ± 0.0592
	$18-20$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.2647 ± 0.0180
	$20-22$	$[-1,1]$	$m_\rho \pm 2\Gamma_\rho$	0.1092 ± 0.0481

TABLE I. Experimental partial branching fractions used as input. The different q^2, x_l and $\sqrt{p_\rho^2}$ intervals are shown. Belle's original ρ^0 data in Ref. [8] is shown multiplied by the factor $2\tau_0/\tau_+$. Both CLEO and Belle use $\sqrt{p_\rho^2}$ in the interval $m_\rho \pm 2\Gamma_\rho$

B. χ^2 definition

The χ^2 function we use for the fit is

$$\chi^2 = \sum_{j,k=1}^{115} [(Q_j^{\text{input}} - Q_j^{\text{fit}})C_{jk}^{-1}(Q_k^{\text{input}} - Q_k^{\text{fit}})] \tag{28}$$

where Q_j^{input} represents any of the input quantities and Q_j^{fit} is the corresponding value obtained in our calculation. In order to construct the C covariant matrix we have not considered any correlation between data from different experiments or between different theoretical calculations, or between experimental and theoretical inputs. C is then block diagonal. CLEO and BaBar collaborations provide statistical and systematic correlation matrices and in these two cases their corresponding blocks in C are constructed as

$$C_{jk} = \sigma_j \sigma_k \mathcal{C}_{jk}^{\text{stat}} + \epsilon_j \epsilon_k \mathcal{C}_{jk}^{\text{sys}}. \tag{29}$$

with $\mathcal{C}^{\text{stat/sys}}$ the statistical/systematic correlation matrices. The Belle Collaboration [8] also provides two independent statistical correlation matrices, one for ρ^+ data and one for ρ^0 data, so that we build two independent blocks as

$$C_{jk} = \sigma_j \sigma_k \mathcal{C}_{jk}^{\text{stat}}. \tag{30}$$

For the block corresponding to UKQCD data we use

$$C_{jk} = \sigma_j^2 \delta_{jk} + \epsilon_j \epsilon_k, \tag{31}$$

	q^2 [GeV^2]	V	A_1	A_2
LCSR [13]	0	0.324 ± 0.032	0.240 ± 0.024	0.221 ± 0.022
	1	0.343 ± 0.035	0.247 ± 0.025	0.232 ± 0.024
	2	0.364 ± 0.038	0.254 ± 0.026	0.244 ± 0.025
	3	0.387 ± 0.041	0.261 ± 0.028	0.257 ± 0.027
	4	0.412 ± 0.045	0.269 ± 0.029	0.271 ± 0.029
	5	0.440 ± 0.049	0.277 ± 0.031	0.286 ± 0.032
	6	0.471 ± 0.053	0.286 ± 0.032	0.302 ± 0.034
	7	0.506 ± 0.058	0.295 ± 0.034	0.320 ± 0.037
	8	0.546 ± 0.064	0.305 ± 0.036	0.339 ± 0.040
	9	0.590 ± 0.070	0.316 ± 0.038	0.360 ± 0.043
	10	0.641 ± 0.078	0.327 ± 0.040	0.384 ± 0.047
SPQcdR [14]	10.69	0.51 ± 0.26	0.354 ± 0.085	0.38 ± 0.26
	12.02	0.61 ± 0.28	0.384 ± 0.087	0.49 ± 0.30
	13.35	0.74 ± 0.30	0.421 ± 0.089	0.65 ± 0.35
	14.68	0.93 ± 0.31	0.465 ± 0.092	0.93 ± 0.41
	16.01	1.20 ± 0.32	0.519 ± 0.097	1.41 ± 0.56
	17.34	1.61 ± 0.33	0.588 ± 0.108	2.39 ± 1.23
	18.67	2.26 ± 0.55	0.678 ± 0.134	4.7 ± 4.1
UKQCD [15]	12.67	$0.684 \pm 0.162^{+0.00}_{-0.56}$	$0.439 \pm 0.067^{+0.000}_{-0.080}$	$0.70 \pm 0.49^{+0.08}_{-0.03}$
	13.01	$0.714 \pm 0.162^{+0.00}_{-0.50}$	$0.448 \pm 0.065^{+0.000}_{-0.079}$	$0.71 \pm 0.46^{+0.08}_{-0.03}$
	13.51	$0.763 \pm 0.155^{+0.00}_{-0.40}$	$0.460 \pm 0.063^{+0.000}_{-0.075}$	$0.72 \pm 0.43^{+0.10}_{-0.02}$
	14.02	$0.818 \pm 0.147^{+0.00}_{-0.31}$	$0.472 \pm 0.059^{+0.000}_{-0.073}$	$0.73 \pm 0.42^{+0.12}_{-0.01}$
	14.52	$0.883 \pm 0.141^{+0.00}_{-0.24}$	$0.485 \pm 0.055^{+0.000}_{-0.070}$	$0.76 \pm 0.42^{+0.14}_{-0.03}$
	15.03	$0.967 \pm 0.137^{+0.00}_{-0.20}$	$0.498 \pm 0.051^{+0.000}_{-0.068}$	$0.78 \pm 0.46^{+0.16}_{-0.05}$
	15.53	$1.057 \pm 0.134^{+0.00}_{-0.19}$	$0.513 \pm 0.049^{+0.000}_{-0.067}$	$0.81 \pm 0.54^{+0.18}_{-0.06}$
	16.04	$1.164 \pm 0.150^{+0.10}_{-0.21}$	$0.529 \pm 0.047^{+0.000}_{-0.066}$	$0.84 \pm 0.71^{+0.20}_{-0.07}$
	16.54	$1.296 \pm 0.184^{+0.21}_{-0.25}$	$0.544 \pm 0.043^{+0.000}_{-0.062}$	$0.87 \pm 0.97^{+0.23}_{-0.08}$
	17.05	$1.46 \pm 0.26^{+0.34}_{-0.30}$	$0.560 \pm 0.043^{+0.000}_{-0.059}$	$0.90 \pm 1.35^{+0.27}_{-0.07}$
	17.55	$1.67 \pm 0.40^{+0.49}_{-0.36}$	$0.577 \pm 0.043^{+0.000}_{-0.058}$	$0.90 \pm 1.89^{+0.33}_{-0.03}$
	16.54	$2.02 \pm 0.68^{+0.73}_{-0.48}$	$0.599 \pm 0.052^{+0.000}_{-0.058}$	$0.9 \pm 2.9^{+0.4}_{-0.1}$

TABLE II. Theoretical form factor inputs used for the fit. SPQcdR and UKQCD data taken from Table 2 in Ref. [3]. Concerning UKQCD data see text for details.

that assumes independent statistical uncertainties and fully correlated systematic errors. Finally for LCSR and SPQcdR results we use

$$C_{jk} = \sigma_j^2 \delta_{jk}. \tag{32}$$

IV. RESULTS AND DISCUSSION

Best fit results are compiled in Table III. The fit has $\chi^2/$d.o.f. $= 1.4$ for a total of 105 degrees of freedom. The corresponding Gaussian correlation matrix is given in Appendix B. As we see, the extracted $|V_{ub}|$ increases, as expected, and now it is in excellent agreement with the determination from $B \to \pi$ decays given by $(3.23 \pm 0.31) \times 10^{-3}$ [1]. In Fig. 2 we show the form factors, together with their 68% confidence level bands, that result from the fit, and we compare them to the different input theoretical input. Finally in Fig. 2 (bottom-right panel) we also present our prediction for $\frac{10^4}{\Gamma(B^0)} \frac{d\Gamma}{dq^2}$ and compare it to data by the Belle [8], BaBar [2], and CLEO [10] collaborations. In this case, we show two predictions corresponding to the two $\sqrt{p_\rho^2}$ ranges used by the different experiments. The largest discrepancy occurs for CLEO data where the experimental distribution peaks a significantly smaller q^2 values than the theoretical distribution. This seems to be incompatible with the theoretical form factor predictions at low q^2

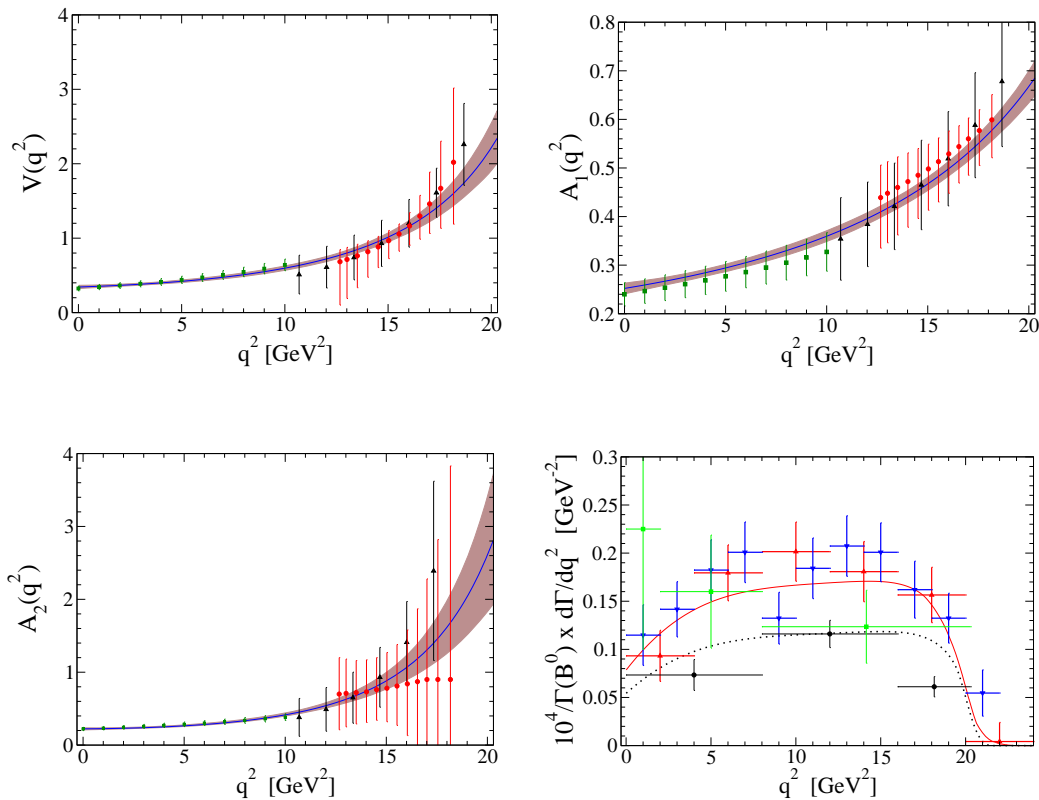|	
$|V_{ub}|$	$(3.40 \pm 0.15) \times 10^{-3}$
$V(0)$	0.343 ± 0.022
$V(2q_{\max}^2/3)$	0.848 ± 0.043
$V(q_{\max}^2)$	2.34 ± 0.35
$A_1(0)$	0.252 ± 0.011
$A_1(2q_{\max}^2/3)$	0.429 ± 0.013
$A_1(q_{\max}^2)$	0.684 ± 0.036
$A_2(0)$	0.223 ± 0.014
$A_2(2q_{\max}^2/3)$	0.683 ± 0.064
$A_2(q_{\max}^2)$	2.80 ± 0.85

TABLE III. Best fit parameters of the global fit.

FIG. 2. Top panels and left-bottom panel: Form factor obtained from the fit (solid line) together with their corresponding 68% confidence level band. We show the predictions from LCSR [13] (squares), and lattice QCD from the SPQcdR [14] (up-triangles) and UKQCD [15] (circles) collaborations. Right-bottom panel: $\frac{10^4}{\Gamma(B^0)} \frac{d\Gamma}{dq^2}$[GeV^{-2}]. Solid line: $m_\rho - 2\Gamma_\rho < \sqrt{p_\rho^2} < m_\rho + 2\Gamma_\rho$. Dotted line: $0.65\,\text{GeV} < \sqrt{p_\rho^2} < 0.85\,\text{GeV}$. Up-triangles, down-triangles, circles, and squares stand respectively for Belle ρ^+ and ρ^0 data [8], BaBar data [2], and CLEO data [10].

obtained in LCSR. Belle and BaBar results agree better with our analysis. However one clearly sees in Fig. 2 that BaBar data would prefer a smaller $|V_{ub}|$ value, whereas Belle data would be better reproduced with a higher $|V_{ub}|$ value.

The recent data by the Belle Collaboration [8] present two good features. First, as in the case of CLEO data [10], the experiment considers a broad range of the ρ meson invariant masses ($m_\rho \pm 2\Gamma_\rho$). Second, Belle gives results for

smaller q^2 bins which means more data and then the possibility for more stringent constraints on theoretical models. In this respect it is worth making a fit just to Belle's data together with the form factors. In this case one gets $|V_{ub}| = (3.62 \pm 0.17) \times 10^{-3}$ which is in perfect agreement with the analyses in Ref. [8] where other sets of form factors were used.

The total decay rate from BaBar extracted in Ref. [2] is some 15% smaller that the one provided in their earlier measurement of Ref. [20] and used in [3]. Also, the expectation in Ref. [9] that, once the effects of the finite ρ-meson invariant mass range were taken into account, the $|V_{ub}|$ value extracted from Ref. [2] would increase from $|V_{ub}| = (2.75 \pm 0.24) \times 10^{-3}$ [2] to $|V_{ub}| \approx 3.6 \times 10^{-3}$ is not met by the data. A fit to the form factors and to the BaBar data of Ref. [2] alone would give $|V_{ub}| = (3.03 \pm 0.23) \times 10^{-3}$. These two problems seem to come mainly from the last BaBar data point that, as we see in the bottom-right panel of Fig. 2, is well below our global fit. In fact, a fit in which we only include the form factors and the two first BaBar data points gives $|V_{ub}| = (3.42 \pm 0.25) \times 10^{-3}$, a larger value that is in agreement with the determination using Belle data alone and within the expectations in Ref. [9].
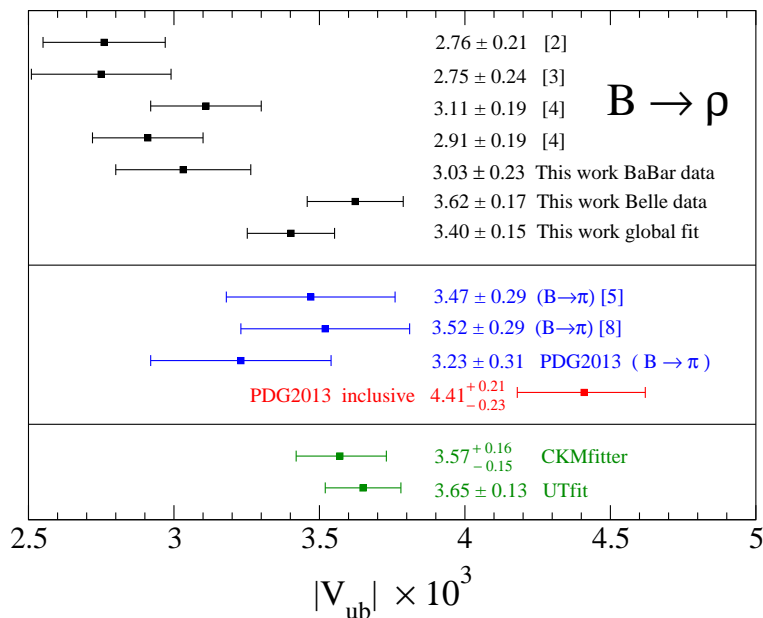

FIG. 3. Different $|V_{ub}|$ values obtained in $B \to \rho$ decay analyses. We also show for comparison the $|V_{ub}|$ determination from the $B \to \pi$ decay in Refs. [5, 8], the PDG exclusive and inclusive 2013 average updates [1], and the fits from the CKMfitter [21] and UTfit [22] Groups.

In Fig. 3 we show different $|V_{ub}|$ values obtained in $B \to \rho$ decay analyses. A comparison of the results in Refs. [2–4] and the present calculation shows, as it was pointed out in Ref. [9], the relevance of taking into account the ρ meson broad-width effects. Our global fit result is now in good agreement with the determination in Ref. [5] from $B \to \pi$ exclusive decay and the average value quoted in the PDG 2013 update [1] also from the same reaction. The results by the CKMfitter [21] and UTfit [22] Groups are in very good agreement with our determination using only the recent Belle data. One should conclude that the disagreement between the $|V_{ub}|$ values extracted from $B \to \rho$ and $B \to \pi$ reactions came mostly from an incorrect treatment of the former in previous analyses, where the effects of the ρ meson width were neglected. However, as seen in Fig. 3, there still persist the large discrepancy between inclusive and exclusive determinations of $|V_{ub}|$, with the global fits by the CKMfitter [21] and UTfit [22] groups being in better agreement with the latter.

Appendix A: Helicity amplitudes

In this appendix we shall write the product $\mathcal{L}^{\alpha\alpha'}(\tilde{p}_l, \tilde{p}_\nu)\mathcal{H}_{\alpha\alpha'}(\Lambda p_B, \Lambda p_\rho)$ in terms of helicity amplitudes. For that purpose we use that

$$g^{\mu\nu} = \sum_{r=t,\pm1,0} g_{rr}\, \epsilon_r^\mu(\Lambda q)\epsilon_r^{\nu\,*}(\Lambda q) = \sum_{r=t,\pm1,0} g_{rr}\, \epsilon_r^{\mu\,*}(\Lambda q)\epsilon_r^{\nu}(\Lambda q), \tag{A1}$$

with $g_{tt} = -g_{+1+1} = -g_{-1-1} = -g_{00} = 1$ and

$$\epsilon_t(\Lambda q) = \frac{\Lambda q}{\sqrt{q^2}} = (1,0,0,0), \tag{A2}$$

$$\epsilon_{+1}(\Lambda q) = \left(0, \frac{-1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}, 0\right), \tag{A3}$$

$$\epsilon_{-1}(\Lambda q) = \left(0, \frac{1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}, 0\right), \tag{A4}$$

$$\epsilon_0(\Lambda q) = (0,0,0,1). \tag{A5}$$

Then,

$$\mathcal{L}^{\alpha\alpha'}(\tilde{p}_l, \tilde{p}_\nu)\, \mathcal{H}_{\alpha\alpha'}(\Lambda p_B, \Lambda p_\rho) = \sum_{r=t,\pm1,0}\sum_{s=t,\pm1,0} g_{rr}g_{ss}\, \mathcal{L}_{rs}(\tilde{p}_l, \tilde{p}_\nu)\, \mathcal{H}_{rs}(\Lambda p_B, \Lambda p_\rho), \tag{A6}$$

where we have defined the hadronic and leptonic helicity amplitudes

$$\mathcal{H}_{rs}(\Lambda p_B, \Lambda p_\rho) = \epsilon_r^{\alpha\,*}(\Lambda q)\, \mathcal{H}_{\alpha\alpha'}(\Lambda p_B, \Lambda p_\rho)\, \epsilon_s^{\alpha'}(\Lambda q), \tag{A7}$$

$$\mathcal{L}_{rs}(\tilde{p}_l, \tilde{p}_\nu) = \epsilon_r^{\beta}(\Lambda q)\, \mathcal{L}_{\beta\beta'}(\tilde{p}_l, \tilde{p}_\nu)\, \epsilon_s^{\beta'\,*}(\Lambda q). \tag{A8}$$

As

$$\mathcal{H}_{\alpha\alpha'}(\Lambda p_B, \Lambda p_\rho) = \sum_{u=\pm1,0} h_{\alpha\sigma}(\Lambda p_B, \Lambda p_\rho)\epsilon_u^{\sigma\,*}(\Lambda p_\rho)h^*_{\alpha'\sigma'}(\Lambda p_B, \Lambda p_\rho)\epsilon_u^{\sigma'}(\Lambda p_\rho), \tag{A9}$$

we will have

$$\mathcal{H}_{rs}(\Lambda p_B, \Lambda p_\rho) = \sum_{u=\pm1,0} \epsilon_r^{\alpha\,*}(\Lambda q)h_{\alpha\sigma}(\Lambda p_B, \Lambda p_\rho)\epsilon_u^{\sigma\,*}(\Lambda p_\rho)\epsilon_s^{\alpha'}(\Lambda q)h^*_{\alpha'\sigma'}(\Lambda p_B, \Lambda p_\rho)\epsilon_u^{\sigma'}(\Lambda p_\rho) \tag{A10}$$

$$= \sum_{u=\pm1,0} h_{ru}(\Lambda p_B, \Lambda p_\rho)h^*_{su}(\Lambda p_B, \Lambda p_\rho), \tag{A11}$$

with

$$h_{ru}(\Lambda p_B, \Lambda p_\rho) = \epsilon_r^{\alpha\,*}(\Lambda q)h_{\alpha\sigma}(\Lambda p_B, \Lambda p_\rho)\epsilon_u^{\sigma\,*}(\Lambda p_\rho). \tag{A12}$$

Using

$$\epsilon_{+1}(\Lambda p_\rho) = \left(0, \frac{-1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}, 0\right), \tag{A13}$$

$$\epsilon_{-1}(\Lambda p_\rho) = \left(0, \frac{1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}, 0\right), \tag{A14}$$

$$\epsilon_0(\Lambda p_\rho) = \left(\frac{\lambda^{1/2}(q^2, m_B^2, p_\rho^2)}{2\sqrt{q^2}\sqrt{p_\rho^2}}, 0, 0, -\frac{m_B^2 - q^2 - p_\rho^2}{2\sqrt{q^2}\sqrt{p_\rho^2}}\right). \tag{A15}$$

we can evaluate the h_{ru} quantities. The nonzero ones are

$$h_{t0} = \frac{\lambda^{1/2}(m_B^2, q^2, p_\rho^2)}{2\sqrt{p_\rho^2}\sqrt{q^2}} \left[-iA_1(q^2)\left(m_B + \sqrt{p_\rho^2}\right) + iA_2(q^2)\left(m_B - \sqrt{p_\rho^2}\right) - i2A(q^2)\sqrt{p_\rho^2} \right],$$

$$h_{+1-1} = -iA_1(q^2)\left(m_B + \sqrt{p_\rho^2}\right) + iV(q^2)\frac{\lambda^{1/2}(m_B^2, q^2, p_\rho^2)}{m_B + \sqrt{p_\rho^2}},$$

$$h_{-1+1} = -iA_1(q^2)\left(m_B + \sqrt{p_\rho^2}\right) - iV(q^2)\frac{\lambda^{1/2}(m_B^2, q^2, p_\rho^2)}{m_B + \sqrt{p_\rho^2}},$$

$$h_{00} = \frac{1}{2\sqrt{p_\rho^2}\sqrt{q^2}} \left[-iA_1(q^2)\left(m_B + \sqrt{p_\rho^2}\right)(m_B^2 - p_\rho^2 - q^2) + i\frac{A_2(q^2)\lambda(m_B^2, q^2, p_\rho^2)}{m_B + \sqrt{p_\rho^2}} \right]. \tag{A16}$$

From these values we get the following nonzero hadronic helicity amplitudes

$$\mathcal{H}_{tt} = \frac{\lambda(m_B^2, q^2, p_\rho^2)}{4\, p_\rho^2\, q^2} \left[-A_1(q^2)\left(m_B + \sqrt{p_\rho^2}\right) + A_2(q^2)\left(m_B - \sqrt{p_\rho^2}\right) - 2A(q^2)\sqrt{p_\rho^2} \right]^2,$$

$$\mathcal{H}_{t0} = \mathcal{H}_{0t} = \frac{\lambda^{1/2}(m_B^2, q^2, p_\rho^2)}{4\, p_\rho^2\, q^2} \left[-A_1(q^2)\left(m_B + \sqrt{p_\rho^2}\right)(m_B^2 - p_\rho^2 - q^2) + \frac{A_2(q^2)\lambda(m_B^2, q^2, p_\rho^2)}{m_B + \sqrt{p_\rho^2}} \right]$$

$$\times \left[-A_1(q^2)\left(m_B + \sqrt{p_\rho^2}\right) + A_2(q^2)\left(m_B - \sqrt{p_\rho^2}\right) - 2A(q^2)\sqrt{p_\rho^2} \right],$$

$$\mathcal{H}_{00} = \frac{1}{4\, p_\rho^2\, q^2} \left[-A_1(q^2)\left(m_B + \sqrt{p_\rho^2}\right)(m_B^2 - p_\rho^2 - q^2) + \frac{A_2(q^2)\lambda(m_B^2, q^2, p_\rho^2)}{m_B + \sqrt{p_\rho^2}} \right]^2,$$

$$\mathcal{H}_{+1+1} = \left[A_1(q^2)\left(m_B + \sqrt{p_\rho^2}\right) - V(q^2)\frac{\lambda^{1/2}(m_B^2, q^2, p_\rho^2)}{m_B + \sqrt{p_\rho^2}} \right]^2,$$

$$\mathcal{H}_{-1-1} = \left[A_1(q^2)\left(m_B + \sqrt{p_\rho^2}\right) + V(q^2)\frac{\lambda^{1/2}(m_B^2, q^2, p_\rho^2)}{m_B + \sqrt{p_\rho^2}} \right]^2. \tag{A17}$$

The corresponding leptonic helicity amplitudes are given by

$$\mathcal{L}_{tt} = (q^2 - m_l^2)\frac{m_l^2}{2\, q^2},$$

$$\mathcal{L}_{t0} = \mathcal{L}_{0t} = -(q^2 - m_l^2)x_l\frac{m_l^2}{2\, q^2},$$

$$\mathcal{L}_{00} = (q^2 - m_l^2)\frac{1}{2}\left[1 - x_l^2 + x_l^2\frac{m_l^2}{q^2}\right],$$

$$\mathcal{L}_{+1+1} = (q^2 - m_l^2)\frac{1}{4}\left[(1 \mp x_l)^2 + (1 - x_l^2)\frac{m_l^2}{q^2}\right],$$

$$\mathcal{L}_{-1-1} = (q^2 - m_l^2)\frac{1}{4}\left[(1 \pm x_l)^2 + (1 - x_l^2)\frac{m_l^2}{q^2}\right], \tag{A18}$$

where the upper (lower) sign corresponds to $l^+\nu_l$ $(l^-, \bar{\nu}_l)$ decays.

Appendix B: Gaussian correlation matrix

The Gaussian correlation matrix corresponding to the best fit parameters in Table III reads

$$
\begin{pmatrix}
1.0000 & -0.0134 & -0.3492 & -0.3044 & -0.1912 & -0.6601 & -0.2927 & 0.1443 & 0.3952 & 0.3175 \\
 & 1.0000 & -0.2166 & 0.3346 & -0.0457 & 0.0275 & -0.0663 & 0.0084 & -0.0155 & -0.0094 \\
 & & 1.0000 & 0.4671 & 0.0578 & 0.3140 & 0.2498 & -0.0201 & 0.1446 & 0.1598 \\
 & & & 1.0000 & -0.0126 & 0.3321 & 0.0780 & -0.0041 & 0.1166 & 0.1476 \\
 & & & & 1.0000 & -0.1681 & 0.3890 & 0.4046 & -0.2209 & -0.1240 \\
 & & & & & 1.0000 & 0.3033 & -0.1671 & 0.1307 & 0.0624 \\
 & & & & & & 1.0000 & 0.1829 & 0.1971 & 0.3665 \\
 & & & & & & & 1.0000 & 0.0989 & 0.3729 \\
 & & & & & & & & 1.0000 & 0.8706 \\
 & & & & & & & & & 1.0000
\end{pmatrix}
\tag{B1}
$$

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