Charmed and Bottom Baryons: a Variational Approach based on Heavy Quark Symmetry.

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The use of Heavy Quark Symmetry to study bottom and charmed baryons leads to important simplifications of the non-relativistic three body problem, which turns out to be easily solved by a simple variational ansatz. Our simple scheme reproduces previous results (baryon masses, charge and mass radii, $\cdots$) obtained by solving the Faddeev equations with simple non-relativistic quark–quark potentials, adjusted to the light and heavy–light meson spectra. Wave functions, parameterized in a simple manner, are also given and thus they can be easily used to compute further observables. Our method has been also used to find the predictions for strangeness-less baryons of the SU(2) chirally inspired quark-quark interactions. We find that the one pion exchange term of the chirally inspired interactions leads to relative changes of the Λ_b and Λ_c binding energies as large as 90%.

I. INTRODUCTION

Prior, and at around, the time the charmed baryons existence was confirmed [1] a great theoretical activity developed in order to understand and predict their masses [2, 3, 4], magnetic moments [5], or decay properties [6, 7, 8]. This activity continued in the following decades. The discovery of the Λ_b baryon at CERN [9], the discovery of most of the charmed baryons of the SU(3) multiplet on the second level of the SU(4) lowest 20-plet [10], and the claims of indirect evidences for the semileptonic decays of Λ_b and Ξ_b [11] renewed the interest in the spectroscopy and weak decays of heavy baryons. There are eight lowest-lying baryons containing one heavy and two light quarks (up, down, or strange). The quantum numbers of the charmed and bottom baryons are listed in Table I.

Heavy Quark Symmetry (HQS) has proved to be a useful tool to understand the bottom and charm physics [13, 14, 15], and it has been extensively used to describe the dynamics of systems containing a heavy quark c or b. For instance, all lattice QCD simulations rely on HQS to describe bottom systems [16]. HQS is an approximate $SU(N_F)$ symmetry of QCD , being N_F the number of heavy flavors: c, b, $\cdots$, which appear in systems containing heavy quarks with masses that are much larger than the typical quantities ($q = \Lambda_{QCD}$, $m_u, m_d, m_s \cdots$) which set up the energy scale of the dynamics of the remaining degrees of freedom. HQS has some resemblances, in atomic physics, to the approximate independence of the electron properties from the nuclear spin and mass, for a fixed nuclear charge. Up to corrections of the order[1] $\mathcal{O}(q/m_Q)$, HQS guaranties that the heavy baryon light degrees of freedom quantum numbers, compiled in Table I, are always well defined. The symmetry also predicts that the pair of baryons Σ and Σ^* or the pair Ξ' and Ξ^* or the pair Ω and Ω^* become degenerated for an infinitely massive heavy quark, since both baryons have the same cloud of light degrees of freedom.

However, HQS has not been systematically employed in the context of non-relativistic quark models. Non-relativistic quark models, based upon simple quark-quark potentials, partially inspired by Quantum Chromodynamics (QCD), lead to reasonably good descriptions of hadrons as bound states of constituent quarks and also of the general features of the baryon-baryon interaction [17]–[19]. Most of the quark-quark interactions include a term with a shape and a color structure determined from the One Gluon Exchange (OGE) contribution [20] and a confinement potential. The force which confines the quarks is still not well understood, although it is assumed to come from long-range nonperturbative features of QCD [21]. Those terms do not incorporate another important feature of QCD: Chiral Symmetry (CS) and its pattern of spontaneous breaking. Quark–quark interaction terms derived from Spontaneous Chiral Symmetry Breaking (SCSB) have been taken into account for the description of the nucleon–nucleon system and/or the light baryon spectra [22, 23, 24, 25, 26]

In this work, we develop a variational approach for the solution of the non-relativistic three-body problem in baryons with a heavy quark. Thanks to HQS, the method we propose turns out to be quite simple, and leads to simple and manageable wave functions[2]. We consider several simple phenomenological quark–quark interactions [27, 28] which free parameters have been adjusted in the meson sector and are then free of three-body ambiguities.

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[1] The quantities q and m_Q are a typical energy scale relevant for the light degrees of freedom and the mass of the heavy quark, respectively.
[2] We use a family of wave functions for which the light degrees of freedom quantum numbers are well defined.

Baryon	S	J^P	I	S_{light}^{π}	Quark content	$M_{exp.}$ [10] [MeV]	$M_{Latt.}$ [12] [MeV]
Λ_c	0	$\frac{1}{2}^+$	0	0^+	udc	2285 ± 1	2270 ± 50
Σ_c	0	$\frac{1}{2}^+$	1	1^+	llc	2452 ± 1	2460 ± 80
Σ_c^*	0	$\frac{3}{2}^+$	1	1^+	llc	2518 ± 2	2440 ± 70
Ξ_c	-1	$\frac{1}{2}^+$	$\frac{1}{2}$	0^+	lsc	2469 ± 3	2410 ± 50
Ξ_c'	-1	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	lsc	2576 ± 3	2570 ± 80
Ξ_c^*	-1	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	lsc	2646 ± 2	2550 ± 80
Ω_c	-2	$\frac{1}{2}^+$	0	1^+	ssc	2698 ± 3	2680 ± 70
Ω_c^*	-2	$\frac{3}{2}^+$	0	1^+	ssc		2660 ± 80
Λ_b	0	$\frac{1}{2}^+$	0	0^+	udb	5624 ± 9	5640 ± 60
Σ_b	0	$\frac{1}{2}^+$	1	1^+	llb		5770 ± 70
Σ_b^*	0	$\frac{3}{2}^+$	1	1^+	llb		5780 ± 70
Ξ_b	-1	$\frac{1}{2}^+$	$\frac{1}{2}$	0^+	lsb		5760 ± 60
Ξ_b'	-1	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	lsb		5900 ± 70
Ξ_b^*	-1	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	lsb		5900 ± 80
Ω_b	-2	$\frac{1}{2}^+$	0	1^+	ssb		5990 ± 70
Ω_b^*	-2	$\frac{3}{2}^+$	0	1^+	ssb		6000 ± 70

TABLE I: Summary of the quantum numbers, experimental and lattice QCD masses of the baryons containing a single heavy quark. I, and S_{light}^{π} are the isospin, and the spin parity of the light degrees of freedom and S, J^P are strangeness and the spin parity of the baryon (l denotes a light quark of flavor u or d). Experimental masses are taken from Ref. [10] and the isospin averaged masses are quoted, with errors counting for the mass differences between the members of the same isomultiplet. Errors on the lattice QCD masses have been obtained by adding in quadratures the statistical and systematic errors given in Ref. [12].

With those potentials we compute the ground states of some heavy flavor hyperons and show how our simple approach reproduces the results obtained in Ref. [28] by solving involved Faddeev type equations. Besides, we also study the Σ^*, Ξ', Ξ^* and Ω^* baryon ground states, which were not considered in the work of Ref. [28]. After that, and within our variational framework, we work out the predictions for the strangeness-less charmed and bottom baryons of the SU(2) chirally inspired quark-quark interaction of Ref. [22], applied with great success to the meson sector in Ref. [29], and study the effects in these systems of including a pattern of SCSB. Some preliminary results have been presented in Ref. [30].

The paper is organized as follows. After this introduction, we study the hamiltonian of the system, different inter-quark interactions (Sect. II) and the general structure of the variational baryon wave–function (Sect. III). Some static properties (mass and charge densities, $\cdots$) are revisited in the Sect. IV and finally in the Sects. V and VI we present our results and compile the main conclusions of this work, respectively.

II.   THREE BODY PROBLEM

A.   Intrinsic Hamiltonian

In the Laboratory (LAB) frame (see Fig. 1), the Hamiltonian (H) of the three quark (q, q', Q, with $q, q' = l$ or s and $Q = c$ or b) system reads:

$$H = \sum_{i=q,q',Q} \left(m_i - \frac{\vec{\nabla}_{x_i}^2}{2m_i} \right) + V_{qq'} + V_{Qq} + V_{Qq'} \tag{1}$$

where $m_q, m_{q'}$ and m_Q are the quark masses, and the quark-quark interaction terms, V_{ij}, depend on the quark spin-flavor quantum numbers and the quark coordinates ($\vec{x}_1, \vec{x}_2$ and $\vec{x}_h$ for the q, q' and Q quarks respectively). The nabla operators in the kinetic energy stand for derivatives with respect to the spatial variables $\vec{x}_1, \vec{x}_2$ and $\vec{x}_h$. To separate the Center of Mass (CM) free motion, we go to the heavy quark frame ($\vec{R}, \vec{r}_1, \vec{r}_2$),

$$\begin{aligned}
\vec{R} &= \frac{m_q \vec{x}_1 + m_{q'} \vec{x}_2 + m_Q \vec{x}_h}{m_q + m_{q'} + m_Q} \\
\vec{r}_1 &= \vec{x}_1 - \vec{x}_h \\
\vec{r}_2 &= \vec{x}_2 - \vec{x}_h
\end{aligned} \tag{2}$$

where $\vec{R}$ and $\vec{r}_1$ ($\vec{r}_2$) are the CM position in the LAB frame and the relative position of the quark q (q') with respect to the heavy quark Q. The hamiltonian now reads

$$H = -\frac{\vec{\nabla}_{\vec{R}}^2}{2M} + H^{\text{int}} \tag{3}$$

$$H^{\text{int}} = -\frac{\vec{\nabla}_1^2}{2\mu_1} - \frac{\vec{\nabla}_2^2}{2\mu_2} - \frac{\vec{\nabla}_1 \cdot \vec{\nabla}_2}{m_Q} + V_{qq'}(\vec{r}_1 - \vec{r}_2, spin) +$$
$$+ V_{Qq}(\vec{r}_1, spin) + V_{Qq'}(\vec{r}_2, spin) + \sum_{i=q,q',Q} m_i \tag{4}$$

where $M = (m_q + m_{q'} + m_Q)$, $\mu_{1,2} = (1/m_{q,q'} + 1/m_Q)^{-1}$ and $\vec{\nabla}_{1,2} = \partial/\partial_{\vec{r}_1, \vec{r}_2}$. The intrinsic hamiltonian H^{int} describes the dynamics of the baryon and we will use a variational approach to solve it. It can be rewritten as the sum of two single particle hamiltonians (h_i^{sp}), which describe the dynamics of the light quarks in the mean field created by the heavy quark, plus the light–light interaction term, which includes the Hughes-Eckart term ($\vec{\nabla}_1 \cdot \vec{\nabla}_2$).

$$H^{\text{int}} = \sum_{i=q,q'} h_i^{sp} + V_{qq'}(\vec{r}_1 - \vec{r}_2, spin) - \frac{\vec{\nabla}_1 \cdot \vec{\nabla}_2}{m_Q} + \sum_{i=q,q',Q} m_i \tag{5}$$

$$h_i^{sp} = -\frac{\vec{\nabla}_i^2}{2\mu_i} + V_{Qi}(\vec{r}_i, spin), \quad i = q, q' \tag{6}$$

B. Quark–Quark Interactions

- *Phenomenological Interactions:* We compile here some phenomenological quark–antiquark potentials fitted to a large sample of meson states in every flavor sector. The general structure is as follows ($i, j = l, s, c, b$):

$$V_{ij}^{q\bar{q}}(r) = -\frac{\kappa\left(1 - e^{-r/r_c}\right)}{r} + \lambda r^p - \Lambda + \left\{ a_0 \frac{\kappa}{m_i m_j} \frac{e^{-r/r_0}}{r r_0^2} + \frac{2\pi}{3 m_i m_j} \kappa'\left(1 - e^{-r/r_c}\right) \frac{e^{-r^2/x_0^2}}{\pi^{\frac{3}{2}} x_0^3} \right\} \vec{\sigma}_i \vec{\sigma}_j \tag{7}$$

with $\vec{\sigma}$ the spin Pauli matrices, m_i the quark masses and

$$x_0(m_i, m_j) = A\left(\frac{2 m_i m_j}{m_i + m_j}\right)^{-B} \tag{8}$$

We have examined five different interactions, one suggested by Bhaduri and collaborators [27] (BD) and four suggested by B. Silvestre-Brac and C. Semay [28, 31] (AL1, AL2, AP1 y AP2). The parameters of the different
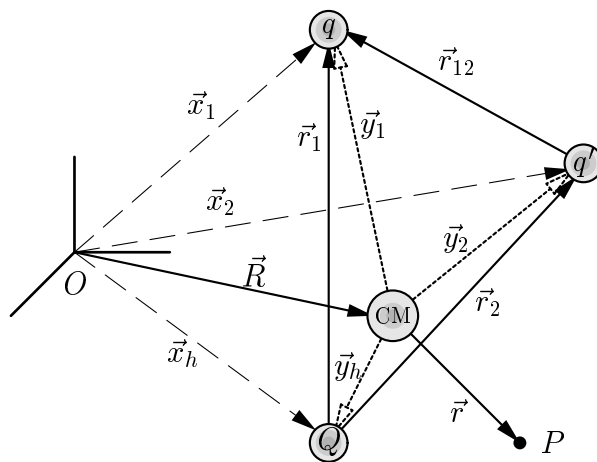

FIG. 1: Definition of different coordinates used through this work.

	BD	AL1	AL2	AP1	AP2
κ	0.52	0.5069	0.5871	0.4242	0.5743
$r_c[\text{GeV}^{-1}]$	0.	0.	0.1844	0.	0.3466
p	1	1	1	2/3	2/3
$\lambda\,[\text{GeV}^{(1+p)}]$	0.186	0.1653	0.1673	0.3898	0.3978
$\Lambda\,[\text{GeV}]$	0.9135	0.8321	0.8182	1.1313	1.1146
a_0	1	0	0	0	0
$r_0\,[\text{GeV}^{-1}]$	2.305	–	–	–	–
κ'	0.	1.8609	1.8475	1.8025	1.8993
$m_u = m_d\,[\text{GeV}]$	0.337	0.315	0.320	0.277	0.280
$m_s\,[\text{GeV}]$	0.600	0.577	0.587	0.553	0.569
$m_c\,[\text{GeV}]$	1.870	1.836	1.851	1.819	1.840
$m_b\,[\text{GeV}]$	5.259	5.227	5.231	5.206	5.213
B	–	0.2204	0.2132	0.3263	0.3478
$A\,[\text{GeV}^{B-1}]$	–	1.6553	1.6560	1.5296	1.5321

TABLE II: Parameters, from Refs. [27, 28], of different phenomenological potentials (Eqs. (7) and (8)).

potentials are compiled in Table II. The potentials considered differ in the form factors used for the hyperfine terms, the power of the confining term ($p = 1$, as suggested by lattice QCD calculations [32], or $p = 2/3$ which for mesons gives the correct asymptotic Regge trajectories [33]), or the use of a form factor in the OGE Coulomb potential.

The usual $V_{ij}^{qq} = V_{ij}^{q\bar{q}}/2$ prescription, coming from a $\vec{\lambda}_i \vec{\lambda}_j$ color dependence ($\vec{\lambda}$ are the Gell-Mann matrices) of the whole potential, has been assumed to obtain the quark–quark interactions from those given in Eq. (7).

All those interactions were used in Ref [28] to obtain, in a Faddeev calculation, the spectrum and static properties of heavy baryons. Our purpose is to show that our simpler variational method gives equally good results for all observables analyzed in [28] and, besides, it provides us with manageable wave functions that can be used for the evaluation of further observables.

- *Chiral Quark Cluster Interactions:* Working in the context of the SU(2) linear sigma model, Fernández and collaborators have derived in Ref. [22] a quark–quark interaction that, apart from the effective OGE and confining terms, contains a pseudoscalar (V^{PS}) and a scalar (V^{S}) potentials provided by the exchange of Goldstone bosons. This model was used in Ref. [29] to analyze the strangeness-less meson spectrum with good results. We analyze this potential in order to check the predictions of the SU(2) chirally inspired quark–quark interactions in the strangeness-less heavy baryon sector. The study of baryons with strangeness would require to extend the model of Ref. [22] to three flavors. A linear realization of SCSB will involve, not only the exchange of new Goldstone bosons, as kaon or eta mesons, but also more than one scalar meson. We are not aware of any published potential with such characteristics and that performs well in the meson sector. Therefore, we prefer to work in this latter case with baryons without strange quark content.

In the quark–quark sector for u and d flavors, the potential reads:

$$V_{ij}^{qq} = V_{ij}^{\text{OGE}} + V_{ij}^{\text{CON}} + V_{ij}^{\text{PS}} + V_{ij}^{\text{S}}, \qquad i,j = u,d \tag{9}$$

$$V_{ij}^{\text{OGE}}(r) = \frac{\alpha_s}{4}\vec{\lambda}_i\vec{\lambda}_j \left\{ \frac{1}{r} - \frac{\pi}{m_i m_j}\left(1 + \frac{2}{3}\vec{\sigma}_i\vec{\sigma}_j\right)\frac{e^{-r/r0}}{4\pi r_0^2 r} \right\} \tag{10}$$

$$V_{ij}^{\text{CON}}(r) = \left(-a_c r + a_b\right)\vec{\lambda}_i\vec{\lambda}_j \tag{11}$$

$$V_{ij}^{\text{PS}}(r) = \frac{\alpha_{ch} m_\pi}{1 - m_\pi^2/\Lambda_{\text{CSB}}^2}\frac{1}{3}\left\{ Y(m_\pi r) - \frac{\Lambda_{\text{CSB}}^3}{m_\pi^3}Y(\Lambda_{\text{CSB}} r) \right\}(\vec{\sigma}_i\vec{\sigma}_j)(\vec{\tau}_i\vec{\tau}_j) \tag{12}$$

$$V_{ij}^{\text{S}}(r) = -\frac{4\alpha_{ch} m_i m_j m_\sigma}{m_\pi^2}\frac{1}{1 - m_\sigma^2/\Lambda_{\text{CSB}}^2}\left\{ Y(m_\sigma r) - \frac{\Lambda_{\text{CSB}}}{m_\sigma}Y(\Lambda_{\text{CSB}} r) \right\} \tag{13}$$

with $\vec{\tau}$ the isospin Pauli matrices, $Y(x) = e^{-x}/x$ and $\left(\vec{\lambda}_i\vec{\lambda}_j\right)$ takes the value $-8/3$ for qq pairs in a baryon.

The parameters given in Refs. [22, 29] are: $r_0 = 0.145$ fm, $\alpha_s = 0.7$, $\Lambda_{\text{CSB}} = 3.15$ fm^{-1}, $a_c = 140$ MeV/fm, $m_u = m_d = 313$ MeV, $\alpha_{ch} = 0.027569$, $m_\pi = 138$ MeV and $m_\sigma = 675$ MeV. To reproduce the experimental $\rho - \pi$ mass splitting ($\approx 771 - 138 = 633$ MeV), the parameter r_0 needs to be slightly reduced to 0.1419 fm.

Besides, the overall origin of energies, a_b, takes the value of 122.1 MeV. Note that $\left(\vec{\lambda}_i \vec{\lambda}_j\right)$ takes the value $-16/3$ for $q\bar{q}$ pairs in a meson and that the isospin structure $(\vec{\tau}_i \vec{\tau}_j)$ should be multiplied by a factor -1 in the $q\bar{q}$ case.

To use the above interaction to predict strangeness-less heavy baryon masses, such potential needs to be supplemented by interactions between heavy quarks (c and b) and light quarks (u and d). If we look at the phenomenological potentials discussed in Eq. (7) and Table II, we see the AL1 interaction uses a light quark mass very close to that used in Refs. [22, 29] (315 versus 313 MeV) and furthermore in both cases a linear confining potential is used. Besides, the AL1 potential provides masses for the D, D^*, B, B^* quite close to the experimental values, and π and ρ masses (138 and 770 MeV respectively) in good agreement with the values obtained from the interaction of Eq. (9). Thus, we will compute the masses of the $\Lambda_{c,b}, \Sigma_{c,b}$ and $\Sigma_{c,b}^*$ baryons with a new potential, which we call AL1χ, defined as follows:

- The light–light quark (u and d) interactions will be described by the potential of Eq. (9), which incorporates a pattern of SCSB, and the parameters given below Eq. (13), but with $m_u = m_d = 315$ MeV, $r_0 = 0.1414$ fm and $a_b = 122.4$ MeV. These two latter parameters have been very slightly modified to obtain 138 and 770 MeV (values provided by the AL1 potential) for the π and ρ masses respectively, when a u, d quark mass of 315 MeV is used. The new set of parameters provides states with masses of 138, 1236 and 1918 (770, 1558 and 2173) MeV with π (ρ) quantum numbers, in perfect agreement with the results of Ref. [29].

- To describe the interaction between the heavy quarks and the u and d quarks we will adopt the AL1 model.

In this way, and by comparing results obtained from both the AL1 and AL1χ interactions, we will be able to test the effects of the inclusion of a SCSB pattern to describe the light–light quark interaction in the u, d sector. Such a study, though it has already been performed in other systems, as light meson or baryon spectrum, etc, has not been ever carried out for heavy baryons. It is of great interest since the quantum numbers of the light degrees of freedom in a $\Lambda-$ or $\Sigma-$type heavy baryon have a clear correspondence to those of the π or ρ meson.

III.   VARIATIONAL WAVE FUNCTIONS

In a baryon, the singlet color wave function is completely anti-symmetric under the exchange of any of the three quarks. Within the SU(3) quark model, we assume a complete symmetry of the wave function under the exchange of the two light quarks (u, d, s) flavor, spin and space degrees of freedom. On the other hand, for the interactions described in the previous subsection, we have that both the total spin of the baryon, $\vec{S}_{\rm B} = \left(\vec{\sigma}_q + \vec{\sigma}_{q'} + \vec{\sigma}_Q\right)/2$, and the orbital angular momentum of the light quarks with respect to Q, $\vec{L}$, defined as

$$\vec{L} = \vec{l}_1 + \vec{l}_2, \qquad \text{with } \vec{l}_k = -\mathrm{i} \ \vec{r}_k \times \vec{\nabla}_k, \quad k = 1,2 \tag{14}$$

commute with the intrinsic hamiltonian. We will assume that the ground states of the baryons are in s–wave, $L = 0$. This implies that the spatial wave function can only depend on the relative distances r_1, r_2 and $r_{12} = |\vec{r}_1 - \vec{r}_2|$. Note that when the heavy quark mass is infinity ($m_Q \to \infty$), the total spin of the light degrees of freedom, $\vec{S}_{\rm light} = \left(\vec{\sigma}_q + \vec{\sigma}_{q'}\right)/2$, commutes with the intrinsic hamiltonian, since the $\vec{\sigma}_Q \cdot \vec{\sigma}_{q,q'}/(m_Q m_{q,q'})$ terms vanish in this limit. With all these ingredients, and taking into account the quantum numbers of the light degrees of freedom for each baryon, compiled in Table I and that in general are always well defined in the static limit mentioned above, we have used the following wave functions in our variational approach[3]

- $\Lambda-$ type baryons: $I = 0, \ S_{\rm light} = 0$

$$|\Lambda_Q; J = \frac{1}{2}, M_J \rangle \ = \ \left\{|00\rangle_I \otimes |00\rangle_{S_{\rm light}}\right\} \Psi_{ll}^{\Lambda_Q}(r_1, r_2, r_{12}) \otimes |Q; M_J\rangle \tag{15}$$

where $\Psi_{ll}^{\Lambda_Q}(r_1, r_2, r_{12}) = \Psi_{ll}^{\Lambda_Q}(r_2, r_1, r_{12})$ to guarantee a complete symmetry of the wave function under the exchange of the two light quarks (u, d) flavor, spin and space degrees of freedom, and finally M_J is the baryon

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[3] An obvious notation has been used for the isospin–flavor ($|I, M_I\rangle_I$, $|ls\rangle$ or $|sl\rangle$) and spin ($|S, M_S\rangle_{S_{\rm light}}$) wave functions of the light degrees of freedom.

total angular momentum third component. Note, that SU(3) flavor symmetry (SU(2), in the case of the Λ_Q baryon) would also allow for a component in the wave function of the type

$$\sum_{M_S M_Q} (\frac{1}{2}1\frac{1}{2}|M_Q M_S M_J)\left\{|00\rangle_I \otimes |1M_S\rangle_{S_{\text{light}}}\right\}\Theta_{ll}^{\Lambda_Q}(r_1, r_2, r_{12}) \otimes |Q; M_Q\rangle \tag{16}$$

with $\Theta_{ll}^{\Lambda_Q}(r_1, r_2, r_{12}) = -\Theta_{ll}^{\Lambda_Q}(r_2, r_1, r_{12})$ (for instance terms of the type $r_1 - r_2$) and, the real numbers $(j_1 j_2 j|m_1 m_2 m) = \langle j_1 m_1 j_2 m_2|jm\rangle$ are Clebsh-Gordan coefficients. This component is forbidden by HQS in the limit $m_Q \to \infty$, where S_{light} turns out to be well defined and set to zero for Λ_Q−type baryons. The most general SU(2) Λ_Q wave function will involve a linear combination of the two components, given in Eqs. (15) and (16). Neglecting $\mathcal{O}(q/m_Q)$, HQS imposes an additional constraint, which justifies the use of a wave function of the type of that given in Eq. (15) with the obvious simplification of the three body problem.

- Σ *and* Σ^*-*type baryons:* $I = 1$, $S_{\text{light}} = 1$.

$$\begin{aligned}
|\Sigma_Q; J = \frac{1}{2}, M_J; M_T \rangle &= \sum_{M_S M_Q} (\frac{1}{2}1\frac{1}{2}|M_Q M_S M_J)\left\{|1M_T\rangle_I| \otimes |1M_S\rangle_{S_{\text{light}}}\right\} \\
&\times \Psi_{ll}^{\Sigma_Q}(r_1, r_2, r_{12}) \otimes |Q; M_Q\rangle
\end{aligned} \tag{17}$$

$$\begin{aligned}
|\Sigma_Q^*; |J = \frac{3}{2}, M_J; M_T \rangle &= \sum_{M_S M_Q} (\frac{1}{2}1\frac{3}{2}|M_Q M_S M_J)\left\{|1M_T\rangle_I \otimes |1M_S\rangle_{S_{|\text{light}}}\right\} \\
&\times \Psi_{ll}^{\Sigma_Q^*}(r_1, r_2, r_{12}) \otimes |Q; M_Q\rangle
\end{aligned} \tag{18}$$

with M_T and M_Q, the baryon isospin and heavy quark spin third components, respectively.

As in the Λ_Q case, and based on the HQS predictions, here we have also neglected components constructed out of the spin–singlet ($|00\rangle_{S_{\text{light}}}$) and anti-symmetric spatial wave functions, since they are suppressed by powers of $1/m_Q$. HQS leads to similar simplifications for the rest of baryons studied below and we will omit any further comment in what follows.

- $\Xi-$*type baryons:* $I = \frac{1}{2}, S_{\text{light}} = 0$

$$|\Xi_Q; J = \frac{1}{2}, M_J; M_T \rangle = \frac{1}{\sqrt{2}}\left\{|ls\rangle\Psi_{ls}^{\Xi_Q}(r_1, r_2, r_{12}) - |sl\rangle\Psi_{sl}^{\Xi_Q}(r_1, r_2, r_{12})\right\} \otimes |00\rangle_{S_{\text{light}}} \otimes |Q; M_J\rangle \tag{19}$$

where the isospin third component of the baryon, M_T, is that of the light quark l ($1/2$ or $-1/2$ for the u or the d quark, respectively).

- Ξ' *and* Ξ^*-*type baryons:* $I = \frac{1}{2}$, $S_{\text{light}} = 1$.

$$\begin{aligned}
|\Xi_Q'; J = \frac{1}{2}, M_J; M_T \rangle &= \sum_{M_S M_Q} (\frac{1}{2}1\frac{1}{2}|M_Q M_S M_J)\frac{1}{\sqrt{2}} \\
&\times \left\{|ls\rangle\Psi_{ls}^{\Xi_Q'}(r_1, r_2, r_{12}) + |sl\rangle\Psi_{sl}^{\Xi_Q'}(r_1, r_2, r_{12})\right\} \otimes |1M_S\rangle_{S_{\text{light}}} \otimes |Q; M_Q\rangle
\end{aligned} \tag{20}$$

$$\begin{aligned}
|\Xi_Q^*; J = \frac{3}{2}, M_J; M_T \rangle &= \sum_{M_S M_Q} (\frac{1}{2}1\frac{3}{2}|M_Q M_S M_J)\frac{1}{\sqrt{2}} \\
&\times \left\{|ls\rangle\Psi_{ls}^{\Xi_Q^*}(r_1, r_2, r_{12}) + |sl\rangle\Psi_{sl}^{\Xi_Q^*}(r_1, r_2, r_{12})\right\} \otimes |1M_S\rangle_{S_{\text{light}}} \otimes |Q; M_Q\rangle
\end{aligned} \tag{21}$$

where the isospin third component of the baryon, M_T, is that of the light quark l.

- Ω *and* Ω^*-*type baryons:* $I = 0$, $S_{\text{light}} = 1$.

$$|\Omega_Q; J = \frac{1}{2}, M_J \rangle = \sum_{M_S M_Q} (\frac{1}{2}1\frac{1}{2}|M_Q M_S M_J)|1M_S\rangle_{S_{\text{light}}}\Psi_{ss}^{\Omega_Q}(r_1, r_2, r_{12}) \otimes |Q; M_Q\rangle \tag{22}$$

$$|\Omega_Q^*; J = \frac{3}{2}, M_J \rangle = \sum_{M_S M_Q} (\frac{1}{2}1\frac{3}{2}|M_Q M_S M_J)|1M_S\rangle_{S_{\text{light}}}\Psi_{ss}^{\Omega_Q^*}(r_1, r_2, r_{12})| \otimes |Q; M_Q\rangle \tag{23}$$

where $\Psi_{ss}^{\Lambda_Q}(r_1, r_2, r_{12}) = \Psi_{ss}^{\Lambda_Q}(r_2, r_1, r_{12})$.

The spatial wave function, $\Psi_{qq'}^{B_Q}$, will be determined by the variational principle: $\delta\langle B_Q|H^{\text{int}}|B_Q\rangle = 0$. For simplicity, we will use a family of functions with free parameters, which will be determined by minimizing the mass of the baryon. The OGE has a regularized delta function, which provides strong short range repulsions/attractions, specially between the light quarks[4]. A similar situation appears in the context of double Λ hypernuclei [34], which in a very good approximation can be also reduced to a three body problem. In this context, it was shown that the use of a Jastrow–type functional form for the spatial wave functions leads to excellent results, and to notably simpler wave functions than other functional forms, as for instance a series of standard Hylleraas type wave functions [35]. Thus, we take

$$\Psi_{qq'}^{B_Q}(r_1, r_2, r_{12}) = N F^{B_Q}(r_{12})\phi_q^Q(r_1)\phi_{q'}^Q(r_2) \tag{24}$$

where N is a constant, which is determined from normalization[5]

$$1 = \int d^3r_1 \int d^3r_2 \left|\Psi_{qq'}^{B_Q}(r_1, r_2, r_{12})\right|^2 = 8\pi^2 \int_0^{+\infty} dr_1\, r_1^2 \int_0^{+\infty} dr_2\, r_2^2 \int_{-1}^{+1} d\mu \left|\Psi_{qq'}^{B_Q}(r_1, r_2, r_{12})\right|^2 \tag{25}$$

where μ is the cosine of the angle between the vectors $\vec{r}_1$ and $\vec{r}_2$, being $r_{12} = (\, r_1^2 + r_2^2 - 2r_1 r_2\mu)^{\frac{1}{2}}$. For simplicity, we do not entirely determine the functions ϕ_q^Q and $\phi_{q'}^Q$ from the variational principle, but we rather fix the bulk of these functions to the $s-$wave ground states ($\varphi_{i=q,q'}^Q$) of the single particle hamiltonians, $h_{i=q,q'}^{sp}$, defined in Eq. (6), and modify their behavior at large distances. Thus, we take

$$\phi_q^Q(r_1) = (1 + \alpha_q r_1)\varphi_q^Q(r_1)$$
$$\phi_{q'}^Q(r_2) = (1 + \alpha_{q'} r_2)\varphi_{q'}^Q(r_2) \tag{26}$$

with only one (two) free parameter for a ll or ss (ls) baryon light quark content. Besides, we construct the light–light correlation function, F^{B_Q}, from a linear combination of gaussians,

$$F^{B_Q}(r_{12}) = f^{B_Q}(r_{12})\sum_{j=1}^4 a_j e^{-b_j^2(r_{12}+d_j)^2}, \quad a_1 = 1 \tag{27}$$

$$f^{B_Q}(r_{12}) = \begin{cases} 1 - e^{-cr_{12}} & \text{if } V_{qq'}^B(r_{12}=0) \gg 0 \\ 1 \ (c \to +\infty) & \text{if } V_{qq'}^B(r_{12}=0) \le 0 \end{cases} \tag{28}$$

where $V_{qq'}^B$ denotes the light–light interaction projected onto the spin and isospin quantum numbers of the baryon B. The correlation function, F^{B_Q}, should vanish at large distances because of the confinement potential[6]. The function f^{B_Q} vanishes at the origin in those cases where $V_{qq'}^B$ is highly repulsive a short distances (this is the case for instance for the AL1χ interaction and $S_{\text{light}} = 1$, i.e. Σ, Σ^*-type baryons). Note that, one of the a_j parameters can be absorbed into the normalization constant N, thus besides the c and $\alpha_{q,q'}$ parameters, there are a total of eleven (we fix $a_1 = 1$) free parameters to be determined by the variational principle and the mass of the baryon is just the expected value of the intrinsic hamiltonian. Note that all interactions of Subsect. II B are diagonal in the flavor space, except for the pion exchange term of the AL1χ one, and that the following relationships are of interest to compute the kinetic energy expected values

$$\vec{\nabla}_1\left[\phi_1(r_1)\phi_2(r_2)F(r_{12})\right] = \frac{\vec{r}_1}{r_1}\frac{d\phi_1(r_1)}{dr_1}\phi_2(r_2)F(r_{12}) + \frac{\vec{r}_1 - \vec{r}_2}{r_{12}}\phi_1(r_1)\phi_2(r_2)\frac{dF(r_{12})}{dr_{12}}$$
$$\vec{\nabla}_2\left[\phi_1(r_1)\phi_2(r_2)F(r_{12})\right] = \frac{\vec{r}_2}{r_2}\phi_1(r_1)\frac{d\phi_2(r_2)}{dr_2}F(r_{12}) - \frac{\vec{r}_1 - \vec{r}_2}{r_{12}}\phi_1(r_1)\phi_2(r_2)\frac{dF(r_{12})}{dr_{12}} \tag{29}$$

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[4] The size of this term of the OGE interaction is proportional to the inverse of the masses of the involved quarks.

[5] Note that, the Jastrow form assumes a factorization of the wave function in three terms, each of them depends only on one of the involved variables r_1, r_2 and r_{12}. Dependences of the type $r_1 - r_2$, mentioned in the discussion of Eq. (16), cannot be accommodated with factorizable functions of this type.

[6] The confinement potential is also responsible for the non–vanishing values of the parameters α_q and α_q' in Eq. (26). Indeed, if at large distances the light–light interaction vanishes ($V_{qq'}^B(r_{12} \gg 1) \to 0$) then the product of single particle wave functions $\varphi_q^Q(r_1)\, \varphi_{q'}^Q(r_2)$ will provide the correct spatial dependence, at large distances, of the solution of the three body problem.

The expected value of the Hughes-Eckart contribution $(\vec{\nabla}_1 \cdot \vec{\nabla}_2)$ to the kinetic energy is readily obtained from the above equations after integrating by parts.

Given the spatial part of the baryon wave function, $\Psi_{qq'}^{B_Q}(r_1, r_2, r_{12})$, the probability, $\mathcal{P}_l$, of finding each of the two light quarks with angular momentum l, referred to the heavy quark, and coupled to $L = 0$ is given by

$$\mathcal{P}_l = 4\pi^2(2l+1) \int_0^{+\infty} dr_1 \, r_1^2 \int_0^{+\infty} dr_2 \, r_2^2 \left| \int_{-1}^{+1} d\mu \, P_l(\mu) \Psi_{qq'}^{B_Q}(r_1, r_2, r_{12}) \right|^2, \tag{30}$$

where P_l is the Legendre Polynomial of order l.

IV.   STATIC PROPERTIES: MASS AND CHARGE DENSITIES AND ORBITAL MAGNETIC MOMENTS

Since the charge operator is diagonal in the spin-flavor space, the baryon charge density at the point P (coordinate vector $\vec{r}$ in the CM frame, see Fig. 1) is given by:

$$\begin{aligned}
\rho_e^{B_Q}(\vec{r}) &= \int d^3R \, d^3r_1 d^3r_2 \left| \frac{e^{i\vec{P}_{CM}\vec{R}}}{\sqrt{V}} \Psi_{qq'}^{B_Q}(r_1, r_2, r_{12}) \right|^2 \left\{ e_Q \delta^3(\vec{r} - \vec{y}_h) + e_q \delta^3(\vec{r} - \vec{y}_1) + e_{q'} \delta^3(\vec{r} - \vec{y}_2) \right\} \\
&= \int d^3r_1 d^3r_2 \left| \Psi_{qq'}^{B_Q}(r_1, r_2, r_{12}) \right|^2 \left\{ e_Q \delta^3(\vec{r} - \vec{y}_h) + e_q \delta^3(\vec{r} - \vec{y}_1) + e_{q'} \delta^3(\vec{r} - \vec{y}_2) \right\} \\
&\equiv \rho_e^{B_Q}(\vec{r})\big|_Q + \rho_e^{B_Q}(\vec{r})\big|_q + \rho_e^{B_Q}(\vec{r})\big|_{q'}
\end{aligned} \tag{31}$$

where $\delta(\cdots)$ and $\vec{P}_{CM}$ are the three-dimensional Dirac's delta and the total baryon momentum respectively, V is the interacting volume[7], $e_{q,q',Q}$ are the quark charges, in proton charge units (e), and from Fig. 1 we have[8], $\vec{y}_h = -(m_q \vec{r}_1 + m_{q'} \vec{r}_2)/M$, $\vec{y}_1 = \vec{y}_h + \vec{r}_1$ and $\vec{y}_2 = \vec{y}_h + \vec{r}_2$. The charge density is spherically symmetric ($\rho_e^{B_Q}(\vec{r}) = \rho_e^{B_Q}(|\vec{r}|)$), since the wave function only depend on scalars (r_1, r_2 and $\vec{r}_1 \cdot \vec{r}_2$) for $L = 0$ baryons. The charge form factor is defined as usual

$$\mathcal{F}_e^{B_Q}(\vec{q}) = \int d^3r \, e^{i\vec{q}\cdot\vec{r}} \rho_e^{B_Q}(r) \tag{32}$$

and it only depends on $|\vec{q}|$. The charge mean square radii are defined

$$\langle r^2 \rangle_e^{B_Q} = \int d^3r r^2 \rho_e^{B_Q}(r) = 4\pi \int_0^{+\infty} dr r^4 \rho_e^{B_Q}(r) \tag{33}$$

The baryon mass density, $\rho_m^{B_Q}(r)$ is readily obtained from Eq. (31) with the obvious substitutions $(e_q, e_{q'}, e_Q) \to (m_q/M, m_{q'}/M, m_Q/M)$.

$$\begin{aligned}
\rho_m^{B_Q}(\vec{r}) &= \int d^3r_1 d^3r_2 \left| \Psi_{qq'}^{B_Q}(r_1, r_2, r_{12}) \right|^2 \left\{ \frac{m_Q}{M} \delta^3(\vec{r} - \vec{y}_h) + \frac{m_q}{M} \delta^3(\vec{r} - \vec{y}_1) + \frac{m_{q'}}{M} \delta^3(\vec{r} - \vec{y}_2) \right\} \\
&\equiv \rho_m^{B_Q}(\vec{r})\big|_Q + \rho_m^{B_Q}(\vec{r})\big|_q + \rho_m^{B_Q}(\vec{r})\big|_{q'}
\end{aligned} \tag{34}$$

where we have normalized $\rho_m^{B_Q}(r)$ to 1. The mass mean square radii are defined

$$\langle r^2 \rangle_m^{B_Q} = \int d^3r r^2 \rho_m^{B_Q}(r) = 4\pi \int_0^{+\infty} dr r^4 \rho_m^{B_Q}(r) \tag{35}$$

The intrinsic orbital magnetic moment is defined in terms of the velocities $\vec{v}_{h,1,2}$ of the quarks Q, q and q', with respect to the position of the CM, and it reads

$$\begin{aligned}
\vec{\mu}^{B_Q} &= \int d^3R \, d^3r_1 d^3r_2 \frac{e^{-i\vec{P}_{CM}\cdot\vec{R}}}{\sqrt{V}} \left(\Psi_{qq'}^{B_Q}(r_1, r_2, r_{12}) \right)^* \left\{ \frac{e_Q}{2m_Q}(\vec{y}_h \times m_Q \vec{v}_{y_h}) \right. \\
&\quad \left. + \frac{e_q}{2m_q}(\vec{y}_1 \times m_q \vec{v}_{y_1}) + \frac{e_{q'}}{2m_{q'}}(\vec{y}_2 \times m_{q'} \vec{v}_{y_2}) \right\} \frac{e^{i\vec{P}_{CM}\cdot\vec{R}}}{\sqrt{V}} \Psi_{qq'}^{B_Q}(r_1, r_2, r_{12})
\end{aligned} \tag{36}$$

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[7] It cancels out with $\int d^3R$.
[8] There exists the obvious restriction $m_Q \vec{y}_h + m_q \vec{y}_1 + m_{q'} \vec{y}_2 = 0$.

with[9] $m_q\vec{v}_{y_1} = -i\vec{\nabla}_1$, $m_{q'}\vec{v}_{y_2} = -i\vec{\nabla}_2$ and $m_Q\vec{v}_{y_h} = i\left(\vec{\nabla}_1 + \vec{\nabla}_2\right)$. Since the wave function only depends on scalars for a $L = 0$ baryon, the intrinsic orbital magnetic moment vanishes. Note that the intrinsic orbital magnetic moment is zero despite the quark pairs (Q, q) and (Q, q') are not in relative $s-$waves, as mentioned above in the discussion of Eq. (30). Thus, the magnetic moment of the baryon is entirely due to the spin contribution and since we are adopting HQS wave functions where the light degrees of freedom spin wave function factorizes, the magnetic moment should coincide to that obtained in the naive quark-model treatment of the baryon.

Finally and as a further test of our variational wave–functions, we compute some quantities which in Ref. [28] are denoted by $\rho_i(0)$ and called wave function at the origin for the (jk) pair, where (i, j, k) is a cyclic permutation of (Q, q, q'). In this reference the following Jacobi coordinates

$$\vec{s}_Q(\vec{r}_1, \vec{r}_2) = \vec{r}_1 - \vec{r}_2$$

$$\vec{s}_q(\vec{r}_1, \vec{r}_2) = a_q\vec{r}_2, \quad a_q = \sqrt{\frac{m_Q(m_q + m_{q'})}{m_q(m_Q + m_{q'})}}$$

$$\vec{s}_{q'}(\vec{r}_1, \vec{r}_2) = a_{q'}\vec{r}_1, \quad a_{q'} = -\sqrt{\frac{m_Q(m_q + m_{q'})}{m_{q'}(m_Q + m_q)}} \tag{37}$$

and correlation functions[10]

$$\eta_i^{B_Q}(s) = \frac{1}{s^2}\int d^3r_1 d^3r_2 \left|\Psi_{qq'}^{B_Q}(r_1, r_2, r_{12})\right|^2 \delta\left(s - \left|\vec{s}_i(\vec{r}_1, \vec{r}_2)\right|\right), \quad i = Q, q, q' \tag{38}$$

are introduced. Thus, the wave functions at the origin are defined by [28]

$$\eta_Q^{B_Q}(s = 0) = 16\pi^2 \int_0^{+\infty} dt t^2 \left|\Psi_{qq'}^{B_Q}(r_1 = t, r_2 = t, r_{12} = 0)\right|^2$$

$$\eta_q^{B_Q}(s = 0) = \frac{16\pi^2}{a_q^3}\int_0^{+\infty} dt t^2 \left|\Psi_{qq'}^{B_Q}(r_1 = t, r_2 = 0, r_{12} = t)\right|^2$$

$$\eta_{q'}^{B_Q}(s = 0) = \frac{16\pi^2}{|a_{q'}^3|}\int_0^{+\infty} dt t^2 \left|\Psi_{qq'}^{B_Q}(r_1 = 0, r_2 = t, r_{12} = t)\right|^2 \tag{39}$$

V.   RESULTS

In Table III we present variational results for the charmed and bottom baryon masses obtained with each of the six quark–quark interactions considered in this work. We use an integration step of 10^{-2} fm and we estimate in 0.5 MeV the numerical error of the variational masses given in the table. On the other hand, by including a higher number of gaussians[11] in the light–light correlation function (Eq. (27)), or quadratic or cubic terms in the one body wave–functions (Eq. (26)), the variational energies cannot be lowered in more than 0.5 MeV. The corresponding wave functions can be reconstructed from the parameters given in Tables X and XI in the Appendix. We use a *Simplex* algorithm to find out the position of the minima [36] and typically for each baryon the minimization procedure, including the construction of the wave function, takes around twenty minutes in a 2 GHz Pentium IV processor.

Besides, in Table III, we compare, when possible, with the Faddeev results of Ref. [28]. We find an excellent agreement between our variational results and those of Ref. [28]. In most cases we find discrepancies smaller[12] than 3 MeV, and only in three cases (BD Ω_c, Σ_b and AL2 Λ_b baryons) Faddeev and variational masses differ in more than 5 MeV. Faddeev masses are usually smaller than variational ones, but in some cases the variational approach presented here provides lower values, and thus better estimates of the baryon masses (see f.i. AL2 Λ_b, where we find a mass of

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[9] Note that the classical kinetic energy has a term on $\vec{v}_{y_1} \cdot \vec{v}_{y_2}$ and then the operator $m_q\vec{v}_{y_1}$ is not proportional to $-i\overrightarrow{\nabla}_{y_1}$, but it is rather given by $m_q\vec{v}_{y_1} = \frac{M - m_q}{M}(-i\overrightarrow{\nabla}_{y_1}) - \frac{m_{q'}}{M}(-i\overrightarrow{\nabla}_{y_2}) = (-i\overrightarrow{\nabla}_1)$. Similarly $m_{q'}\vec{v}_{y_2} = (-i\overrightarrow{\nabla}_2)$.

[10] These correlation functions satisfy $\int_0^{+\infty} ds s^2 \eta_i^{B_Q}(s) = 1$.

[11] In the Appendix (Table IX), we illustrate, for the Λ_c baryon with an AL1 inter-quark interaction, the dependence of the VAR results on the number of gaussians.

[12] Such differences are certainly small fractions of the baryon binding energies.

Baryon		Charm							Bottom						
		BD	AL2	AP1	AP2	AL1	AL1χ	Exp	BD	AL2	AP1	AP2	AL1	AL1χ	Exp
Λ	FAD	2321	2302	2311	2313	2296	–		5660	5637	5659	5643	5643	–	
	VAR	2320	2305	2308	2310	2295	2140	2285 ± 1	5663	5629	5659	5643	5643	5475	5624 ± 9
Σ	FAD	2494	2474	2484	2491	2466	–		5865	5844	5868	5859	5849	–	
	VAR	2498	2479	2484	2491	2469	2457	2452 ± 1	5871	5844	5870	5861	5851	5841	$5770^{\dagger} \pm 70$
Σ*	VAR	2570	2559	2560	2571	2548	2535	2518 ± 2	5900	5874	5899	5893	5882	5869	$5780^{\dagger} \pm 70$
Ξ	FAD	2502	2480	2485	2488	2473	–		5830	5803	5819	5805	5808	–	
	VAR	2506	2480	2485	2488	2474	–	2469 ± 3	5832	5800	5820	5807	5808	–	$5760^{\dagger} \pm 70$
Ξ′	VAR	2606	2584	2587	2592	2578	–	2576 ± 3	5963	5939	5959	5946	5946	–	$5900^{\dagger} \pm 70$
Ξ*	VAR	2671	2661	2664	2672	2655	–	2646 ± 2	5989	5970	5987	5979	5975	–	$5900^{\dagger} \pm 80$
Ω	FAD	2707	2686	2678	2682	2678	–		6048	6029	6033	6020	6035	–	
	VAR	2713	2683	2682	2684	2681	–	2698 ± 3	6050	6030	6036	6023	6033	–	$5990^{\dagger} \pm 70$
Ω*	VAR	2770	2759	2759	2765	2755	–	$2660^{\dagger} \pm 80$	6074	6061	6068	6056	6063	–	$6000^{\dagger} \pm 70$

TABLE III: Masses (in MeV) of the ground states of different baryons with the six potentials considered in this work. Variational and Faddeev results are given in the rows denoted by VAR and FAD, respectively. The latter ones are taken from Ref. [28], but the effect of the three body forces considered in that reference, $V_{123} = \text{constant}/m_q m_{q'} m_Q$, has been eliminated. Experimental masses are taken from Ref. [10] when possible, and the lattice QCD estimates of Ref. [12] are quoted for those cases (†) where the experimental masses are not known.

around 8 MeV smaller than that quoted in Ref. [28]). Having in mind that the Faddeev masses suffer from numerical uncertainties of the order of 5 MeV [28], we can safely conclude that our variational approach reproduces the ground state masses quoted in Ref. [28]. Thus, corrections to the leading HQS wave functions (Eq. (15) and Eqs. (17)–(23)) should provide quite small (let us say, smaller than 5 MeV) changes in ground state masses, even for the charm sector.

We have also computed the Σ^*, Ξ', Ξ^* and Ω^* ground state masses, which were not evaluated in the work of Ref. [28]. Note that, the $\vec{\sigma}_q \cdot \vec{\sigma}_Q/m_q m_Q$ OGE term breaks[13] the mass degeneration between the S_{light} triplet states (Σ-Σ^*, Ξ'-Ξ^* and Ω-Ω^*). The five phenomenological interactions studied in Ref. [28] lead to ground state masses which compare reasonably well with the existing experimental data and lattice QCD mass estimates for all baryons.

Besides, we see that while the AL1 and AL1χ interactions lead to similar masses (differences of the order of 10 MeV) for the Σ_Q and Σ_Q^* baryons, the chirally inspired potential, AL1χ, predicts masses for the Λ_Q baryons of about 155 (charm sector) and 168 (bottom sector) MeV smaller than those obtained from the phenomenological AL1 interaction. In terms of binding energies ($B_Q = M[\Lambda_Q] - m_Q - m_u - m_d$), we have relative changes as large as 90%,

$$\left| \frac{B_c^{\text{AL1}} - B_c^{\text{AL1}\chi}}{B_c^{\text{AL1}}} \right| = 0.9, \qquad \left| \frac{B_b^{\text{AL1}} - B_b^{\text{AL1}\chi}}{B_b^{\text{AL1}}} \right| = 0.7 \qquad (40)$$

Such big changes are due to the different behaviour of the One Pion Exchange (OPE) potential in the light quark–quark and quark–antiquark sectors. In the first case we have, from Eq. (12), the operator $(\vec{\sigma}_i \vec{\sigma}_j)(\vec{\tau}_i \vec{\tau}_j)$ which provides a factor 9 (1) for the light quantum numbers of the Λ_Q (Σ_Q or Σ_Q^*) baryon. The spin–isospin operator turns out to be $-(\vec{\sigma}_i \vec{\sigma}_j)(\vec{\tau}_i \vec{\tau}_j)$ for the case of quark–antiquark interactions, and thus one gets a factor 3 (-1) for the light quantum numbers of the π (ρ) meson. In averaged, both potentials, AL1 and AL1χ, are strongly attractive (moderately repulsive) for the π (ρ) quantum numbers, and have been adjusted to provide the same ground state pion (rho) mass. For baryons, the AL1 potential gets reduced its strength by a factor of two (because of the $V_{ij}^{qq} = V_{ij}^{q\bar{q}}/2$ prescription, coming from a $\vec{\lambda}_i \vec{\lambda}_j$ color dependence), however this is not the case for the light sector of the AL1χ potential. In this latter case, the OGE and confinement terms follow the former prescription, but while the scalar exchange remains unaltered, the OPE term is three times more attractive for the Λ_Q baryon than in the pion. The resulting effect is that the light sector of the AL1χ interaction in the Λ_Q baryon gets reduced its strength by a factor significantly smaller than 2. The OPE exchange plays a minor role in the spin triplet states[14] (ρ, Σ_Q and Σ_Q^*), and that explains why the AL1 and AL1χ potentials predict similar Σ_Q and Σ_Q^* masses.

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[13] For s–wave baryons, the spin operator $\vec{\sigma}_q \cdot \vec{\sigma}_Q$ vanishes for $S_{\text{light}} = 0$ wave functions and for spin triplet states, $S_{\text{light}} = 1$, takes the values 1 and -2, for baryon total angular momentum $J = \frac{3}{2}$ and $\frac{1}{2}$, respectively.

[14] For instance, while the OPE is responsible for around 150 MeV of the binding energy of the pion, it contributes only about 8 MeV to the rho meson mass.

Baryon		Charm						Bottom					
		BD	AL2	AP1	AP2	AL1	AL1χ	BD	AL2	AP1	AP2	AL1	AL1χ
Λ	FAD	0.097	0.101	0.109	0.103	0.104	–	0.043	0.044	0.046	0.044	0.045	–
	VAR	0.100	0.101	0.108	0.106	0.106	0.101	0.045	0.043	0.048	0.045	0.045	0.042
Σ	FAD	0.111	0.117	0.128	0.121	0.121	–	0.051	0.053	0.056	0.053	0.054	–
	VAR	0.112	0.118	0.127	0.118	0.123	0.123	0.051	0.053	0.055	0.053	0.057	0.059
Σ^*	VAR	0.122	0.131	0.138	0.131	0.135	0.139	0.052	0.056	0.058	0.056	0.060	0.061
Ξ	FAD	0.097	0.100	0.107	0.101	0.104	–	0.045	0.046	0.048	0.046	0.048	–
	VAR	0.099	0.103	0.110	0.104	0.105	–	0.047	0.046	0.050	0.047	0.049	–
Ξ'	VAR	0.106	0.110	0.122	0.111	0.119	–	0.053	0.053	0.059	0.052	0.060	–
Ξ^*	VAR	0.115	0.123	0.133	0.126	0.123	–	0.052	0.057	0.060	0.054	0.059	–
Ω	FAD	0.100	0.103	0.110	0.102	0.108	–	0.050	0.052	0.055	0.051	0.054	–
	VAR	0.096	0.101	0.110	0.103	0.108	–	0.050	0.054	0.057	0.053	0.057	–
Ω^*	VAR	0.107	0.117	0.122	0.113	0.120	–	0.051	0.055	0.061	0.056	0.059	–

TABLE IV: Mass mean square radii in fm^2 units, see Eq. (35). Variational and Faddeev [28] results are denoted by VAR and FAD, respectively.

Baryon		Charm						Bottom					
		BD	AL2	AP1	AP2	AL1	AL1χ	BD	AL2	AP1	AP2	AL1	AL1χ
Λ_c^+ , Λ_b^0	FAD	0.117	0.124	0.147	0.139	0.129		0.115	0.123	0.148	0.140	0.128	
	VAR	0.120	0.125	0.145	0.142	0.131	0.124	0.121	0.122	0.152	0.143	0.127	0.119
Σ_c^0 , Σ_b^-	FAD	−0.224	−0.245	−0.304	−0.287	−0.256		−0.280	−0.305	−0.369	−0.347	−0.318	
	VAR	−0.226	−0.248	−0.299	−0.279	−0.261	−0.260	−0.279	−0.304	−0.357	−0.341	−0.332	−0.345
Σ_c^+ , Σ_b^0	FAD	0.134	0.145	0.174	0.164	0.151		0.138	0.151	0.183	0.172	0.157	
	VAR	0.135	0.147	0.172	0.160	0.153	0.154	0.138	0.150	0.176	0.169	0.164	0.171
Σ_c^{++} , Σ_b^+	FAD	0.494	0.535	0.652	0.615	0.557		0.555	0.607	0.734	0.692	0.633	
	VAR	0.497	0.541	0.643	0.599	0.568	0.567	0.555	0.603	0.710	0.679	0.659	0.687
Σ_c^{*0} , Σ_b^{*-}	VAR	−0.246	−0.274	−0.323	−0.307	−0.283	−0.293	−0.289	−0.322	−0.376	−0.364	−0.349	−0.358
Σ_c^{*+} , Σ_b^{*0}	VAR	0.148	0.163	0.187	0.177	0.168	0.173	0.143	0.159	0.186	0.180	0.172	0.177
Σ_c^{*++} , Σ_b^{*+}	VAR	0.541	0.599	0.697	0.661	0.619	0.639	0.574	0.639	0.747	0.725	0.694	0.711
Ξ_c^0 , Ξ_b^-	FAD	−0.145	−0.154	−0.183	−0.171	−0.161		−0.193	−0.203	−0.234	−0.219	−0.212	
	VAR	−0.148	−0.156	−0.182	−0.174	−0.163		−0.198	−0.201	−0.240	−0.220	−0.213	
Ξ_c^+ , Ξ_b^0	FAD	0.160	0.170	0.206	0.196	0.177		0.151	0.161	0.197	0.187	0.168	
	VAR	0.158	0.170	0.198	0.196	0.179		0.150	0.151	0.189	0.180	0.158	
$\Xi_c^{\prime 0}$, $\Xi_b^{\prime -}$	VAR	−0.164	−0.179	−0.213	−0.196	−0.192		−0.228	−0.235	−0.285	−0.248	−0.267	
$\Xi_c^{\prime +}$, $\Xi_b^{\prime 0}$	VAR	0.174	0.199	0.234	0.218	0.207		0.184	0.188	0.234	0.209	0.214	
Ξ_c^{*0} , Ξ_b^{*-}	VAR	−0.177	−0.196	−0.232	−0.217	−0.198		−0.224	−0.250	−0.287	−0.259	−0.266	
Ξ_c^{*+} , Ξ_b^{*0}	VAR	0.190	0.218	0.254	0.241	0.213		0.179	0.201	0.236	0.221	0.212	
Ω_c^0 , Ω_b^-	FAD	−0.111	−0.117	−0.131	−0.120	−0.124		−0.164	−0.173	−0.191	−0.174	−0.183	
	VAR	−0.108	−0.115	−0.131	−0.120	−0.124		−0.163	−0.177	−0.196	−0.180	−0.189	
Ω_c^{*0} , Ω_b^{*-}	VAR	−0.118	−0.132	−0.143	−0.131	−0.138		−0.167	−0.182	−0.210	−0.189	−0.196	

TABLE V: Charge mean square radii in $e\mathrm{fm}^2$ units, see Eq. (33). Variational and Faddeev [28] results are denoted by VAR and FAD, respectively.

We have also computed mass (Table IV) and charge (Table V) mean square radii, charge form factors (Figs. 2–3) and quark mass densities (Figs. 4–5), for all baryons and the six inter-quark interactions considered in this work. In Tables IV and V, when possible, we compare our variational results with those quoted in Ref. [28] and find an excellent agreement, with tiny differences which at most are of the order of 3-4%. Differences between AL1 and AL1χ potentials predictions, in Tables IV and V and in Figs. 2–5, are also quite small and it will be quite difficult to use those to experimentally disentangle among both interactions.

To further test the goodness of our variational baryon wave functions, we have followed Ref. [28] and we have also computed the so called wave functions at the origin for the different quark pairs inside the heavy baryon (see Eq. (39)). It seems [28] that the absolute value of this quantity depends dramatically on the numerical procedure

used to solve the three body problem[15] and then is a perfect observable to check the validity of our approach. Results for the different inter-quark interactions and heavy baryons can be found in Table VI, where, when possible, we compare our variational results with those quoted in Ref. [28], as well. For $\Lambda-$, $\Sigma-$ and $\Omega-$type baryons, we find that our variational results and those obtained from the Faddeev calculation of Ref. [28], agree[16] within a 10 or 20%. This constitutes a remarkable result, since, as it is mentioned in Ref. [28], it is not surprising to have approximate treatments of the three body problem giving values for the wave function at the origin off by almost an order of magnitude.

However, variational and Faddeev results are in total disagreement for the Ξ baryons (discrepancies of about a factor three or larger). We have no explanation for this fact, beyond that saying that we might have misunderstood the definition of Ref. [28] of the wave functions at the origin and thus Eq. (39) is wrong or that our $\Xi-$wave functions at the origin are wrong. We very much doubt this latter possibility, and we would like to point out that the Faddeev values for the $\Xi-$baryons are abnormally big (about a factor four), when compared to those quoted for the rest of the baryons Λ, Σ and Ω. We are not aware of any reason to support/explain this fact and indeed we do not observe such a situation in the variational results.

To finish this section we would like to devote a few words to the multipolar content of the variational wave functions. In Tables VII and VIII we show the probability $\mathcal{P}_l$, defined in Eq. (30), of finding each of the two light–quarks with angular momentum l and coupled to $L = 0$ for the Λ and Σ baryons. Results obtained with two different interactions (AL1 and AL1χ) are compiled, but the gross features of the multipolar decomposition do not depend significantly on the selected interaction. Only multipoles up to the waves $l = 3$ or $l = 4$ significantly contribute[17], being $l = 0$ the dominant one.

VI.   CONCLUSIONS AND OUTLOOK

We have developed a variational scheme to describe the charm and bottom baryons compiled in Table I. We compute baryon masses and wave functions from the variational principle applied to a family of Jastrow type functions constrained by HQS. For several inter-quark interactions and baryons, we reproduce the Faddeev results of Ref. [28] for masses, charge and mass radii, and wave functions at the origin. Thus, we conclude that the finite heavy quark mass corrections to the wave-functions should be small, even for charmed baryons. Besides of giving results for baryons not studied in Ref. [28] (Σ^*, Ξ', Ξ^* and Ω^*), we provide, in Tables X and XI in the Appendix, wave functions, parameterized in a simple manner, for all baryons compiled in Table I and six different inter-quark interactions. Thanks to HQS, the baryon wave functions are significantly simpler and more manageable than those obtained from Faddeev calculations and they can be used by the community to compute further observables, without having to solve the involved three body problem. Hence, we have shown in this context, once more, that HQS is a useful tool to understand the bottom and charm physics. We have also worked out the predictions for the strangeness-less baryons of the SU(2) chirally inspired quark-quark interactions developed in Ref. [22, 29]. The chirally inspired potential, AL1χ, predicts masses for the Λ_Q baryons of about 155 (charm sector) and 168 (bottom sector) MeV smaller than those obtained from the phenomenological AL1 interaction. Assuming that the inclusion of a pattern of SCSB should lead to a more theoretically founded scenario to address hadron spectroscopy, the findings of this work might hint the existence of sizeable three quarks forces with the Λ_Q quantum numbers. New SU(3) chirally inspired quark-quark interactions, valid also for the s quark (exchanges of eta and kaon mesons are also introduced) are being developed by the Salamanca group [37]. It would be interesting to work out the predictions for the strangeness baryons with this new set of chirally inspired interactions, once the potential becomes available.

Using the semi–analytical wave functions found in this work, we plan to compute the semileptonic decay of the bottom baryons into the charmed ones. The mixing parameter $|V_{cb}|$ of the Cabibbo-Kobayashi-Maskawa matrix can be determined from the study of these processes. Furthermore, the Isgur-Wise form-factor, which is a universal

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[15] Indeed, the wave function at the origin is especially sensitive to short-range correlations which are not precisely taken into account by approximate treatments.

[16] Note that for Σ baryons and when the BD interaction is used, there exists a serious discrepancy between Faddeev and variational predictions for the ll pair wave function at the origin. Indeed, the Faddeev prediction is non-zero, while it is zero within the variational approach. This is because our variational scheme assumes a pure $S_{\text{light}} = 1$ configuration for this baryon. It turns out, that the spin triplet light quark–light quark BD potential is infinitely repulsive for $r_{12} = 0$ and conversely the variational wave function vanishes at the origin (because of the correlation function f^{BQ}, in Eq. (28)). However in the Faddeev calculation of Ref. [28], the HQS suppressed $S_{\text{light}} = 0$ component of the Σ wave function is kept, and since for the spin singlet configuration the BD potential becomes attractive, a non-vanishing wave function at the origin for the ll pair is found in Ref. [28].

[17] Note that we show probabilities: the components of the wave function for each value of l are given, up to a sign, by the squared root of the numbers presented in the tables.

Baryon	Pair		Charm						Bottom					
			BD	AL2	AP1	AP2	AL1	AL1χ	BD	AL2	AP1	AP2	AL1	AL1χ
	ll	FAD	23.1	15.0	12.1	11.9	14.8	–	23.3	15.2	12.2	12.1	14.9	–
	ll	VAR	22.8	18.5	14.0	13.9	18.6	54.0	19.5	19.1	14.4	14.8	18.3	74.7
	Ql	FAD	8.8	7.7	6.1	6.4	7.4	–	8.8	7.7	6.1	6.4	7.4	–
Λ	Ql	VAR	7.6	7.0	5.6	5.5	6.5	6.0	7.5	7.9	4.9	5.6	6.9	7.0
	ll	FAD	6.9	6.6	5.0	5.6	6.1	–	6.8	6.4	4.9	5.4	6.0	–
	ll	VAR	0.0	6.8	5.2	6.2	5.7	0.0	0.0	6.9	5.5	5.3	6.7	0.0
	Ql	FAD	8.2	6.7	5.3	5.5	6.5	–	7.1	5.9	4.7	4.9	5.7	–
Σ	Ql	VAR	7.3	6.0	4.6	5.2	5.9	5.5	6.7	6.1	3.9	4.5	4.8	4.2
	ll	VAR	0.0	6.6	4.7	5.3	5.9	0.0	0.0	6.3	5.1	5.2	6.4	0.0
Σ^*	Ql	VAR	5.7	4.3	3.5	3.8	4.4	4.1	6.4	5.2	3.3	3.5	4.0	3.9
	ls	FAD	70.5	54.6	51.0	52.1	53.5	–	93.1	72.0	66.8	68.4	70.2	–
	ls	VAR	21.1	21.9	17.0	18.1	19.3	–	23.8	20.4	15.6	17.2	20.2	–
	Ql	FAD	40.0	37.0	34.4	37.0	35.5	–	51.4	48.1	44.6	48.0	46.1	–
	Ql	VAR	11.7	9.5	8.3	8.7	9.1	–	11.6	11.9	8.7	10.2	10.0	–
	Qs	FAD	30.8	27.7	25.3	26.4	27.1	–	39.9	35.5	32.4	33.4	34.9	–
Ξ	Qs	VAR	8.8	7.5	6.1	6.8	7.8	–	8.7	8.3	6.1	7.2	7.3	–
	ls	VAR	12.5	10.3	8.4	9.2	9.6	–	11.9	10.1	8.2	9.1	9.2	–
	Ql	VAR	11.3	9.5	7.1	9.0	8.2	–	9.8	9.5	5.8	8.6	6.7	–
Ξ'	Qs	VAR	8.7	8.2	5.8	6.9	6.5	–	8.2	7.3	4.7	6.3	5.5	–
	ls	VAR	12.1	8.8	6.8	7.9	8.9	–	12.1	9.3	7.9	8.4	9.4	–
	Ql	VAR	8.5	6.4	5.8	6.0	7.7	–	10.4	8.0	5.7	7.7	6.8	–
Ξ^*	Qs	VAR	6.7	5.7	4.8	4.6	6.1	–	8.4	6.3	4.7	6.0	5.5	–
	ss	FAD	16.5	15.5	13.8	15.3	14.6	–	16.7	15.6	13.7	15.3	14.6	–
	ss	VAR	17.7	17.2	14.8	16.9	15.7	–	19.9	15.8	13.1	16.9	15.5	–
	Qs	FAD	17.8	15.4	14.4	15.0	15.1	–	16.0	13.8	12.8	13.3	13.5	–
Ω	Qs	VAR	17.0	14.5	12.3	13.5	13.0	–	15.3	12.5	10.8	11.6	11.1	–
	ss	VAR	18.7	13.5	13.5	14.9	13.0	–	18.4	15.7	12.5	15.0	14.3	–
Ω^*	Qs	VAR	12.6	9.9	9.4	10.2	9.7	–	15.0	11.7	9.4	10.7	10.4	–

TABLE VI: Wave function at the origin, $\eta_i^{B_Q}(0)$ in fm^{-3} units, see Eq. (39). In the second column, ll and ss pairs stand for $\eta_Q^{B_Q^{ll}}(0)$ and $\eta_Q^{B_Q^{ss}}(0)$. Besides for strange [strangeness-less] baryons, the pairs Ql and Qs stand for $\eta_{q'=s}^{B_Q^{ls}}(0)$ $\left[\eta_{q=l}^{B_Q^{ll}}(0) = \eta_{q'=l}^{B_Q^{ll}}(0) \right]$ and $\eta_{q=l}^{B_Q^{ls}}(0)$. Variational and Faddeev [28] results are denoted by VAR and FAD, respectively. For the Σ and Σ^* baryons, our variational scheme assumes a pure $S_{\text{light}} = 1$ configuration. The VAR estimates for $\eta_Q^{B_Q^{ll}}(0)$, and BD and AL1χ interactions, vanish for these baryons, since the spin triplet light quark–light quark forces are infinitely repulsive at $r_{12} = 0$, for these potentials.

	Charm							Bottom						
l	0	1	2	3	4	5	6	0	1	2	3	4	5	6
AL1χ–$\mathcal{P}_l$	0.75495	0.18706	0.04066	0.01103	0.00368	0.00143	0.00062	0.79079	0.16186	0.03237	0.00894	0.00324	0.00139	0.00067
$\sum_{k=0}^{l} \mathcal{P}_k$	0.7550	0.9420	0.9827	0.9937	0.9974	0.9988	0.9994	0.7908	0.9526	0.9850	0.9940	0.9972	0.9986	0.9993
AL1–$\mathcal{P}_l$	0.87389	0.10972	0.01346	0.00227	0.00052	0.00015	0.00005	0.87368	0.11207	0.01221	0.00169	0.00032	0.00008	0.00003
$\sum_{k=0}^{l} \mathcal{P}_k$	0.8739	0.9836	0.9971	0.9993	0.9999	1.0000	1.0000	0.8737	0.9857	0.9980	0.9996	1.0000	1.0000	1.0000

TABLE VII: Probabilities $\mathcal{P}_l$ (defined in Eq. (30)) for several waves. Results were obtained for the Λ baryon and AL1 and AL1χ inter-quark interaction. The errors are always less than one unit in the last digit.

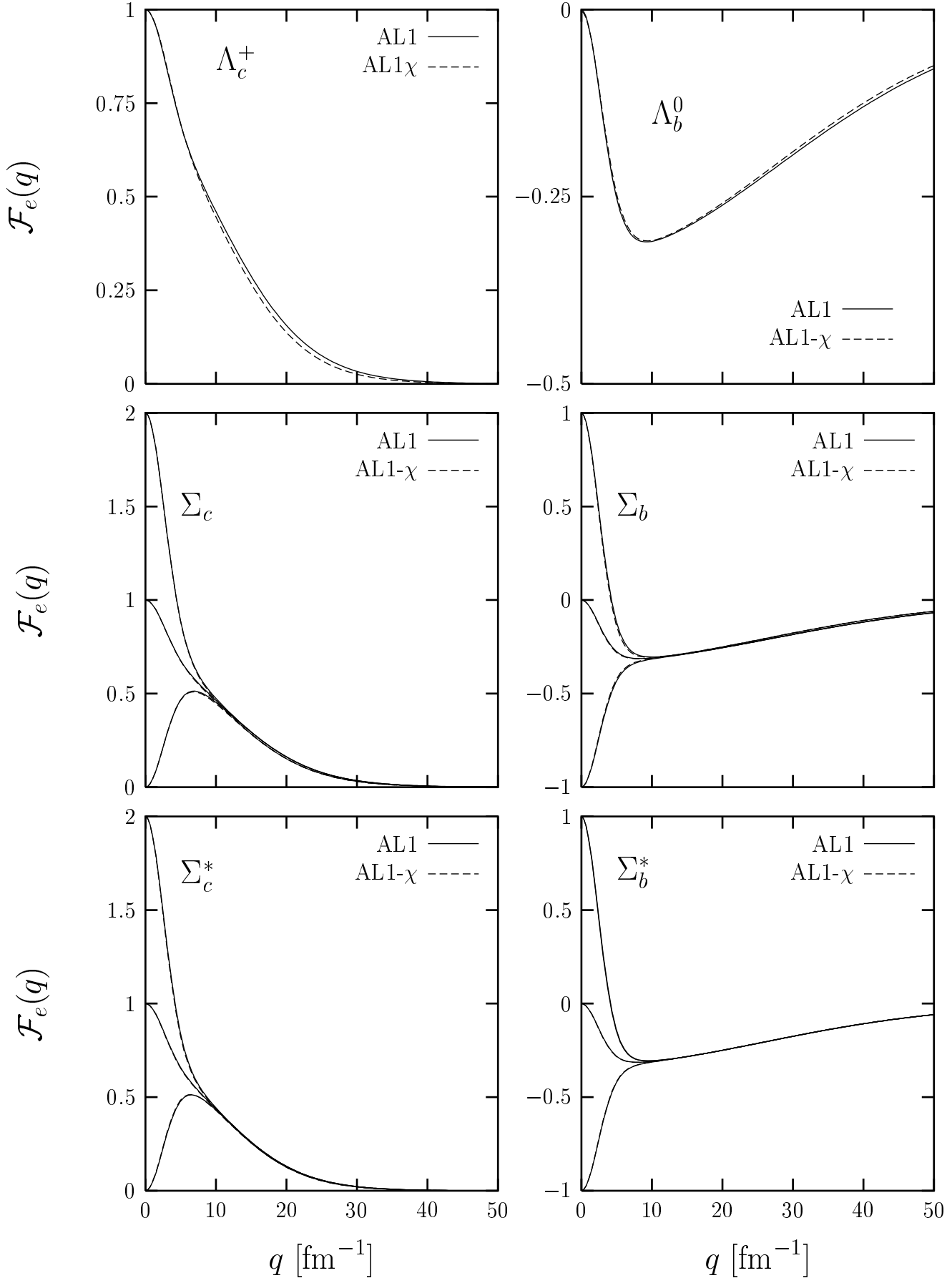|	Charm							Bottom						
l	0	1	2	3	4	5	6	0	1	2	3	4	5	6
AL1χ–$\mathcal{P}_l$	0.95074	0.04711	0.00210	0.00014	0.00002	0.00000	0.00000	0.95010	0.04777	0.00204	0.00013	0.00002	0.00000	0.00000
$\sum_{k=0}^{l} \mathcal{P}_k$	0.9507	0.9978	0.9999	1.0000	1.0000	1.0000	1.0000	0.9501	0.9979	0.9999	1.0000	1.0000	1.0000	1.0000
AL1–$\mathcal{P}_l$	0.96542	0.03377	0.00091	0.00003	0.00000	0.00000	0.00000	0.96109	0.03734	0.00153	0.00011	0.00001	0.00000	0.00000
$\sum_{k=0}^{l} \mathcal{P}_k$	0.9654	0.9992	1.0000	1.0000	1.0000	1.0000	1.0000	0.9611	0.9984	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE VIII: Same as Table VII for the Σ baryon.

FIG. 2: Charge form factors (Eq. (32)) of strangeness-less charmed and bottom baryons obtained with AL1, AL1χ interactions. The only appreciable difference, as it is also the case for the spectrum, appears for the Λ_c and Λ_b baryons. Note that, $\mathcal{F}_e(0)$ determines the charge of the baryon.
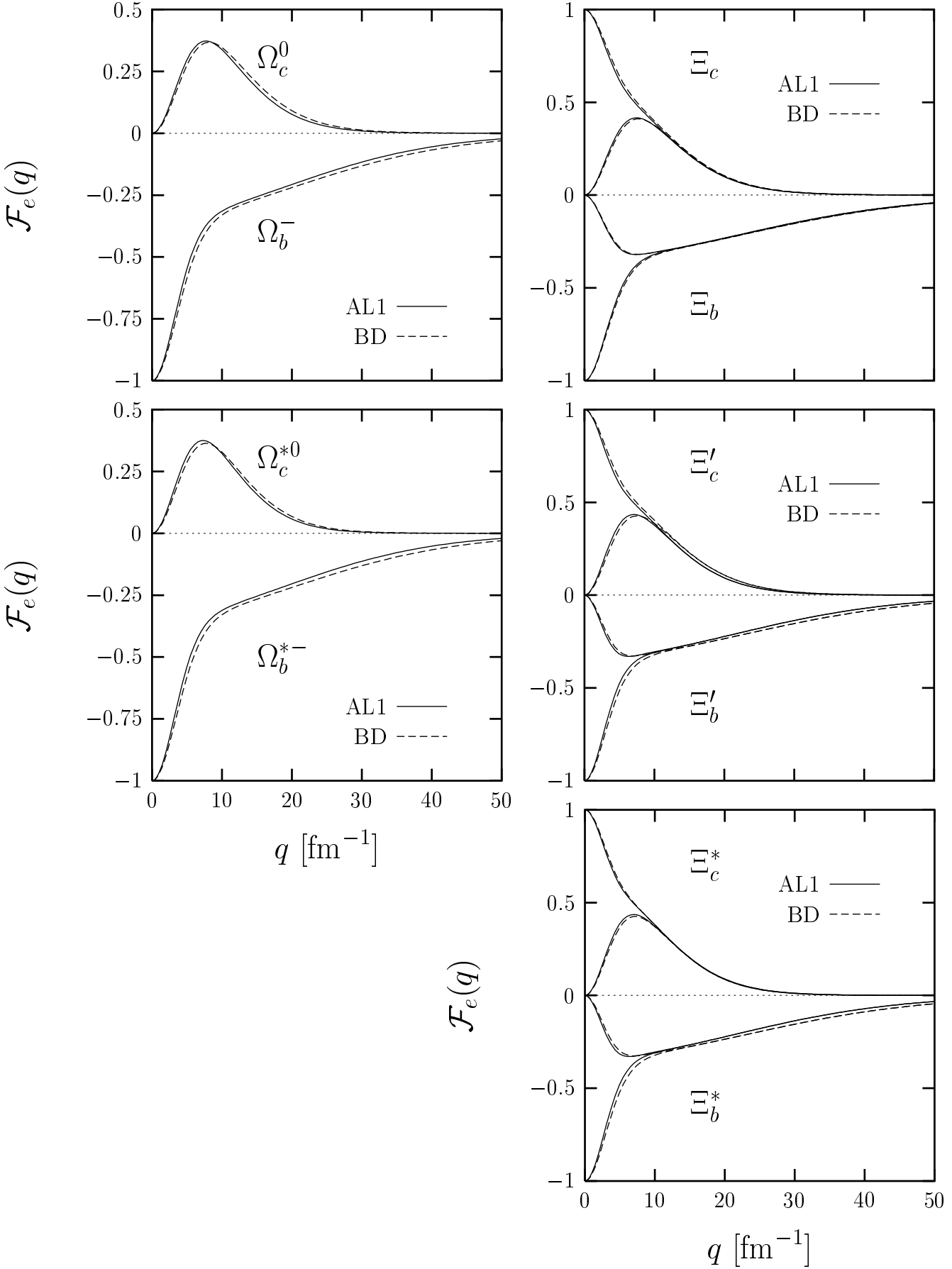
FIG. 3: Charge form factors (Eq. (32)) of strangeness charmed and bottom baryons obtained with AL1 and BD quark–quark interactions. The charge of the baryon is given, in units of the proton charge, by the value of the form factor at the origin.
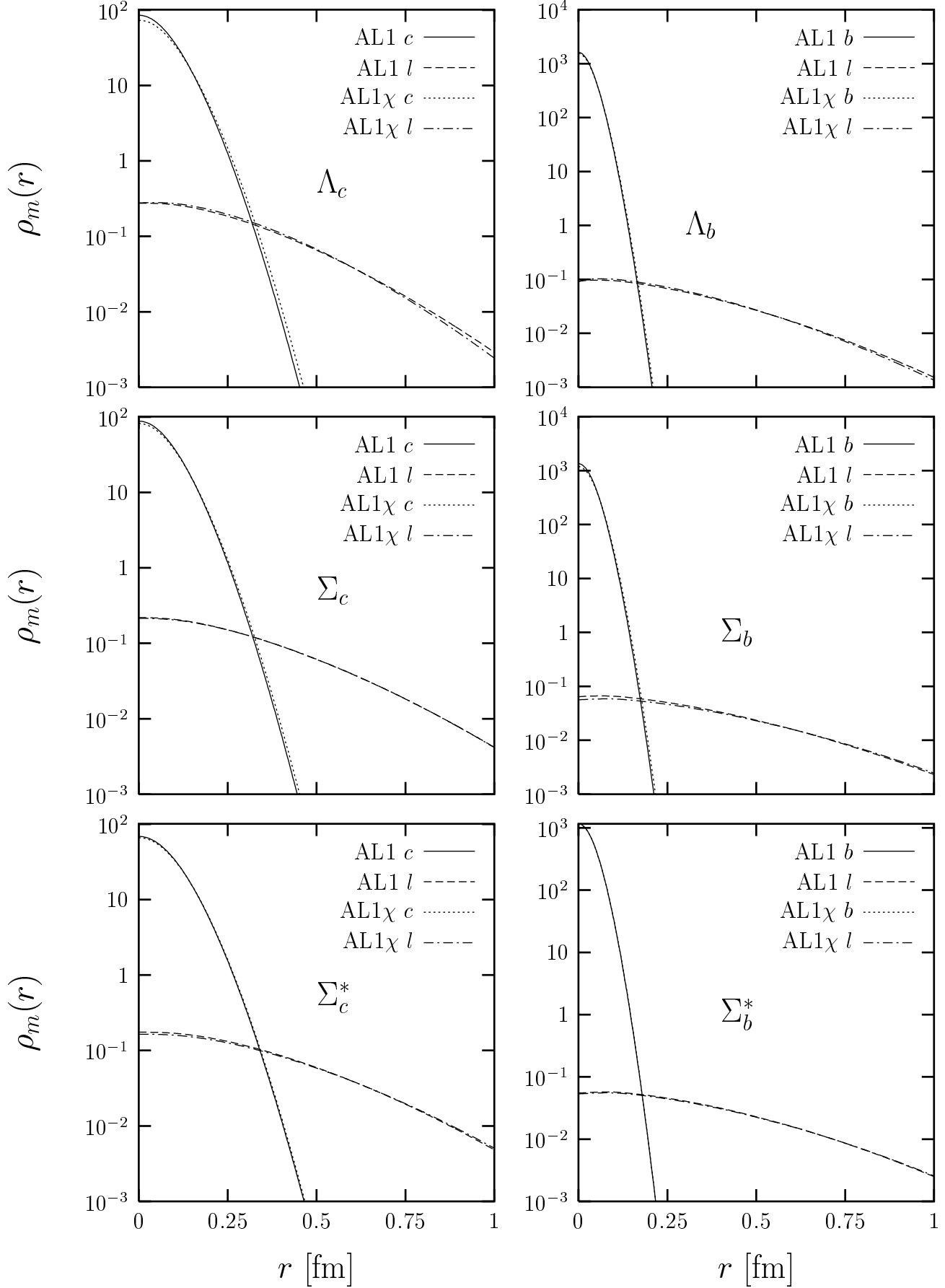
FIG. 4: Quark mass densities [fm^{-3}], $\rho_m^{B_Q}(\vec{r})\big|_{Q=c,b}$ and $\rho_m^{B_Q}(\vec{r})\big|_{q=l}$ (Eq. (34)), of strangeness-less charmed and bottom baryons obtained with AL1 and AL1χ quark–quark interactions.
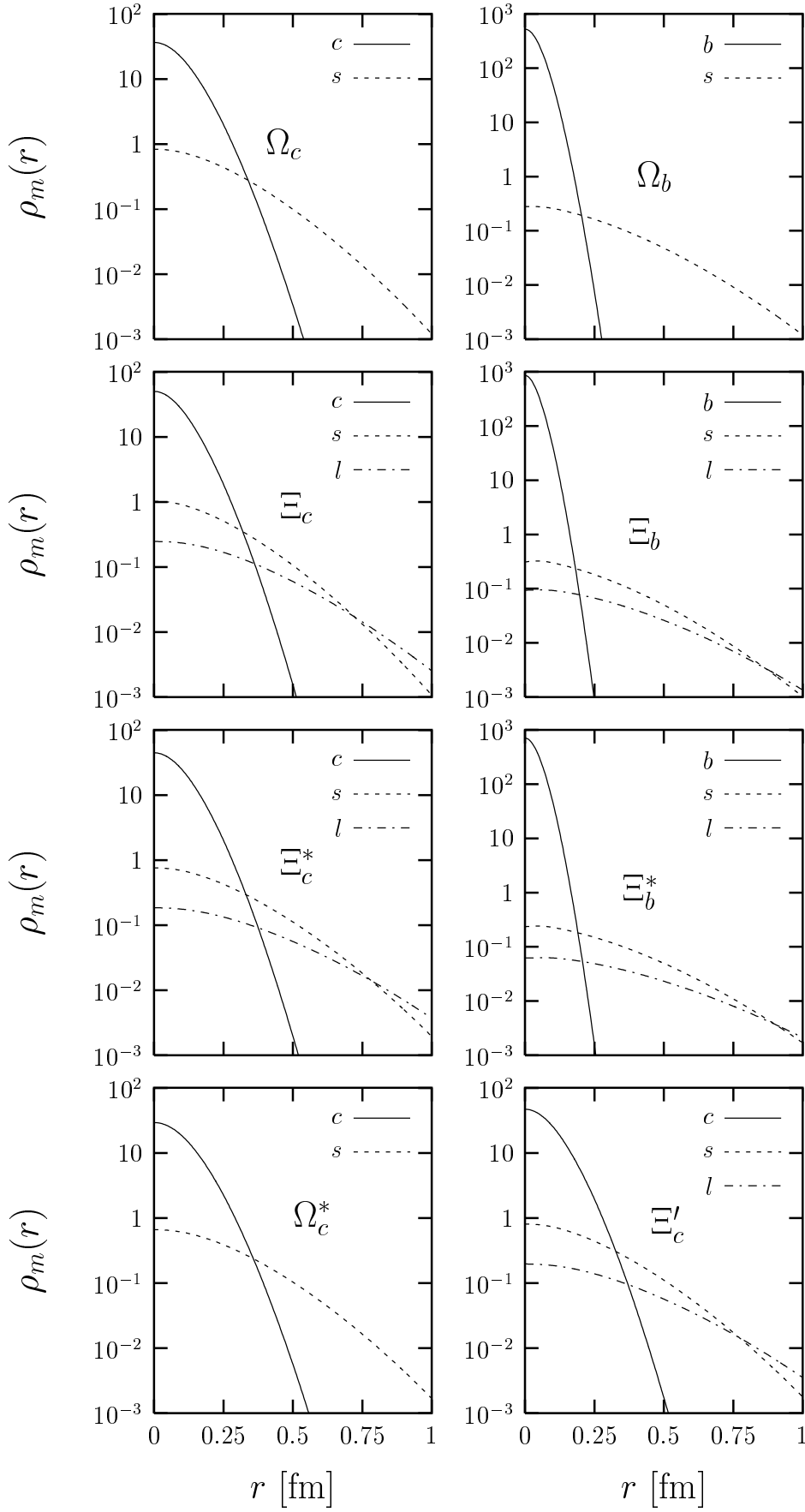
FIG. 5: Quark mass densities [fm^{-3}], $\rho_m^{B_Q}(\vec{r})\big|_{Q=c,b}$ and $\rho_m^{B_Q}(\vec{r})\big|_{q=l,s}$ (Eq. (34)), of strangeness charmed and bottom baryons obtained with the AL1 quark–quark interactions.

Number of gaussians	Λ_c Mass [MeV]	$\eta_{ll}^{\Lambda_c}(0)$ [fm^{-3}]	$\eta_{cl}^{\Lambda_c}(0)$ [fm^{-3}]
1	2302.1	16.4	5.2
2	2295.0	17.0	5.7
3	2294.8	18.6	6.2
4	2294.6	18.6	6.5
FAD	2296	14.8	7.4

TABLE IX: Λ_c baryon mass and wave functions at the origin (see Eq. (39)) as a function of the number of gaussians included in the light–light correlation function. Results have been obtained with the AL1 inter–quark interaction and for comparison, the Faddeev results of Ref.[28] (denoted by FAD) have been also compiled.

function that governs all the exclusive $b \to c$ semileptonic decays in the $m_Q \to \infty$ limit, could be obtained and more importantly the size of the $1/m_Q$ corrections in these decays would be also estimated. We will compare our results to those obtained in lattice QCD simulations, both from the meson and baryon sectors [38].

Acknowledgments

VII.   APPENDIX: VARIATIONAL WAVE FUNCTION PARAMETERS

All VAR results presented in the paper have been obtained with light-light correlation functions, F^{B_Q}, constructed from a linear combination of four gaussians (see eq. (27)). In Table IX and for the Λ_c baryon with an AL1 inter-quark interaction, we present the dependence of the mass and the value of the wave function at the origin on the number of gaussians used to build up the correlation function. Finally, we present in Tables X and XI the variational parameters of the charmed and bottom baryon wave functions for the different quark-quark interactions analyzed in this work. The wave functions are easy to handle and can be used to evaluate further observables.

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		Charm											
		b_1	d_1	a_2	b_2	d_2	a_3	b_3	d_3	a_4	b_4	d_4	α_q $\alpha_{q'}$
	AL1	0.51	0.76	0.67	0.73	0.73	0.56	0.85	0.76	0.77	1.12	0.84	0.19 0.19
	AL1χ	0.57	1.51	0.68	1.11	1.02	1.31	1.49	1.10	0.82	1.27	0.95	0.39 0.39
	AL2	1.07	0.37	0.62	0.65	0.93	0.51	0.99	0.49	1.13	0.45	0.94	0.18 0.18
	AP1	0.56	0.64	0.69	0.53	0.62	0.64	0.79	0.68	0.92	0.92	0.76	0.13 0.13
	AP2	0.72	0.77	0.79	0.72	0.50	0.78	0.46	0.65	0.72	0.81	0.77	0.20 0.20
Λ	BD	1.08	0.86	0.34	0.45	0.61	0.79	0.56	0.83	1.28	0.87	0.74	0.22 0.22
	AL1	0.55	−0.12	−0.61	0.62	0.01	0.52	0.63	0.24	0.77	0.50	0.27	0.06 0.06
	AL1χ	0.46	0.44	0.63	0.51	0.50	0.54	0.50	0.45	0.61	0.54	0.62	0.15 0.15
	AL2	0.71	−0.12	1.00	0.56	0.50	0.51	0.44	0.26	1.17	0.43	0.68	0.13 0.13
	AP1	0.43	0.42	0.75	0.51	0.39	0.53	0.38	0.37	0.62	0.49	0.48	0.10 0.10
	AP2	0.42	0.43	0.80	0.42	0.47	0.54	0.52	0.38	0.68	0.55	0.70	0.07 0.07
Σ	BD	0.58	0.90	0.99	0.48	0.67	0.76	0.33	0.71	1.08	0.54	0.65	0.08 0.08
	AL1	0.49	0.46	0.61	0.50	0.45	0.63	0.53	0.40	0.69	0.43	0.52	0.33 0.33
	AL1χ	0.46	0.45	0.62	0.45	0.46	0.69	0.54	0.38	0.72	0.43	0.57	0.37 0.37
	AL2	0.67	0.44	0.92	0.49	0.70	0.86	0.53	0.05	0.90	0.44	0.48	0.41 0.41
	AP1	0.42	0.43	0.64	0.48	0.42	0.64	0.46	0.38	0.69	0.49	0.53	0.32 0.32
	AP2	0.45	0.51	0.63	0.50	0.47	0.64	0.51	0.39	0.62	0.41	0.53	0.32 0.32
Σ^*	BD	0.75	0.87	0.93	0.45	0.68	0.75	0.51	0.58	1.22	0.41	0.65	0.27 0.27
	AL1	0.65	0.86	0.35	0.73	0.56	0.69	0.50	0.81	0.68	1.06	0.62	0.34 0.15
	AL2	0.78	0.49	0.82	0.52	0.47	1.06	0.69	0.73	1.65	1.12	0.62	0.37 0.25
	AP1	1.10	0.45	0.09	0.63	0.47	0.55	0.74	0.51	0.65	0.55	0.28	0.28 0.25
	AP2	0.92	0.71	0.01	0.69	0.49	0.51	0.87	0.37	0.54	0.48	0.43	0.32 0.20
Ξ	BD	0.82	0.82	0.45	0.33	0.57	0.77	0.60	0.75	2.10	0.86	0.55	0.23 0.18
	AL1	0.53	0.51	0.64	0.58	0.54	0.60	0.60	0.43	0.71	0.51	0.59	0.24 0.22
	AL2	0.61	0.46	1.00	0.67	0.47	0.77	0.50	0.39	0.99	0.48	0.41	0.16 0.04
	AP1	0.50	0.51	0.66	0.57	0.52	0.60	0.60	0.43	0.69	0.50	0.56	0.23 0.20
	AP2	0.61	0.45	0.44	0.47	0.39	0.74	0.42	0.49	0.69	0.57	0.52	0.08 0.08
Ξ'	BD	0.58	0.78	0.75	0.53	0.61	1.01	0.52	0.62	1.35	0.59	0.61	0.12 0.11
	AL1	0.55	0.47	0.60	0.55	0.47	0.59	0.54	0.46	0.66	0.53	0.57	0.28 0.28
	AL2	0.53	0.18	0.71	0.82	0.31	0.71	0.59	0.37	1.36	0.69	0.16	0.54 0.40
	AP1	0.49	0.47	0.57	0.53	0.44	0.62	0.54	0.38	0.64	0.48	0.50	0.35 0.36
	AP2	0.74	0.38	0.21	0.57	0.19	0.68	0.57	0.21	0.64	0.47	0.20	0.44 0.50
Ξ^*	BD	0.61	0.83	0.69	0.49	0.69	1.09	0.64	0.74	1.48	0.52	0.67	0.39 0.36
	AL1	0.66	0.43	0.10	0.59	0.58	0.92	0.56	0.55	0.75	0.71	0.65	0.17 0.17
	AL2	0.71	0.46	0.86	0.75	0.43	0.80	0.55	0.42	0.83	0.69	0.50	0.13 0.13
	AP1	0.78	0.14	0.97	0.50	0.97	−0.04	0.57	0.80	0.83	1.16	1.33	0.19 0.19
	AP2	0.63	0.50	0.58	0.60	0.60	0.52	0.67	0.53	0.76	0.65	0.64	0.19 0.19
Ω	BD	1.02	0.20	0.75	0.51	0.22	0.68	0.58	0.33	1.37	0.90	0.21	0.04 0.04
	AL1	0.74	0.49	0.46	0.59	0.49	0.63	0.66	0.38	0.74	0.52	0.45	0.44 0.44
	AL2	0.86	0.17	0.43	0.50	0.47	0.38	0.48	0.53	0.70	0.67	0.62	0.50 0.50
	AP1	0.70	0.61	0.40	0.61	0.54	0.75	0.60	0.49	0.72	0.54	0.60	0.46 0.46
	AP2	0.67	0.49	0.25	0.77	0.61	0.55	0.74	0.45	0.69	0.56	0.43	0.47 0.47
Ω^*	BD	0.46	1.10	0.64	0.61	0.77	1.15	0.73	0.75	1.32	0.63	0.77	0.34 0.34

TABLE X: Variational parameters ($a's$ are dimensionless, $d's$ have dimensions of fm and $b's$ have dimensions of fm^{-1}) of the baryon three body wave function (Eqs. (26) and (27)), in the charm sector and for different inter-quark interactions. Besides for the BD and AL1χ interactions, the c parameter (Eq. (28)) has been set to 200 fm^{-1} for the Σ and Σ^* baryons. In the rest of cases, c is set to $+\infty$.

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		b_1	d_1	a_2	b_2	d_2	a_3	b_3	d_3	a_4	b_4	d_4	α_q	$\alpha_{q'}$
							Bottom							
	AL1	0.60	0.54	0.58	0.71	0.76	0.46	0.75	0.72	0.78	0.84	0.77	0.00	0.00
	AL1χ	0.48	2.67	1.09	1.67	1.40	1.74	1.54	1.04	0.66	1.30	1.59	0.09	0.09
	AL2	0.45	0.29	1.21	0.63	0.91	0.96	0.97	0.62	1.73	0.88	0.63	−0.08	−0.08
	AP1	0.56	0.67	0.72	0.53	0.65	0.66	0.80	0.70	0.94	0.91	0.78	0.09	0.09
	AP2	0.96	0.68	0.41	0.70	0.47	0.55	0.72	0.42	0.55	0.43	0.58	0.04	0.04
Λ	BD	0.78	0.95	0.99	0.43	0.76	1.22	0.73	0.79	1.24	0.77	0.82	0.04	0.04
	AL1	0.38	0.18	1.17	0.48	0.44	0.69	0.58	0.33	0.86	0.59	0.74	0.08	0.08
	AL1χ	0.49	0.44	0.68	0.47	0.50	0.59	0.54	0.38	0.65	0.46	0.67	0.21	0.21
	AL2	0.65	0.06	0.95	0.56	0.71	0.29	0.67	0.19	1.31	0.22	0.58	−0.08	−0.08
	AP1	0.43	0.39	1.09	0.51	0.44	0.99	0.39	0.41	1.09	0.54	0.55	0.09	0.09
	AP2	0.40	0.46	0.71	0.45	0.41	0.71	0.50	0.29	0.75	0.37	0.53	0.02	0.02
Σ	BD	0.64	0.50	1.07	0.58	0.40	1.71	0.47	0.35	1.77	0.37	0.54	−0.05	−0.05
	AL1	0.44	0.45	0.63	0.52	0.48	0.58	0.49	0.41	0.68	0.54	0.57	0.23	0.23
	AL1χ	0.39	0.51	0.69	0.55	0.40	0.60	0.46	0.41	0.60	0.52	0.45	0.23	0.23
	AL2	0.56	0.12	0.98	0.45	0.72	0.48	0.55	0.31	1.14	0.32	0.61	0.04	0.04
	AP1	0.39	0.51	0.70	0.55	0.40	0.59	0.46	0.41	0.60	0.52	0.45	0.22	0.22
	AP2	0.38	0.50	0.70	0.54	0.39	0.60	0.45	0.41	0.59	0.52	0.45	0.23	0.23
Σ*	BD	0.71	0.74	0.87	0.31	0.62	0.83	0.40	0.60	1.30	0.44	0.65	−0.014	−0.014
	AL1	0.62	0.61	0.60	0.75	0.72	0.54	0.82	0.62	0.76	0.71	0.70	0.10	0.06
	AL2	0.87	0.40	0.97	0.63	0.58	0.69	0.48	0.56	0.93	0.97	0.73	−0.03	−0.03
	AP1	0.92	0.63	0.47	0.64	0.50	0.71	0.74	0.40	0.68	0.46	0.54	0.06	0.04
	AP2	0.95	0.74	0.50	0.68	0.48	0.81	0.71	0.40	0.59	0.43	0.56	0.00	−0.05
Ξ	BD	0.84	0.85	1.13	0.52	0.86	0.18	0.73	1.06	1.10	1.46	1.12	0.06	0.01
	AL1	0.50	0.51	0.66	0.57	0.52	0.60	0.60	0.43	0.69	0.50	0.56	0.23	0.20
	AL2	0.69	0.21	1.12	0.53	0.42	0.58	0.41	0.21	1.29	0.48	0.68	−0.03	−0.03
	AP1	0.49	0.51	0.67	0.54	0.52	0.63	0.57	0.46	0.69	0.51	0.59	0.23	0.20
	AP2	0.49	0.48	0.57	0.52	0.43	0.64	0.53	0.38	0.64	0.47	0.49	−0.04	−0.03
Ξ'	BD	0.74	0.92	0.90	0.33	0.62	0.89	0.50	0.63	1.26	0.49	0.72	0.02	−0.05
	AL1	0.50	0.51	0.66	0.57	0.52	0.60	0.60	0.51	0.69	0.50	0.56	0.22	0.20
	AL2	0.85	0.06	1.00	0.59	0.50	0.48	0.44	0.26	0.87	0.33	0.67	0.11	0.11
	AP1	0.49	0.52	0.68	0.51	0.51	0.65	0.53	0.48	0.70	0.52	0.62	0.23	0.20
	AP2	0.57	0.48	0.62	0.40	0.39	0.80	0.44	0.40	0.74	0.48	0.58	0.02	0.00
Ξ*	BD	0.77	0.86	0.90	0.33	0.63	0.86	0.46	0.63	1.28	0.50	0.71	−0.03	−0.07
	AL1	0.60	0.45	0.58	0.61	0.58	0.49	0.55	0.50	0.72	0.66	0.66	0.13	0.13
	AL2	0.62	0.31	0.67	0.65	0.56	0.50	0.59	0.32	0.88	0.65	0.47	0.06	0.06
	AP1	0.72	0.26	0.69	0.66	0.32	0.51	0.62	0.22	0.75	0.57	0.26	0.08	0.08
	AP2	0.63	0.45	0.78	0.62	0.58	0.49	0.55	0.50	0.72	0.66	0.66	0.13	0.13
Ω	BD	0.82	0.85	0.89	0.43	0.73	0.73	0.54	0.69	1.29	0.56	0.74	−0.07	−0.07
	AL1	0.58	0.48	0.60	0.59	0.56	0.55	0.57	0.49	0.71	0.61	0.63	0.18	0.18
	AL2	0.60	0.44	0.62	0.62	0.57	0.53	0.60	0.42	0.77	0.62	0.56	0.13	0.13
	AP1	0.57	0.49	0.66	0.50	0.50	0.57	0.57	0.48	0.73	0.59	0.62	0.20	0.20
	AP2	0.58	0.48	0.60	0.59	0.56	0.55	0.57	0.49	0.71	0.61	0.63	0.18	0.18
Ω*	BD	0.79	0.83	0.84	0.39	0.69	0.75	0.51	0.67	1.32	0.55	0.72	−0.07	−0.07

TABLE XI: Same as Table X, for the bottom sector.

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