Heavy Antiquark–Diquark Symmetry and Heavy Hadron Molecules: Are There Triply Heavy Pentaquarks?

Feng-Kun Guo,[1,*] Carlos Hidalgo-Duque,[2,†] Juan Nieves,[2,‡] and Manuel Pavón Valderrama[3,§]

[1] *Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany*

[2] *Instituto de Física Corpuscular (IFIC), Centro Mixto CSIC-Universidad de Valencia, Institutos de Investigación de Paterna, Aptd. 22085, E-46071 Valencia, Spain*

[3] *Institut de Physique Nucléaire, Université Paris-Sud, IN2P3/CNRS, F-91406 Orsay Cedex, France*

We explore the consequences of heavy flavour, heavy quark spin and heavy antiquark-diquark symmetries for hadronic molecules within an effective field theory framework. Owing to heavy antiquark-diquark symmetry, the doubly heavy baryons have approximately the same light-quark structure as the heavy antimesons. As a consequence, the existence of a heavy meson-antimeson molecule implies the possibility of a partner composed of a heavy meson and a doubly-heavy baryon. In this regard, the $D\bar{D}^*$ molecular nature of the $X(3872)$ will hint at the existence of several baryonic partners with isospin $I = 0$ and $J^P = \frac{5}{2}^-$ or $\frac{3}{2}^-$. Moreover, if the $Z_b(10650)$ turns out to be a $B^*\bar{B}^*$ bound state, we can be confident of the existence of $\Xi^*_{bb}\bar{B}^*$ hadronic molecules with quantum numbers $I(J^P) = 1(\frac{1}{2}^-)$ and $I(J^P) = 1(\frac{3}{2}^-)$. These states are of special interest since they can be considered to be triply-heavy pentaquarks.

PACS numbers: 03.65.Ge,13.75.Lb,14.40Pq,14.40Rt

The spectroscopic properties of bound states tell us a great deal about the symmetries and underlying dynamics of their components. For instance, the hydrogen atom has been an extraordinary source of information about several aspects of quantum electrodynamics, from the accidental SO(4) symmetry in the spectrum to the vacuum polarization, radiative corrections and renormalization that are necessary to explain the Lamb shift. The classification of hadrons according to isospin, SU(3) flavour and so on reveals the underlying strong dynamics binding the hadrons and has been instrumental in the past for the development of quantum chromodynamics (QCD). Conversely a deeper understanding of QCD and its symmetries — such as heavy quark symmetries — will eventually give new insights into the hadron spectrum.

Heavy hadron molecules are a type of exotic hadron theorized more than thirty years ago [1, 2]. Their main component is a pair of heavy hadrons instead of a quark–antiquark pair. The experimental advances in heavy quarkonium spectroscopy have identified several molecular candidates among the recently observed XYZ states. The most promising ones are the $X(3872)$ [3] and the twin $Z_b(10610)$ and $Z_b(10650)$ states, to be called Z_b and Z_b', respectively [4, 5]. The $Y(4260)$ [6] might have finally revealed its (so far conjectural [7, 8]) molecular nature [9] by being an intermediate step in the production of the recent $Z_c(3900)$ [10, 11] (which may also be molecular [9, 12, 13]). There are always competing explanations such as conventional heavy quarkonia, tetraquarks or hybrid states which have the same quantum numbers. It is thus challenging to distinguish the hadronic molecules from other possibilities. In this work, we will explore a model-independent approach which leads to unique predictions for hadronic molecules. A set of triply-heavy pentaquark-like molecules can be predicted as partners of heavy mesonic molecules as a consequence of heavy quark symmetries.

Heavy hadronic molecules are very interesting objects because they have an exceptionally high degrees of symmetry stemming from their combined light and heavy quark content [12, 14–20]. While the presence of heavy quarks imply that molecular states are subject to heavy quark symmetries, the light quarks allow to classify the molecular states in isospin and SU(3) flavor multiplets [20]. Among the manifestations of heavy quark symmetries, we can count heavy quark spin symmetry (HQSS), which implies that molecular states may appear in HQSS multiplets, for instance (but not limited to) the $D^*_{s0}(2317)$ and $D_{s1}(2460)$ [14], the Z_b and Z_b' [15–17, 19]. From heavy flavor symmetry (HFS), we know that the interaction among heavy hadrons is roughly independent on whether they contain a charm or a bottom quark. In this regard, the recently discovered $Z_c(3900)$ could very well be a heavy flavour partner of the Z_b [12]. Last, there is a less explored type of heavy quark symmetry that is going to have particularly interesting consequences: heavy antiquark-diquark symmetry (HADS) [21].

HADS states that the two heavy quarks within a doubly heavy baryon behave approximately as a heavy antiquark. The heavy diquark component of the baryon forms a color anti-triplet with a characteristic length scale of $1/(m_Q v)$, where m_Q is the mass of the heavy quarks and v their velocity. The length scale of the diquark is smaller than the typical QCD length scale $1/\Lambda_{\rm QCD}$ and hence we can treat the diquark as point-like if the quarks are heavy enough. The consequence is that the light-quark cloud surrounding the heavy diquark in a heavy baryon would be similar to the one around the

heavy antiquark in a heavy antimeson. We expect violations of the order of $\Lambda_{\rm QCD}/(m_Q v)$, instead of $\Lambda_{\rm QCD}/m_Q$ as in HQSS and HFS. This translates into a $30-40\%$ uncertainty in the charm sector and $15-20\%$ in the bottom one. Yet even with this limitation HADS can be useful in suggesting the possibility of new charmed molecules (see also Ref. [22] for a discussion of this symmetry in the charm sector), while for bottom ones concrete predictions can be made, as we will show in what follows.

We consider a state molecular if its most important component is the set of hadrons conforming the molecule, where the other components such as compact multiquarks play a minor role in its description. In general this is only true if the separation among the hadrons is big enough as for them to retain their individual character. This suggests that genuine molecular states show a clear separation of scales between their long and short range structure and are thus amenable to an effective field theory (EFT) treatment [17, 18, 23, 24] (analogous to nuclear EFT, see Refs. [25, 26] for reviews). Among the theoretical advantages of EFT a very interesting one is power counting: we can expand any physical quantity as a power series in terms of the expansion parameter (Q/M), where Q is a typical low energy scale (for instance, the inverse of the size of the molecular state) and M the high energy scale at which the EFT description stops being valid (the inverse size of the hadrons). The heavy hadrons are nonrelativistic: thus we can define a hadron-hadron potential that admits the low energy expansion $V_{HH}^{\rm EFT} = V_{HH}^{\rm LO} + V_{HH}^{\rm NLO} + \ldots$, where LO stands for "leading order", NLO for "next-to-leading order" and so on. In addition, the interaction among the heavy hadrons forming a molecule is nonperturbative so that we have to iterate the LO (i.e. the most important) piece of the EFT potential. Due to the exploratory character of this work, we will not go beyond LO.

Now we will consider the case of heavy meson-antimeson molecules with the light and heavy quarks being $q = u, d$ and $Q = c, b$. The EFT potential is subjected to the constraints of chiral symmetry and heavy quark symmetries. At LO there are only contact range interactions, with pion exchanges entering as a correction at NLO, for more details, see [18] [1]. The application of HQSS leads to the following LO potential [19, 20].

$$V_{P\bar{P}}^{\rm LO}(\vec{q}, 0^{++}) = C_{Ia}\,, \tag{1}$$

$$V_{P^*\bar{P}/P\bar{P}^*}^{\rm LO}(\vec{q}, 1^{+-}) = V_{P^*\bar{P}^*}^{\rm LO}(\vec{q}, 1^{+-}) = C_{Ia} - C_{Ib}\,, \tag{2}$$

$$V_{P^*\bar{P}/P\bar{P}^*}^{\rm LO}(\vec{q}, 1^{++}) = V_{P^*\bar{P}^*}^{\rm LO}(\vec{q}, 2^{++}) = C_{Ia} + C_{Ib}\,, \tag{3}$$

$$V_{P^*\bar{P}^*}^{\rm LO}(\vec{q}, 0^{++}) = C_{Ia} - 2\,C_{Ib}\,, \tag{4}$$

where the subscript indicates the particle channel ($P = D, \bar{B}$ and $P^* = D^*, \bar{B}^*$), $\vec{q}$ is the momentum exchanged by the heavy meson and antimeson and J^{PC} indicates the quantum numbers. The subscript $I = 0, 1$ indicates the isospin of the molecule (unless stated otherwise, we are working in the isospin symmetric limit). For each isospin, the LO potential depends on two parameters, C_{Ia} and C_{Ib}, that determine the mass of up to six states.

In turn, HFS implies that the previous potential does not depend on the flavor of heavy quarks contained in the mesons [12]. As a consequence of this symmetry we can expect a particular pattern of states in the charm sector to repeat itself in the bottom one, though the binding energies will be different.

Finally, we consider the interaction between doubly-heavy baryons $\Xi_{Q_1Q_2}, \Xi^*_{Q_1Q_2}$ ($Q_{1,2} = c, b$, total spin of the heavy pair $s_{Q_1Q_2} = 1$ and $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$, respectively) or the $J^P = \frac{1}{2}^+$ Ξ'_{bc} ($s_{bc} = 0$), and a heavy meson $P^{(*)}$. HADS allows us to write the LO $\Xi^{(*)}_{Q_1Q_2}P^{(*)}$ potential in terms of the same counter-terms C_{Ia} and C_{Ib} that appear in the LO meson-antimeson potential. The analysis of the light quark components in the heavy baryon-meson system leads to potentials listed in the Table I, from which we can derive the spectrum of the heavy baryon-meson molecules.

To estimate the binding energies of the molecules we solve the Lippmann-Schwinger equation and look for the poles of the T-matrix. The EFT potential is singular when iterated: we have to regularize and renormalize the potential to make predictions. For the renormalization we employ a gaussian regulator with the cut-offs $\Lambda = 0.5\,\text{GeV}$ and $1\,\text{GeV}$, and the couplings C_{Ia} and C_{Ib} will depend on Λ. The complete procedure and the choice of the cut-off window is explained in detail in Refs. [19, 20]. For the meson masses, we take isospin averaged values $M_D = 1867.24$ MeV, $M_{D^*} = 2008.63$ MeV, $M_B = 5279.34$ MeV, $M_{B^*} = 5325.1$ MeV and $M_X = 3871.68\,\text{MeV}$ [27]. The mass of the $Z_b^{(\prime)}$ reported in Ref. [4] ([28]) is 1σ higher (overlaps) with the corresponding $B^{(*)}\bar{B}^*$ threshold. However, the location of the Z_b's may depend on the parametrization employed for them, as shown in Ref. [29]. Therefore, we simply assume that the binding energy of the Z_b is $2.0 \pm 2.0\,\text{MeV}$, as in Ref. [12]. The doubly charmed baryons were only reported by the SELEX Collaboration [30–32]. However, the measured masses are lower than expectations in most of the model and lattice calculations, and the observed isospin splittings seem too large to be accommodated in QCD [33]. Thus, we will use a recent lattice calculation for the masses, $M_{\Xi_{cc}} = 3606 \pm 22$ MeV and $M_{\Xi^*_{cc}} = 3706 \pm 28$ MeV [34]. For the doubly bottom baryons, there is no experimental observation yet, and the lattice QCD predictions are $M_{\Xi_{bb}} = 10127 \pm 13^{+12}_{-26}$ MeV and $M_{\Xi^*_{bb}} = 10151 \pm 14^{+16}_{-25}$ MeV [35]. Their ventral values will be used. Furthermore, we take constituent quark

[1] An exception are isoscalar bottom molecules, for which the strength of the OPE potential is considerable and hence should be included at LO unless the molecular state is very shallow.

TABLE I. LO potentials and quantum numbers for various doubly-heavy baryon–heavy meson systems.

States	$\Xi_{Q_1Q_2}P$	$\Xi_{Q_1Q_2}P^*$	$\Xi_{Q_1Q_2}P^*$	$\Xi^*_{Q_1Q_2}P$	$\Xi^*_{Q_1Q_2}P^*$	$\Xi^*_{Q_1Q_2}P^*$	$\Xi^*_{Q_1Q_2}P^*$	$\Xi'_{bc}P$	$\Xi'_{bc}P^*$	$\Xi'_{bc}P^*$
J^P	$\frac{1}{2}^-$	$\frac{1}{2}^-$	$\frac{3}{2}^-$	$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{3}{2}^-$	$\frac{5}{2}^-$	$\frac{1}{2}^-$	$\frac{1}{2}^-$	$\frac{3}{2}^-$
$V^{\rm LO}$	C_{Ia}	$C_{Ia}+\frac{2}{3}C_{Ib}$	$C_{Ia}-\frac{1}{3}C_{Ib}$	C_{Ia}	$C_{Ia}-\frac{5}{3}C_{Ib}$	$C_{Ia}-\frac{2}{3}C_{Ib}$	$C_{Ia}+C_{Ib}$	C_{Ia}	$C_{Ia}-2C_{Ib}$	$C_{Ia}+C_{Ib}$

model predictions for the Ξ'_{bc} and Ξ^*_{bc} masses, 6958 and 6996 MeV, respectively [36]. Predictions will be made for the binding energies instead of masses to avoid introducing the lattice QCD errors of the baryon masses into the results. Finally, the HQSS/HFS uncertainty in the counter-terms is assumed to be 20%(7%) in the charm (bottom sector), while for HADS we use 40% (20%). They are assumed to be uncorrelated. We will also use a 30% HADS uncertainty for the Ξ'_{bc} systems. We will not show them explicitly when writing down the value of the counter-terms, yet we will take them into account.

We begin by considering the $X(3872)$ as a pure isoscalar 1^{++} $D\bar{D}^*$ molecule as in [19]. The LO potential is given by the counter-term combination $C_{0X}\equiv C_{0a}+C_{0b}$, which is identical to the one appearing in the family of $\Xi^*_{Q_1Q_2}P^*$ with $J=\frac{5}{2}^-$ and the $\Xi'_{bc}P^*$ systems with $J^P=\frac{3}{2}^-$. We have $C_{0X}=-1.94\,(-0.79)\,\text{fm}^2$ for $\Lambda=0.5\,(1)\,\text{GeV}$ [19]. Bound state solutions are found in all the considered systems, though the $\Xi^*_{cc}D^*$ system can be very loosely bound due to the large uncertainty of the LO potential, and the predictions can be found in Table II. In addition, it is more than probable that the isoscalar $\Xi^{(*\prime)}_{bc}B^*$ and $\Xi^*_{bb}\bar{B}^*$ molecules require nonperturbative OPE owing to their heavy reduced mass. Though, the nonperturbative OPE will modify the binding energies, we expect, however, that these systems will remain still bound.

Now we continue with what can be deduced from the $Z_b^{(\prime)}$ as isovector 1^{+-} $B^{(*)}\bar{B}^*$ molecular states. As can be seen from Table I, there is no exact match among the LO potential for the Z_b's, $C_{1Z}\equiv C_{1a}-C_{1b}$, and the six possible $\Xi^{(*)}_{Q_1Q_2}P^{(*)}$ configurations. Yet, the $\frac{3}{2}^-$ $\Xi^*_{Q_1Q_2}P^*$ configuration has coupling: $C_{1a}-\frac{2}{3}C_{1b}=C_{1Z}+\frac{1}{3}C_{1b}$. As far as the relative contribution of the C_{1b} coupling is not excessive, a hadronic molecule, either as a bound or virtual state, looks probable. Other two interesting configurations are the $\frac{1}{2}^-$ $\Xi^*_{Q_1Q_2}P^*$ and $\frac{3}{2}^-$ $\Xi_{Q_1Q_2}P^*$ systems, for which the couplings read $C_{1Z}\mp\frac{2}{3}C_{1b}$. Depending on the sign and size of C_{1b} at least one of the two configurations should bind.

All this indicates that the isospin-1 doubly-heavy baryon–meson molecules are probable, but a further assessment requires the determination of both C_{1a} and C_{1b}. From the Z_b's we obtain [12] $C_{1Z}=-\left(0.75^{+0.15}_{-0.28}\right)\,[-\left(0.30^{+0.03}_{-0.07}\right)]\,\text{fm}^2$ for $\Lambda=0.5\,\text{GeV}\,[1\,\text{GeV}]$, where the errors come from the uncertainties in the binding energy. But for disentangling the C_{1a} and C_{1b} couplings a second source of information is necessary. For that we will use the isospin symmetry breaking of the $X(3872)$, which offers a window into the interaction in the isovector 1^{++} $D\bar{D}^*$ channel [20]. The decay of the $X(3872)$ into the isovector $J/\psi 2\pi$ channel indicates that the X is not a pure isoscalar state, but contains a small isovector component. The branching ratio of the isovector $J/\psi 2\pi$ to the isoscalar $J/\psi 3\pi$ decays constrains the size of this component and hence the strength of the interaction in the isovector channel [20]. We find $C_{1X}=-(0.13\pm0.40)\,[-(0.39\pm0.09)]\,\text{fm}^2$ for $\Lambda=0.5\,\text{GeV}\,[1\,\text{GeV}]$, where the errors reflect the experimental uncertainty in the branching ratio.[2] Using the formulas $C_{1a}=(C_{1X}+C_{1Z})/2$ and $C_{1b}=(C_{1X}-C_{1Z})/2$, we obtain $C_{1a}=-(0.44\pm0.24)\,[-(0.35\pm0.06)]\,\text{fm}^2$ and $C_{1b}=(0.31\pm0.24)\,[-(0.05\pm0.06)]\,\text{fm}^2$ (the errors shown are for guidance only and have been obtained by adding in quadratures those quoted for C_{1X} and C_{1Z}). We see that C_{1b} is either positive or, if negative, extremely small and that $|C_{1b}|<|C_{1a}|$, which already contains a lot of information about the possible molecular states. We show the predictions in Table II, where the uncertainties in the binding energies come from the errors in C_{1X} and C_{1Z}, the additional HQSS/HFS 20% error (as part of the information comes from the charm sector) and from the expected 20% violation of HADS.

In the isovector sector, all configurations are plausible molecular candidates. However, when we take into account the various uncertainties of the current approach, we cannot discard in all cases the appearance of virtual states instead of proper bound molecules. The most promising predictions are the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ $\Xi^*_{bb}\bar{B}^*$ molecules, for which binding is moderately robust against the different error sources.

To confirm these states from the theoretical side we need to pinpoint the value of C_{1b} more accurately. This could be done either by more accurate measurements of the X isospin breaking ratio or, better yet, by the eventual discovery of HQSS partners of the Z_b's, the W_b states proposed in Ref. [16]. Notice that all the isospin-1 triply-heavy molecules are very interesting in the sense that they have a non-trivial pentaquark component. We point out that though heavy pentaquarks have been predicted

[2] The central value of C_{1X} differs from that quoted in [20] by an amount that is around 10% of its error. This is because of the use of different values for the X resonance mass.

TABLE II. Predictions of the doubly-heavy baryon–heavy meson molecules. The isoscalar states are related to the $X(3872)$, and the error in their binding energies is a consequence of the approximate nature of HADS. The isovector states are determined by the $Z_b(10610, 10650)$ and the isovector component of the X. In this part, different error sources have been taken into account: the uncertainty in the Z_b binding, in the isospin breaking decays of the X and in the HADS breaking. For simplicity, we only show an unique error obtained by adding in quadratures all the previous ones. Here, $M_{\rm th}$ represents the threshold, and all masses are given in units of MeV. When we decrease the strength of the potential to account for the various uncertainties, in some cases (marked with † in the table) the bound state pole reaches the threshold and the state becomes virtual. The cases with a virtual state pole at the central value are marked by [V], for which †† means that the pole evolves into a bound state one and N/A means that the pole is far from the threshold with a momentum larger than 1 GeV so that it is both undetectable and beyond the EFT range.

State	$I(J^P)$	$V^{\rm LO}$	Thresholds	Mass ($\Lambda = 0.5$ GeV)	Mass ($\Lambda = 1$ GeV)
$\Xi_{cc}^* D^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	5715	$(M_{\rm th} - 10)^{+10}_{-15}$	$(M_{\rm th} - 19)^{\dagger}_{-44}$
$\Xi_{cc}^* \bar{B}^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	9031	$(M_{\rm th} - 21)^{+16}_{-19}$	$(M_{\rm th} - 53)^{+45}_{-59}$
$\Xi_{bb}^* D^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	12160	$(M_{\rm th} - 15)^{+9}_{-11}$	$(M_{\rm th} - 35)^{+25}_{-31}$
$\Xi_{bb}^* \bar{B}^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	15476	$(M_{\rm th} - 29)^{+12}_{-13}$	$(M_{\rm th} - 83)^{+38}_{-40}$
$\Xi_{bc}' D^*$	$0(\frac{3}{2}^-)$	$C_{0a} + C_{0b}$	8967	$(M_{\rm th} - 14)^{+11}_{-13}$	$(M_{\rm th} - 30)^{+27}_{-40}$
$\Xi_{bc}' \bar{B}^*$	$0(\frac{3}{2}^-)$	$C_{0a} + C_{0b}$	12283	$(M_{\rm th} - 27)^{+15}_{-16}$	$(M_{\rm th} - 74)^{+45}_{-51}$
$\Xi_{bc}^* D^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	9005	$(M_{\rm th} - 14)^{+11}_{-13}$	$(M_{\rm th} - 30)^{+27}_{-40}$
$\Xi_{bc}^* \bar{B}^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	12321	$(M_{\rm th} - 27)^{+15}_{-16}$	$(M_{\rm th} - 74)^{+46}_{-51}$
$\Xi_{bb} \bar{B}$	$1(\frac{1}{2}^-)$	C_{1a}	15406	$(M_{\rm th} - 0.3)^{\dagger}_{-2.5}$	$(M_{\rm th} - 12)^{+11}_{-15}$
$\Xi_{bb} \bar{B}^*$	$1(\frac{1}{2}^-)$	$C_{1a} + \frac{2}{3} C_{1b}$	15452	$(M_{\rm th} - 0.9)[{\rm V}]^{\rm N/A}_{\dagger\dagger}$	$(M_{\rm th} - 16)^{+14}_{-17}$
$\Xi_{bb} \bar{B}^*$	$1(\frac{3}{2}^-)$	$C_{1a} - \frac{1}{3} C_{1b}$	15452	$(M_{\rm th} - 1.2)^{\dagger}_{-2.9}$	$(M_{\rm th} - 10)^{+9}_{-13}$
$\Xi_{bb}^* \bar{B}$	$1(\frac{3}{2}^-)$	C_{1a}	15430	$(M_{\rm th} - 0.3)^{\dagger}_{-2.4}$	$(M_{\rm th} - 12)^{+11}_{-13}$
$\Xi_{bb}^* \bar{B}^*$	$1(\frac{1}{2}^-)$	$C_{1a} - \frac{5}{3} C_{1b}$	15476	$(M_{\rm th} - 8)^{+8}_{-7}$	$(M_{\rm th} - 5)^{\dagger}_{-8}$
$\Xi_{bb}^* \bar{B}^*$	$1(\frac{3}{2}^-)$	$C_{1a} - \frac{2}{3} C_{1b}$	15476	$(M_{\rm th} - 2.5)^{\dagger}_{-3.6}$	$(M_{\rm th} - 9)^{+9}_{-11}$
$\Xi_{bb}^* \bar{B}^*$	$1(\frac{5}{2}^-)$	$C_{1a} + C_{1b}$	15476	$(M_{\rm th} - 4.3)[{\rm V}]^{\rm N/A}_{+3.3}$	$(M_{\rm th} - 18)^{+17}_{-19}$

in the literature on the basis of several arguments [37–39], this is the first prediction of a triply heavy one.

To summarize, we have studied the implications of HADS (plus HQSS and HFS) for heavy hadronic molecules. As a consequence of this symmetry, we can be confident about the existence of doubly-heavy baryon–heavy meson (and eventually doubly-heavy baryon–antibaryon: $\Xi_{Q_1Q_2}^{(*)}$–$\Xi_{\bar{Q}_1\bar{Q}_2}^{(*)}$) partners of heavy meson–antimeson molecules. From the assumption that the $X(3872)$ and the $Z_b(10610/10650)$ are molecular states we can predict the existence of the exotic pentaquark-like partners of these states. We notice that phase space forbids any of the predicted molecules to decay through the strong decays of their components. One of the possible strong decay channels is a triply-heavy baryon plus one or more pions. Such a decay involves exchanging a heavy quark and a light quark so that it would have a small partial width. The $\Xi_{Q_1Q_2}P$ in a D wave could be the dominant decay channel of the $\Xi_{Q_1Q_2}^* P^*$ states with $J^P = \frac{3}{2}^-$ and $\frac{5}{2}^-$. However, if the binding energy is so small that the binding momentum is much smaller than the pion mass, the predicted state should be quite stable. It would be intriguing if any of the predicted states can be found in high-energy hadron colliders and heavy ion collisions.

We would like to thank Ulf-G. Meißner for a careful reading of the manuscript. FKG acknowledges the Theory Division of IHEP in Beijing, where part of the work was done, for the hospitality. This work is supported in part by the DFG and the NSFC through funds provided to the Sino-German CRC 110 "Symmetries and the Emergence of Structure in QCD", by the NSFC (Grant No. 11165005), by the Spanish Ministerio de Economía y Competitividad and European FEDER funds under the contract FIS2011-28853-C02-02 and the Spanish Consolider-Ingenio 2010 Programme CPAN (CSD2007-00042), by Generalitat Valenciana under contract PROMETEO/2009/0090 and by the EU HadronPhysics2 project, grant no. 227431.

* fkguo@hiskp.uni-bonn.de
† carloshd@ific.uv.es
‡ jmnieves@ific.uv.es
§ pavonvalderrama@ipno.in2p3.fr

[1] M. Voloshin and L. Okun, JETP Lett. **23**, 333 (1976)
[2] A. De Rujula, H. Georgi, and S. Glashow, Phys.Rev.Lett. **38**, 317 (1977)
[3] S. K. Choi *et al.* (Belle), Phys. Rev. Lett. **91**, 262001 (2003), arXiv:hep-ex/0309032

[4] A. Bondar *et al.* (Belle Collaboration), Phys.Rev.Lett. **108**, 122001 (2012), arXiv:1110.2251 [hep-ex]

[5] I. Adachi *et al.* (Belle Collaboration)(2012), arXiv:1207.4345 [hep-ex]

[6] B. Aubert *et al.* (BABAR Collaboration), Phys.Rev.Lett. **95**, 142001 (2005), arXiv:hep-ex/0506081 [hep-ex]

[7] G.-J. Ding, Phys.Rev. **D79**, 014001 (2009), arXiv:0809.4818 [hep-ph]

[8] M.-T. Li, W.-L. Wang, Z.-Y. Zhang, and Y.-B. Dong(2013), arXiv:1303.4140 [nucl-th]

[9] Q. Wang, C. Hanhart, and Q. Zhao(2013), arXiv:1303.6355 [hep-ph]

[10] M. Ablikim *et al.* (BESIII Collaboration)(2013), arXiv:1303.5949 [hep-ex]

[11] Z. Liu *et al.* (Belle Collaboration)(2013), arXiv:1304.0121 [hep-ex]

[12] F.-K. Guo, C. Hidalgo-Duque, J. Nieves, and M. Pavon Valderrama(2013), arXiv:1303.6608 [hep-ph]

[13] E. Wilbring, H. W. Hammer, and U. G. Meißner(2013), arXiv:1304.2882 [hep-ph]

[14] F.-K. Guo, C. Hanhart, and U.-G. Meißner, Phys. Rev. Lett. **102**, 242004 (2009), arXiv:0904.3338 [hep-ph]

[15] A. Bondar, A. Garmash, A. Milstein, R. Mizuk, and M. Voloshin, Phys.Rev. **D84**, 054010 (2011), arXiv:1105.4473 [hep-ph]

[16] M. Voloshin, Phys.Rev. **D84**, 031502 (2011), arXiv:1105.5829 [hep-ph]

[17] T. Mehen and J. W. Powell, Phys.Rev. **D84**, 114013 (2011), arXiv:1109.3479 [hep-ph]

[18] M. Pavon Valderrama, Phys.Rev. **D85**, 114037 (2012), arXiv:1204.2400 [hep-ph]

[19] J. Nieves and M. Pavon Valderrama, Phys.Rev. **D86**, 056004 (2012), arXiv:1204.2790 [hep-ph]

[20] C. Hidalgo-Duque, J. Nieves, and M. Pavon Valderrama, Phys.Rev. **D87**, 076006 (2012), arXiv:1210.5431 [hep-ph]

[21] M. J. Savage and M. B. Wise, Phys.Lett. **B248**, 177 (1990)

[22] T. D. Cohen and P. M. Hohler, Phys.Rev. **D74**, 094003 (2006), arXiv:hep-ph/0606084 [hep-ph]

[23] E. Braaten and M. Kusunoki, Phys.Rev. **D69**, 074005 (2004), arXiv:hep-ph/0311147 [hep-ph]

[24] S. Fleming, M. Kusunoki, T. Mehen, and U. van Kolck, Phys. Rev. **D76**, 034006 (2007), arXiv:hep-ph/0703168

[25] E. Epelbaum, H.-W. Hammer, and U.-G. Meißner, Rev. Mod. Phys. **81**, 1773 (2009), arXiv:0811.1338 [nucl-th]

[26] R. Machleidt and D. Entem, Phys.Rept. **503**, 1 (2011), arXiv:1105.2919 [nucl-th]

[27] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012)

[28] I. Adachi *et al.* (Belle Collaboration)(2012), arXiv:1209.6450 [hep-ex]

[29] M. Cleven, F.-K. Guo, C. Hanhart, and U.-G. Meißner, Eur.Phys.J. **A47**, 120 (2011), arXiv:1107.0254 [hep-ph]

[30] M. Mattson *et al.* (SELEX Collaboration), Phys.Rev.Lett. **89**, 112001 (2002), arXiv:hep-ex/0208014 [hep-ex]

[31] M. Moinester *et al.* (SELEX), Czech.J.Phys. **53**, B201 (2003), arXiv:hep-ex/0212029 [hep-ex]

[32] A. Ocherashvili *et al.* (SELEX Collaboration), Phys.Lett. **B628**, 18 (2005), arXiv:hep-ex/0406033 [hep-ex]

[33] S. J. Brodsky, F.-K. Guo, C. Hanhart, and U.-G. Meißner, Phys.Lett. **B698**, 251 (2011), arXiv:1101.1983 [hep-ph]

[34] Y. Namekawa *et al.* (PACS-CS Collaboration)(2013), arXiv:1301.4743 [hep-lat]

[35] R. Lewis and R. Woloshyn, Phys.Rev. **D79**, 014502 (2009), arXiv:0806.4783 [hep-lat]

[36] C. Albertus, E. Hernandez, and J. Nieves, Phys.Lett. **B683**, 21 (2010), arXiv:0911.0889 [hep-ph]

[37] M. Genovese, J. Richard, F. Stancu, and S. Pepin, Phys.Lett. **B425**, 171 (1998), arXiv:hep-ph/9712452 [hep-ph]

[38] I. W. Stewart, M. E. Wessling, and M. B. Wise, Phys.Lett. **B590**, 185 (2004), arXiv:hep-ph/0402076 [hep-ph]

[39] T. D. Cohen, P. M. Hohler, and R. F. Lebed, Phys.Rev. **D72**, 074010 (2005), arXiv:hep-ph/0508199 [hep-ph]