

# Renormalization of the $^1S_0$ One-Pion-Exchange NN Interaction in Presence of Derivative Contact Interactions

J. Nieves<sup>1</sup>

<sup>1</sup>*Departamento de Física Moderna, Universidad de Granada, E-18071 Granada, Spain.*

We use standard distorted wave theory techniques and dimensional regularization to find out solutions of the nucleon-nucleon Lippman-Schwinger equation with a potential that includes one-pion exchange and additional contact terms with derivatives. Though for simplicity, we restrict the discussion to the  $^1S_0$  channel and to contact terms containing up to two derivatives, the generalization to higher waves and/or number of derivatives is straightforward. The undetermined low energy constants emerging out of the renormalization procedure are fitted to data.

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## I. INTRODUCTION

Over the last years, Effective Field Theory (EFT) methods have become the standard tool to deal with strong interactions in the non-perturbative regime. In the form of Chiral Perturbation Theory (ChPT), they have been used with some success in both the mesonic and single nucleon sectors. The EFT of nuclear forces based on a chiral expansion was originally suggested by Weinberg [1], and since then a lot of work has been devoted to gain a better understanding of the two-nucleon interaction at low and intermediate energies [2]–[17]. While at the beginning, these studies did not aim at substituting the highly successful *realistic* potentials built from meson exchanges (Bonn-Jülich, Nijmegen, Argonne,  $\dots$  potentials), the importance of uncovering such an EFT cannot be ignored, since this theory will allow for rigorous calculations of both elastic and inelastic processes in systems with two or more nucleons, in a framework consistent with the Standard Model of strong and electroweak interactions. Furthermore, the latter EFT works provide an accurate description of nucleon-nucleon (NN) phase-shifts for several partial waves and in a wide range of energies [16]. Weinberg's original proposal was to determine the NN potentials using the organizational principles of ChPT and then to insert these potentials into the Lippman-Schwinger Equation (LSE) to solve for NN scattering amplitudes. Though such a scheme has proved to be successful [2], [3], [7], it suffers from formal inconsistencies, in particular, divergences that arise at a given order in the chiral expansion cannot be absorbed by terms of the same order in the Lagrangian [4],[5]. The trouble arises, since Weinberg's power counting assumes that the coefficients of the higher order operators were set by a *typical* Quantum Chromo Dynamics (QCD) scale, but in practice it turns out that there appear explicit  $M$  factors (being  $M$  the nucleon mass) in the needed counter-terms to renormalize the amplitudes, which formally decrease the degree of the operator in the Weinberg's counting. This was partially solved by Kaplan, Savage and Wise (KSW) who introduced a new scheme where pions are treated perturbatively [5]. However, it turns out that the KSW expansion converges slowly in the  $^1S_0$  ( $^{2I+1}L_J$ ) channel and does not converge at all in the  $^3S_1$ – $^3D_1$  coupled channels [6], [10]. Very recently, a so called *expansion about the chiral limit* has been proposed in Ref. [13], which seems to be equivalent to KSW power counting in the  $^1S_0$  channel and to Weinberg power counting in the  $^3S_1$ – $^3D_1$  coupled channels.

In this work we adopt the following point of view. For a scattering process involving external momenta  $< q$ , one should only consider a Lagrangian/potential which explicitly includes light degrees of freedom for which  $m < q$ . The effects of heavy virtual particles appear as an infinite number of non-renormalizable operators suppressed by some mass scale relevant to the degrees of freedom excluded from the theory. At low-intermediate momenta, below twice the pion mass, it should be enough to include explicitly one-pion-exchange, and simulate the rest of the physical contributions by a tower on non-renormalizable contact interactions, organized as a derivative expansion. Given a potential, which includes contact terms up to some order, the LSE performs an infinite sum of diagrams, which in general would require to be renormalized. After renormalization the coefficients of the contact terms would contain, in general, contributions from all orders in  $m_\pi$  (pion mass).

Most of the formal problems appearing within the Weinberg's scheme are linked to the use of a somehow restrictive scheme to renormalize the LSE [12]–[14]. Such a restricted conception of the renormalization of the LSE would also lead to unexpected consequences in other scenarios, as for example  $\pi\pi$  scattering in the  $\rho$ -channel [18]. Among the abundant literature on the subject, the framework presented in Ref. [8] constitutes a first step towards the renormalization scheme proposed in this work. There, a subtracted LSE is derived, and the numerical values of the fitted Low Energies Constants (LEC's) are not translated to values for any Ultraviolet (UV) cutoff.

The aim of this paper is to present a renormalization scheme of the LSE for NN scattering by a combination of known long-range and unknown short-range potentials. For simplicity, we restrict the discussion to the  $^1S_0$  channel, where

we show that the proposed scheme describes phase-shifts up to Center of Mass (CM) nucleon momenta of the order of 260 MeV. This renormalization scheme is based on previous findings on  $\pi\pi$  [18] and meson-baryon scattering [19]–[22], it leads to renormalized amplitudes which fulfill exact elastic unitarity, and it is easily generalized to study higher waves and/or number of derivatives in the contact part of the interaction.

## II. EFFECTIVE POTENTIALS AND THE LSE

After projecting into the  $^1S_0$  partial wave, the NN LSE for a CM nucleon kinetic energy  $E$ , reads:

$$T(E; p, p') = V(p, p') + \int_0^{+\infty} dq q^2 \frac{V(p, q)T(E; q, p')}{2mE - q^2 + i\epsilon} \quad (1)$$

with  $m = 469.46$  MeV the NN reduced mass, for which we take that of the neutron-proton system,  $p$  and  $p'$  are the initial and final relative momenta of the two nucleons, and  $V(p, p')$  the  $^1S_0$  NN potential. The normalization of the scattering amplitude  $T(E; p, p')$  is such that on the mass shell,  $p = p' = \sqrt{2mE} \equiv k$ , it is related to the phase shifts,  $\delta$ , by:

$$T(k) = -\frac{2}{\pi} \frac{e^{2i\delta(k)} - 1}{2ik} \quad (2)$$

The potential consists of the one-pion-exchange potential,  $V_\pi$ , plus contact terms,  $V_s$ , with up to two derivatives [4]:

$$\begin{aligned} V(p, p') &= V_s(p, p') + V_\pi(p, p') \\ V_s(p, p') &= g_0 + g_1(p^2 + p'^2) \\ V_\pi(p, p') &= -\frac{2m\alpha_\pi}{\pi} \int_{-1}^{+1} \frac{dx}{p^2 + p'^2 - 2pp'x + m_\pi^2} = \frac{m\alpha_\pi}{\pi} \frac{1}{pp'} \log \frac{p_-^2 + m_\pi^2}{p_+^2 + m_\pi^2}, \quad \text{with } \alpha_\pi = \frac{g_A^2 m_\pi^2}{16\pi f_\pi^2} \end{aligned} \quad (3)$$

with  $p_\pm = p \pm p'$ ,  $m_\pi = 138$  MeV and  $f_\pi = 93$  MeV the pion mass and weak decay constant and finally  $g_A = 1.25$  the axial coupling constant. Note that the contact term includes the  $\delta^3(\vec{r})$  contribution from one pion exchange, and it also does the leading and sub-leading contributions in the derivative expansion of all shorter distance effects, such as Two Pion Exchange (TPE)<sup>1</sup>, intermediate  $\Delta$ 's,  $\omega$  exchange, etc  $\dots$ . Such a procedure suffers from some limitations, for instance, since the TPE potential has not been explicitly included one cannot relate NN scattering to other processes as pion-nucleon, pion-deuteron, etc  $\dots$  scattering. The LEC's  $g_0$  and  $g_1$  are not fixed by chiral symmetry and have to be determined by a fit to the phase shifts, as we will see below. Scattering by short-range interactions in the presence of a known long-range potential,  $V_\pi$ , can be treated by Distorted Wave Theory (DWT) [15]. We write the full scattering matrix as<sup>2</sup>

$$T = T_\pi + (I + T_\pi G_0) \hat{T}_s (I + G_0 T_\pi) \quad (4)$$

with  $I$  the identity,  $G_0$  the free nucleon Green function and  $T_\pi$  the scattering matrix for  $V_\pi$  alone. Besides,  $\hat{T}_s$  describes the scattering between distorted waves of  $V_s$ . It satisfies the LSE

$$\hat{T}_s = V_s + V_s G_\pi \hat{T}_s \quad (5)$$

where  $G_\pi$  is the nucleon Green function in the presence of  $V_\pi$ , i.e.,  $G_\pi = G_0 + G_0 T_\pi G_0$ . To solve the LSE of Eq. (4), the full off-shell scattering matrix  $T_\pi(E; p, p')$  is required, which is determined by solving the LSE of Eq. (1) with the obvious substitution of  $V \rightarrow V_\pi$ . This equation is finite (it corresponds to the scattering by the usual Yukawa force studied in most books of Quantum Mechanics) and does not require to be renormalized, being then possible

<sup>1</sup> Note that, the full structure of the logarithmic terms, stemming from the pion loops implicit in the TPE contribution, cannot be entirely accounted for the contact terms. Indeed, for CM transferred momenta above  $2m_\pi$ , TPE contributions would have to be treated in the same footing as the one-pion exchange ones.

<sup>2</sup> Since the angular integrals have been already performed when projecting into the  $^1S_0$  wave and taking into account that do not lead to UV divergences, in what follows we will assume that the measure in momentum space and the free nucleon propagator are given by  $\int_0^{+\infty} dq q^2$  and  $G_0(E; p, p') = \frac{\delta(p-p')}{p^2} \frac{1}{2mE - p^2 + i\epsilon}$ , respectively.

a numerical evaluation. We have obtained  $T_\pi$  by discretizing the momentum space and using the inverse matrix algorithm. Some results are shown in the top panel of Fig. 1. To obtain  $\hat{T}_s(E; p, p')$ , we solve

$$\hat{T}_s(E; p, p') = V_s(p, p') + \int_0^{+\infty} dq q^2 \frac{V_s(p, q) \hat{T}_s(E; q, p')}{k^2 - q^2 + i\epsilon} + \int_0^{+\infty} dq dq' \frac{q^2 q'^2 V_s(p, q) T_\pi(E; q, q') \hat{T}_s(E; q', p')}{(k^2 - q^2 + i\epsilon)(k^2 - q'^2 + i\epsilon)} \quad (6)$$

The above equation can be reduced to a linear algebraic system of equations, which solution is straightforward,

$$\hat{T}_s(E, p, p') = \alpha + \beta(p^2 + p'^2) + \gamma p^2 p'^2, \quad (7)$$

with the energy dependent functions

$$\begin{aligned} \alpha &= \frac{g_0 + g_1^2 K_4}{\Delta}, & \beta &= \frac{g_1 - g_1^2 K_2}{\Delta}, & \gamma &= \frac{g_1^2 K_0}{\Delta} \\ \Delta &= 1 - g_0 K_0 - 2g_1 K_2 + g_1^2 (K_2^2 - K_0 K_4). \end{aligned} \quad (8)$$

The UV divergent integrals  $K_n(E) = I_n(E) + J_n(E)$  are defined by:

$$I_n(E) = \int_0^\infty dq \frac{q^{n+2}}{k^2 - q^2 + i\epsilon} \quad (9)$$

$$J_n(E) = \int_0^\infty \frac{dq dq' q^a q'^b T_\pi(E; q, q')}{(k^2 - q^2 + i\epsilon)(k^2 - q'^2 + i\epsilon)} \quad (10)$$

and the pair of integers  $(a, b)$  take the values (2,2), (4,2) and (4,4) for the  $n = 0, 2$  and 4 cases respectively.

We use the Dimensional Regularization (DR) scheme, since it preserves chiral and gauge symmetry and Galilean invariance, which makes the integrals relatively simple to evaluate. DR discards all power-law divergences of the type  $\int dq q^n$ , which makes finite all the  $I_n$  integrals define above,

$$I_n(E) = k^n \times i \left( -\frac{\pi k}{2} \right) \quad (11)$$

To simplify the  $J_n$  integrals it is useful to realize that in DR the linearly UV divergent integral  $\int_0^{+\infty} dq q^2 V_\pi(q, p)$  is finite ( $= 2m\alpha_\pi m_\pi$ ) and independent of  $p$  [4]. Making use of the LSE satisfied by  $T_\pi$  to get expressions involving the above integral, the  $J_2$  and  $J_4$  integrals can be related to the  $J_0$  one,

$$\frac{J_n(E)}{2m} = (E - m_\pi \alpha_\pi) J_{n-2}(E) + i \frac{\pi}{2} m_\pi \alpha_\pi k^{n-1} \quad (12)$$

for  $n = 2, 4$ . Besides,  $J_0(E)$  is logarithmically divergent and it only requires one subtraction, i.e.,  $\bar{J}_0(E) = J_0(E) - J_0(0)$  is finite and can be numerically evaluated. Plugging Eq. (7) into Eq. (4) one gets for the on-shell scattering amplitude  $T(E; k, k) \equiv T(k)$  :

$$T(k) = T_\pi(k) + \hat{T}_s(k) + 2 \{ (\alpha + \beta k^2) L_0 + (\beta + \gamma k^2) L_2 \} + \alpha L_0^2 + 2\beta L_0 L_2 + \gamma L_2^2 \quad (13)$$

with  $T_\pi(k)$  and  $\hat{T}_s(k)$  the long and short range on-shell matrices and

$$L_n(k) = \int_0^\infty \frac{dq q^{n+2} T_\pi(E; k, q)}{(k^2 - q^2 + i\epsilon)} \quad (14)$$

The integral  $L_0$  is finite and the UV divergent integral  $L_2$  in DR becomes finite, i.e.,  $L_2(k) = 2m \times \{ (E - \alpha_\pi m_\pi) L_0(k) - \alpha_\pi m_\pi \}$ . With all above results one gets

$$T(k) = T_\pi(k) + (1 + L_0(k))^2 \hat{T}_s(E; \hat{k}, \hat{k}) = T_\pi(k) + \frac{(1 + L_0(k))^2}{V_s^{-1}(\hat{k}, \hat{k}) + i\pi k/2 - \bar{J}_0(E) - J_0(0)} \quad (15)$$

with  $\hat{k}^2 = k^2 - 2m\alpha_\pi m_\pi$ . Elastic unitarity is exactly fulfilled thanks to a Watson's type relation satisfied by  $L_0(k)^3$ ,

$$L_0(k) = (l_0(k) - i\pi k/2) T_\pi(k), \quad l_0 \in \mathbb{R} \quad (16)$$

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<sup>3</sup> This relation is easily obtained from the Optical Theorem satisfied by  $T_\pi$ , i.e.,  $2\text{Im}T_\pi(E; p_1, p_2) = -\pi k T_\pi(E; p_1, k) T_\pi^*(E; p_2, k)$ .

and that

$$\text{Im}\bar{J}_0(k) = - \{(\pi k/2)^2 - l_0(l_0 + 2\text{Re}T_\pi^{-1}(k))\} \text{Im}T_\pi(k). \quad (17)$$

Indeed, the on shell scattering matrix,  $T$ , can be re-written in a form where unitarity is manifest,

$$T^{-1}(k) = T_\pi^{-1}(k) - \frac{(l_0(k) + \text{Re}T_\pi^{-1}(k))^2}{\text{Re}T_\pi^{-1}(k) + g(k)} \quad (18)$$

$$g(k) = V_s^{-1}(\hat{k}, \hat{k}) - \text{Re}J_0(k) + \frac{\text{Im}\bar{J}_0(k)}{\tan \delta_\pi(k)} \quad (19)$$

Above threshold, the function  $g(k)$  is real, and  $\delta_\pi(k)$  are the phase shifts deduced from  $T_\pi(k)$ . The functions  $l_0(k)$  and  $\text{Re}\bar{J}_0(k)$  and the phase shifts  $\delta_\pi$  are plotted in the top panel of Fig. 1. Note that in the complex  $E$  plane,  $g(k)$  does not have right-hand-cut, but it does have a left-hand-cut due to pion exchange. Before going ahead we should study how to get rid of the UV divergences.

### III. RENORMALIZATION SCHEME.

The DR scheme has led to a drastic reduction of UV divergences and thus we are just left with the logarithmic divergent integral<sup>4</sup>:  $J_0(0)$ , which should be absorbed into a redefinition of the coupling constants of the potential, provided one included in the potential all terms consistent with the symmetry principles. However, the amplitude cannot be made finite by simply redefining the couplings  $g_0$  and  $g_1$ . This problem arises because we have not included operators with more than two derivatives in the potential, which are needed as counter-terms to render multiple insertions of the two derivative operator finite. Indeed, the divergent constant  $J_0(0)$  appears in the function  $g(k)$  defined above, and it seems natural to define a renormalized contact potential  $V_s^{R-1}(\hat{k}, \hat{k}) = V_s^{-1}(\hat{k}, \hat{k}) - J_0(0)$  which leads to an infinite series of powers of  $k^2$  for  $V_s^R$ . Since the LSE performs a non-perturbative resummation of diagrams, in principle one should also add a series of counter-terms to the bare amplitude such that the sum of both becomes finite. At each order in the  $k^2$  (or derivative) expansion, the divergent part of the counter-term series is completely determined. However, the finite part remains arbitrary. It means that the coefficients of each of the terms in the  $k^2$  series, implicit in the definition of  $V_s^{R-1}$ , become undetermined and have to be fitted to data or, if possible, evaluated in QCD. Thus, the scenario might look like pessimistic, and because of the appearance of divergences,  $g(k)$  turns out to be a completely undetermined function. This situation has some resemblances to ChPT in the meson-meson and meson-baryon sectors. For simplicity, let us pay attention to elastic  $SU(2)$   $\pi\pi$  scattering. It is well known [23] that the divergences arising at one loop,  $\mathcal{O}(p^4)$ , cannot be absorbed into a redefinition of the leading terms<sup>5</sup>,  $\mathcal{O}(p^2)$ , of the Lagrangian. New counter-terms,  $l_i$ , higher in the chiral expansion are needed to absorb the divergences, which finite parts  $\bar{l}_i$ , remains undetermined, cannot be fixed by chiral symmetry and have to be fitted to data. These LEC's are fundamental parameters of the EFT, which contain the contribution at low energies of higher degrees of freedom, which have been integrated out. The evaluation of the divergent parts of the amplitudes with an UV cut-off does not necessarily provide a reasonable estimate for them<sup>6</sup>. To be predictive, one can adopt here a renormalization scheme, similar to that used in Refs. [18]–[20] to renormalize the Bethe Salpeter equation for meson-meson and meson-baryon scattering, which reduces the enormous freedom discussed above. Since for small momenta, higher derivative operators should have a tiny influence, we choose to reduce all this proliferation of LEC's, by imposing relations, among all LEC's associated to counter-terms with a number of derivatives higher than the terms included in the potential, in such a way that the renormalized amplitude can be cast, again, as in Eq. (18) and therefore it fulfills elastic unitarity. This amounts in practice, to interpret the previously divergent quantity  $J_0(0)$  as a renormalized

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<sup>4</sup> The divergent part of  $J_0(0)$  is given by the integral

$$\int_0^{+\infty} dq dp V_\pi(q, p) = -2m\alpha_\pi \int_0^{+\infty} \frac{dq}{q} \left( \frac{\pi}{2} - \arctan \frac{m_\pi}{q} \right). \quad (20)$$

<sup>5</sup> Power divergences can be absorbed into a redefinition of the leading terms, but logarithmic ones cannot, and require the inclusion of higher order structures.

<sup>6</sup> If an UV cut-off  $\Lambda$  is employed, the  $\mathcal{O}(p^4)$  contributions in all isospin-angular momentum elastic  $\pi\pi$  scattering channels will be determined just by one parameter,  $\Lambda$ , while at this order there are four independent parameters,  $\bar{l}_{1,2,3,4}$ , which, for instance, incorporate the effect of the  $\rho$  and other resonances in the amplitudes (see for instance discussion in Section 3 (pages 63–70) of second entry of Ref. [18]).

undetermined parameter. After having renormalized, we add a superscript  $R$  to differentiate between the previously divergent,  $J_0(0)$ , and now finite quantity<sup>7</sup>,  $J_0^R(0)$ . This parameter and therefore the renormalized amplitude can be expressed in terms of physical (measurable) magnitudes. The estimate given by means of a *reasonable* UV cut-off for the numerical value of  $J_0^R(0)$  might not be good. For instance, Eqs. (A15) of second entry of Ref. [18] illustrate this point, if an UV cutoff is employed, the divergent integrals appearing there will be independent of the  $IJ$  channel, and to get a reasonable value for  $I_0^{R,I=1}$  unrealistic scales or cut-offs of the order of 300 GeV will be needed. This is due to the special nature of the  $\rho$ -resonance, and that is not the case, for instance, for the  $s$ -waves.

Thus, we are proposing an expansion of the short range part of the potential in powers of  $k^2$ , i.e., delta function and its derivatives in coordinate space, and for a given potential, to compute  $T$  to all orders in the  $k^2$  expansion to restore exact unitarity. Thus, the contact terms effectively account for degrees of freedom higher than the pion and one should expect the scheme to work up to the first threshold, likely two pion production ( $k_{LAB} \approx 2m_\pi$ ), which is around the normal nuclear matter Fermi momentum. A word of caution must be said here; this expansion is not equivalent to a chiral expansion in  $m_\pi$ , since the coefficients of the contact interaction would contain contributions from all orders in  $m_\pi$ , as it was firstly pointed out in Ref. [4].

#### IV. RESULTS AND CONCLUDING REMARKS

After the above discussion, it is clear we have three undetermined LEC's:  $g_0$ ,  $g_1$  and  $J_0^R(0)$ , which we obtain from a  $\chi^2$ -fit to Nijmegen phase-shifts [24]. To perform the fits, we assume errors on the phase shifts,  $\delta$  (in degrees), given by  $0.5 + \text{abs}(\delta) \times \max(0.1, E_{Lab}[\text{MeV}]) / 100$ , where  $E_{Lab}$  is the kinetic energy in the laboratory frame. We fit data from threshold up to a certain value,  $k_{\text{max}}$ , of the CM nucleon momentum. We determine  $k_{\text{max}}$ , by studying the dependence of  $\chi^2/dof$  on it, and fix it in the region of 260 MeV, since the inclusion of simply a new datum of higher energy would double the value of  $\chi^2/dof$ . Thus, we have fitted 62 phase shift values for which  $E_{Lab}$  has varied from 0.01 to 142 MeV, and results are shown (solid line) in the bottom panel of Fig. 1. Best fit parameters are

$$g_0 m = -0.276(6), \quad g_1 m^3 = 0.347(14), \quad \frac{J_0^R(0)}{m} = -3.21(8), \quad (21)$$

with  $\chi^2/dof = 0.016$ . In the above equation, statistical errors have been shown in brackets, affect to the last digit of the parameters and are given by the square root of the diagonal elements of the covariance matrix,  $v_{ij} = \left[ \left( \frac{1}{2} \frac{\partial \chi^2}{\partial b_i \partial b_j} \right)^{-1} \right]_{ij}$ ,

with  $b_i$  any of the three fitted parameters. It is worth to mention that from the above matrix, one also learns that the pairs  $g_0, g_1$  and  $g_0, J_0^R(0)$  are highly anticorrelated, with correlation coefficients ( $r_{ij} = v_{ij} / \sqrt{v_{ii} v_{jj}}$ ) smaller than  $-0.99$ . This means that effectively there is just one independent parameter. We will come back to this point below.

If the derivative terms of the contact interaction are set to zero,  $g_1 = 0$ , the UV divergence can be absorbed in  $g_0$  [4] and thus one is just left with one parameter, which can be fitted to data. Yet, though  $g_1 \neq 0$ , one can arbitrarily set the renormalized coefficients of the higher order terms to zero, i.e., take  $J_0^R(0) = 0$ , and fit the non-zero LEC's,  $g_0$  and  $g_1$ , to data. Both procedures are also shown in Fig. 1, the first one leads to the scheme developed in Ref. [8] and the second one was studied for the very first time in Ref. [4]. As seen in the figure, these two schemes do not work for momenta higher than 10 and 40 MeV respectively, while the solid line provides a good description of data up to momenta of the order of 260 MeV. Indeed, from Eq. (21) we have  $\sqrt{|g_0/2g_1|} \approx 295$  MeV, while such ratio takes a value around of 70 MeV when  $J_0^R(0)$  is set to zero.

The scattering length,  $a$ , and the effective range,  $r_0$ , are defined from the effective range expansion,

$$T^{-1}(k) = -\frac{\pi}{2} \left( -\frac{1}{a} + \frac{1}{2} r_0 k^2 + \dots - ik \right) \quad (22)$$

Fitting the inverse of our amplitude, up to  $k = 48.5$  MeV, to the above formula, we get  $a = -23.65$  fm and  $r_0 = 2.63$  fm, in reasonable agreement with the Nijmegen results [24] ( $-23.7$  fm and  $2.73$  fm, respectively) also obtained in this way.

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<sup>7</sup> Thus, in the calculation presented in this work, we relate the finite parts of the  $k^6$ ,  $k^8$ , etc..., contributions to that of the  $k^4$  one (note that, the  $k^4$  contribution shows up at the one-loop level induced by contact terms quadratic in momenta:  $p^2$  and  $p'^2$ ), which is determined by  $J_0^R(0)$ . The relation is such that elastic unitarity is restored, and thus this scheme differs from those where the higher order terms are set to zero. Indeed, setting to zero the higher order terms is as arbitrary as setting them to any other value. At next order in this derivative expansion, it is to say when terms of the type  $p^2 p'^2$ ,  $p^4$  or  $p'^4$  are included explicitly in the potential, the LSE would generate terms of order  $k^6$  and higher in the amplitude. Thus, the expansion based on this renormalization scheme is systematically improvable.

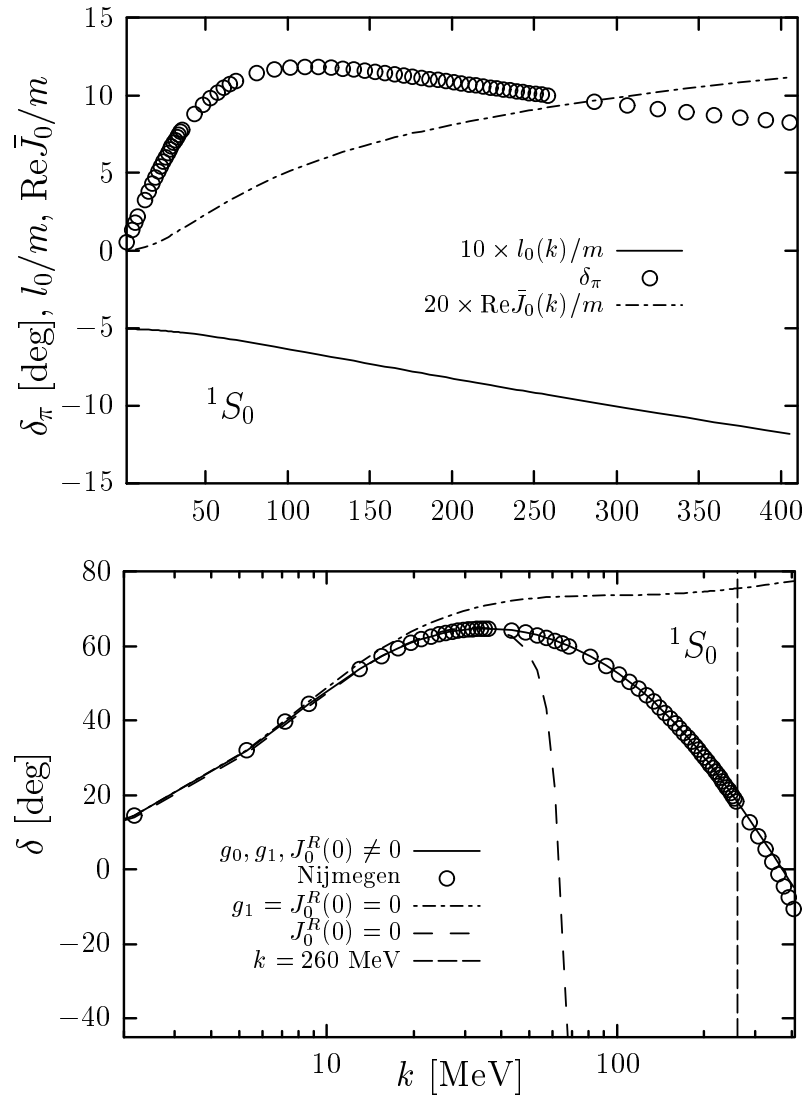


FIG. 1: Top panel: Several quantities extracted from  $T_\pi$ : phase shifts and functions  $l_0(k)$  and  $\text{Re}\bar{J}_0(k) = \text{Re}J_0(k) - J_0(0)$  defined in Eqs. (12) and (16), respectively. From the energy dependence of  $\delta_\pi(k)$ , we deduce the effective range parameters (see Eq. (22))  $a_\pi = -0.88$  fm and  $r_{0\pi} = 12.38$  fm. Bottom panel:  $^1S_0$   $np$  phase shifts in degrees plotted versus CM momentum (log scale). Solid line stands for the results of this work (Eq. (21)), while other two approaches with  $J_0^R(0) = 0$  and  $J_0^R(0) = g_1 = 0$  respectively, are also shown. Data are taken from the phase-shift analysis of Ref. [24], and data for momenta above the vertical line (260 MeV) have not been included in the fit.

To finish, we would like just to summarize the results obtained in this work. We advocate for an expansion of the short range part of the potential in powers of  $k^2$  and computing  $T$  to all orders in the  $k^2$  expansion to restore exact elastic unitarity. DWT techniques and DR have been used to solve the  $^1S_0$  NN LSE with a potential, which consists of pion exchange and contact terms with up to two derivatives. The procedure of solving the LSE is quite simple and an explicit expression, where exact elastic unitarity is manifest, has been given as well (see Eq. (18)). A particular renormalization scheme has also been discussed. It is based on previous findings on meson-meson and meson-baryon systems and its main ingredient is to realize that an EFT is not a renormalizable field theory in the sense of QCD, i.e., with a finite number of counter-terms. Thus, to keep finite the amplitude, obtained after performing the non-perturbative resummation implicit in the LSE, would require, in general, the addition of an infinite set of counter-terms in the short distance potential ( $k^2$  series). The finite parts of the coefficients of the series would remain undetermined (LEC's) and encode the contribution of higher degrees of freedom, not explicitly included. An UV cut-off can effectively account for this freedom only up to some momentum scale, which will depend on the terms explicitly included in the potential<sup>8</sup>. For a contact potential including up to  $k^2$  terms, we find a good description of

<sup>8</sup> The EFT only becomes cut-off independent when all counter-terms compatible with the underlying symmetry are included in the

phase shifts up to CM NN momenta of the order of 260 MeV with the inclusion of just one additional parameter,  $J_0^R(0)$ .

In Ref. [7], an UV cut-off is used to regularize the amplitudes, and with reasonable values of it, in the range 0.6 – 1 GeV, a good description of data is found. Note, however, that this does not have always to be the case and depends, as we mentioned above, of the physical system and of the order of the expansion included in the potential. Thus, to describe the  $\rho$ -resonance in  $\pi\pi$  scattering, one is left to deal with UV cut-off's of the order of 300 GeV [18],[21]. Unrealistic values of the UV cutoff are also needed to account for the  $N(1650)$ -resonance in meson baryon scattering [19],[20], but this is not the case, for instance, for the  $\sigma$  and  $f_0(980)$  resonances in  $\pi\pi$  scattering [21] or the  $\Lambda(1405)$  resonance in meson baryon scattering [22].

Besides, we find extremely big anticorrelations between  $J_0^R(0)$  and the coefficients,  $g_0$  and  $g_1$ , of the iterated short distance potential and among the two later ones, as well. This indicates that the higher derivative operators in the EFT, generated in this scheme by the inclusion of  $J_0^R(0)$ , are actually highly correlated. This might support the idea that though, the higher derivative operators are controlled by a scale that diverges as  $|a| \rightarrow \infty$ , thanks to these high correlations, the effects that diverge with  $a$  cancel, as it was pointed out in Ref. [4].

To include more derivatives is trivial and would result, besides the left hand cut due to pion exchange and accounted for by means of  $T_\pi$ , in a higher order *padè* approximant for the function  $g(k)$  in Eq. (18). The extension of the procedure to higher partial waves is also straightforward although cumbersome and will be presented elsewhere [25].

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- [1] S. Weinberg, Phys. Lett. **B251** (1990) 288; Nucl. Phys. **B363** (1991)2; Phys. Lett. **B295** (1992) 114.
  - [2] C. Ordóñez and U. van Kolck, Phys. Lett. **B291** (1992) 459; U. van Kolck, Phys. Rev. **C49** (1994) 2932; Prog. Part. Nucl. Phys. **43** (1999) 337; C. Ordóñez, L. Ray and U. van Kolck, Phys. Rev. Lett. **72** (1994) 459; Phys. Rev. **C53** (1996) 2086.
  - [3] T.Park, D.Min and M. Rho, Phys. Rev. Lett. **74** (1995) 4153; Nucl. Phys. **A596** (1996) 515.
  - [4] D. Kaplan, M. Savage and M. Wise, Nucl. Phys. **B478** (1996) 629.
  - [5] D. Kaplan, M. Savage and M. Wise, Phys. Lett. **B424** (1998) 390; Nucl. Phys. **B534** (1998) 329.
  - [6] T. Cohen and J. Hansen, Phys. Lett. **B440** (1998) 233; Phys. Rev. **C59** (1999) 13; Phys. Rev. **C59** (1999) 3047; nucl-th/9908049.
  - [7] E. Epelbaum, W. Glöckle and U. Meißner, Nucl. Phys. **A637** (1998) 107; Nucl. Phys. **A671** (2000) 295.
  - [8] T. Frederico, V.S. Timóteo and L. Tomio, Nucl. Phys. **A653** (1999) 209.
  - [9] D. R. Phillips, G. Rupak, M. Savage, Phys. Lett. **B473** (2000) 209.
  - [10] S. Fleming, T. Mehen and I. Stewart, Nucl. Phys. **A677** (2000) 313; Phys. Rev. **C61** (2000) 044005.
  - [11] M. Lutz, Nucl. Phys. **A677** (2000) 241.
  - [12] J. Gegelia, Phys. Lett. **B463** (1999) 133; J. Gegelia and G. Japaridze, Phys. Lett. **B517** (2001) 476.
  - [13] S. Beane, P. Bedaque, M. Savage, U. van Kolck, Nucl.Phys. **A700** (2002) 377.
  - [14] D. Eiras and J. Soto, nucl-th/0107009.
  - [15] T. Barford and M.C. Birse, hep-ph/0206146.
  - [16] D.R. Entem and R. Machleidt, Phys. Lett. **B524** (2002) 93.
  - [17] J. Oller, nucl-th/0207086.
  - [18] J. Nieves and E. Ruiz Arriola, Phys. Lett. **B455** (1999) 30; Nucl. Phys. **A679** (2000) 57.
  - [19] J. Nieves and E. Ruiz Arriola, Phys. Rev. **D64** (2001) 116008; C. García-Recio, J. Nieves, E. Ruiz Arriola and M. J. Vicente-Vacas, Phys. Rev. **D67** (2003) 076009.
  - [20] T. Inoue, E. Oset and M.J. Vicente-Vacas, Phys. Rev. **C65** (2002) 035204.
  - [21] J.A. Oller and E. Oset, Nucl. Phys. **A620** (1997) 438; J.A. Oller, E. Oset and J.R. Peláez, Phys. Rev. Lett. **80** (1998) 3452; Phys. Rev. **D59** (1991) 074001.
  - [22] E. Oset and A. Ramos, Nucl. Phys. **A 635** (1998) 99.
  - [23] J. Gasser and H. Leutwyler, Ann. of Phys., NY **158** (1984) 142.
  - [24] V. Stocks, R.Klomp, M. Rentmeester and J. de Swart, Phys. Rev. **C48** (1993) 792.
  - [25] J. Nieves in preparation.