

**PION DOUBLE CHARGE EXCHANGE REACTIONS
LEADING TO DEEPLY BOUND DOUBLE PIONIC ATOMS.**

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Abstract

We study theoretically pion double charge exchange reactions leading to deeply bound double pionic atoms. The reaction cross section with two pions in the 1s pionic orbit in ^{208}Pb is calculated with realistic pionic atom wave functions and distortion effects and found to be $d^2\sigma/dE d\Omega \sim 10^{-4} - 10^{-5}$ $\mu\text{b}/\text{sr MeV}$, which is only a small fraction of the double charge exchange background.

Deeply bound pionic atoms in heavy nuclei are getting increasing attention from both experimentalists and theoreticians, since these states were found quasi-stable and hence accessible experimentally with some suitable methods¹⁾. Pion transfer reactions such as (n,p) and $(d,^2\text{He})$ were suggested to look for these states^{1,2)}. Successively, (n,p) experiments were performed at TRIUMF³⁾ and $(d,^2\text{He})$ at SATURN⁴⁾ without getting clear signals. More accurate calculations accounting for the distortion of the nucleon wave functions were performed and the signals were indeed found to be small^{3,5)}. Other reactions have been investigated theoretically by several groups: $(\gamma, \pi^+)^{6)}$, $(e, e')^{7,8)}$, $(\pi^-, \pi^+)^{9)}$, $(n, d)^{10,11)}$ and resonant compton scattering¹²⁾. Experimental efforts are being undertaken using the above mentioned possibilities and others at several institutes. We have hence, a hope that deeply bound pionic atoms will be found in a near future.

Deeply bound pionic atoms have an interesting structure. The bulk part of the pion-nucleus interaction is strongly repulsive and it pushes out a pion from the nucleus, while the Coulomb force attracts the pion to form a bound state. Hence, the pion forms a pionic halo around the nucleus. Binding energies of 1s states in nuclei like Pb are of the order of 5-10 MeV, which is the energy scale of Nuclear Physics. This feature motivates us to think how a pionic system with many pions would look like. It would be very interesting to make an object with more than two bosons in a simple orbit. A new field of multi-bosons with multi-fermions interacting strongly with each other will be open if we are able to create such states and study them experimentally¹³⁾.

Multiple pionic atoms are, for sure, produced in high energy heavy ion collisions^{14,15)}. It is however, very difficult to identify them experimentally. In addition, we cannot study the structure of multiple pionic

atoms by heavy ion collisions.

In this paper, we would like to study a clear method, motivated by all the theoretical efforts made up to now in looking for deeply bound pionic atoms, to produce double pionic atoms. The idea is the use of direct reactions, in which a particle is bombarded on a target nucleus and an outgoing particle is measured. The signature for forming pionic atoms is a peak structure in the excitation function just below the two pion production threshold. As the first trial, we look into (π^-, π^+) reactions leading to double pionic atoms in 208Pb . This is $\pi^- + 208\text{Pb} \rightarrow \pi^+ + (208\text{Pb} \pi^- \pi^+)$ with π^- a bound pion. The purpose of doing this investigation is to see what is the size of the formation cross sections and what considerations are needed in order to find better methods.

The (π^-, π^+) reaction leading to double pionic atoms in a nucleus A is sketched in Fig. 1. The differential cross section leading to pionic atoms with two pions in orbits (n_1, l_1) and (n_2, l_2) is written as

$$\frac{d\sigma}{d\Omega} = \frac{|\vec{k}_f|}{|\vec{k}_i|} \frac{1}{(2\pi)^2} \frac{1}{2\omega_{n_1 l_1} 2\omega_{n_2 l_2}} \frac{1}{4} \sum_{m_1 m_2} N_f |\Gamma|^2, \quad (1)$$

where \vec{k}_i, \vec{k}_f are the momenta of incoming and outgoing pions, $\omega_{n_1 l_1}$ and $\omega_{n_2 l_2}$ are the energies of bound pions in orbits $n_1 l_1$ and $n_2 l_2$. N_f is the symmetry factor; e. g. $N_f = 1/2$ for $(n_1, l_1, m_1) = (n_2, l_2, m_2)$ and 1 otherwise. If we want to take care of the widths of pionic atom states, we would replace the energy conserving δ -function using the prescription,

$$2\pi\delta(E_i - E_f) \rightarrow -2\text{Im} \frac{1}{E_i - E_f + i\Gamma/2} \quad (2)$$

and do not make the energy integration to get the double differential cross section. The width Γ is $\Gamma = \Gamma_{n_1 l_1} + \Gamma_{n_2 l_2}$. The resulting expression for $d^2\sigma/d\Omega dE$ is then given by multiplying the expression in eq. (1) by

$$\frac{1}{2\pi} \frac{\Gamma_1 + \Gamma_2}{(E_1 - E_f)^2 + ((\Gamma_1 + \Gamma_2)/2)^2} \quad (3)$$

The T-matrix may be calculated by taking the 4π interaction of Weinberg⁽⁶⁾, which is written as

$$\mathcal{L}_{2\pi} = -\frac{1}{4f_\pi^2} \left[\vec{\Phi}^2 \partial^\mu \vec{\Phi} \partial_\mu \vec{\Phi} - \frac{1}{2} \left(1 - \frac{1}{2} \xi\right) m_\pi^2 \vec{\Phi}^4 \right] \quad (4)$$

Here, $\vec{\Phi}$ is the pion field with the arrow indicating a vector in the isospin space and m_π the pion mass, f_π is the pion decay constant, $f_\pi = 93$ MeV. ξ is a chiral symmetry breaking parameter, which is expected to be zero from QCD based quark models. Setting the pion momenta as denoted in Fig. 1, the T-matrix for creation of two pions in the orbits n_1, l_1, m_1 and n_2, l_2, m_2 is written as

$$T = \left[d^3r \chi_f^*(\vec{r}) \Phi_{n_1, l_1, m_1}^*(\vec{r}) \Phi_{n_2, l_2, m_2}^*(\vec{r}) \frac{(-i)}{f_\pi^2} \right. \\ \left. \left[(k_1 - k_f)(q_1 + q_2) - 2\left(1 - \frac{1}{2}\xi\right)m_\pi^2 \right] \chi_1(\vec{r}) \right] \quad (5)$$

Here, χ_1 and χ_f are distorted waves of incoming and outgoing pions, respectively and Φ_s^* are the pionic atom wave functions with n, l, m being the node, orbital angular momentum and magnetic quantum numbers. In the above expression, k and q are meant to be four momenta corresponding to each pion (Fig. 1). The bracket in eq. (5) is approximated by neglecting the three momentum components $\vec{q}_1 + \vec{q}_2$ which are much smaller than $q_1^0 + q_2^0 \approx 2m_\pi$ and partly cancel when integrating over angles. Thus we have

$$T \approx -2i \frac{m_\pi^2}{f_\pi^2} \left[d^3r \chi_f^*(\vec{r}) \Phi_{n_1, l_1, m_1}^*(\vec{r}) \Phi_{n_2, l_2, m_2}^*(\vec{r}) \chi_1(\vec{r}) \right] \quad (6)$$

where we have set $\xi = 0$.

We shall first estimate the cross sections by taking the non-relativistic hydrogen-like 1s pionic atom wave functions and assuming also plane waves for χ_s^* . The 1s wave function is

$$\Phi_{1s}(r) = (\pi a^3)^{-1/2} e^{-r/a}, \quad \text{where } a = 1/Z m_\pi \alpha.$$

$$\chi_f(\vec{r}) = e^{+i\vec{k}_f \cdot \vec{r}} \quad \text{and} \quad \chi_1(\vec{r}) = e^{+i\vec{k}_1 \cdot \vec{r}}$$

The T-matrix is

$$T = -2i \frac{m_\pi^2}{f_\pi^2} \int d^3r e^{i\vec{q} \cdot \vec{r}} (\pi a^3)^{-1} e^{-2r/a} \\ = -2i \frac{m_\pi^2}{f_\pi^2} \left[1 + \left(\frac{a q}{2}\right)^2 \right]^{-2} \quad (7)$$

Since, a typical momentum transfer is $q \sim 2m_\pi$, the momentum dependent term dominates in the denominator as far as $(Z\alpha)^{-1} = 137/Z \gg 1$ and, hence we can drop the 1 in the denominator. The resulting cross section is written as

$$\frac{d\sigma}{d\Omega} \sim 8 \times 10^{-3} \mu\text{b/sr} \left(\frac{Z}{40}\right)^8 \quad (8)$$

For this estimate we have taken $k_f/k_i = 1/2$ and $\omega = m_\pi$. The cross section increases very rapidly with the charge number as Z^8 . This is because the higher Z is the higher are the momentum components in the pionic atom wave function. Hence, heavy atoms are preferred by this method. If we put $Z = 82$ for ^{208}Pb in eq. (8), we get $d\sigma/d\Omega \sim 3 \mu\text{b/sr}$. This value is quite encouraging. We should, however, consider realistic wave functions for bound pions and distorted waves for in and out-going pions.

We take 1s wave functions by solving the Klein-Gordon equation

with the optical potential of Seki and Masutani¹⁷⁾. Concerning the distortion, we take the eikonal approximation with the pion-nucleon cross section modified by the nuclear medium effect;

$$e^{i\vec{k}_1 \cdot \vec{r}} e^{-i\vec{k}_f \cdot \vec{r}} \rightarrow e^{i\vec{k}_1 \cdot \vec{r}} \exp\left(-i \int_{-\infty}^z \frac{1}{2k_i} \Pi(k_i, \rho(\vec{b}, z')) dz'\right) \quad (9)$$

$$e^{-i\vec{k}_f \cdot \vec{r}} \exp\left(-i \int_0^{\infty} \frac{1}{2k_f} \Pi(k_f, \rho(\vec{r}')) dt\right)$$

$$\text{with } \vec{r}' = \vec{r} + \frac{\vec{k}_f}{|\vec{k}_f|}$$

where $\Pi(k, \rho)$ is the pion selfenergy ($\Pi \approx 2\omega V_{\text{opt}}$) and \vec{b} is the impact parameter.

In order to minimize the distortion effects on the incoming and outgoing pions, we take $\omega_j \sim 500$ MeV and hence $\omega_f \approx 200$ MeV, so we are above and below the delta resonance peak in both cases. For the pion selfenergy we use the parametrization of Oset et al.¹⁸⁾, who have calculated Π microscopically around the resonance region.

The 1s wave function in momentum space is shown in Fig. 3 of ref.¹⁾. Because of nuclear finite size effects and the fact that the pion is pushed out from the nuclear interior by the strong interaction, the high momentum components are reduced from the non-relativistic hydrogen-like wave function values. At the value of interest $q \sim m_\pi$, $\Phi_{1,s}(q)$ is reduced by an order of magnitude. Because the (π^-, π^+) cross section goes as $|\Phi_{1,s}(q)|^4$, assuming the momentum transfer equally shared by the two bound pions, the use of the realistic wave function, makes the cross section $d\sigma/d\Omega \sim 10^{-4}$ $\mu\text{b}/\text{sr}$. We should still include the effect of the distortion of the external pions. We show the calculated results on (π^-, π^+) double

differential cross sections at the peak energy as a function of the scattering angle in Fig. 2. The double differential cross sections with the plane wave approximation are of the order of 10^{-4} – 10^{-5} $\mu\text{b}/\text{sr}/\text{MeV}$ in the forward angles. These results agree with the rough estimate given above by looking at the pion wave function at $q \sim m_\pi$, provided that the width of the 1s state used in the calculation is $\Gamma_{1,s} = 0.66$ MeV. The distorted wave results have a large angle dependence. At $\Theta = 0^\circ$, the DW cross section is slightly larger than the PW one.

The (π^-, π^+) cross sections leading to the 1s double pionic atom in 208Pb are found quite small $d^2\sigma/dE d\Omega \sim 10^{-4}$ – 10^{-5} $\mu\text{b}/\text{sr}/\text{MeV}$. One might try still the pion double charge exchange reactions experiments, if the background for the inclusive (π^-, π^+) reaction were relatively low. We have calculated the background double differential cross sections in the region of interest by using the pion cascade model developed in ref.¹⁹⁾ which was used to evaluate all the inclusive pion nuclear reaction cross sections. The application to the particular case of double charge exchange was done in ref.²⁰⁾

The double differential cross sections are found to be $d^2\sigma/dE d\Omega \sim 2\mu\text{b}/\text{sr}/\text{MeV}$. Hence, the (π^-, π^+) reactions seem not to be suited for double pionic atoms.

We have studied the (π^-, π^+) reactions for formation of deeply bound double pionic atoms in heavy nuclei. The cross sections increase very rapidly with the nuclear charge. Even for 208Pb , however, the cross sections are found $d^2\sigma/d\Omega dE \sim 10^{-4}$ – 10^{-5} $\mu\text{b}/\text{sr}/\text{MeV}$. The peak cross sections are much too small as compared to the background cross section estimated by the pion cascade model.

We hope our estimate of the formation cross sections of deeply bound pionic atoms has made the conditions to be considered clear. The cross sections increase with the charge Z very rapidly, which is related with the large momentum transfer inherent to coherent pion transfer processes. Although we have coherence in the pionic atom formation, we might rather consider nuclear transfer reactions such as (π^-, p) in order to minimize the

momentum carried by pions. Such a consideration is under way.

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FIGURE CAPTIONS.

Fig. 1.— A sketch of (π^-, π^+) reactions leading to double pionic atoms in a nucleus A.

Fig. 2.— (π^-, π^+) double differential cross sections leading to the 1s double pionic atom in 200pb at $\omega_1 = 500$ MeV as a function of the scattering angle. The solid curve is obtained by using the distorted waves for in and outgoing pions, while the dashed one uses plane waves.

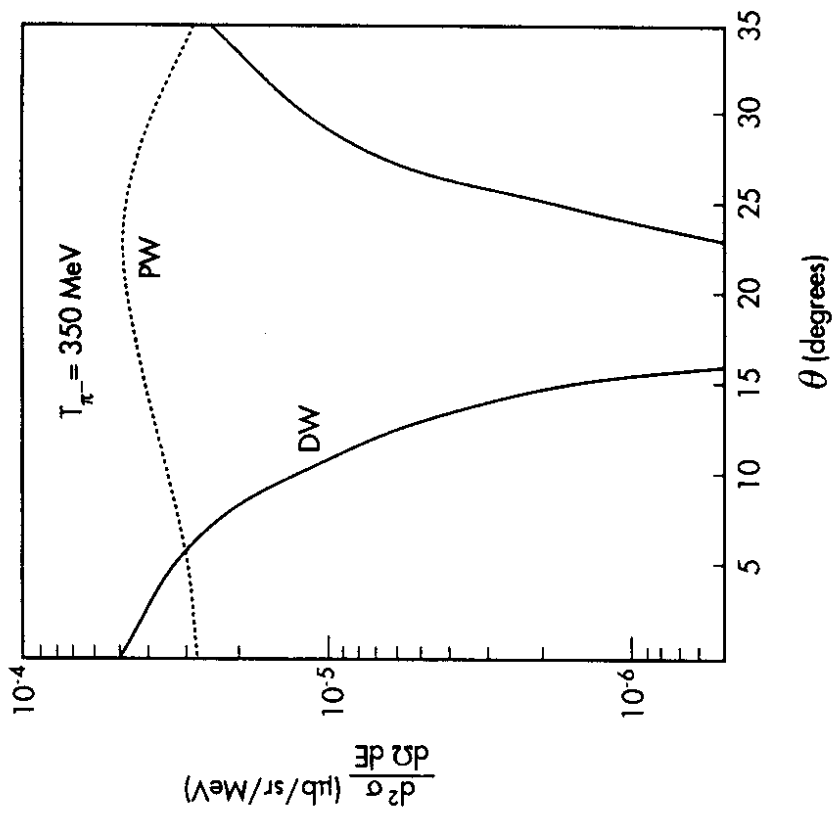


Fig. 2

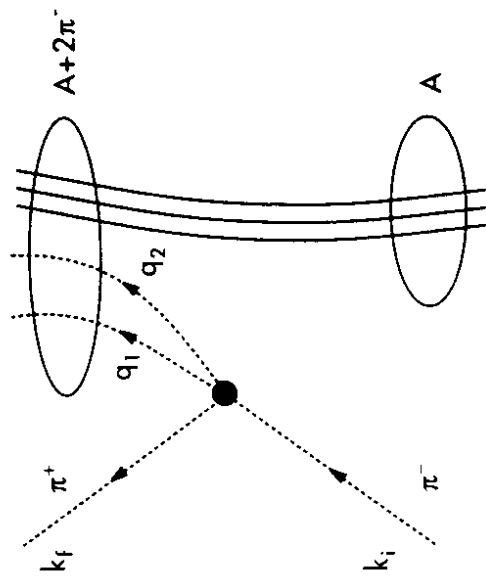


Fig. 1