

η bound states in nuclei

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Abstract

The energies and widths of bound states of the η meson in different nuclei are obtained using the results for its selfenergy in a nuclear medium, which is evaluated in a selfconsistent manner using techniques of unitarized chiral perturbation theory. We find bound states in all studied nuclei (from ^{12}C on) and the half widths obtained are larger than the separation of the levels, what makes the experimental observation of peaks unlikely. We have paid a special attention to the region of nuclei where only the $1s$ state appears and the binding energies are of the order of magnitude of the half width, which would magnify the chances that some broad peak could be observed. This is found in the region of ^{24}Mg with a binding energy around 12.6 MeV and half width of 16.7 MeV. In heavy nuclei like ^{208}Pb there are many bound states which would be difficult to disentangle and the deepest state has a binding energy about 21 MeV and half width around 16 MeV. Such an overlapping accumulation of states could be seen as an extension of the continuum of η strength into the bound region in η production experiments.

1 Introduction

The η nucleus optical potential has been a subject of intense study for a long time linked to the existence of η bound states in nuclei [1]-[13]. This latter topic has been the object of research lately and several reactions, $^7\text{Li}(d, ^3\text{He})_\eta^6\text{He}$, $^{12}\text{C}(d, ^3\text{He})_\eta^{11}\text{B}$ and $^{27}\text{Al}(d, ^3\text{He})_\eta^{26}\text{Mg}$ are being investigated at GSI [14] and others are being carried out or proposed in other laboratories [15]. With the exception of the η in the two nucleon system where two independent works seem to rule out a bound state [16, 17], all the other works predict bound η states from ^4He up. Experimentally there is no evidence that bound η states exist and their observation might be problematic since, even if there are bound states, their widths could be very large compared to the separation of the levels. This was indeed the case in the potential derived in Ref. [4]. However, as shown in this latter reference, there were large uncertainties in the potential, and hence in the energy and width of the bound states, mostly tied to the uncertainty in the binding potential of the $N^*(1535)$ resonance in the nucleus. This has been an open problem for years and only recently was it possible to give a credible answer to this question. The answer has come from the study of Refs. [18, 19] where the πN and coupled channels scattering was investigated up to energies above

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the $N^*(1535)$ resonance, including the ηN elastic scattering of relevance for the η interaction in nuclei. The study was done using techniques of unitarized chiral perturbation theory adapted to the meson baryon interaction [20, 21] which followed and extended the works using chiral Lagrangians and the Lippmann-Schwinger equation initiated in Refs. [23, 24]. In such schemes the $N^*(1535)$ resonance is generated dynamically from the multiple scattering of the mesons implied in the Lippmann-Schwinger equation of Refs. [23, 24], the Bethe-Salpeter equation of Refs. [18, 20] or in the N/D unitarization method used in Ref. [21]. This is important, since by implementing systematically medium corrections to the scattering equation, one can see how the pole of the $N^*(1535)$ resonance is changed in the nuclear medium and thus eliminate the main source of uncertainty of earlier evaluations of the η nucleus optical potential. This latter work was conducted in Ref. [25], using the vacuum amplitude of Ref. [19], and the η nucleus optical potential was evaluated in a selfconsistent way, as it was done for the antikaon case in Ref. [26]. Among other results, in Ref. [25], it was shown that the $N^*(1535)$ resonance barely moved in the nuclear medium with respect to the free-space, although its width was certainly changed. Besides in that reference, the η selfenergy in the nuclear medium was evaluated as a function of the η energy, its momentum and the nuclear density. In this work, we use the optical potential developed there in order to calculate η -nucleus bound states, with the aim of finding the optimal nuclei to search for such states. This condition is met when there is a minimum overlap between the states, and the half widths are not much bigger than the separation of the levels. We find this situation for medium size nuclei rather than in the light ones, which have been mostly used or suggested up to now in the search for such states.

2 The η nucleus optical potential

In Ref. [25] the self-energy of the η meson is evaluated in nuclear matter at various densities ρ , as a function of the η energy, k^0 , and its momentum, \vec{k} , in the nuclear matter frame. It is calculated by means of

$$\Pi_\eta(k^0, \vec{k}; \rho) = 4 \int^{k_F} \frac{d^3 \vec{p}_n}{(2\pi)^3} T_{\eta n}(P^0, \vec{P}; \rho) \quad (1)$$

where \vec{p}_n and k_F are the momentum of the neutron and the Fermi momentum at nuclear density ρ respectively, and $T_{\eta n}(P^0, \vec{P}; \rho)$ is the η -neutron in-medium s -wave interaction, with the total four-momentum of the system (P^0, \vec{P}) in the nuclear matter frame, namely $P^0 = k^0 + E_n(\vec{p}_n)$ and $\vec{P} = \vec{k} + \vec{p}_n$. Here, the isospin symmetry, $T_{\eta p} = T_{\eta n}$, is assumed and the amplitude is summed over nucleons in the Fermi sea. Since we are interested in finding bound states, we shall be concerned about the s -wave η self-energy which around the $N^*(1535)$ region is largely the most relevant, given the large s -wave coupling of this resonance to the ηN states.

The in-medium interaction $T_{\eta n}$ is obtained by basing on the model of Ref. [19] for πN and coupled channels scattering. That model reproduces the experimental data of πN scattering up to energies above the $N^*(1535)$ region. A similar chiral approach which covers a much wider energy range in isospin 1/2, although with more free parameters, is also done in Ref. [18].

In Ref. [19], the Bethe-Salpeter equation is considered with eight coupled channels including two $\pi\pi N$ states, namely $\{\pi^- p, \pi^0 n, \eta n, K^0 \Lambda, K^+ \Sigma^-, K^0 \Sigma^0, \pi^0 \pi^- p, \pi^+ \pi^- n\}$. The kernels for the meson-baryon two-body sector are taken from the lowest order chiral Lagrangians and improved by applying a form factor corresponding to a vector meson exchange in the t -channel. The kernels for $\pi N \leftrightarrow \pi\pi N$ transitions are determined so that they account for both the πN elastic and $\pi N \rightarrow \pi\pi N$ processes. That model reproduces well the the πN scattering amplitudes, especially

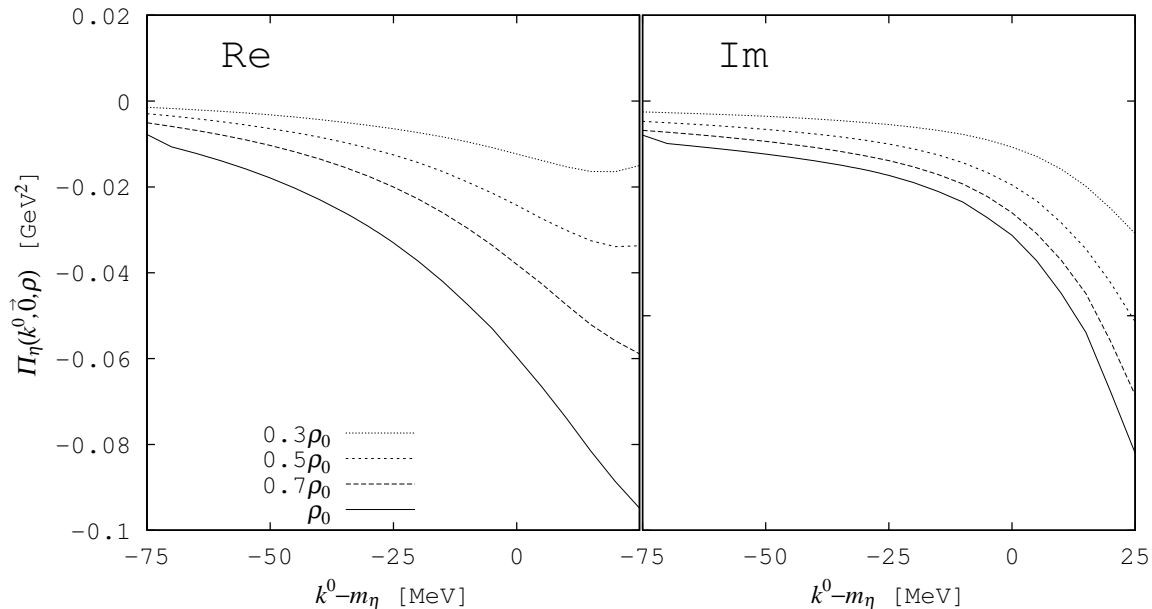


Figure 1: η self-energy for zero momentum as a function of energy, for four different densities. Both the Pauli blocking and hadron dressing are taken into account.

in isospin 1/2, for the center of mass energy energies from threshold to 1600 MeV. It reproduces also the $\pi^- p \rightarrow \eta n$ cross section at the region where the p -wave contribution is negligible. In this coupled channels approach, the model also provides the ηN interaction in free space and generates dynamically the $N^*(1535)$ resonance providing the width and branching ratios for its decay in good agreement with experiment, among them the ηN branching ratio which is quite large for that resonance. The agreement of the model with the different available data around the $N^*(1535)$ resonance region and the adequate description of the properties of the resonance, in particular the strong coupling to the ηN state, give us confidence that the model is rather accurate to make predictions on the $\eta N \rightarrow \eta N$ interaction and η nucleus interaction.

The medium effects in the ηN scattering amplitude which are considered in Ref. [25] are: 1) The Pauli blocking of the intermediate nucleon states appearing in the Bethe–Salpeter equation. 2) The selfenergy of the mesons (pions, kaons and eta) in the intermediate states, with the η selfenergy considered selfconsistently. 3) The baryon selfenergy of the intermediate states (N, Λ and Σ).

The results obtained for the η selfenergy can be seen in Fig. 1. There one can see that the η selfenergy around zero η energy and negative values is strongly energy dependent, both for the real and the imaginary parts. In order to have a feeling of the strength of the potential we can evaluate the optical potential by means of

$$U_\eta(\rho) = \frac{\Pi_\eta(m_\eta, \vec{0}, \rho)}{2m_\eta}. \quad (2)$$

This simple prescription gives the potential of $(-54 - i29)$ MeV at normal nuclear matter density. This means that one can expect bound states with around 50 MeV binding and a width of about 60 MeV. However, the energy dependence of the selfenergy is quite strong as one can see from Fig. 1 and, hence, a realistic determination of the η bound states should take that into account. For such purpose the results of Ref. [25] were parametrized in terms of analytical functions in the

energy range $-50 \text{ MeV} < k^0 - m_\eta < 0$ as

$$\begin{aligned} \text{Re}[\Pi_\eta(k^0, \vec{0}; \rho)] &= a(\rho) + b(\rho)(k^0 - m_\eta) + c(\rho)(k^0 - m_\eta)^2 + d(\rho)(k^0 - m_\eta)^3 \\ \text{Im}[\Pi_\eta(k^0, \vec{0}; \rho)] &= e(\rho) + f(\rho)(k^0 - m_\eta) + g(\rho)(k^0 - m_\eta)^2 + h(\rho)(k^0 - m_\eta)^3 \end{aligned} \quad (3)$$

with

$$\begin{aligned} a(\rho) &= (-36200.3 \rho/\rho_0 - 24166.6 \rho^2/\rho_0^2) \text{ MeV}^2 \\ b(\rho) &= (-1060.05 \rho/\rho_0 - 326.803 \rho^2/\rho_0^2) \text{ MeV} \\ c(\rho) &= -13.2403 \rho/\rho_0 - 0.154177 \rho^2/\rho_0^2 \\ d(\rho) &= (-0.0701901 \rho/\rho_0 + 0.0173533 \rho^2/\rho_0^2) \text{ MeV}^{-1} \\ e(\rho) &= (-43620.9 \rho/\rho_0 + 11408.4 \rho^2/\rho_0^2) \text{ MeV}^2 \\ f(\rho) &= (-1441.14 \rho/\rho_0 + 511.247 \rho^2/\rho_0^2) \text{ MeV} \\ g(\rho) &= -27.6865 \rho/\rho_0 + 10.0433 \rho^2/\rho_0^2 \\ h(\rho) &= (-0.221282 \rho/\rho_0 + 0.0840531 \rho^2/\rho_0^2) \text{ MeV}^{-1} \quad . \end{aligned} \quad (4)$$

This potential is evaluated in infinite nuclear matter. In finite nuclei we use the local density approximation, substituting ρ by $\rho(r)$, the local density at each point in the nucleus which we take from experiment. For the s -wave that we use here, it was shown in Ref. [27] that the local density approximation (LDA) gave the same results as a direct finite nucleus calculation.

In the next section we solve the Klein-Gordon equation with the two potentials: *i*) the energy dependent one, defined in Eqs. (3) and (4), and *ii*) an energy independent potential obtained from the latter one by taking $k^0 = m_\eta$ in Eq. (3). Finally, we discuss the implications of our results in the practical search for these η bound states.

3 Results

To compute de η -nucleus bound states, we solve the Klein-Gordon equation (KGE) with the strong LDA η -selfenergy, $\Pi_\eta(k^0, r) \equiv \Pi_\eta(k^0, \vec{0}, \rho(r))$. We have then:

$$\left[-\vec{\nabla}^2 + \mu^2 + \Pi_\eta(\text{Re}[E], r) \right] \Psi = E^2 \Psi \quad (5)$$

where μ is the η -nucleus reduced mass, the real part of E is the total meson energy, including its mass, and the imaginary part of E , with opposite sign, is the half-width $\Gamma/2$ of the state. The binding energy $B < 0$ is defined as $B = \text{Re}[E] - m_\eta$. As mentioned above two different η -selfenergies are being considered.

In order to solve the KGE in coordinate space we use a numerical algorithm which has been extensively tested in the similar problems of pionic [27] and antikaonic [28] atomic states and in the search of possible antikaon-nucleus bound states [28]. Charge densities are taken from Ref. [29]. For each nucleus, we take the neutron matter density approximately equal to the charge one, though we consider small changes, inspired by Hartree-Fock calculations with the DME (density-matrix expansion) [30] and corroborated by pionic atom analysis [31]. In Table 1 of Ref. [28] all the densities used throughout this work can be found. However, charge (neutron matter) densities do not correspond to proton (neutron) ones because of the finite size of the proton (neutron). We take that into account following the lines of Ref. [27] and use the proton (neutron) densities in our approach.

Table 1: $(B, -\Gamma/2)$ for η -nucleus bound states calculated with the energy dependent potential.

	^{12}C	^{24}Mg	^{27}Al	^{28}Si	^{40}Ca	^{208}Pb
1s	(-9.71, -17.5)	(-12.57, -16.7)	(-16.65, -17.98)	(-16.78, -17.93)	(-17.88, -17.19)	(-21.25, -15.88)
1p			(-2.90, -20.47)	(-3.32, -20.35)	(-7.04, -19.30)	(-17.19, -16.58)
1d						(-12.29, -17.74)
2s						(-10.43, -17.99)
1f						(-6.64, -19.59)
2p						(-3.79, -19.99)
1g						(-0.33, -22.45)

Table 2: $(B, -\Gamma/2)$ for η -nucleus bound states calculated with the energy independent potential.

	^{12}C	^{24}Mg	^{27}Al	^{28}Si	^{40}Ca	^{208}Pb
1s	(-17.71, -25.42)	(-22.69, -25.78)	(-33.80, -30.63)	(-34.01, -30.36)	(-35.42, -30.12)	(-39.71, -28.65)
1p			(-5.28, -23.20)	(-6.07, -23.45)	(-13.02, -25.19)	(-31.97, -27.61)
1d						(-22.69, -26.30)
2s						(-19.11, -25.55)
1f						(-12.16, -24.69)
2p						(-6.81, -23.12)
1g						(-0.60, -22.74)

Our results are shown in Tables 1 and 2 for the energy dependent and energy independent potentials respectively. We also show them in graphic form in Fig. 2. As an average we can see that the binding energies are about double with the energy independent potential than with the energy dependent one, which shows the importance of taking into account this effect. On the other hand we see that the half widths of the states are large, larger in fact than the binding energies or the separation energies between neighboring states. The widths are also smaller with the energy dependent potential, but comparatively to the size of the binding energies they are a little bit larger.

With the results obtained here it looks like the chances to see distinct peaks corresponding to η bound states are not too big. However, the systematics carried out with the study of different nuclei is instructive. Indeed, we see that the tendency for nuclei lighter than ^{24}Mg is that the binding energy becomes smaller but the half width stays comparatively more stable, such that the ratio of the width to binding energy becomes bigger for lighter nuclei and hence the chances to see these bound states become smaller. This tendency seems to be telling us that the light nuclei is definitively not the place where one should search for η bound states. On the other hand, if we go to heavy nuclei, like Pb, there is a superposition of many bound states and the separation of the levels is small compared to the half width of these states. The chances to see peaks there are null. On the other hand there is a region, which, even with difficulty, still provides the optimum ground to see the η bound states. This appears around the Mg region where the binding energy is similar to the half width and there is only one bound state. If one goes to heavier nuclei the ratio of binding energy versus width become bigger, a welcome feature for the observation of the bound states, but simultaneously there appear new bound states, as one can see in ^{27}Al , such that

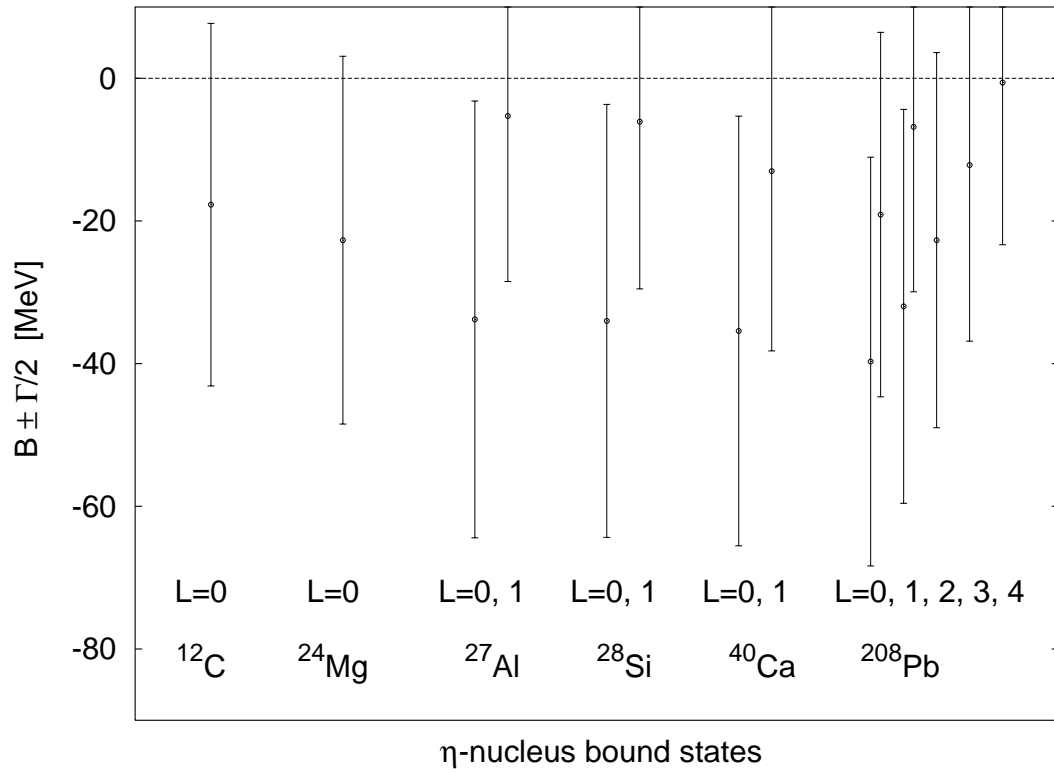
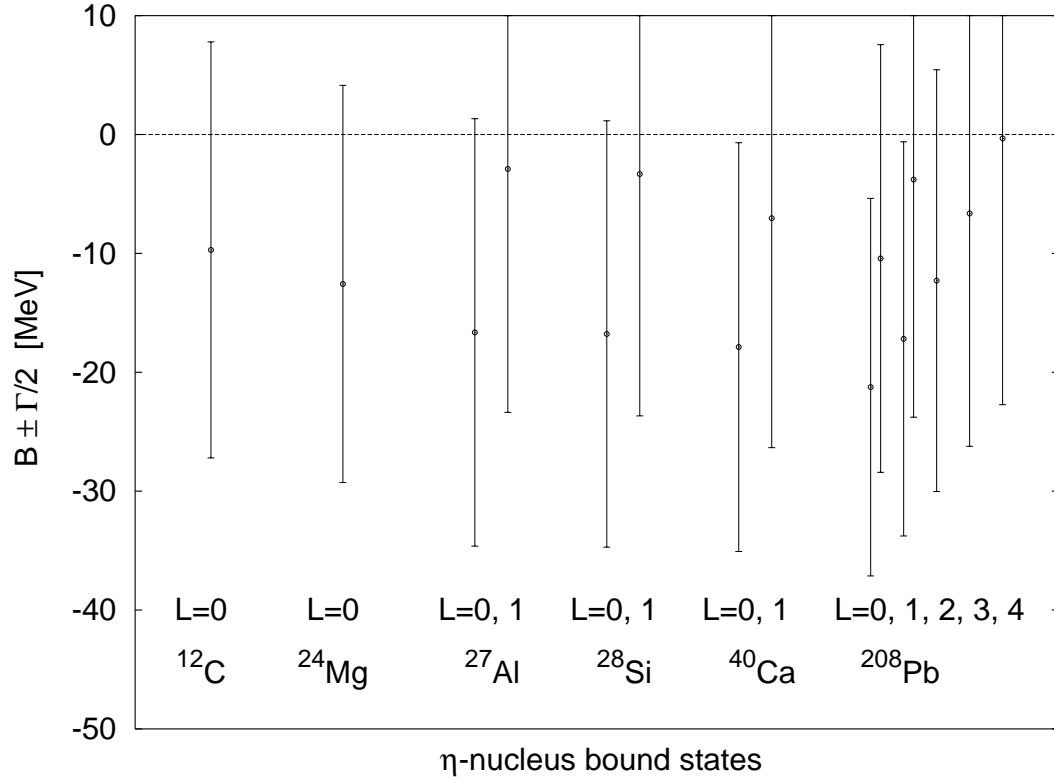


Figure 2: Top panel: Binding energies and widths for different nuclei obtained using the strong energy dependent η -selfenergy *i*). Bottom panel: Results obtained with the energy independent η -selfenergy *ii*).

the half width of the states is much bigger than the separation between the levels. With all these considerations one comes to the conclusion that the region of nuclei around ^{24}Mg would offer the best chances to see the η bound states. In this case, with a binding energy of around 12.5 MeV and a half width of 16.7 MeV one could see still some broad bump.

On the other hand one can look at the results with a more optimistic view if one simply takes into account that experiments searching for these states might not see them as peaks, but they should see some clear strength below threshold in the η production experiments. The range by which this strength would go into the bound region would measure the combination of half width and binding energy. Even if this is less information than the values of the energy and width of the states, it is by all means a relevant information to gain some knowledge on the η nucleus optical potential.

4 Conclusions

We have used a recent η nucleus optical potential, evaluated within a unitarized chiral perturbative approach, in order to find η bound states. The potential is attractive and produces bound states for all nuclei which we study from ^{12}C up. On the other hand it also produces large widths, with the half widths slightly larger than the binding energy, which makes the observation of these states unlikely. We calculated the results using an energy independent potential, which is obtained taking the value of the potential for an η energy equal to its mass. The second, more realistic potential, takes into account the strong energy dependence of the potential due to the fact that it is much influenced by the $N^*(1535)$ resonance appearing above the ηN threshold. This potential reduces the strength of both the real and imaginary parts of the potential for energies below the η threshold and leads to substantially narrower states, but at the same time with smaller binding energies, such that the half widths of the states are still larger than the binding energies. We found that light nuclei are not the best ones to search for η bound states since the widths become comparatively much larger than the binding energies. The heavy nuclei accommodate many bound states and the separation of the levels is much smaller than the half width of the states. The best chances for observation of bound states are in the region of ^{24}Mg where there is only one bound state and the half width is only a little bigger than the binding energy. In any case it was stressed that, even if no broad bumps are found in the experiments, they should find some strength in the bound region stretching in energy down to the sum of the binding energy plus the half width of the bound states. Short of having the values for the binding energy and width of the states, this more limited information is still very valuable to gain some knowledge on the η nucleus optical potential and it should stimulate experiments in this direction.

Acknowledgments

This work has been partly supported by the Spanish Ministry of Education in the program “Estancias de Doctores y Tecnólogos Extranjeros en España”, by the DGICYT contract number BFM2000-1326, the DGES contract number PB98-1367, the EU TMR network Eurodaphne, contact no. ERBFMRX-CT98-0169, and by the Junta de Andalucía.

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