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## PRODUCTION OF PIONIC ATOMS WITH THE $(e, e')$ REACTION

J. Nieves and E. Oset

*Departamento de Física Teórica and IFIC,  
Centro Mixto Universidad de Valencia - CSIC,  
Facultad de Física, Burjassot (Valencia) Spain.*

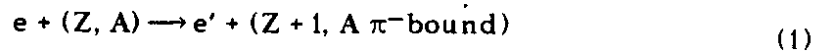
### ABSTRACT

We study the reaction  $e+A \rightarrow e' + (A\pi^-)$  with the  $\pi^-$  bound in the nucleus as a means of producing deeply bound pionic states so far unobserved. Although the cross sections found can be rather large, they are overwhelmed by the background from the inclusive  $(e, e')$  reaction, which makes the reaction not too suited for the investigation of these pionic states.

Except for the anomalous pionic atoms<sup>1)</sup>, the bulk of pionic atom states are well described in terms of standard optical potentials of the Ericson-Ericson type<sup>2)</sup>. The traditional X ray method to measure energies and widths of pionic states fails to produce the deeply bound states of heavy nuclei, like the 1s or 2p states of nuclei in the Pb region. However, such states are predicted by the standard potentials and the widths are fairly smaller than the separation between the levels<sup>3-5)</sup>. This is particularly true in the potential of ref.<sup>5)</sup> where an acceptable solution to the anomalies is found. Although some suggestions to measure these states with different techniques have been occasionally done<sup>6)</sup>, the real experimental push was given recently using the (n, p) reaction producing a  $\pi^-$  bound<sup>7)</sup>, which was suggested in ref.<sup>8)</sup>. The reaction has failed so far to show any clear signal of these bound states.

The interest in finding these elusive states advises the exploration of different reactions which can lead to their successful detection. One such possibility is offered by the (e, e') reaction producing a  $\pi^-$  bound. This reaction offers one incentive over the (n, p) because there is no distortion of the electron waves. Instead, in the (n, p) reaction there is a substantial reduction of the cross section due to the distortion of the incoming and outgoing nucleons<sup>9,10)</sup> which makes the cross sections fairly smaller than the first estimates of ref.<sup>8)</sup>. It has however a drawback with respect to the (n, p) reaction because it changes the charge of one nucleon in the nucleus and then, because of Pauli blocking, only the valence nucleons will participate, while in the (n, p) reaction there is a coherent contribution of all the nucleons to the amplitude.

The reaction has to be seen relative to the background. These two reactions are



While in the reaction (2) there will be a distribution of energies and angles of the outgoing electron, in reaction (1), given an angle in the lab system, for the outgoing electron, the energy of the nuclear system is fixed. Hence we shall see some peaks corresponding to the reaction (1) at particular energies given by the energies of the pionic atom state.

The cross section for the reaction (1) is

$$\frac{d\sigma}{d\Omega dE_{out}} = \frac{|\vec{p}_{out}|}{|\vec{p}_{in}|} m_e^2 \frac{1}{2\omega_\pi} \frac{1}{(2\pi)^3} \frac{\Gamma}{E_{in} + M_A - E_{out} - E(A' \pi^-) + \Gamma^2/4} \cdot \sum_{\Sigma} |T|^2 \quad (3)$$

where  $E_{in}$ ,  $\vec{p}_{in}$ ,  $E_{out}$ ,  $\vec{p}_{out}$  are the energy and momentum for the incoming and outgoing electrons  $m_e$  the electron mass,  $M_A$  the initial nucleus mass,  $E(A' \pi^-)$  the energy of the outgoing nucleus with the bound pion, including recoil,  $\omega_\pi$  the pion energy including its mass and  $\Gamma$  the width of the pionic atom state. We will calculate explicitly the cross section for the reaction  $e^- + {}^{209}\text{Pb} \rightarrow e^- + ({}^{209}\text{Bi} \pi^-)$

In order to construct a model for the T matrix in eq. (3) we look at the diagram in fig. 1. The vertex of the right corresponds to the  $\gamma n \rightarrow \pi \pi^-$  reaction, which is well described by a model including the Kroll Ruderman and pion pole terms plus additional nucleon and  $\Delta$  pole terms<sup>12,13</sup>. However, since we are involved with small pion momentum components in the pionic wave function, a very good approximation is given by the Kroll Ruderman plus pion pole terms which we obtain from the diagrams of fig. 2 by neglecting the pion momentum and using a relativistic nucleon propagator in fig. 2a. As a consequence the T matrix can be written as

$$-iT = +ie \bar{u}(\vec{p}_{out}) \gamma^\mu u(\vec{p}_{in}) \frac{-i g_{\mu\nu}}{q^2 + i\epsilon} e \sqrt{2} \frac{g}{2M - q^0} \cdot J^\nu \quad (4)$$

$$J^\nu = \langle n' l' j' m' | \left\{ \begin{array}{l} \partial_{\vec{q}} \left( \frac{1}{2M} + \frac{q_0}{q^0 + \vec{q}^2} \frac{2M - q^0}{2M} \right) \\ \sigma^i - \frac{\partial_{\vec{q}}}{q^0 + \vec{q}^2} \frac{2M - q^0}{2M} q^i \end{array} \right\} \cdot e^{i\vec{q}\vec{r}} \Phi_{n_1 l_1 m_1}^*(\vec{r}) | n l j m \rangle \quad (5)$$

where  $q = p_{in} - p_{out}$ ,  $M$  the nucleon mass,  $g$  the  $\pi NN$  coupling constant ( $g/2M = f/m_\pi$ ;  $f^2/4\pi = 0.08$ )  $e$  the electron charge ( $e^2/4\pi = \alpha = 1/137$ ),  $\Phi_{n_1 l_1 m_1}(\vec{r})$  the pionic wave function and  $n' l' j' m'$ ,  $n l j m$  the final and initial nuclear states for the valence nucleons. We follow the conventions of Bjorken Drell<sup>14</sup> for the normalization of the spinors etc. Gauge invariance is preserved in these approximations and one can easily see that  $q_\mu J^\mu = 0$ .

The trace involved in  $\bar{\Sigma} \Sigma |T|^2$  is easily evaluated and we find

$$\bar{\Sigma} \Sigma |T|^2 = \left(\frac{f}{m_\pi}\right)^2 e^4 2 \left(\frac{2M}{2M-q^0}\right)^2 \left(\frac{1}{q^2}\right)^2 \frac{1}{2} \frac{1}{m_e^2} \frac{1}{2j+1} \sum_m \sum_{m'} \sum_{m_1} \left[ 2|p_i n^\mu J_\mu|^2 + \frac{q^2}{2} J_\mu^* J^\mu \right] \quad (6)$$

where we have used the gauge condition  $q_\mu J^\mu = 0$ . In order to obtain the value of  $J^\mu$  we must evaluate the following nuclear matrix element

$$M(\vec{k}) = \langle n' l' j' m' | \vec{\sigma} \cdot \vec{k} e^{i\vec{q}\cdot\vec{r}} \Phi_{n_1 l_1 m_1}^*(r) | n l j m \rangle \quad (7)$$

which using Racah algebra can be expressed as

$$\begin{aligned} M(\vec{k}) &= \sqrt{4\pi} (-1)^{m_1} |\vec{k}| \left[ \frac{(2l+1)(2l_1+1)}{(2l'+1)} \right]^{1/2} \\ &\sum_{\mu} \sum_{\mu'} C(l+1/2 j; \mu, m-\mu) C(l'+1/2 j'; \mu', m'-\mu') \\ &C(1/2 \ 1 \ 1/2; m-\mu, m'-\mu'+\mu-m) Y_{1, m'-\mu'+\mu-m}^*(\hat{k}) \\ &\sum_{\lambda} (i)^\lambda Y_{\lambda, \mu'-\mu+m_1}^*(\hat{q}) (2\lambda+1)^{1/2} n' l' F_{\lambda}^{n l} (q, n_1 l_1) \\ &\sum_{\lambda} C(\lambda l_1 \lambda'; \mu'-\mu+m_1, -m_1) C(1\lambda' l'; \mu, \mu'-\mu) \\ &C(\lambda l_1 \lambda'; 000) C(1\lambda' l'; 000) \end{aligned} \quad (8)$$

and

$$n' l' F_{\lambda}^{n l} (q, n_1 l_1) = \int_0^\infty r^2 R_{n' l'}(r) R_{n l}(r) j_\lambda(qr) R_{n_1 l_1}^*(r) dr \quad (9)$$

where  $R$  and  $R'$  are the radial wave functions for the nucleon and the pion respectively and  $j_\lambda$  the Bessel function.

The section under study can give rise to big cross sections because if we look only at small scattering angles the photon exchanged in fig. 1 is nearly on shell and the photon propagator can be made very large. However one does not have to forget that the signal is to be seen over the background of the inclusive  $(e, e')$  reaction, which also contains the same photon

propagator. This inclusive cross section is very well known<sup>15)</sup> and is given in terms of the longitudinal and transverse response functions by

$$\frac{d\sigma}{d\Omega dE_{out}} = \left(\frac{2\alpha}{q^2}\right)^2 E_{out}^2 \cos^2 \frac{\Theta}{2} \cdot \quad (10)$$

$$\left[ \left(\frac{q^2}{\vec{q}^2}\right)^2 R_L(q^0, |\vec{q}|) + \frac{1}{2} R_T(q^0, |\vec{q}|) \left( \frac{4(p_{in} \cdot p_{out} - 2m_e^2)}{q^2 + 4E_{in} E_{out}} - \frac{q^2}{\vec{q}^2} \right) \right]$$

where  $R_L$ ,  $R_T$  are the longitudinal and transverse response functions given in terms of the structure functions by

$$\begin{aligned} R_T(q^0, |\vec{q}|) &= 2W_1(q^0, |\vec{q}|) \\ R_L(q^0, |\vec{q}|) &= W_1(q^0, |\vec{q}|) \frac{\vec{q}^2}{q^2} + W_2(q^0, |\vec{q}|) \left(\frac{\vec{q}^2}{q^2}\right)^2 \end{aligned} \quad (11)$$

In ref. <sup>15)</sup>  $W_1$ ,  $W_2$  are related to the longitudinal and transverse cross sections by

$$\begin{aligned} W_1(q^0, |\vec{q}|) &= \frac{2Mq^0 + q^2}{8\pi^2\alpha M} \sigma_T \\ W_2(q^0, |\vec{q}|) &= \frac{2Mq^0 + q^2}{8\pi^2\alpha M} \frac{1}{1 - q^0^2/q^2} (\sigma_L + \sigma_T) \end{aligned} \quad (12)$$

In the limit of  $q^2 \rightarrow 0$  (real photons)  $\sigma_L \rightarrow 0$  and hence, from eq. (11),  $R_L \rightarrow 0$ . In this limit  $\sigma_T$  becomes the real photon cross section. Note that because we can go with the formulas to very small angles we have not neglected the electron mass in eqs. (10), and (11) as in usually done in the literature.

For small angles where the photon is very close to on shell, the transverse part in eq. (10) dominate over the longitudinal part and from eqs. (11) (12) we have that

$$R_T(q^0, |\vec{q}| = q^0) = \frac{q^0}{2\pi^2\alpha} \sigma_T(q^0) \quad (13)$$

with  $\sigma_T$  the total photon cross section. We take  $\sigma_T(q^0) = 65 \mu b \times A$  from experiment in the region of interest<sup>16,17)</sup> and with these ingredients we evaluate the background cross section of eq. (10).

In table 1 we show the results for  $d\sigma/d\Omega dE$  in the maximum of the peak

for the excitation of the 1s and 2p states of  $^{209}\text{Pb}$  compared to the background contribution from eq. (10), calculated at  $E_{in} = 200 \text{ MeV} + m_e$ .

As we can see the cross sections for the excitation of the discrete pionic states can be large but the ratio of this cross section to the background is very small. We should also note that for technical reasons the forward angles are not reachable. We have also evaluated the cross sections at  $E_{in} = 500 \text{ MeV}$ . At these energies  $q^2$  is very close to zero for forward angles and the cross sections increase. However, the ratio of signal to background is about the same as for  $E_{in} = 200 \text{ MeV}$ . Note that as  $q \rightarrow 0$  the behaviour of eqs. (6) and (10) is not just given by the photon propagator since the square bracket in eq. (10) and  $p_{in}^\mu J_\mu$  from eq. (6) both go to zero.

We have also evaluated the results for the  $e^- + ^{41}\text{Ca} \rightarrow e^- + (^{41}\text{Sc} \pi^-)$  for the 1s state. The ratio of the cross section at the peak with the background is of the order of  $10^{-2}$  (bigger than for  $^{209}\text{Bi}$ ). However one should also note that one of the reasons for the increased signal is the fact that the width of the  $^{41}\text{Sc}$  pionic state is very small (80 KeV versus 350 KeV for  $^{209}\text{Bi}$ ). A measurement with less resolution than this width would spread the contribution and reduce accordingly the cross section at the peak.

The numbers obtained indicate that the reaction is not very suited to produce the pionic states. The main reason is the lack of coherence of the reaction, since only the valence nucleon contributes to the amplitude as we indicated. Looking simultaneously for signals of the creation or decay of the pionic atoms can help to reduce the background. Hence taking into account that the  $\pi^-$  is predominantly absorbed by pn pairs, the detection of two neutrons back to back sharing the energy of the pion would be an indication that the pionic state was created. This signal would be blurred by final state interaction of the nucleons, but the nucleons from the background would be less correlated. There could be confusion with the nucleons coming from direct  $\gamma$  absorption by two nucleons in the background process, but here again the  $\gamma$  is mostly absorbed by np pairs<sup>13)</sup> in the region of interest and we would see, up to final state interactions, np pairs (not nn pairs) back to back. This is only approximative because the nucleons should match the photon momentum plus the Fermi momentum of the initial nucleon pair. In ref.<sup>18)</sup> where the  $\gamma + A \rightarrow (\pi^- A')$  reaction was explored experimentally, the radiative pion capture of the pionic atom was instead chosen as a test.

One of the findings of this work is that one should try to look for reactions giving a chance to the nucleons to contribute coherently to the cross section in order to take advantage of the  $A^2$  factor in the cross section. In this sense the  $(n, p)$  reactions and related ones, or the  $(\gamma, \pi^+)$  reaction<sup>19)</sup>, could stand a better chance than the one studied here or the related one of ref.<sup>18)</sup>.

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## FIGURE CAPTIONS

**Fig. 1:** Feynman diagram for the  $(e, e' \pi^-)$  reaction on a nucleon.

**Fig. 2:** Model chosen for the  $\gamma N \rightarrow \pi N$  vertex: **a)** gives rise to the Kroll Ruderman terms when  $p_{\pi^-} \rightarrow 0$ . **b)** Pion pole term.

## TABLE CAPTIONS

**Table 1:**  $d^2\sigma/d\Omega dE_{out}$  for the excitation of the 1s and 2p states  $^{209}\text{Bi}$  through the reaction of eq. (4), together with the background from the inclusive  $(e, e')$  reaction on  $^{209}\text{Pb}$  different scattering angles and  $E_{in} = 200.5$  MeV. The valence proton in  $^{209}\text{Bi}$ , is taken in the  $1h_{9/2}$  shell and the valence neutron in  $^{209}\text{Pb}$  in the  $2g_{9/2}$  shell. The oscillator parameter taken is  $\alpha = 0.407 \text{ fm}^{-1}$  (the radial part is  $\exp(-\alpha^2 r^2/2)$ ).

**TABLE 1**

$d^2/d\Omega dE_{out} [\mu\text{b}/\text{sr}/\text{MeV}]$ ,  $E_{in} = 200.5 \text{ MeV}$

$\theta_{lab}$	1s	2p	background	$q^2 [\text{MeV}^2]$
0°	0.23	0.45	600	-0.3
2.5°	$7.3 \cdot 10^{-3}$	$1.5 \cdot 10^{-2}$	20	-26.0
5.0°	$1.9 \cdot 10^{-3}$	$3.7 \cdot 10^{-3}$	5	-103
10.0°	$0.46 \cdot 10^{-3}$	$0.9 \cdot 10^{-3}$	1.4	-410

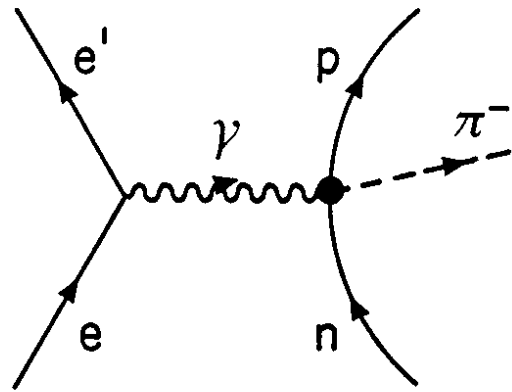


fig. 1

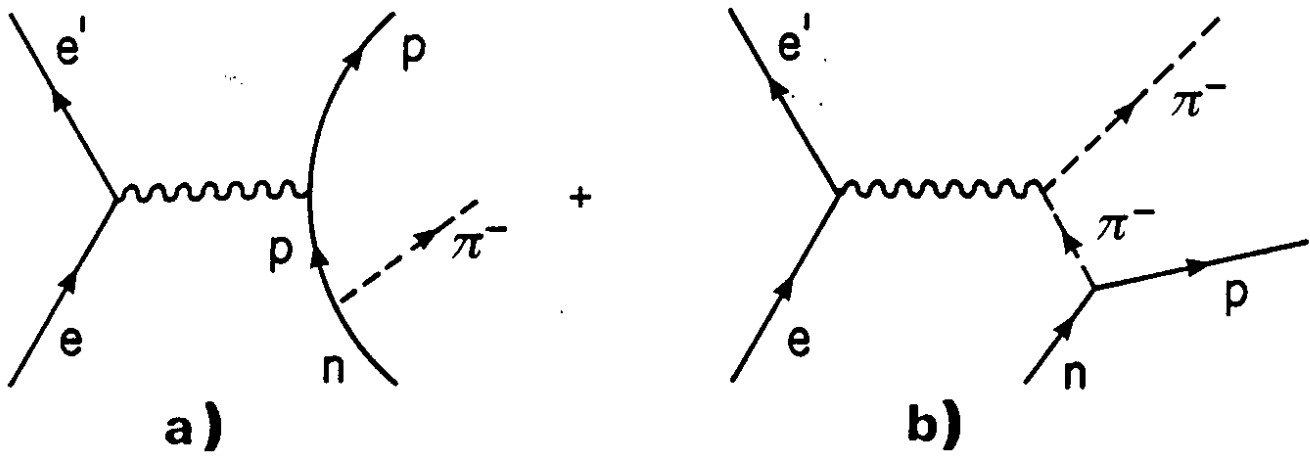


fig. 2