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Form Factors for Semileptonic $B \to \pi$ and $D \to \pi$ Decays from the Omnès Representation

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Abstract

We use the Omnès representation to obtain the q^2 dependence of the form factors $f^{+,0}(q^2)$ for semileptonic $H \to \pi$ decays from elastic $\pi H \to \pi H$ scattering amplitudes, where H denotes a B or D meson. The amplitudes used satisfy elastic unitarity and are obtained from two-particle irreducible amplitudes calculated in tree-level heavy meson chiral perturbation theory (HMChPT). The q^2-dependences for the form factors agree with lattice QCD results when the HMChPT coupling constant, g, takes values smaller than 0.32, and confirm the milder dependence of f^0 on q^2 found in sumrule calculations.

1 Introduction

In this letter we present a description of the form factors f^+ and f^0 describing semileptonic $H \to \pi$ decays, where H denotes a D or B meson. For the B meson this exclusive semileptonic decay can be used to determine the magnitude of the CKM matrix element V_{ub}, currently the least well-known entry in the CKM matrix. Ultimately, experimental measurements of $f_B^+(q^2)$ for given momentum-transfer q will be compared directly to theoretical determinations at the same q^2 values to determine $|V_{ub}|$. In the interim, it may be helpful to consider the decay rate integrated partially or completely over q^2, but this requires knowledge of the q^2 dependence of the form factors. Lattice calculations and sumrule calculations apply in (different) restricted ranges of q^2 while dispersion relations may be used to bound the form factors over the whole q^2 range [1, 2], or as a basis for models [3]. A variety of models exists for the whole range of q^2. One can ensure that general kinematic relations and the demands of heavy quark symmetry (HQS) are satisfied, but an ansatz, such as pole, dipole or other forms, is still required [4, 5].

Here we use the Omnès representation to obtain the full q^2 dependence of these form factors from the elastic $\pi H \to \pi H$ scattering amplitudes. For our application we have an isospin-1/2 channel, with angular momentum $J = 1$ or 0 for f^+ and f^0 respectively. We rely on the following description of the (inverse) amplitude for elastic $\pi H \to \pi H$ scattering in the isospin I, angular momentum J, channel, with centre-of-mass squared-energy s and masses m and M respectively [6],

$$T_{IJ}^{-1}(s) = -\overline{I}_0(s) - C_{IJ} + 1/V_{IJ}(s), \tag{1}$$

where V_{IJ} is the two-particle irreducible scattering amplitude and C_{IJ} is a constant. C_{IJ} and V_{IJ} are real in the scattering region. This description implements elastic unitarity automatically. Equation (1) is justified by a dispersion relation for T^{-1}, where the contributions of the left hand cut and the poles (if any) are contained in $-C_{IJ} + 1/V_{IJ}$. $\overline{I}_0$ gives the exact contribution from the right hand cut, after any necessary subtractions[1]. The description of equation (1) may also be justified by an approach using the Bethe–Salpeter equation.

Once T_{IJ} is known, we can compute the corresponding phase shift δ_{IJ}. In turn, δ_{IJ} can be used in an Omnès representation [8] giving $f_{IJ}(q^2)/f_{IJ}(0)$ in terms of an integral involving the phase shift, assuming that at threshold the phase shift should be $n\pi$, where n is the number of bound states in the particular channel considered, and $\delta_{IJ}(\infty) = k\pi$, where k is the number of zeros of the scattering amplitude on the physical sheet (this is Levinson's theorem [9]).

We determine V_{IJ} from tree level heavy meson chiral perturbation theory (HMChPT) [10], which implements HQS and is a double expansion in powers of $1/M$ and momenta, where M is the heavy meson mass. The parameter C_{IJ} in equation (1) partially accounts for higher order contributions in the expansion [6].

We find consistency of our description with lattice results for the $D \to \pi$ [11, 12] and $B \to \pi$ [12, 13] form factors if we set the HMChPT coupling, g, to values smaller than 0.32. This upper bound is in reasonable agreement with other determinations, but g is not very well known [14, 15].

Our model and the Omnès representation are not guaranteed at high energies where inelasticities become important. However, our hypothesis is that only the low-lying states and energies should influence the form factors we consider.

A dispersive approach to the f^+ form factor was taken by Burdman and Kambor [3], who also used HMChPT to calculate the phase shift in $\pi H \to \pi H$ scattering. Here by working with the *inverse* amplitude we can ensure that Watson's theorem and elastic unitarity are satisfied exactly. Moreover, we compute f^+ and f^0 together to examine whether different behaviours in q^2 are found, consistent with lattice QCD results and allowing extra information from f^0 to be used to constrain f^+.

2 Scattering Amplitudes and Form Factors

We compute $V_{1/2}$, the two-particle irreducible amplitude for πH scattering in the isospin 1/2 channel, $\pi(p_1)H(Mv) \to \pi(p_2)H(Mv + q_2)$. Here, v is the four-velocity of the initial heavy meson of mass M. The pion mass is m. We use the direct tree level interaction from the lowest order HMChPT lagrangian, together with tree diagrams for H^* exchange which involve the leading interaction term with coupling g [10, 14]. The result is,

$$V_{1/2} = -\frac{M}{f^2}\left\{3v{\cdot}p_1 + v{\cdot}p_2 + g^2(p_1{\cdot}p_2 - v{\cdot}p_1 v{\cdot}p_2)\left(\frac{3}{v{\cdot}p_1 - \Delta} + \frac{1}{v{\cdot}p_2 + \Delta}\right)\right\}. \qquad (2)$$

Here, $f = 130.7\,\mathrm{MeV}$ is the pion decay constant and $\Delta = (M_*^2 - M^2)/2M \approx M_* - M$, where M_* is the heavy vector meson mass. We subsequently project $V_{1/2}$ onto the angular momentum 0 and 1 channels.

The full scattering amplitude at centre of mass energy-squared s, in the isospin I and angular momentum J channel, is obtained in our approach from equation (1). The phase

[1] $\overline{I}_0$ is calculated from a one-loop 'bubble' diagram. In the notation of reference [7], $\overline{I}_0(s) = T_G((m + M)^2) - T_G(s)$, where M and m are the masses of the two propagating particles

shift δ_{IJ} is then obtained from,

$$T_{IJ}(s) = \frac{8\pi i s}{\lambda^{1/2}(s, M^2, m^2)} \left(e^{2i\delta_{IJ}(s)} - 1\right), \tag{3}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$ is the usual kinematic function.

Once the phase shift is known, we use the Omnès representation to obtain the q^2 dependence of the form factors as follows:

$$\frac{f(q^2)}{f(0)} = \exp\left[\frac{q^2}{\pi}\int_{(m+M)^2}^{\infty} \frac{\delta_{IJ}(s)\, ds}{s(s-q^2)}\right]. \tag{4}$$

In this work, we always have $I = 1/2$. The form factor f^+ is obtained when $J = 1$ and depends on $J^P = 1^-$ resonances, while f^0 is obtained when $J = 0$ and depends on $J^P = 0^+$ resonances. We perform the integral numerically, taking the upper limit as 100 times the lower limit[2]. The form factors are equal at $q^2 = 0$: $f^+(0) = f^0(0)$.

- For $J^P = 1^-$ we take $C = 0$ for the D decay because the D^* resonance is so close to threshold that we expect it to saturate all the counterterms in HMChPT (compare to vector meson dominance in $\pi\pi$ scattering in ordinary chiral perturbation theory). Calculating C in this case reveals the value $C = 8 \times 10^{-6}$. We still have the freedom to vary the lowest order coupling constant g in HMChPT. For the B meson decay, we set $C = -0.0014$ to keep the B^* pole at its correct mass.

- For $J^P = 0^+$ we ignore D^* and B^* s-channel exchanges, which have the wrong quantum numbers to contribute in this case. These exchanges only contribute because of the heavy meson mass expansion implicit in HMChPT. Instead we keep C non-zero, setting $C = -0.0051$ for the D-physics case to get a resonance at about 2350 MeV, and $C = -0.0016$ for B-physics to get a resonance at about 5660 MeV [16].

The values of C are determined by demanding that T^{-1} ($\mathrm{Re}\, T^{-1}$) vanishes at the position of a pole (resonance). For the $J = 1$ channels, V^{-1} vanishes by construction at the positions of the D^* or B^*, and so, from equation (1), C is independent of g. In the $J = 0$ channels, g-dependence enters in V^{-1}, but only through the t-channel tree graphs, and is very weak. C varies by less than 0.5% for $0 < g < 0.45$ in the D-meson case and the dependence is even weaker for the B-meson case.

We noted that in using the Omnès representation [8] of equation (4), the phase shift at threshold should be $n\pi$, where n is the number of bound states in the channel under consideration. Thus $n = 0$ in all channels used here except for $J^P = 1^-$ in the B case where $n = 1$ to account for the B^*. In fact, our model also gives a bound state in the 0^+ channel in the B case, which we ignore. One could try to improve the model to avoid this unphysical bound state by replacing C with a function of q^2 (the function should have no right hand cut).

[2] $f^+_{D\pi}(q^2_{\rm max})$, where $q^2_{\rm max} = (m_D - m_\pi)^2$, varies by less than 1% as the upper limit of integration varies from 50 to 200 times the lower limit, and the variation is smaller at lower q^2.

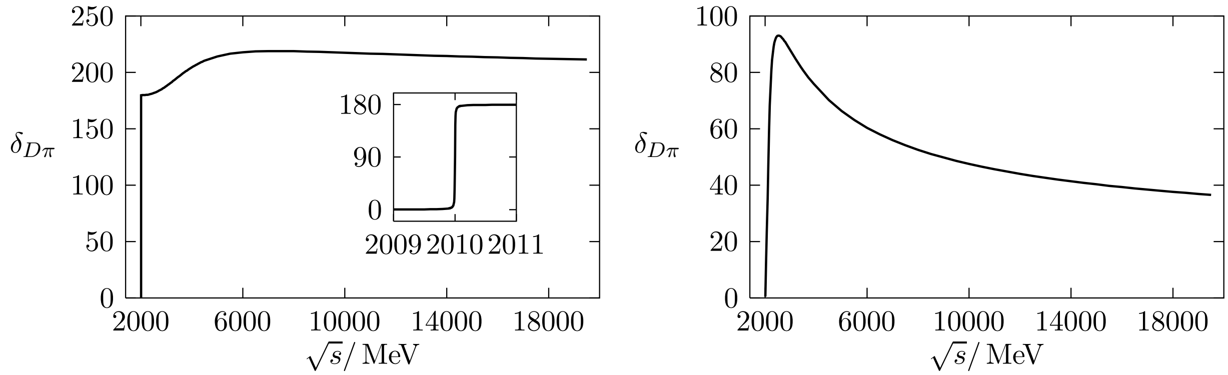

Figure 1 Phase $\delta_{D\pi}$ for the $IJ = 1/2, 1$ (left) and $1/2, 0$ (right) channels in $D\pi$ scattering. The inset on the left shows the resonance at $\sqrt{s} = m_{D^*} = 2010\,\mathrm{MeV}$. Phases are calculated with $g = 0.21$.

3 Semileptonic Decays

The process $D^* \to D\pi$ is kinematically allowed, so the D^* is a resonance in $D\pi$ scattering. In HMChPT the decay rates of D^{*+} to $D^0\pi^+$ and $D^+\pi^0$ are given to lowest order by,

$$\Gamma(D^{*+} \to D^0\pi^+) = \frac{g^2 p^3}{6\pi f^2}, \qquad \Gamma(D^{*+} \to D^+\pi^0) = \frac{g^2 p^3}{12\pi f^2}. \tag{5}$$

The sum of these rates can also be obtained from the slope of the phase shift at the resonance mass. We find that these two methods agree for a range of g values.

The D^* exchange is included in our tree level amplitude, and we expect it to saturate the counterterms in HMChPT, so in calculating $T_{1/2,1}$ we set $C = 0$ as noted above. Figure 1 (left) shows the phase shift obtained for $J = 1$. With input masses, $m_D = 1864.5\,\mathrm{MeV}$ and $m_{D^*} = 2010\,\mathrm{MeV}$, the D^* resonance shows up as the jump of π in the phase at the D^* mass. In the $J = 0$ channel, we tune C to produce a resonance at the expected mass of the D_0^* at $2350\,\mathrm{MeV}$ [16]. The $J = 0$ phase shift is shown on the right in figure 1.

In the B case, the decay process $B^* \to B\pi$ is not kinematically allowed and the B^* meson is a pole, sitting between the maximum physical q^2 value for the form factor, $q^2_{\mathrm{max}} = (m_B - m_\pi)^2$, and the start of the physical cut at $q^2 = (m_B + m_\pi)^2$. Again we use the physical pseudoscalar and vector meson masses as inputs, $m_B = 5278.9\,\mathrm{MeV}$, $m_{B^*} = 5324.8\,\mathrm{MeV}$. The phase shift for the $J = 1$ case is shown on the left in figure 2. The appearance of the B^* as a bound state between q^2_{max} and $q^2 = (m_B + m_\pi)^2$ is signalled by the vanishing of $T^{-1}_{1/2,1}(s)$ at $\sqrt{s} = m_{B^*}$. The $J = 0$ phase shift appears on the right of figure 2.

From the phase shifts we find the form factors f^+ and f^0. We perform a simultaneous three-parameter fit to the UKQCD and APE lattice results [11, 12, 13] for the form factors $f^+(q^2)$ and $f^0(q^2)$ which determine the B and D semileptonic decays. The free parameters are the HMChPT coupling constant g and the form factors at $q^2 = 0$: $f_B(0)$ for $B \to \pi$ decays and $f_D(0)$ for $D \to \pi$ decays[3]. The best fit parameters with 39 degrees of freedom are

$$g = 0.21\,{}^{+\,0.11}_{-\,0.21}, \quad f_B(0) = 0.39 \pm 0.02, \quad f_D(0) = 0.60 \pm 0.02 \quad \text{with} \quad \chi^2/\mathrm{dof} = 0.34. \tag{6}$$

[3] To use the results in [11], we take $Z_V^{\mathrm{eff}} = 0.88$ for the vector renormalisation constant connecting lattice and continuum results.

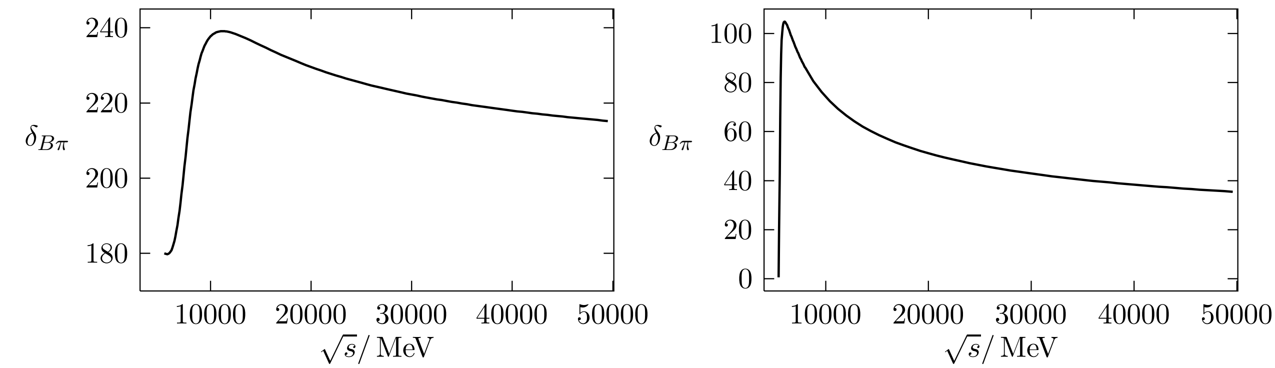

Figure 2 Phase $\delta_{B\pi}$ for the $IJ = 1/2, 1$ (left) and $IJ = 1/2, 0$ (right) channels in $B\pi$ scattering. Phases are calculated with $g = 0.21$.

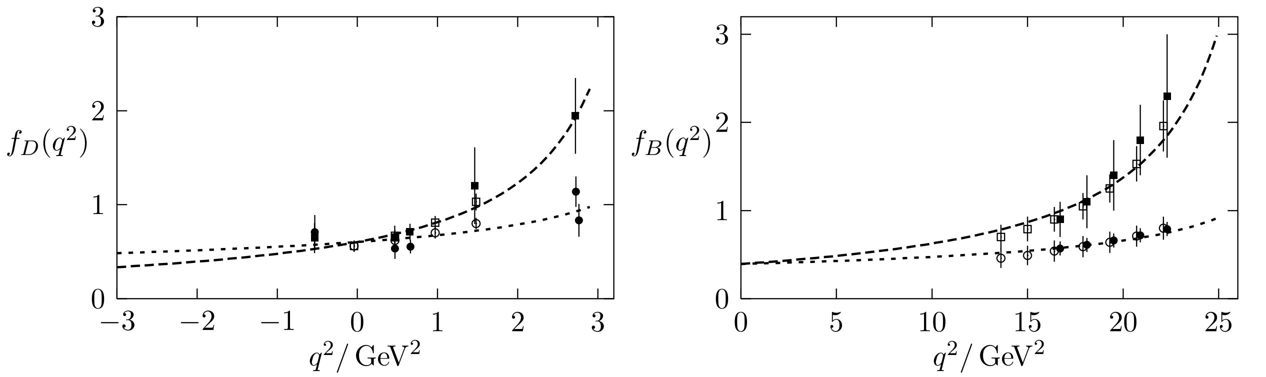

Figure 3 Form factors in $D \to \pi$ (left) and $B \to \pi$ (right) semileptonic decays. The squares (circles) denote f^+ (f^0) from lattice calculations, while the long-dashed (short-dashed) lines denote the fitted curves for f^+ (f^0). Solid symbols are results from UKQCD [11, 13], open symbols are results from APE [12].

Results can be seen in figure 3. Errors in the fitted parameters are statistical and have been obtained by increasing the value of the total χ^2 by one unit. A word of caution must be stated about the results for the HMChPT coupling constant g. Scalar channels are almost insensitive to this parameter. For the vector channels, in the case of D meson decay, the resonance is so close to threshold that it completely dominates the process, independent of the value of g, as long as the resonant contribution is more important than the background. This turns out to be true as long as g is greater than 0.001, thus the smallest value g can take is 0.001 and not zero as can be inferred from equation (6). To clarify the dependence of our results on g, we show in figure 4 both χ^2 and $f_B(0)$, $f_D(0)$ versus g, for $g \geq 0.001$. In the first figure the line at $\chi^2 = 13.28$ shows the minimum value of χ^2, while the line at $\chi^2 = 14.28$ determines the upper error. We also show best fit values, with fixed g, of $f_B(0)$ and $f_D(0)$ versus g. The points with errors correspond to the results quoted in equation (6).

We note that f_D^+ is well-approximated by a simple pole form with the D^* giving the pole mass, while f_D^0 is noticeably 'flatter' in q^2. This is consistent with lattice results. For the $B \to \pi$ case, f_B^+ is well-approximated by a pole form with the pole mass of order the B^* meson mass. The f_B^0 form factor has much less q^2 dependence, consistent with the behaviour found in lattice calculations.

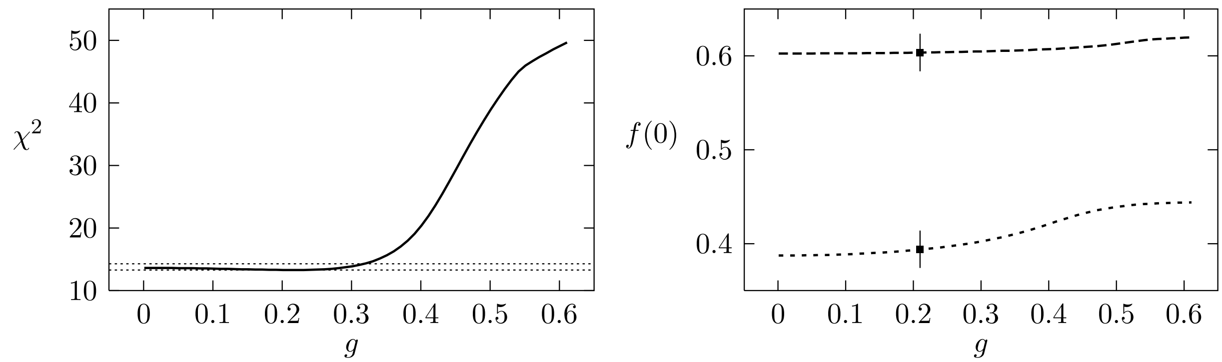

Figure 4 Left: chi-squared for the fit described in the text as a function of the HMChPT coupling g, for $g \geq 0.001$. Right: values of $f_B^{+,0}(0)$ (lower curve) and $f_D^{+,0}(0)$ (upper curve) as functions of g. The points with errors on the right are the best fit values of equation (6) at $g = 0.21$.

We have also determined the coupling g and form factors at $q^2 = 0$ separately for D and B decays using independent fits to the UKQCD and APE lattice data for D and B. The best fit values turn out to be the same as in equation (6), although g_D can be as large as 0.46 while still giving an acceptable chi-squared. To compare with light cone sumrule (LCSR) results, we take the LCSR values $f_{D^*} g_{D^* D\pi} = 2.7 \pm 0.8\,\text{GeV}$ and $f_{B^*} g_{B^* B\pi} = 4.4 \pm 1.3\,\text{GeV}$ [17], and combine with lattice calculations of the vector meson decay constants from Becirevic et al [18] and UKQCD [19], to yield

$$g_D = \begin{cases} 0.35 \begin{smallmatrix} +0.12 \\ -0.11 \end{smallmatrix} & f_{D^*} \text{ from [18]} \\ 0.39 \pm 0.12 & f_{D^*} \text{ from [19]} \end{cases} \qquad g_B = \begin{cases} 0.23 \pm 0.08 & f_{B^*} \text{ from [18]} \\ 0.28 \begin{smallmatrix} +0.10 \\ -0.09 \end{smallmatrix} & f_{B^*} \text{ from [19]} \end{cases} \tag{7}$$

The values are quite compatible in the B case, less so for D decays, although, as noted above, our fit for g_D allowed a large variation above the best-fit value. The value of $f_D(0)$ found here agrees well with the LCSR result $f_D^+(0) = 0.65 \pm 0.11$ [17], while $f_B(0)$ in equation (6) is higher than the LCSR value $f_B^+(0) = 0.28 \pm 0.05$ [17]. In the D case, the D^* resonance is only a few MeV above threshold and the range of q^2 for the semileptonic decay is not large, so one expects a simple pole form for f^+ to work well. For B physics, the effects of higher resonances and continuum states are evidently more important: such effects are incorporated in LCSR calculations but are not present in the very simple model used here. We address this issue in section 4 below.

Heavy quark symmetry (HQS) is an input in HMChPT. The HQS scaling relations for the B decay form factors at q^2_{max} are preserved because $f^+(q^2_{\text{max}})/f^0(q^2_{\text{max}})$ is proportional to

$$\exp\left(\frac{q^2_{\text{max}}}{\pi} \int_{(M+m)^2}^{\infty} \frac{\delta^+ - \delta^0}{s(s - q^2_{\text{max}})}\, ds\right). \tag{8}$$

The above result relies on the equality of the form factors at $q^2 = 0$, $f^+(0) = f^0(0)$. If $\delta^+ - \delta^0 = \pi$, which we see is satisfied by our phase shifts at large $\sqrt{s}$, then the ratio $f^+(q^2_{\text{max}})/f^0(q^2_{\text{max}}) = 1/(1 - q^2_{\text{max}}/(M+m)^2)$ as demanded by HQS, where M and m are the masses of the heavy meson and the pion respectively.

We have applied the same approach to describe semileptonic $D \to K$ decays. Here, it gives form factors flatter than lattice results [11] and the experimental evidence [20]. However, corrections of both types m_K/m_D and $m_K^2/(4\pi f_\pi)^2$ to the tree level HMChPT results used here are expected to be sizeable in this case.

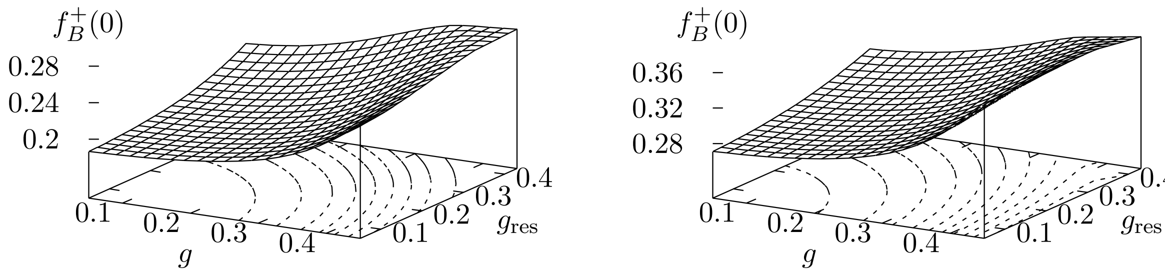

Figure 5 Variation of $f_B^+(0)$ with g and $g_{\rm res}$ where the B^* coupling is g and a second resonance is added in the $J = 1$ channel with coupling $g_{\rm res}$. On the left the resonance mass is $m_{\rm res} = 6100\,\mathrm{MeV}$ and contours are plotted from $f_B^+(0) = 0.20$ to 0.28 in increments of 0.01; on the right $m_{\rm res} = 8100\,\mathrm{MeV}$ and contours are plotted from $f_B^+(0) = 0.28$ to 0.37 in increments of 0.01.

4 Extra Resonances

We noted above that our result for $f_B(0)$ in equation (6) is higher than the LCSR value of around 0.28, while our fit for $f_B^+(q^2)$ is well-approximated by a simple pole form with pole mass of order m_{B^*}. This suggests that deviations from B^* pole dominance can become significant at low q^2. This phenomenon was also noted by Burdman and Kambor [3] who implemented a constrained dispersive model for f_B^+. Likewise, lattice results have favoured dipole forms in fits to f_B^+ [4, 12, 13].

To address this issue we have added a second resonance of mass $m_{\rm res}$ by hand in the 2PI $J = 1$ amplitude $V_{1/2,1}$, coupling it like the H^* but with its own coupling strength $g_{\rm res}$. In the D-meson case, we already had a good fit to the lattice results and a consistent value for $f_D(0)$. If $m_{\rm res}$ is large enough the extra resonance does not disturb this picture. In the B case, we can easily make $f_B(0)$ smaller while still fitting lattice results at large q^2. In figure 5 we show $f_B(0)$ as a function of the couplings g and $g_{\rm res}$ for two choices of the extra resonance mass, $m_{\rm res} = 6100\,\mathrm{MeV}, 8100\,\mathrm{MeV}$. The problem in this case is that it is not possible to make a statistically acceptable fit to f_B^+ and f_B^0 simultaneously. One could try to add an extra resonance in the $J = 0$ channel also, but while our choice of $m_{\rm res} = 6100\,\mathrm{MeV}$ for $J = 1$ may be motivated by potential models [21] or lattice results [22], we do not know whether or how to set the mass for additional $J = 0$ resonances, having already set the C values to account for rather poorly known resonances. This emphasises the importance of looking at f^+ and f^0 together, even though f^+ is the experimentally accessible form factor.

5 Conclusion

Our model is extremely simple, using only tree level HMChPT information for the two particle irreducible amplitude V_{IJ}, thereby incorporating only the first excited hadron state. Furthermore we fix to a constant an allowed polynomial in q^2 muliplying the Omnès exponential factor in equation (4). Thus, deviations from LCSR results for f^+ are not unexpected because those calculations incorporate effects of higher resonances and continuum states. Taking our model beyond leading order is not possible at present because of the proliferation of undetermined parameters which would appear in the next order of HMChPT and the lack of experimental data to fix them. This is a standard difficulty in using effective theories at higher orders.

The simple model presented here gives an excellent description of semileptonic D-decays. For B-decays it gives a good description of the lattice data near $q^2_{\rm max}$ and is also compatible within two standard deviations with LCSR predictions at $q^2 = 0$. Moreover, it provides a framework compatible with heavy quark symmetry, naturally accomodating pole-like behaviour for f^+ and, simultaneously, non-constant behaviour for f^0. Previously, as pointed out in [4, 2], a difficulty for form factor models with pole-type behaviour for f^+ was fixing a behaviour for f^0 which satisfied both the relation $f^0(0) = f^+(0)$ and the requirements of heavy quark symmetry. Pole-like behaviour of f^+ turns out again to be feasible in our model, thanks to the fact that the B^* is a bound state rather than a πB resonance.

Qualitatively, the results found here are encouraging. However, the larger value found for $f_B^+(0)$ compared to that from LCSR calculations would lead to appreciably smaller values for $|V_{ub}|$. We caution the reader that this should not be taken to indicate a large theoretical spread in the value of $|V_{ub}|$ from exclusive semileptonic $B \to \pi$ decays: one should bear in mind the simplicity of the model used. We indicated how a second resonance in the $J = 1$ channel can restore compatibility with both LCSR and lattice results for f_B^+, although this shifts the problem to making f_B^0 compatible with the lattice data in a combined fit and emphasises the importance of using information from both form factors.

Acknowledgements

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References

[1] CG Boyd, B Grinstein and RF Lebed, Phys. Rev. Lett. 74 (1995) 4603, hep-ph/9412324

[2] L Lellouch, Nucl. Phys. B 479 (1996) 353, hep-ph/9509358

[3] G Burdman and J Kambor, Phys. Rev. D55 (1997) 2817, hep-ph/9602353

[4] UKQCD Collaboration, DR Burford et al., Nucl. Phys. B 447 (1995) 425, hep-lat/9503002

[5] D Becirevic and AB Kaidalov, Phys. Lett. B 478 (2000) 417, hep-ph/9904490

[6] J Nieves and E Ruiz Arriola, Phys. Lett. B455 (1999) 30; Nucl. Phys. A679 (2000) 57

[7] C Itzykson and JB Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980) p308

[8] R Omnès, Nuovo Cimento 8 (1958) 316; NI Mushkelishvili, *Singular Integral Equations* (Noordhoff, Groningen, 1953)

[9] AD Martin and TD Spearman, *Elementary Particle Theory* (North-Holland, Amsterdam, 1970) p401

[10] MB Wise, Phys. Rev. D45 (1992) 2188

[11] UKQCD Collaboration, KC Bowler et al, Phys. Rev. D51 (1995) 4905, hep-lat/9410012

[12] A Abada et al, preprint CERN–TH 99–186, LPT–Orsay 00/86, RM3–TH/00–18, ROMA 1304/00, hep-lat/0011065

[13] UKQCD Collaboration, KC Bowler et al, Phys. Lett. B 486 (2000) 111, hep-lat/9911011

[14] R Casalbuoni, A Deandrea, N Di Bartolomeo, F Feruglio, R Gatto and G Nardulli, Phys. Rept. 281 (1997) 145, hep-ph/9605342

[15] UKQCD collaboration, GM de Divitiis, L Del Debbio, M Di Pierro, JM Flynn, C Michael and J Peisa, JHEP 9810 (1998) 010, hep-lat/9807032

[16] V Ciulli, talk presented at 8th Int. Symp. on Heavy Flavour Physics, University of Southampton, July 1999, JHEP Conf. Proc. PRHEP–hf8/044, hep-ex/9911044

[17] A Khodjamirian, R Rückl, S Weinzierl, CW Winhart and O Yakovlev, Phys. Rev. D62 (2000) 114002, hep-ph/0001297

[18] D Becirevic et al., Phys. Rev. D60 (1999) 074501

[19] UKQCD Collaboration, KC Bowler et al., preprint, Edinburgh 2000/14 IFUP–TH/2000–17 JLAB–THY–00–25 SHEP–00–08, hep-lat/0007020

[20] CLEO Collaboration, A Bean et al., Phys. Lett. B317 (1993) 647

[21] E Eichten, CT Hill and C Quigg, *Spectra of Heavy-Light Mesons*, preprint FERMILAB–CONF–94/117–T, in Proc. workshop on *The Future of High Sensitivity Charm Experiments*, eds D Kaplan and S Kwan, Batavia, Illinois, 1994 (FERMILAB–CONF–94/190 p345)

[22] J Hein et al., Phys. Rev. D62 (2000) 074503, hep-ph/0003130