

Chiral restoration from pionic atoms?

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July 26, 2013

Abstract

We evaluate widths and shifts of pionic atoms using a theoretical microscopical potential in which the pion decay constant f_π is changed by an in-medium density dependent one ($f_\pi(\rho)$), predicted by different partial Chiral restoration calculations. We show that the results obtained for shifts and widths are worse than if this modification were not implemented. On the other hand, we argue that in microscopic many body approaches for the pion selfenergy, based on effective Lagrangians, the mechanisms responsible for the change of f_π in the medium should be automatically incorporated. Therefore, the replacement of f_π by $f_\pi(\rho)$ in the many body derivation of the microscopic potential would be inappropriate.

1 Introduction

The value of the quark condensate $\langle \bar{q}q \rangle$ plays an important role in chiral dynamics [1]. In the presence of a nuclear medium the value of the condensate drops as a function of the density and the linear terms in the nuclear density ρ have been derived with different formalisms. In Refs. [2,3] the Hellmann–Feynman theorem is used, a mean field approach is used in Refs. [4,5] and the Nambu–Jona–Lasinio model is used in Refs. [6–9]. The different formalisms lead to identical results in the terms linear in the baryon density, ρ , giving at zero temperature

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_{\rho=0}} = 1 - \frac{\sigma_N}{m_\pi^2 f_\pi^2} \rho + \dots \quad (1)$$

where σ_N is the pion-nucleon sigma term.

Assuming that the Gellmann–Oakes–Renner (GOR) relationship holds, at zero temperature, for finite baryon density [5], [10], [11], one gets for the in-medium pion decay

constant, $f_\pi(\rho)$, defined from the time component of the axial current¹,

$$f_\pi^2(\rho) = -\frac{m_q}{m_\pi^{*2}} \langle \bar{q}q \rangle_\rho + \dots \quad (2)$$

to leading order in the average quark mass $m_q = \frac{1}{2}(m_u + m_d)$, where $\langle \bar{q}q \rangle_\rho$ now stands for the ρ -dependent condensate $\langle \bar{u}u + \bar{d}d \rangle_\rho$. Besides m_π^* stands for the pion mass in the medium. Thus, the dropping of the condensate for finite densities is interpreted as a dropping of the pion decay constant $f_\pi(\rho)$, leading to the phenomenon of partial chiral restoration [7] in the nuclear medium.

This dropping of the pion decay constant was used in Ref. [12] to suggest that it could solve the long standing puzzle of the missing repulsion in the s -wave pion selfenergy: it might account for the discrepancy between theoretical predictions for that part of the pion optical potential and the strength demanded by fits to pionic atoms data [13]. The idea behind Ref. [12] is that the isovector πN scattering length, b_1 , involving the factor f_π^{-2} , will be enhanced in the nucleus², renormalizing both the isovector part of the s -wave pion selfenergy, proportional to b_1 , as well as the isoscalar part from the Pauli blocking rescattering correction, [15], which is proportional to $b_0^2 + 2b_1^2$.

A more detailed study was done recently in Ref. [16] where conducting fits to the pionic atoms data and re-scaling the b_1 free parameter by the ratio $(f_\pi/f_\pi(\rho))^2$, an improved agreement with the data was obtained. The agreement became excellent, without invoking any other extra repulsion, when the relativistic corrections of Ref. [17] were also included. However, the relativistic corrections of Ref. [17] were found to be ambiguous in Ref. [18] and another study [19] showed that they were a consequence of approximations which broke the exact cancellation of some large terms, and no correction was found when the exact calculation was done. Leaving apart these relativistic corrections, the fact still would remain that the renormalization of b_1 would generate a considerable part of the missing repulsion, leading the author of Ref. [16] to claim that the data of pionic atoms offered an evidence of partial chiral restoration in nuclei.

The problem is more subtle than just replacing the value of b_1 . Actually, changing the s -wave real part of the optical potential is justified if one might argue that the replacement of f_π by $f_\pi(\rho)$ is done at the level of the in-medium Chiral Lagrangian. But, then the changes in the f_π parameter should be implemented in a theoretical calculation wherever this parameter appears. This reminds us that this parameter actually appears not only in the $b_0\rho$, $b_1\delta\rho$ and the Pauli blocking rescattering terms mentioned in Ref. [12], but in all terms of the optical potential, including the p -wave part of the potential, and more importantly, all the absorption terms³, which go like f_π^{-6} . We will show that this scenario

¹Note that there exists a breaking of covariance because of the nuclear medium which leads to different renormalization of the space and time components of the axial vector current.

²Similar effect will also occur for the isoscalar πN scattering length, b_0 , but in Ref. [12] such an effect is neglected because of the very small value of b_0 deduced from pionic hydrogen and deuterium [14].

³There is also another point worth mentioning since in the pion absorption terms [20–22] the πN amplitude appears half off-shell, with the off-shell pion with an energy of $m_\pi/2$ and a momentum of $\sqrt{m_\pi M_N} = 360$ MeV approximately. Although we do not expect drastic changes with this moderate off-

is strongly disfavoured by the pionic atom data. These findings are corroborated by the recent works of Ref. [24], where a new mode of chiral restoration is suggested in which the longitudinal ρ would be the chiral partner of the pion and both would become massless in the limit of chiral symmetry restoration. These works show that the parameter f_π appearing in Eq. (2) and the one that appears in the chiral Lagrangians when performing perturbation theory calculations with them, let us call it \hat{f}_π , are not the same object. They would be the same at tree level, but, as soon as perturbation theory is performed, \hat{f}_π becomes explicitly dependent on the scale of renormalization and is not equal to zero even in the limit of chiral restoration where f_π appearing in Eq. (2) would vanish. This line of thought about the inadequacy of using the constant f_π that one induces from the GOR relation as the coupling constant in a perturbative approach, is in accordance with our ideas expressed below, where we give different arguments on why the identification of these two objects in a many body microscopic calculation would lead to doublecounting.

In the present work we shall explore the consequences of replacing f_π by $f_\pi(\rho)$ in all the terms where it appears in a theoretical evaluation of the pion selfenergy⁴. For the purpose of completeness we shall also investigate what happens in the case where the modifications are done in the s -wave but not in the p -wave parts. However, while the idea of changing f_π by its medium value, everywhere that it appears, seems reasonable with the caveats pointed above, it is also true that one has to look in detail in the elements of the theoretical derivation of the potential to avoid double-counting in the case that the many body approach already contains the renormalization mechanisms that would lead to the quenching of the f_π parameter⁵.

The point we would like to make here is that while in the chiral studies the interpretation of the renormalization of the axial vector current as a change of the pion decay constant in the medium is a valid option, the standard many body approach in which the currents are renormalized using effective Lagrangians and the pions are renormalized in the medium using the same Lagrangians is also a valid option, but then one cannot reinterpret the renormalization of the axial vector current in terms of a change of the pion decay constant,

shellness, the renormalization of b_1 in this amplitude could be different than that for on-shell pions [23]. However, this problem also appears in the Pauli corrected rescattering term mentioned above, which provides the main source of s -wave repulsion when the renormalization of b_1 is done. Indeed, the derivation of this term in a many body framework was done in [21], Fig. 22 and Eq. (A7), and it was shown to come from rescattering terms implicit in a Lippmann Schwinger equation when the Pauli blocking in the intermediate nucleon states was considered. In this rescattering term one can see (Eq. (A7) of [21]) that the term also involves the half off-shell πN amplitude with the off-shell pion with a four-momentum (q^0, q) such that $m_\pi - E_F < q^0 < m_\pi + E_F$ and $0 < q < 2k_F$, where E_F and k_F are the Fermi energy and momentum respectively. One can thus see that the level of off-shellness is similar in the rescattering term and in the absorption terms.

⁴ At this point, it is of interest to remind that the derivation of the renormalization of f_π is linked to the renormalization of the axial vector current. The links to the renormalization of f_π appearing in the s - and p -waves of the πN scattering amplitudes are not so clear from the way the derivation of $f_\pi(\rho)$ is done, and does not have to be the same for s - or p -waves [23].

⁵ It is worth mentioning that studies of the renormalization of the axial current in nuclei, using standard many body theory with effective Lagrangians, have been done both for the space component [25, 26] as well as for the time component [27, 28].

recast the terms obtained for the pion selfenergy in terms of this decay constant and change it in all terms where it appears. A consistent many body approach using effective Lagrangians, as done for instance in [20–22, 29, 30] would be a valid approach by itself.

In this sense it is also interesting to mention that in the recent study of Ref. [11] the corrections to the space part of the pion decay constant were related to Δ excitations induced from the axial current, thus connecting with the findings of Refs. [25, 26] where a conventional many body expansion, including nucleon and isobar degrees of freedom, was done. These are also the degrees of freedom used in [20, 21] to evaluate the pion selfenergy.

All this said, the purpose of the present paper is to investigate, in the line of Ref. [16], what would happen if we ignore the points discussed above and simply change f_π by $f_\pi(\rho)$ in all the terms of the microscopic many body calculation of Ref. [20]. The exercise is illustrative because although there might seem that pionic atoms are just governed mostly by the s -wave optical potential, this is not actually the case and the p -wave potential, as well as the absorption terms, responsible for the width of the pionic states, play also a very important role [31].

2 Results

We analyze in this section, the partial chiral restoration effect on the theoretical description of the pionic atom data. We base this study on our previous and detailed work of Ref. [20] and use the same set of experimental pionic shifts and widths as in this reference. There is a total of 61 piece of data. We present in Table 1 the values of χ^2/N , where $N = 61$ is the number of data, obtained with two different optical potentials (**II** and **III**) modified to somehow account for the change of the pion decay constant in the medium, together with the value obtained with the theoretical potential (**I**) developed in Ref. [20].

More specifically, potential **I** in Table 1 corresponds to the potential TH of Ref. [20], defined in Eqs. (20-30) and (34-36) of that reference for the p - and s -wave parts of the optical potential. For the s -wave term the TH potential uses

$$b_0 = -0.013m_\pi^{-1} \quad b_1 = -0.092m_\pi^{-1} \quad \text{Im}B_0 = 0.041m_\pi^{-4} \quad (3)$$

where b_0 and b_1 were taken from the experimental analysis of Ref. [32] and $\text{Im}B_0$ computed in Ref. [21]. The potential (TH) has been developed microscopically and it contains the ordinary lowest order optical potential pieces constructed from the s - and p -wave πN amplitudes. In addition second order terms in both s - and p -waves, responsible for pion absorption, are also considered. Standard corrections, as second-order Pauli re-scattering term, ATT term, Lorentz-Lorenz effect and long and short range nuclear correlations, are also taken into account. This theoretical potential reproduces fairly well the data of pionic atoms (binding energies and strong absorption widths) [20] and low energy π -nucleus scattering [33].

In the potential **II** we take for the p -wave part that of potential **I**, but the s -wave part is modified following the prescription of Ref. [12]. Thus, we replace b_0, b_1 and $\text{Im}B_0$

Potential	χ^2/N	$\chi_\epsilon^2/N_\epsilon$	χ_Γ^2/N_Γ
I	5	5	5
II	47	64	30
III	67	16	112

Table 1: Second column: values for $\chi^2/(\text{num.data} = 61)$ obtained for the potentials **I**, **II** and **III** defined in the text, when the set of experimental pionic shifts and widths, used in Ref. [20] are considered. Third and fourth columns: Shift ($\chi_\epsilon^2 = \sum_\epsilon [(\epsilon^{\text{pot}} - \epsilon^{\text{exp}})/\delta\epsilon^{\text{exp}}]^2$) and width ($\chi_\Gamma^2 = \sum_\Gamma [(\Gamma^{\text{pot}} - \Gamma^{\text{exp}})/\delta\Gamma^{\text{exp}}]^2$) contributions to $\chi^2 (= \chi_\epsilon^2 + \chi_\Gamma^2)$, divided by the number of shifts ($N_\epsilon = 29$) and widths ($N_\Gamma = 32$). The s -wave part of the optical potentials uses the parameters given in Eq. (3).

by $F_\chi \times b_0$, $F_\chi \times b_1$ and $F_\chi^3 \times \text{Im}B_0$ respectively, where F_χ can be deduced from Eqs. (1) and (2),

$$F_\chi = F_\chi(\rho) = \left(\frac{f_\pi}{f_\pi(\rho)} \right)^2 \approx \frac{1}{1 - \sigma_N \rho / (f_\pi^2 m_\pi^2)} = \frac{1}{1 - 2.3 \text{fm}^3 \rho} \quad (4)$$

for $\sigma_N = 50$ MeV and neglecting the in-medium pion mass change because of its Goldstone boson nature. Note that $F_\chi(\rho)$ depends on the spatial coordinate \vec{r} in the local density approach which we use for the optical potential. In potential **III** we modified both s - and p -wave parts of the pion-nucleus optical potential. Thus, in addition to the changes mentioned above for the s -wave, in the computation of the p -wave part we have replaced the πNN and $\pi N\Delta$ coupling constants, f and f^* in the notation of Ref. [20], by $\sqrt{F_\chi} \times f$ and $\sqrt{F_\chi} \times f^*$ respectively⁶. Note that, because of the non-local nature of the p -wave part, first and second order derivatives of $F_\chi(\rho)$ are needed. Neglecting those derivatives lead to negative widths of the pionic atoms.

The results of Table 1 clearly contradict the expectations of Ref. [12], corroborated in Ref. [16], of finding a signal of partial chiral restoration in nuclei from pionic atom data.

To better understand the results presented in Table 1, one should bear in mind that the real part of the s - and p -wave contributions to the pion-nucleon optical potential are repulsive and attractive, respectively, and the fact that F_χ is a number greater than one for all densities, i.e., the effect of the partial chiral restoration is to increase, in absolute value, the size of each part (s - and p -waves) of the potential. Thus, the potential **II** is more repulsive than the potential **I**. The main source of the enhancement of the repulsion in the s -wave part of the potential is through the increase of the Pauli blocking rescattering term, which is of isoscalar nature and goes, as quoted above, as $b_0^2 + 2b_1^2$. Besides, the imaginary part of the potential **II** is substantially bigger in absolute value than that of

⁶For the case of the in medium Δ -selfenergy, Σ_Δ , for which no explicit expression in terms of the f and f^* coupling constants is given in Ref. [20], we have corrected only the leading ρ behaviour, this is to say, we have multiplied Σ_Δ by F_χ^2 . Thus, for the imaginary part, we have not modified the saturation coefficient appearing in the argument of the arc-tan parametrization of Eq. (11) of [20].

the potential **I**, since it gets multiplied by a factor F_χ^3 . Thus, we see in the table that the shifts of potential **II** are clearly worse than those obtained with the potential **I**. The same occurs for the widths, those of potential **II** are bigger than the experimental ones, but the effect is not as drastic as one might expect since the potential **II** is more repulsive than the potential **I** and thus the effective densities seen by the pion for the potential **II** case are smaller than those relevant when the interaction **I** is used. When the partial chiral restoration effects are also incorporated to the p -wave, potential **III**, the shifts are improved respect to the potential **II** case. This is because there is a cancellation between the increase of repulsion and attraction generated by the inclusion of the chiral effects in each wave of the optical potential. However, the potential **III** has an imaginary part too big, a fact which is clearly appreciated in the table ($\chi_\Gamma^2/N_\Gamma = 112$).

However, a word of caution must be said now. The recent determination of the isovector and isoscalar πN scattering lengths, b_1 and b_0 , in Ref. [14]

$$b_0 = -0.0001_{-0.0021}^{+0.0009} m_\pi^{-1} \quad b_1 = -0.0885_{-0.0021}^{+0.0010} m_\pi^{-1} \quad (5)$$

is incompatible, specially for b_0 , with that of Ref. [32] quoted in Eq. (3). When one uses the central values of b_1 and b_0 given above, to re-compute $\text{Im}B_0$, following the lines of Ref. [21], one gets

$$\text{Im}B_0 = 0.0345 m_\pi^{-4} \quad (6)$$

These new values (Eqs. (5) and (6)) for the s -wave part of the optical potential lead to the results presented in Tables 2 and 3. Neither potential **I** nor potentials **II** and **III** provide an acceptable description of the data. For the interaction **I** case, the main effect is the important reduction of the s -wave repulsion which leads to a poorer description of binding energies and to greater effective densities felt by the pion than in the case presented in Table 1. This latter effect increases, in absolute value, the imaginary part of the optical potential. On the other hand the smaller value of the parameter $\text{Im}B_0$ in Eq. (6) than in Eq. (3) reduces, in absolute value, the imaginary part. The total effect depends on the pionic level (nl and nucleus) considered. Partial chiral restoration effects incorporated in potential **II** increase the repulsion and lead to an acceptable description of the shifts, but the increase in the imaginary part, in absolute value, produces an unacceptable description of the widths. The results presented in Ref. [16] correspond to a situation like that of the potential **II** in Table 2 but where the imaginary part of the s -wave optical potential is not being affected by this partial chiral restoration. While this could be an acceptable procedure from the fitting point of view, is not satisfactory from a theoretical or microscopical point of view. In any case, we have simulated this scenery by the potential **II***, which only scale by the chiral factor F_χ the dispersive real part of the s -wave potential. This potential **II*** is the one providing the best description without any fitted parameter, as can be seen in Table 2, with also a large value of $\chi^2/N=12$. Finally, potential **III** provides really poor results: imaginary parts, in absolute values, are too big while the potential turns out to be too attractive, once the partial chiral restoration enhancement of the p -wave potential is also considered. We consider adding a phenomenological fitted part to the previous potentials **I** and **III** as done in [20]. The results obtained

Potential	χ^2/N	$\chi_\epsilon^2/N_\epsilon$	χ_Γ^2/N_Γ
I	84	158	16
II	47	4	86
II*	12	16	8
III	207	200	215

Table 2: Same as in Table 1, but for the s -wave part of the optical potentials, the parameters given in Eqs. (5) and (6) have been used. The potential **II*** is like the potential **II** (the s -wave coefficients b_0 and b_1 are replaced by $F_\chi \times b_0$ and $F_\chi \times b_1$), except that the s -wave absorptive part, $\text{Im}B_0$, is kept unchanged as in the potential **I**.

Potential	χ^2/N	$\chi_\epsilon^2/N_\epsilon$	χ_Γ^2/N_Γ
I ^{fit}	1.8	0.8	2.8
III ^{fit}	3.1	1.6	4.4

Table 3: Results from potentials **I**^{fit} and **III**^{fit} obtained from a best fit to the data after adding phenomenological terms to potentials **I** and **III** of table 2.

are shown in rows **I**^{fit} and **III**^{fit} of Table 3. For potential **I**^{fit}, the fitted parameters are $\delta b_0 = -0.0207(10) m_\pi^{-1}$, $\delta b_1 = -0.0163(60) m_\pi^{-1}$, $\delta \text{Im}B_0 = 0.0152(17) m_\pi^{-4}$, $\delta c_0 = 0.045(10) m_\pi^{-3}$, $\delta c_1 = 0.092(51) m_\pi^{-3}$, $\delta \text{Im}C_0 = 0.128(23) m_\pi^{-6}$. For potential **III**^{fit}, the fitted parameters are $\delta b_0 = -0.0235(12) m_\pi^{-1}$, $\delta b_1 = 0.0201(64) m_\pi^{-1}$, $\delta \text{Im}B_0 = -0.0455(24) m_\pi^{-4}$, $\delta c_0 = -0.078(14) m_\pi^{-3}$, $\delta c_1 = -0.040(56) m_\pi^{-3}$, $\delta \text{Im}C_0 = -0.212(36) m_\pi^{-6}$.

The results of the fits are instructive. In both cases we can see that there is a need for extra s -wave repulsion in the demanded value of δb_0 , similar in both cases, and around a value 50 percent bigger than the repulsion provided by the old value of b_0 , Eq. (3). On the other hand, while from the fit **I**^{fit} the data demand a value for b_1 about 16 percent bigger in size than the free value, the results of **III**^{fit} are telling us that the renormalization of f_π provides an effective value of b_1 about 20 percent larger than demanded by the data. The effects on $\text{Im}B_0$ are more striking. While the data would demand a value about 30 percent bigger than the one provided by the theoretical potential **I**, the effectively renormalized value of $\text{Im}B_0$ in potential **III** requires a reduction three times bigger. These results are thus telling us again that largely renormalized values of b_1 are not welcome by the data. Particularly, the values of $\text{Im}B_0$ that are obtained after multiplying by F_χ^3 the theoretical value of $\text{Im}B_0$ are far too large.

3 Conclusions

The idea expressed here is that, without questioning the dropping of the pion decay constant in a nuclear medium, a systematic many body expansion using effective Lagrangians,

consistent with present knowledge of χPT and incorporating explicitly the elements which go into the counter-terms of χPT , (for instance Δh excitations), already incorporates the mechanisms responsible for the dropping of f_π in the medium and, hence, this explicit renormalization should not be considered in addition to avoid double-counting.

It should also be stated that efforts were made to solve the puzzle of the missing repulsion and several small corrections were found to the s -wave selfenergy, among them corrections of second order in the density from dispersion corrections linked to the s -wave pion absorption [21], from the pion scattering with the virtual pion cloud in the nucleus [30, 34–36] and from a Lorentz- Lorenz correction to the s -wave rescattering terms [13], such that taking into account all the different effects and their errors there was only moderate room for extra s -wave repulsion [13]. Of course, the new data for the b_0 parameter have reduced a source of s -wave repulsion provided by the old data of [32] and have worsened the problem, such that we can say that with the values of the scattering lengths provided by the recent data of pionic hydrogen [14] the problem of the missing repulsion in pionic atoms would be reopened.

The fits conducted, before and after the renormalization of f_π is done, are also telling us that the strong renormalization of b_1 provided by this idea is not welcome by the data. Particularly the value for ImB_0 which is multiplied by F_χ^3 .

All together the conclusions of this paper seem clear: First, that more theoretical work needs to be done to understand the meaning of the dropping of the pion decay constant and how it could have a repercussion in the renormalization of the πN , πNN , etc., amplitudes in the nuclear medium within a determined many body scheme. And second, that it is quite important to settle the question of the precise values of the πN scattering lengths, so that the present discrepancy between the results from pionic hydrogen and from scattering data, reminiscent of the one that remained for long in the K^-p problem, and which was finally settled in [37], should be resolved. In this respect it is interesting to mention that efforts in this direction are presently underway. As an example the recent paper [38], which makes a combined analysis of pionic hydrogen and deuterium data, advocates for values of the b_0 parameter of the order of $-0.004 m_\pi^{-1}$, which are much bigger in strength than the value quoted here in Eq. (5). Should however results of the order of those used in Eq. (5) prevail, then it is clear that there is an important missing s -wave repulsion which again will require renewed theoretical efforts to be understood.

Acknowledgments

We would like to acknowledge some useful discussions with J.A. Oller and useful comments from W. Weise and E. Friedman. This work is partly supported by DGICYT contract number BFM2000-1326, by DGES contract number PB98-1367, by the Junta de Andalucia and by the EU TMR network Eurodaphne, contract no. ERBFMRX-CT98-0169.

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