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PRODUCTION OF PIONIC ATOMS WITH THE (π^-, π^+) REACTION

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ABSTRACT

We have evaluated the cross section for the $\pi^- + A \longrightarrow \pi^+ + (A\ \pi^-)$ reaction in heavy nuclei producing deeply bound π^- atom states still unobserved. The cross sections found, although in the measurable range, are only a small fraction of the background from the inclusive (π^-, π^+) reaction, which makes this experiment not very suited for the investigation of these elusive pionic states.

The possibility of producing deeply bound pionic atom states like the 1s, 2p states of ^{208}Pb, so far unreacheable with the traditional X ray technique, has stimulated a new wave of activity. The (n, p) reaction was suggested in ref.[1] as a tool to produce these states, but the experiment conducted at TRIUMF has shown no clear signal so far[2]. The interesting thing, however, is that there is consensus among the practitioners of pionic atoms about the existence of these states and their observability. This is based on the fact that the different existing optical potentials, providing a reasonable fit to the data, give widths for these states which are narrower than the difference of energies between neighbouring states[3,4,5]. This is particularly true of the potential of ref.[5] which gives an acceptable solution to the problem of the pionic anomalous atoms[6]. In view of this it is worth investigating the possibility of producing these states with different reactions. One such reaction is the (π^-, π^+) which has received much attention recently at TRIUMF[7]. It was suggested that this reaction producing an extra π^- bound in the nucleus might yield sufficient cross section as to make these states observable[8].

The reaction under study is

$$\pi^- + (A, Z) \longrightarrow \pi^+ + (A, Z-1, \pi^-) \tag{1}$$

which has the attractive feature that the background for the reaction consists of the inclusive (π^-, π^+) reaction in nuclei, the one with smallest cross section among the different (π, π') reactions. The unwelcome feature is that the nucleus changes charge and because of Pauli blocking only the valence particles will participate in the process. This is a drawback with respect to reactions like (n, p)[1,9] or (γ, π^+)[10] creating a π^- bound, where there is a coherent contribution of all the nucleons to the amplitude.

The cross section for the reaction (1) is given in the lab system by

$$\frac{d\sigma}{d\Omega\, dE(\pi^+)} = \frac{|\vec{p}_{\pi^+}|}{|\vec{k}_{\pi^-}|}\, \frac{1}{4}\, \frac{1}{2\omega_\pi}\, \frac{1}{(2\pi)^3}\, \frac{\Gamma}{[\omega_{in} + M_A - \omega_{out} - E(A', \pi^-)] + \Gamma^2/4} \cdot \overline{\sum} \sum |T|^2 \tag{2}$$

where $\vec{k}_{\pi^-}$, $\vec{p}_{\pi^+}$ are the momenta of the incoming and outgoing pion, ω_{in}, ω_{out} their energies and ω_π the energy of the bound π^-. M_A is the mass of the initial nucleus, $E(A', \pi^-)$ the energy of the final nucleus with the bound pion and Γ the width of the pionic state.

In deducing eq. (2) we have made use of the prescription

$$-\pi\delta(E_{in}-E_{out}) \longrightarrow \mathrm{Im}\,\frac{1}{E_{in}-E_{out}+i\frac{\Gamma}{2}} \tag{3}$$

since the final atomic state is unstable.

We will calculate explicitly the transition

$$\pi^- + {}^{209}\mathrm{Bi} \longrightarrow \pi^+ + ({}^{209}\mathrm{Pb}\,\pi^-) \tag{4}$$

which simplifies a bit the computational task. In this case only the valence proton of ^{209}Bi and the valence neutron of ^{209}Pb participate in the reaction. The T matrix of eq. (2) is then given by

$$-i\,T = \langle n'\,l'\,j'\,m'\,|B\left(1+a\,\frac{T_{\pi^-}-T_\pi^{th}}{T_\pi^{th}}\right)]\,\vec{\sigma}\,.\,\vec{k}_{\pi^-}\,.\,e^{i\vec{q}\,\vec{r}}\;\Phi^*_{n_1\,l_1\,m_1}(\vec{r})\,|\,n\,l\,j\;m\rangle \tag{5}$$

where $\vec{q}=\vec{k}_{\pi^-}-\vec{p}_{\pi^+}$,

T_{π^-} is the pion kinetic energy and T_π^{th} the threshold energy for the pion production in the $\pi N \longrightarrow \pi\pi N$ reaction ($T_\pi^{th} \simeq 170$ MeV). $\Phi_{n_1\,l_1\,m_1}(\vec{r})$ is the wave function for the bound π^- state and $|n\;l\;j\;m\rangle$, $|n'\;l'\;j'\;m'\rangle$ the initial and final nuclear wave functions for the valence nucleon. We have used the parametrization of the $\pi^- p \longrightarrow \pi^+\pi^- n$ amplitude of ref.[11] with

$$B = 2.58\; m_\pi^{-3};\;\; a = 1.02 \tag{6}$$

Using Racah algebra one can perform analytically the angular integrations in eq. (5) and one obtains[12]

$$\begin{aligned} T = i\,B\left[1+a\,\frac{T_{\pi^-}-T_\pi^{th}}{T_\pi^{th}}\right]\sqrt{3}\,(-1)^{m_1}\,|\vec{k}_{\pi^-}|\left[\frac{(2l+1)\;(2l_1+1)}{2l'+1}\right]^{1/2} \\ \sum_\mu C(l\;1/2\;j;\;\mu,\,m-\mu)\;\;C(l'\;1/2\;j';\;m'-m+\mu,\,m-\mu)\;C(1/2\;1\;1/2;\;m-\mu,\,0) \\ \sum_\lambda i^\lambda\,Y^*_{\lambda,\,m'-m+m_1}(\hat{q})\,(2\lambda+1)^{1/2}\;\;{}^{n'\,l'}F_\lambda^{n\,l}(q,\,n_1\,l_1) \\ \sum_{\lambda'} C(l_1\,l\,\lambda';\;-m_1,\,\mu)\;\;C(\lambda'\,\lambda\,l';\;\mu-m_1,\;m'-m+m_1) \\ C(l_1\,l\,\lambda';\;0\;0\;0)\;\;C(\lambda'\,\lambda\,l';\;0\;0\;0) \end{aligned} \tag{7}$$

with ${}^{n'l'}F_{\lambda}^{nl}(q, n, l_1) = \int_0^{\infty} r^2 dr\, R_{n'l'}(r) R_{nl}(r)\, j_{\lambda}(q\,r)\, R'^{*}_{n_1 l_1}(r)$ and R, R' the radial wave functions for the nucleon and pion wave functions. We must then evaluate

$$\overline{\Sigma}\,\Sigma\,|T|^2 = \frac{1}{2j+1} \sum_{m} \sum_{m'} \sum_{m_1} |T|^2 \qquad (8)$$

The m sums can also be done analytically using orthogonality relations but the expression involves as many sums as the direct evaluation of the second member of eq. (8). An alternative formula using 9j symbols is equally length, from the computational point of view.

We have evaluated the cross section $d\sigma/d\Omega\, dE(\pi^+)$ for the reaction (4) at several pion energies and different scattering angles. In Table 1 we show the cross sections as a function of the energy for $\Theta=0°$. The numbers quoted there are the values at the peak of the Lorentzian distribution (eq. (2)). The cross sections are of the order of a 10^{-2} μb/sr/MeV. They grow with the energy due to phase space and the $\pi N \rightarrow \pi\pi N$ amplitude of eq. (5). The parametrization used in eq. (5) has been checked up to $T_\pi = 350$ MeV, but this amplitude is only an approximation to the more realistic one of ref. 13. It reproduces the experimental cross section , but has no dependence on the external pion momenta. Because of that, the numbers for the last energy in Table 1 are somewhat overestimated.

In table 2 we show the results at $T_{\pi^-} = 225$ MeV as a function of the scattering angle. Because of the nuclear form factors appearing in eq. (7) there is a dependence on this angle (or equivalently in the momentum transfer q). For this energy and the 1s state the maximum of the cross section is at 0°, but for the 2p state it appears at 25°. For the last two energies in table 1 we find the maximum for the 1s state at 0° and for the 2p state at 20° and 15° with values 0.05 and 0.18 respectively.

The valence proton in ^{209}Bi is taken in the $1h_{9/2}$ shell and the valence neutron in ^{209}Pb in the $2g_{9/2}$. The oscillator parameter taken is $\alpha = 0.407\ fm^{-1}$ (exponential part, is $\exp(-\frac{1}{2}\alpha^2 r^2)$).

We have also made evaluations of the cross sections for $\pi^- + {}^{41}Sc \longrightarrow \pi^+ + ({}^{41}Ca\ \pi^-)$. The cross sections for the 1s state are of the order of 0.40 μb/sr/MeV at $T_{\pi^-} = 275$ MeV for $\Theta=0°$ and 1.44 μb/sr/MeV at $T_{\pi^-} = 375$ MeV for $\Theta=0°$ but this number is again overstimated. One reason for this bigger numbers is, see eq. (2), the small width of this state ($\Gamma \simeq 80$ KeV). An experiment with less resolution than this width would spread out the contribution and the strength at the peak would be smaller.

Although the small cross sections obtained are observable with present techniques, one must compare these signals with the background from the inclusive (π^-, π^+) reaction. There are experimental results available for this reaction from refs.[14-16], and theoretical results[17] using a microscopic theory which gives simultaneusly all the pion inclusive reactions[18] and elastic scattering[19]. The agreement of the theory with experiment is fairly good except in some punctual cases which include the forward angles in light nuclei and the (π^+, π^-) reaction in heavy nuclei[20]. We have used this theory to deduce the (π^-, π^+) background which is of the order of 2-4 μb/sr/MeV.

The numbers calculated are obtained using plane waves for the incoming π^- and outgoing π^+. Taking into account the distortion of the pion waves should produce a reduction of about a factor 10 or more in the maximum values of the cross section in tables 1 and 2. Thus we are led to cross sections of the order of 10^{-2} - 10^{-3} of the values of the background. With these values the chances to see the signal of the pionic atoms are very small.

One could try to reduce the background by looking at some distintive features of the production or decay of the pionic atom. In this sense we know that a π^- is predominantly absorbed by np pairs and the result would be two neutrons going back to back and sharing equally the energy of the pion (with some broadening because of Fermi motion). There would be final state interactions of the neutrons but one can estimate that in 1/10 or more of the cases the two neutrons would go out without further collisions with other nucleons, giving thus a clear signal. The nucleons from the inclusive (π^-, π^+) reaction would be highly uncorrelated in angle and energy and as a consequence this coincidence test would drastically reduce the background.

The reason for the smallness of the cross section in the reaction (4) is that only one nucleon contributes. One immediately realizes that in a reaction like (π^-, π^o) with production of a π^- bound, the reaction would be coherent. The background would be now bigger because it is given by the inclusive (π^-, π^o) reaction[21], but the coherence of the pionic atom production, with the A^2 factor in the cross section, could easily overcome the increase in the background. Unfortunately the dynamics of the process, with the coupling $\vec{\sigma}\vec{k}$ of eq. (5), is such that the sum of the contributions from the nucleons in a chosed shell vanishes. Thus, once more, only the valence particles would contribute with the inconvenience of having a larger background. However, as we mentioned, the $\vec{\sigma}\,\vec{k}$ dependence of eq. (5) is only approximate. This coupling appears in the non vanishing pieces of the amplitude at threshold but at bigger energies the three point diagrams of the amplitude of ref.[13] contain some pieces which would involve terms of the type $p_{\pi}^{1}(p_{\pi}^{2} \times p_{\pi}^{3})$. Such terms would contribute coherently to the amplitude but, appart from the bound pion momentum which is small they involve this mixed product which makes the amplitude small when evaluated with a pionic wave function.

In conclusion we can say that the cross sections for the production of pionic atoms with the (π^-, π^+) reaction are very small compared with the background, because of Pauli blocking which only allows the valence particles to participate in the process. The (π^-, π^o) reactions stands better chances in principle, because some parts of the amplitude would contribute coherently. Unfortunately the larger pieces in the amplitude are such that they cancell when summed coherently over a spin closed shell. Coherent reactions like (n, p) or the (γ, π^+)[10] stand better chances. The interest in finding these deeply bound pionic states and the difficulties met in their observation should stimulate further research, looking for more reactions. On the other hand, one lesson that we learn from this work is that the coherence of the reaction is of extreme importance in order to maximize the signal for the production of the pionic states with respect to a usually incoherent background.

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TABLE CAPTIONS

Table 1.– Results for $d^2\sigma/d\Omega\, dE(\pi^+)$ for several kinetic energies of the incoming π^- for the 1s and 2p states of the reaction of eq. (4) at $\Theta(\pi^+) = 0°$.

Table 2.– Results for $d^2\sigma/d\Omega\, dE(\pi^+)$ for several scattering angles (or equivalently momentum transfers) for the 1s and 2p states of the reaction of eq. (4) at $T_{\pi^-} = 225$ MeV.

TABLE 1

$d^2\sigma/d\Omega\, dE(\pi^+)$ $[\mu b/sr/MeV]$, $\Theta = 0°$

T_{π^-} [MeV]	1s	2p
175	0.002	0.006
225	0.01	0.01
275	0.03	0.02
375	0.09	0.08

TABLE 2

$d\sigma/d\Omega\, dE(\pi^+)$ $[\mu b/sr/MeV]$, $T_{\pi^-} = 225$ MeV

[π^+]	q[fm^{-1}]	1s	2p
0°	0.784	0.011	0.012
10°	0.814	0.009	0.014
15°	0.850	0.008	0.018
20°	0.897	0.005	0.020
25°	0.954	0.004	0.022
30°	1.018	0.004	0.018
35°	1.089	0.005	0.014
40°	1.163	0.006	0.009
45°	1.240	0.005	0.009
50°	1.319	0.004	0.009