Study of the strong $\Sigma_c \to \Lambda_c\,\pi$, $\Sigma_c^* \to \Lambda_c\,\pi$ and $\Xi_c^* \to \Xi_c\,\pi$ decays in a nonrelativistic quark model

C. Albertus,[1] E. Hernández,[2] J. Nieves,[1] and J. M. Verde-Velasco[2]

[1]*Departamento de Física Moderna, Universidad de Granada, E-18071 Granada, Spain.*
[2]*Grupo de Física Nuclear, Facultad de Ciencias, E-37008 Salamanca, Spain.*

We present results for the strong widths corresponding to the $\Sigma_c \to \Lambda_c\,\pi$, $\Sigma_c^* \to \Lambda_c\,\pi$ and $\Xi_c^* \to \Xi_c\,\pi$ decays. The calculations have been done in a nonrelativistic constituent quark model with wave functions that take advantage of the constraints imposed by heavy quark symmetry. Partial conservation of axial current hypothesis allows us to determine the strong vertices from an analysis of the axial current matrix elements. Our results $\Gamma(\Sigma_c^{++} \to \Lambda_c^+\,\pi^+) = 2.41 \pm 0.07 \pm 0.02\,\mathrm{MeV}$, $\Gamma(\Sigma_c^+ \to \Lambda_c^+\,\pi^0) = 2.79 \pm 0.08 \pm 0.02\,\mathrm{MeV}$, $\Gamma(\Sigma_c^0 \to \Lambda_c^+\,\pi^-) = 2.37 \pm 0.07 \pm 0.02\,\mathrm{MeV}$, $\Gamma(\Sigma_c^{*\,++} \to \Lambda_c^+\,\pi^+) = 17.52 \pm 0.74 \pm 0.12\,\mathrm{MeV}$, $\Gamma(\Sigma_c^{*\,+} \to \Lambda_c^+\,\pi^0) = 17.31 \pm 0.73 \pm 0.12\,\mathrm{MeV}$, $\Gamma(\Sigma_c^{*\,0} \to \Lambda_c^+\,\pi^-) = 16.90 \pm 0.71 \pm 0.12\,\mathrm{MeV}$, $\Gamma(\Xi_c^{*\,+} \to \Xi_c^0\,\pi^+ + \Xi_c^+\pi^0) = 3.18 \pm 0.10 \pm 0.01$ MeV and $\Gamma(\Xi_c^{*\,0} \to \Xi_c^+\,\pi^- + \Xi_c^0\pi^0) = 3.03 \pm 0.10 \pm 0.01$ MeV are in good agreement with experimental determinations.

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I. INTRODUCTION

The nonrelativistic constituent quark model (NRCQM), using QCD-inspired potentials, has proved to be an excellent tool to predict properties of hadrons. In the case of baryons including one heavy quark c or b and two light ones u, d or s, we can take advantage of yet another property of QCD: Heavy quark symmetry (HQS) [1, 2, 3, 4]. This symmetry arises when the heavy quark mass is much larger than the QCD scale (Λ_{QCD}). In that limit the dynamics of the light quark degrees of freedom becomes independent of the heavy quark flavor and spin. The light degrees of freedom are thus well defined and the masses of the baryons depend only on the quark content and on the light-light quantum numbers. This simplification was used in Ref. [5] to solve the three-body problem for the ground state (L=0) of baryons with a heavy quark using a simple variational ansatz. We obtained static properties (masses, mass and charge radii. . .) in good agreement with previous calculations that used more involved Faddeev equations [6]. The advantage of our approach is that we also obtain easy to handle wave functions. Those wave functions were already used in Ref. [7] to study, with good results, the semileptonic decays of Λ_b and Ξ_b baryons.

In this work we shall evaluate strong widths for the $\Sigma_c \to \Lambda_c\,\pi$, $\Sigma_c^* \to \Lambda_c\,\pi$ and $\Xi_c^* \to \Xi_c\,\pi$ decays. Last decade has seen a great progress on charmed baryon physics and now the ground state baryons with a c quark, with the exception of the Ω_c^*, are well established [8], and we have experimental information on the strong one-pion decay widths for the Σ_c [9, 10, 11], Σ_c^* [10, 12] and Ξ_c^* [13, 14]. With very little kinetic energy available in the final state these reactions should be well described in a nonrelativistic approach. Although they have been analyzed before in the framework of the constituent quark model (CQM) [15, 16], no attempt was made there to evaluate the full matrix elements. While there have been dynamical calculations in other models (see references below), to our knowledge, ours is the first dynamical calculation within a nonrelativistic approach. In our calculation we will use the HQS–constrained wave functions that we evaluated in Ref. [5] using different inter-quark interactions, and whose goodness have already been tested in the study of the semileptonic $\Lambda_b \to \Lambda_c$ and $\Xi_b \to \Xi_c$ decays in Ref. [7]. The use of different quark–quark potentials will allow us to obtain theoretical uncertainties on the widths due to the quark-quark interaction. The pion emission amplitude will be obtained in a spectator model (one-quark pion emission) with the use of partial conservation of axial current hypothesis (PCAC).

These reactions, and similar ones, have also been addressed in QCD sum rules (QCDSR) [17, 18], in heavy hadron chiral perturbation theory (HHCPT) [16, 19, 20, 21, 22], and within relativistic quark models like the light-front quark model (LFQM) [23] and the relativistic three-quark model (RTQM) [24].

II. ONE-PION EMISSION AMPLITUDE $\mathcal{A}^{(s,s')}_{BB'\pi}(P_B, P_{B'})$

To determine the pion emission amplitude $\mathcal{A}^{(s,s')}_{BB'\pi}(P_B, P_{B'})$ we shall use PCAC, as we have done in the meson sector, in a previous study of the strong $B^*B\pi$ and $D^*D\pi$ couplings [25]. PCAC allows us to relate that amplitude to the matrix element of the divergence of the axial current. For the emission of a π^+ we have[1]

$$\left\langle B',\ s'\,\vec{P}_{B'}\,|\,q^\mu\,J^{d\,u}_{A\,\mu}(0)\,|\,B,\ s\,\vec{P}_B\right\rangle_{non-pole} \;=\; i\,f_\pi\,\mathcal{A}^{(s,s')}_{BB'\pi^+}(P_B,P_{B'}) \tag{1}$$

where s, s' are the third component of the spin of the B, B' baryons in their respective center of mass systems, $P_B=(E_B(|\vec{P}_B|),\,\vec{P}_B)$, $P_{B'}=(E_{B'}(|\vec{P}_{B'}|),\,\vec{P}_{B'})$ are their respective four–momenta and $q=P_B-P_{B'}$. $J^{d\,u}_{A\,\mu}(0)$ is the axial current for the $u\to d$ transition, and $f_\pi=130.7\,\mathrm{MeV}$ [8] is the pion decay constant. The baryon states are normalized as $\left\langle B,s'\vec{P}'\,|B,s\vec{P}\right\rangle=\delta_{s,s'}\,(2\pi)^3\,2E_B(|\vec{P}\,|)\,\delta^3(\vec{P}-\vec{P}')$. Furthermore we shall use physical masses taken from Ref. [8] in all calculations.

III. DESCRIPTION OF BARYON STATES

We use the following expression for the state of a baryon B with three-momentum $\vec{P}$ and spin projection s in the baryon center of mass

$$\begin{aligned}\left|B,s\,\vec{P}\right\rangle_{NR} = \int d^3Q_1\int d^3Q_2\;\frac{1}{\sqrt{2}}\sum_{\alpha_1,\alpha_2,\alpha_3}\hat{\psi}^{(B,s)}_{\alpha_1,\alpha_2,\alpha_3}(\,\vec{Q}_1,\vec{Q}_2\,)\;\frac{1}{(2\pi)^3\,\sqrt{2E_{f_1}(|\vec{p}_1|)2E_{f_2}(|\vec{p}_2|)2E_{f_3}(|\vec{p}_3|)}}\\ \times\left|\ \alpha_1\ \vec{p}_1=\frac{m_{f_1}}{\overline{M}}\vec{P}+\vec{Q}_1\ \right\rangle\left|\ \alpha_2\ \vec{p}_2=\frac{m_{f_2}}{\overline{M}}\vec{P}+\vec{Q}_2\ \right\rangle\left|\ \alpha_3\ \vec{p}_3=\frac{m_{f_3}}{\overline{M}}\vec{P}-\vec{Q}_1-\vec{Q}_2\ \right\rangle\end{aligned} \tag{2}$$

α_j represents the quantum numbers of spin s, flavor f and color c ($\alpha_j\equiv(s_j,f_j,c_j)$) of the j-*th* quark, while $(E_{f_j}(|\vec{p}_j|),\,\vec{p}_j)$ and m_{f_j} represent its four–momentum and mass. $\overline{M}$ stands for $\overline{M}=m_{f_1}+m_{f_2}+m_{f_3}$. We choose the third quark to be the c quark while the first two will be the light ones. The normalization of the quark states is $\langle\,\alpha'\ \vec{p}\,'\,|\,\alpha\ \vec{p}\,\rangle=\delta_{\alpha',\,\alpha}\,(2\pi)^3\,2E(|\vec{p}|)\,\delta^3(\vec{p}\,'-\vec{p})$. Besides, $\hat{\psi}^{(B,s)}_{\alpha_1,\alpha_2,\alpha_3}(\,\vec{Q}_1,\vec{Q}_2\,)$ is the nonrelativistic momentum space wave function for the internal motion, being $\vec{Q}_1$ and $\vec{Q}_2$ the momenta conjugate to the relative positions $\vec{r}_1$ and $\vec{r}_2$ of the two light quarks with respect to the heavy one. This wave function is antisymmetric under the simultaneous exchange $\alpha_1\longleftrightarrow\alpha_2,\vec{Q}_1\longleftrightarrow\vec{Q}_2$, being also antisymmetric under an overall exchange of the color degrees of freedom. It is normalized such that

$$\int d^3Q_1\int d^3Q_2\sum_{\alpha_1,\alpha_2,\alpha_3}\left(\hat{\psi}^{(B,s')}_{\alpha_1,\alpha_2,\alpha_3}(\,\vec{Q}_1,\vec{Q}_2\,)\right)^*\hat{\psi}^{(B,s)}_{\alpha_1,\alpha_2,\alpha_3}(\,\vec{Q}_1,\vec{Q}_2\,)=\delta_{s',\,s} \tag{3}$$

and, thus, the normalization of our nonrelativistic baryon states is

$$_{NR}\left\langle\,B,s'\,\vec{P}\,'\,|\,B,s\,\vec{P}\,\right\rangle_{NR}=\delta_{s',\,s}\,(2\pi)^3\,\delta^3(\vec{P}\,'-\vec{P}\,) \tag{4}$$

For the particular case of ground state Λ_c, Σ_c, Σ_c^*, Ξ_c and Ξ_c^* we can assume the orbital angular momentum to be zero. We will also take advantage of HQS and assume the light–degrees of freedom quantum numbers are well defined (For quantum numbers see, for instance, Table 1 in Ref [5]). In that case we have[2]

$$\begin{aligned}\hat{\psi}^{(\Lambda_c^+,s)}_{\alpha_1,\alpha_2,\alpha_3}(\,\vec{Q}_1,\vec{Q}_2\,)\;&=\;\frac{\varepsilon_{c_1\,c_2\,c_3}}{\sqrt{3!}}\;(1/2,1/2,0;s_1,s_2,0)\\ &\qquad\times\frac{\delta_{f_3,\,c}\,\delta_{s_3,\,s}}{\sqrt{2}}\left(\delta_{f_1,\,u}\,\delta_{f_2,\,d}\,\widetilde{\phi}^{S_l=0}_{u,\,d,\,c}(\,\vec{Q}_1,\vec{Q}_2\,)\;-\delta_{f_1,\,d}\,\delta_{f_2,\,u}\,\widetilde{\phi}^{S_l=0}_{d,\,u,\,c}(\,\vec{Q}_1,\vec{Q}_2\,)\right)\\ \hat{\psi}^{(\Sigma_c^{++},s)}_{\alpha_1,\alpha_2,\alpha_3}(\,\vec{Q}_1,\vec{Q}_2\,)\;&=\;\frac{\varepsilon_{c_1\,c_2\,c_3}}{\sqrt{3!}}\;\;\widetilde{\phi}^{S_l=1}_{u,\,u,\,c}(\,\vec{Q}_1,\vec{Q}_2\,)\;\delta_{f_1,\,u}\,\delta_{f_2,\,u}\,\delta_{f_3,\,c}\sum_m(1/2,1/2,1;s_1,s_2,m)\;(1,1/2,1/2;m,s_3,s)\\ \hat{\psi}^{(\Sigma_c^{*\,++},s)}_{\alpha_1,\alpha_2,\alpha_3}(\,\vec{Q}_1,\vec{Q}_2\,)\;&=\;\frac{\varepsilon_{c_1\,c_2\,c_3}}{\sqrt{3!}}\;\;\widetilde{\phi}^{S_l=1}_{u,\,u,\,c}(\,\vec{Q}_1,\vec{Q}_2\,)\;\delta_{f_1,\,u}\,\delta_{f_2,\,u}\,\delta_{f_3,\,c}\;\sum_m(1/2,1/2,1;s_1,s_2,m)\;(1,1/2,3/2;m,s_3,s)\end{aligned}$$

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[1] Note that we give the expression corresponding to the non–pole part of the matrix element. If the pion pole contribution is included then the relation is given by $\left\langle B',\ s'\,\vec{P}_{B'}\ \middle|\,q^\mu\,J^{d\,u}_{A\,\mu}(0)\,\middle|\ B,\ s\,\vec{P}_B\right\rangle=-i\,f_\pi\,\frac{m_\pi^2}{q^2-m_\pi^2}\ \mathcal{A}^{(s,s')}_{BB'\pi^+}(P_B,P_{B'})$.

[2] We only give the wave function for the baryons involved in π^+ emission. Wave functions for other isospin states of the same baryons are easily constructed.

$$\hat{\psi}^{(\Xi_c^0,s)}_{\alpha_1,\alpha_2,\alpha_3}(\vec{Q}_1,\vec{Q}_2) = \frac{\varepsilon_{c_1 c_2 c_3}}{\sqrt{3!}} (1/2,1/2,0;s_1,s_2,0) \times \frac{\delta_{s_3,s}\,\delta_{f_3,c}}{\sqrt{2}} \Big(\delta_{f_1,d}\,\delta_{f_2,s}\ \widetilde{\phi}^{S_l=0}_{d,s,c}(\vec{Q}_1,\vec{Q}_2) - \delta_{f_1,s}\,\delta_{f_2,d}\ \widetilde{\phi}^{S_l=0}_{s,d,c}(\vec{Q}_1,\vec{Q}_2)\Big)$$

$$\hat{\psi}^{(\Xi_c^{*+},s)}_{\alpha_1,\alpha_2,\alpha_3}(\vec{Q}_1,\vec{Q}_2) = \frac{\varepsilon_{c_1 c_2 c_3}}{\sqrt{3!}} \sum_m (1/2,1/2,1;s_1,s_2,m)\,(1,1/2,3/2;m,s_3,s) \times \frac{\delta_{f_3,c}}{\sqrt{2}} \Big(\delta_{f_1,u}\,\delta_{f_2,s}\ \widetilde{\phi}^{S_l=1}_{u,s,c}(\vec{Q}_1,\vec{Q}_2) + \delta_{f_1,s}\,\delta_{f_2,u}\ \widetilde{\phi}^{S_l=1}_{s,u,c}(\vec{Q}_1,\vec{Q}_2)\Big) \tag{5}$$

Here $\varepsilon_{c_1c_2c_3}$ is the fully antisymmetric tensor on color indices being $\varepsilon_{c_1c_2c_3}/\sqrt{3!}$ the antisymmetric color wave function, the $(j_1,j_2,j;m_1,m_2,m_3)$ are Clebsch-Gordan coefficients and the $\widetilde{\phi}^{S_l}_{f_1,f_2,f_3}(\vec{Q}_1,\vec{Q}_2)$, with S_l the total spin of the light degrees of freedom, are the Fourier transform of the corresponding normalized coordinate space wave functions obtained in Ref. [5]. Their dependence on momenta is through $|\vec{Q}_1|$, $|\vec{Q}_2|$ and $\vec{Q}_1\cdot\vec{Q}_2$ alone, and they are symmetric under the simultaneous exchange $f_1 \longleftrightarrow f_2, \vec{Q}_1 \longleftrightarrow \vec{Q}_2$.

IV. RESULTS

A. $\Sigma_c \to \Lambda_c\,\pi$ decay

Let us do the $\Sigma_c^{++} \to \Lambda_c^+\,\pi^+$ case. The matrix element of the divergence $q^\mu J^{d\,u}_{A\,\mu}(0)$ of the axial current determines the π^+ emission amplitude as

$$\mathcal{A}^{(s,s')}_{\Sigma_c^{++}\Lambda_c^+\pi^+}(P,P') = \frac{-i}{f_\pi} \left\langle \Lambda_c^+,\ s'\,\vec{P}'\,\middle|\,q^\mu\,J^{d\,u}_{A\,\mu}(0)\,\middle|\,\Sigma_c^{++},\ s\,\vec{P}\right\rangle_{non-pole} = i g_{\Sigma_c^{++}\Lambda_c^+\pi^+}\,\overline{u}_{\Lambda_c^+\ s'}(\vec{P}')\,\gamma_5\,u_{\Sigma_c^{++}\ s}(\vec{P}) \tag{6}$$

where the coupling constant $g_{\Sigma_c^{++}\Lambda_c^+\pi^+}$, in analogy to the pion coupling to nucleons and nucleon resonances, has been chosen to be dimensionless, and $u_{\Sigma_c^{++}\ s}(\vec{P})$, $u_{\Lambda_c^+\ s'}(\vec{P}')$ are Dirac spinors normalized to twice the energy. The width is given by

$$\Gamma(\Sigma_c^{++}\to\Lambda_c^+\pi^+) = \frac{|\vec{q}\,|}{8\pi M^2_{\Sigma_c^{++}}}\ g^2_{\Sigma_c^{++}\Lambda_c^+\pi^+}\Big(\ (M_{\Sigma_c^{++}} - M_{\Lambda_c^+})^2 - m_\pi^2\Big) \tag{7}$$

with $|\vec{q}\,|$ the modulus of the final baryon or pion three-momentum. From Eq. (6), taking $\vec{P}=\vec{0}$, $\vec{P}'=-|\vec{q}\,|\,\vec{k}$ in the z direction, $s=s'=1/2$, and taking into account the different normalization of our nonrelativistic states, we have

$$g_{\Sigma_c^{++}\Lambda_c^+\pi^+} = \frac{-1}{f_\pi}\,\frac{\sqrt{E_{\Lambda_c^+}(|\vec{q}\,|)+M_{\Lambda_c^+}}\sqrt{2M_{\Sigma_c^{++}}2E_{\Lambda_c^+}(|\vec{q}\,|)}}{|\vec{q}\,|\sqrt{2M_{\Sigma_c^{++}}}}\Big((M_{\Sigma_c^{++}} - E_{\Lambda_c^+}(|\vec{q}\,|))\ A^{1/2,1/2}_{\Sigma_c^{++}\Lambda_c^+,\ 0} + |\vec{q}\,|\ A^{1/2,1/2}_{\Sigma_c^{++}\Lambda_c^+,\ 3}\Big) \tag{8}$$

with

$$A^{1/2,1/2}_{\Sigma_c^{++}\Lambda_c^+,\ \mu} = {}_{NR}\Big\langle \Lambda_c^+,\ 1/2\ -|\vec{q}|\,\vec{k}\ \Big|\ J^{d\,u}_{A\,\mu}(0)\ \Big|\ \Sigma_c^{++},\ 1/2\ \vec{0}\Big\rangle_{NR,\ non-pole} \tag{9}$$

The $A^{1/2,1/2}_{\Sigma_c^{++}\Lambda_c^+,\ \mu}$ are easily evaluated using one-body current operators and their expressions can be found in the appendix.

In Table I we present the results for $g_{\Sigma_c^{++}\Lambda_c^+\pi^+}$ and the widths $\Gamma(\Sigma_c^{++}\to\Lambda_c^+\pi^+)$, $\Gamma(\Sigma_c^{+}\to\Lambda_c^+\pi^0)$ and $\Gamma(\Sigma_c^{0}\to\Lambda_c^+\pi^-)$. To get the values for $\Gamma(\Sigma_c^{+}\to\Lambda_c^+\pi^0)$ and $\Gamma(\Sigma_c^{0}\to\Lambda_c^+\pi^-)$ we use $g_{\Sigma_c^{++}\Lambda_c^+\pi^+}$ and make the appropriate mass changes in the rest of factors in Eq. (7). Our results show two types of errors. The second one results from the Monte Carlo evaluation of the integrals needed to obtain the $g_{\Sigma_c^{++}\Lambda_c^+\pi^+}$ coupling constant. The first one, can be considered as a theoretical uncertainty and comes from the use of different quark–quark interactions [3]. The results are in very good

[3] We use five different inter-quark interactions, one suggested by Bhaduri and collaborators [26], and four others suggested by Silvestre-Brac and Semay [6, 27]. All of them contain a confinement term, plus Coulomb and hyperfine terms coming from one-gluon exchange, and differ from one another in the form factors used for the hyperfine terms, the power of the confining term or the use of a form factor in the one gluon exchange Coulomb potential.

	$g_{\Sigma_c^{++}\Lambda_c^+\pi^+}$	$\Gamma(\Sigma_c^{++}\to\Lambda_c^+\pi^+)$ [MeV]	$\Gamma(\Sigma_c^{+}\to\Lambda_c^+\pi^0)$ [MeV]	$\Gamma(\Sigma_c^{0}\to\Lambda_c^+\pi^-)$ [MeV]
This work	$21.73\pm0.32\pm0.08$	$2.41\pm0.07\pm0.02$	$2.79\pm0.08\pm0.02$	$2.37\pm0.07\pm0.02$
Experiment		$2.3\pm0.2\pm0.3$ [9]	< 4.6 (CL=90%) [10]	$2.5\pm0.2\pm0.3$ [9]
		$2.05^{+0.41}_{-0.38}\pm0.38$ [11]		$1.55^{+0.41}_{-0.37}\pm0.38$ [11]
Theory				
CQM		1.31 ± 0.04 [15]	1.31 ± 0.04 [15]	1.31 ± 0.04 [15]
		$2.025^{+1.134}_{-0.987}$ [16]		$1.939^{+1.114}_{-0.954}$ [16]
HHCPT	22, 29.3 [19]	2.47, 4.38 [19]	2.85, 5.06 [19]	2.45, 4.35 [19]
		2.5 [20]	3.2 [20]	2.4 [20]
				1.94 ± 0.57 [21]
LFQM		1.64 [23]	1.70 [23]	1.57 [23]
RTQM		2.85 ± 0.19 [24]	3.63 ± 0.27 [24]	2.65 ± 0.19 [24]

TABLE I: Coupling constant $g_{\Sigma_c^{++}\Lambda_c^+\pi^+}$ and total widths $\Gamma(\Sigma_c^{++}\to\Lambda_c^+\pi^+)$, $\Gamma(\Sigma_c^{+}\to\Lambda_c^+\pi^0)$ and $\Gamma(\Sigma_c^{0}\to\Lambda_c^+\pi^-)$ (See text for details). Experimental data and different theoretical calculations are also shown.

agreement with the experimental data by the CLEO Collaboration in Refs. [9, 10]. The value for $\Gamma(\Sigma_c^{++}\to\Lambda_c^+\pi^+)$ also agrees with the experimental data by the FOCUS Collaboration in Ref. [11]. The agreement with FOCUS data is not good for the $\Gamma(\Sigma_c^{0}\to\Lambda_c^+\pi^-)$ case, although our result is still within experimental errors. Our results show variations as large as ≈17% between different charge configurations. This is due to the little kinetic energy available in the final state that makes the widths very sensitive to the precise masses of the hadrons involved. In this respect there is a new precise determination of the Λ_c^+ mass by the *BABAR* collaboration $M_{\Lambda_c} = 2286.46\pm0.14$ MeV/c^2 [28], which is roughly 1.5 MeV/c^2 above the value quoted by the Particle Data Group in Ref. [8]. With this new value our calculated widths would get reduced by 9%. This reduction comes from phase space factors while the coupling $g_{\Sigma_c^{++}\Lambda_c^+\pi^+}$ changes only at the level of 0.1%.

As for the other theoretical determinations, the CQM calculation in Ref. [15] uses exact $s\longleftrightarrow c$ symmetry to relate the $\Sigma_c\to\Lambda_c\pi$ decay to the non–charmed $\Sigma^*\to\Lambda\pi$ analogue decay. Their results are smaller than the experimental data obtained by the CLEO Collaboration. In Ref. [16] a unique coupling constant is fixed in order to reproduce all experimental information on $\Sigma_c^*\to\Lambda_c\pi$ widths. That coupling is latter used to predict the $\Sigma_c\to\Lambda_c\pi$ widths. This coupling suffers from large uncertainties and thus the theoretical errors on the predicted widths are also very large. In the HHCPT calculation of Ref. [19] a simple CQM argument is used in order to obtain the unknown coupling constant in the HHCPT Lagrangian. Furthermore the authors allow for a renormalization of the axial coupling g_A^{ud} for light quarks. The largest of the two values quoted corresponds to the case where that coupling is unrenormalized and then is given by $g_A^{ud}=1$. The smaller number quoted corresponds to the use of a renormalized value of $g_A^{ud}=0.75$. The case $g_A^{ud}=1$ is the one that compares with our calculation. Their results for the widths almost double ours and are not in agreement with experiment. Their simple determination of the coupling constant can not be correct. The values obtained in the HHCPT calculation of Ref. [20] are closer to our results and experimental data. There the authors determine the needed coupling constants from the analysis of analogue decays involving non–charmed baryons. In Ref. [21], also within the HHCPT approach, and similarly to Ref. [16], the authors fix the unknown coupling in the Lagrangian using the experimental data for the $\Sigma_c^*\to\Lambda_c\pi$ decays. From there they predict the $\Gamma(\Sigma_c^{0}\to\Lambda_c^+\pi^-)$ obtaining a value very close to the one in Ref. [16], and that suffers also from large uncertainties. The two relativistic quark model calculations of Refs. [23, 24] give results that differ by almost a factor of two. Our results are closer to the ones obtained within the RTQM of Ref. [24].

B. $\Sigma_c^*\to\Lambda_c\pi$ decay

Let us analyze the case with a π^+ in the final state, $\Sigma_c^{*\,++}\to\Lambda_c^+\pi^+$. Similarly to the Σ_c decay before we now have

$$\begin{aligned}\mathcal{A}^{(s,s')}_{\Sigma_c^{*\,++}\Lambda_c^+\pi^+}(P,P') &= \frac{-i}{f_\pi}\left\langle \Lambda_c^+,\ s'\,\vec{P}'\,\middle|\, q^\mu\, J^{d\,u}_{A\,\mu}(0)\,\middle|\,\Sigma_c^{*\,++},\ s\,\vec{P}\right\rangle_{non-pole}\\ &= i\frac{g_{\Sigma_c^{*\,++}\Lambda_c^+\pi^+}}{2M_{\Lambda_c^+}}\ q_\nu\ \overline{u}_{\Lambda_c^+\ s'}(\vec{P}'\,)\ u^\nu_{\Sigma_c^{*\,++}\ s}(\vec{P})\end{aligned} \tag{10}$$

	$g_{\Sigma_c^{*\,++}\Lambda_c^+\pi^+}$	$\Gamma(\Sigma_c^{*\,++}\to\Lambda_c^+\pi^+)$ [MeV]	$\Gamma(\Sigma_c^{*\,+}\to\Lambda_c^+\pi^0)$ [MeV]	$\Gamma(\Sigma_c^{*\,0}\to\Lambda_c^+\pi^-)$ [MeV]
This work	$36.20\pm0.75\pm0.13$	$17.52\pm0.74\pm0.12$	$17.31\pm0.73\pm0.12$	$16.90\pm0.71\pm0.12$
Experiment		$14.1^{+1.6}_{-1.5}\pm1.4$ [12]	<17 (CL=90%) [10]	$16.6^{+1.9}_{-1.7}\pm1.4$ [12]
Theory				
QCDSR	$13.8\div24.2$ [17]			
	$32.5\pm2.1\pm6.9$ [18]			
CQM		20 [15]	20 [15]	20 [15]
HHCPT		25 [20]	25 [20]	25 [20]
LFQM		12.84 [23]		12.40 [23]
RTQM		21.99 ± 0.87 [24]		21.21 ± 0.81 [24]

TABLE II: Coupling constant $g_{\Sigma_c^{*\,++}\Lambda_c^+\pi^+}$ and total widths $\Gamma(\Sigma_c^{*\,++}\to\Lambda_c^+\pi^+)$, $\Gamma(\Sigma_c^{*\,+}\to\Lambda_c^+\pi^0)$ and $\Gamma(\Sigma_c^{*\,0}\to\Lambda_c^+\pi^-)$. Experimental data and different theoretical calculations are also shown. Note that in order to compare with our definition of $g_{\Sigma_c^{*\,++}\Lambda_c^+\pi^+}$ we have multiplied the coupling constants evaluated in Refs. [17, 18] by $2M_{\Lambda_c^+}/f_\pi$.

where we have introduced the dimensionless coupling constant $g_{\Sigma_c^{*\,++}\Lambda_c^+\pi^+}$ and $u^\nu_{\Sigma_c^{*\,++}\ s}(\vec P)$ is a Rarita-Schwinger spinor normalized to twice the energy. The width is given by

$$\Gamma(\Sigma_c^{*\,++}\to\Lambda_c^+\pi^+)=\frac{|\vec q\,|^3}{24\pi M^2_{\Sigma_c^{*\,++}}}\ \frac{g^2_{\Sigma_c^{*\,++}\Lambda_c^+\pi^+}}{4M^2_{\Lambda_c^+}}\left((M_{\Sigma_c^{*\,++}}+M_{\Lambda_c^+})^2-m_\pi^2\right) \tag{11}$$

Taking again $\vec P=\vec 0$, $\vec P'=-|\vec q\,|\,\vec k$ in the z direction, and $s=s'=1/2$ we obtain from Eq.(10)

$$g_{\Sigma_c^{*\,++}\Lambda_c^+\pi^+}\ =\ \frac{\sqrt3}{f_\pi\sqrt2}\ \frac{2M_{\Lambda_c^+}\sqrt{2M_{\Sigma_c^{*\,++}}2E_{\Lambda_c^+}(|\vec q\,|)}}{|\vec q\,|\sqrt{2M_{\Sigma_c^{*\,++}}\left(E_{\Lambda_c^+}(|\vec q\,|)+M_{\Lambda_c^+}\right)}}\left((M_{\Sigma_c^{*\,++}}-E_{\Lambda_c^+}(|\vec q\,|))\ A^{1/2,1/2}_{\Sigma_c^{*\,++}\Lambda_c^+,\ 0}+|\vec q\,|\ A^{1/2,1/2}_{\Sigma_c^{*\,++}\Lambda_c^+,\ 3}\right) \tag{12}$$

with

$$A^{1/2,1/2}_{\Sigma_c^{*\,++}\Lambda_c^+,\ \mu}={}_{NR}\left\langle\Lambda_c^+,\ 1/2\ \ -|\vec q\,|\,\vec k\ \left|\ J^{d\,u}_{A\,\mu}(0)\ \right|\ \Sigma_c^{*\,++},\ 1/2\ \vec 0\right\rangle_{NR,\ non-pole} \tag{13}$$

The expressions for $A^{1/2,1/2}_{\Sigma_c^{*\,++}\Lambda_c^+,\ \mu}$ $(\mu=0,3)$ can be found in the appendix.
Results for $g_{\Sigma_c^{*\,++}\Lambda_c^+\pi^+}$ and the total widths $\Gamma(\Sigma_c^{*\,++}\to\Lambda_c^+\pi^+)$, $\Gamma(\Sigma_c^{*\,+}\to\Lambda_c^+\pi^0)$ and $\Gamma(\Sigma_c^{*\,-}\to\Lambda_c^+\pi^-)$ appear in Table II. Our value for the latter two are obtained with the use of $g_{\Sigma_c^{*\,++}\Lambda_c^+\pi^+}$ and with the appropriate mass changes in the rest of factors in Eq. (11). Our central value for $\Gamma(\Sigma_c^{*\,++}\to\Lambda_c^+\pi^+)$ is above the central value of the latest experimental determination by the CLEO Collaboration in Ref. [12]. For some of the potentials used, AP1 and AP2 of Ref. [6], the results obtained are within experimental errors. The central value for $\Gamma(\Sigma_c^{*\,+}\to\Lambda_c^+\pi^0)$ is slightly above the upper experimental bound determined also by the CLEO Collaboration in Ref. [10], but again, we obtain results which are below the experimental bound using the AP1 and AP2 potentials. As for $\Gamma(\Sigma_c^{*\,-}\to\Lambda_c^+\pi^-)$ we get a nice agreement with experiment. Our results for the different charge configurations differ by 4% at most. With the new value for $M_{\Lambda_c^+}$ given by the *BABAR* Collaboration in Ref. [28] they would get reduced by 3%. Our results are globally in better agreement with experiment than the ones obtained by other theoretical calculations [4] with perhaps the exception of the QCDSR calculation of Ref. [18].

C. $\Xi_c^*\to\Xi_c\pi$ decay

Once more we analyze the case with a π^+ in the final state, $\Xi_c^{*\,+}\to\Xi_c^0\pi^+$. What we obtain is

$$\mathcal{A}^{(s,s')}_{\Xi_c^{*\,+}\Xi_c^0\pi^+}(P,P')\ =\ \frac{-i}{f_\pi}\left\langle\Xi_c^0,\ s'\,\vec P'\,|\,q^\mu\,J^{d\,u}_{A\,\mu}(0)\,|\,\Xi_c^{*\,+},\ s\,\vec P\right\rangle_{non-pole}$$

[4] We did not show those cases were data on $\Sigma_c^*\to\Lambda_c\pi$ widths were used to fit parameters of the models.

	$g_{\Xi_c^{*\,+}\Xi_c^0\pi^+}$	$\Gamma(\Xi_c^{*\,+}\to\Xi_c^0\pi^+)$ [MeV]	$\Gamma(\Xi_c^{*\,+}\to\Xi_c^+\pi^0)$ [MeV]	$\Gamma(\Xi_c^{*\,0}\to\Xi_c^+\pi^-)$ [MeV]	$\Gamma(\Xi_c^{*\,0}\to\Xi_c^0\pi^0)$ [MeV]
This work	$-28.83\pm0.50\pm0.10$	$1.84\pm0.06\pm0.01$	$1.34\pm0.04\pm0.01$	$2.07\pm0.07\pm0.01$	$0.956\pm0.030\pm0.007$
Theory					
LFQM		1.12 [23]	0.69 [23]	1.16 [23]	0.72 [23]
RTQM		1.78 ± 0.33 [24]	1.26 ± 0.17 [24]	2.11 ± 0.29 [24]	1.01 ± 0.15 [24]

	$\Gamma(\Xi_c^{*\,+}\to\Xi_c^0\pi^+ + \Xi_c^+\pi^0)$ [MeV]	$\Gamma(\Xi_c^{*\,+}\to\Xi_c^0\pi^+ + \Xi_c^+\pi^0)$ [MeV]
This work	$3.18\pm0.10\pm0.01$	$3.03\pm0.10\pm0.01$
Experiment	< 3.1 (CL=90%) [13]	< 5.5 (CL=90%) [14]
Theory		
CQM	$< 2.3\pm0.1$ [15]	$< 2.3\pm0.1$ [15]
	1.191 – 3.971 [16]	1.230 – 4.074 [16]
HHCPT	2.44 ± 0.85 [21]	2.51 ± 0.88 [21]
LFQM	1.81 [23]	1.88 [23]
RTQM	3.04 ± 0.50 [24]	3.12 ± 0.33 [24]

TABLE III: Values for the coupling $g_{\Xi_c^{*\,+}\Xi_c^0\pi^+}$ and decay widths $\Gamma(\Xi_c^{*\,+}\to\Xi_c^0\pi^+)$, $\Gamma(\Xi_c^{*\,+}\to\Xi_c^+\pi^0)$, $\Gamma(\Xi_c^{*\,0}\to\Xi_c^+\pi^-)$ and $\Gamma(\Xi_c^{*\,0}\to\Xi_c^0\pi^0)$. Experimental upper bounds for the total $\Xi_c^{*\,+}$ and $\Xi_c^{*\,0}$ widths, and different theoretical calculations are also shown.

$$= i\frac{g_{\Xi_c^{*\,+}\Xi_c^{+}\pi^+}}{M_{\Xi_c^+}+M_{\Xi_c^0}}\; q_\nu\; \overline{u}_{\Xi_c^0\;s'}(\vec{P}^{\,\prime})\; u^\nu_{\Xi_c^{*\,+}\;s}(\vec{P}) \tag{14}$$

where again we have introduced a dimensionless coupling $g_{\Xi_c^{*\,+}\Xi_c^0\pi^+}$. The width is given as

$$\Gamma(\Xi_c^{*\,+}\to\Xi_c^0\pi^+) = \frac{|\vec{q}\,|^3}{24\pi M^2_{\Xi_c^{*\,+}}}\;\frac{g^2_{\Xi_c^{*\,+}\Xi_c^0\pi^+}}{(M_{\Xi_c^+}+M_{\Xi_c^0})^2}\left((M_{\Xi_c^{*\,+}}+M_{\Xi_c^0})^2-m_\pi^2\right) \tag{15}$$

Taking now $\vec{P}=\vec{0}$, $\vec{P}^{\,\prime}=-|\vec{q}\,|\,\vec{k}$ in the z direction, and $s=s'=1/2$, $g_{\Xi_c^{*\,+}\Xi_c^0\pi^+}$ is evaluated from Eq.(14) to be

$$g_{\Xi_c^{*\,+}\Xi_c^0\pi^+} = \frac{\sqrt{3}}{f_\pi\sqrt{2}}\;\frac{(M_{\Xi_c^+}+M_{\Xi_c^0})\sqrt{2M_{\Xi_c^{*\,+}}2E_{\Xi_c^0}(|\vec{q}\,|)}}{|\vec{q}\,|\sqrt{2M_{\Xi_c^{*\,+}}\left(E_{\Xi_c^0}(|\vec{q}\,|)+M_{\Xi_c^0}\right)}}\left((M_{\Xi_c^{*\,+}}-E_{\Xi_c^0}(|\vec{q}\,|))\;A^{1/2,1/2}_{\Xi_c^{*\,+}\Xi_c^0,\;0}+|\vec{q}\,|\;A^{1/2,1/2}_{\Xi_c^{*\,+}\Xi_c^0,\;3}\right) \tag{16}$$

with

$$A^{1/2,1/2}_{\Xi_c^{*\,+}\Xi_c^0,\;\mu} = {}_{NR}\left\langle \Xi_c^0,\;1/2\;\; -|\vec{q}\,|\,\vec{k}\;\middle|\;J^{d\,u}_{A\,\mu}(0)\;\middle|\;\Xi_c^{*\,+},\;1/2\;\vec{0}\right\rangle_{NR,\;non-pole} \tag{17}$$

which expressions can be found in the appendix.
Results for the coupling $g_{\Xi_c^{*\,+}\Xi_c^0\pi^+}$, the widths $\Gamma(\Xi_c^{*\,+}\to\Xi_c^0\pi^+)$, $\Gamma(\Xi_c^{*\,+}\to\Xi_c^+\pi^0)$, $\Gamma(\Xi_c^{*\,0}\to\Xi_c^+\pi^-)$ and $\Gamma(\Xi_c^{*\,0}\to\Xi_c^0\pi^0)$, and the total widths $\Gamma(\Xi_c^{*\,+}\to\Xi_c^0\pi^+ + \Xi_c^+\pi^0)$ and $\Gamma(\Xi_c^{*\,0}\to\Xi_c^0\pi^0 + \Xi_c^+\pi^-)$ appear in Table III. Our values for $\Gamma(\Xi_c^{*\,+}\to\Xi_c^+\pi^0)$, $\Gamma(\Xi_c^{*\,0}\to\Xi_c^+\pi^-)$ and $\Gamma(\Xi_c^{*\,0}\to\Xi_c^0\pi^0)$ are obtained with the use of $g_{\Xi_c^{*\,+}\Xi_c^0\pi^+}/(M_{\Xi_c^+}+M_{\Xi_c^0})$, and with the appropriate mass changes in the rest of factors in Eq. (15). For $\Gamma(\Xi_c^{*\,+}\to\Xi_c^+\pi^0)$ and $\Gamma(\Xi_c^{*\,0}\to\Xi_c^0\pi^0)$ an extra 1/2 isospin factor should be included. Our results for $\Gamma(\Xi_c^{*\,+}\to\Xi_c^0\pi^+ + \Xi_c^+\pi^0)$ are slightly above the experimental bound obtained by the CLEO Collaboration [13]. As for the Σ_c^* decay case above, the AP1 and AP2 potentials gives results closer to experiment. For $\Gamma(\Xi_c^{*\,0}\to\Xi_c^+\pi^- + \Xi_c^0\pi^0)$ our result is well below the CLEO Collaboration experimental bound in Ref. [14]. Isospin breaking due to mass effects is clearly seen when comparing the predictions for $\Gamma(\Xi_c^{*\,+}\to\Xi_c^0\pi^-)$ and $\Gamma(\Xi_c^{*\,+}\to\Xi_c^+\pi^0)$. One finds a factor 1.4 difference when a factor of two would be expected from isospin symmetry. Again, the fact that there is little phase space available makes the results very sensitive to the actual mass values. Compared to other theoretical calculations our results agree nicely with the ones obtained within the RTQM of Ref. [24], while they are larger than most other determinations.

V. CONCLUDING REMARKS

We have evaluated the widths for the charmed-baryon decays $\Sigma_c^* \to \Lambda_c\pi$, $\Sigma_c \to \Lambda_c\pi$ and $\Xi_c^* \to \Xi_c\pi$ within the framework of a nonrelativistic quark model. While there have been dynamical calculations of these reactions in other models, to our knowledge this is the first time that such a calculation has been done within a nonrelativistic approach. We have used wave functions constrained by HQS and that were obtained solving the nonrelativistic three–body problem with the help of a simple variational ansazt. For that purpose we took five different nonrelativistic quark–quark interactions that included a confining term plus Coulomb and hyperfine terms coming from one-gluon exchange. To evaluate the pion emission amplitude we have used a spectator or one-quark pion emission model. The amplitude has been obtained with the use of PCAC from the analysis of weak current matrix elements. Our results are rather stable against the quark–quark interaction, with variations in the decay widths at the level of 6–8%. We find an overall good agreement with experiment for the three reactions. This agreement is, in most cases, better than the one obtained by other models.

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APPENDIX A: EXPRESSIONS FOR THE $A^{1/2,1/2}_{BB',\mu}$ MATRIX ELEMENTS

The values for the $A^{1/2,1/2}_{BB',\mu}$ are evaluated using one-body current operators and their expressions are given by

$$\begin{aligned} A^{1/2,1/2}_{\Sigma_c^{++}\Lambda_c^+,\mu} &= \frac{\sqrt{2}}{\sqrt{3}}\int d^3Q_1\ d^3Q_2\ \phi^{S_l=1}_{u,u,c}(\vec{Q}_1,\vec{Q}_2)\left(\phi^{S_l=0}_{d,u,c}(\vec{Q}_1-\frac{m_u+m_c}{\overline{M}_{\Lambda_c^+}}\,|\vec{q}\,|\,\vec{k},\ \vec{Q}_2+\frac{m_u}{\overline{M}_{\Lambda_c^+}}\,|\vec{q}\,|\,\vec{k})\right)^* \\ &\times\sum_{s_1}(1/2,1/2,1;s_1,-s_1,0)\,(1/2,1/2,0;s_1,-s_1,0)\ \frac{\overline{u}_{d\ s_1}(\vec{Q}_1-|\vec{q}\,|\,\vec{k})\ \gamma_\mu\gamma_5\ u_{u\,s_1}(\vec{Q}_1)}{\sqrt{2E_d(|\vec{Q}_1-|\vec{q}\,|\,\vec{k}|)2E_u(|\vec{Q}_1|)}} \end{aligned} \tag{A1}$$

where the quark Dirac spinors are normalized to twice the energy. For $\mu=0,3$ we get the final expressions

$$\begin{aligned} A^{1/2,1/2}_{\Sigma_c^{++}\Lambda_c^+,0} &= \frac{\sqrt{2}}{\sqrt{3}}\int d^3Q_1\ d^3Q_2\ \phi^{S_l=1}_{u,u,c}(\vec{Q}_1,\vec{Q}_2)\left(\phi^{S_l=0}_{d,u,c}(\vec{Q}_1-\frac{m_u+m_c}{\overline{M}_{\Lambda_c^+}}\,|\vec{q}\,|\,\vec{k},\ \vec{Q}_2+\frac{m_u}{\overline{M}_{\Lambda_c^+}}\,|\vec{q}\,|\,\vec{k})\right)^* \\ &\times\sqrt{\frac{\left(E_d(|\vec{Q}_1-|\vec{q}\,|\,\vec{k}|)+m_d\right)\left(E_u(|\vec{Q}_1|)+m_u\right)}{2E_d(|\vec{Q}_1-|\vec{q}\,|\,\vec{k}|)2E_u(|\vec{Q}_1|)}}\ \left(\frac{Q_1^z}{E_u(|\vec{Q}_1|)+m_u}+\frac{Q_1^z-|\vec{q}\,|}{E_d(|\vec{Q}_1-|\vec{q}\,|\,\vec{k}|)+m_d}\right) \end{aligned} \tag{A2}$$

$$\begin{aligned} A^{1/2,1/2}_{\Sigma_c^{++}\Lambda_c^+,3} &= -\frac{\sqrt{2}}{\sqrt{3}}\int d^3Q_1\ d^3Q_2\ \phi^{S_l=1}_{u,u,c}(\vec{Q}_1,\vec{Q}_2)\left(\phi^{S_l=0}_{d,u,c}(\vec{Q}_1-\frac{m_u+m_c}{\overline{M}_{\Lambda_c^+}}\,|\vec{q}\,|\,\vec{k},\ \vec{Q}_2+\frac{m_u}{\overline{M}_{\Lambda_c^+}}\,|\vec{q}\,|\,\vec{k})\right)^* \\ &\times\sqrt{\frac{\left(E_d(|\vec{Q}_1-|\vec{q}\,|\,\vec{k}|)+m_d\right)\left(E_u(|\vec{Q}_1|)+m_u\right)}{2E_d(|\vec{Q}_1-|\vec{q}\,|\,\vec{k}|)2E_u(|\vec{Q}_1|)}}\left(1+\frac{2(Q_1^z)^2-|\vec{Q}_1|^2-Q_1^z|\vec{q}\,|}{\left(E_d(|\vec{Q}_1-|\vec{q}\,|\,\vec{k}|)+m_d\right)\left(E_u(|\vec{Q}_1|)+m_u\right)}\right) \end{aligned} \tag{A3}$$

For $A^{1/2,1/2}_{\Sigma_c^{*\,++}\Lambda_c^+,\mu}$ we just have

$$A^{1/2,1/2}_{\Sigma_c^{*\,++}\Lambda_c^+,\mu} = -\sqrt{2}\ A^{1/2,1/2}_{\Sigma_c^{++}\Lambda_c^+,\mu} \tag{A4}$$

Similarly we get for $A^{1/2,1/2}_{\Xi_c^{*\,+}\Xi_c^0,\,\mu}$, $\mu=0,3$

$$A^{1/2,1/2}_{\Xi_c^{*\,+}\Xi_c^0,\,0}=\frac{\sqrt{2}}{\sqrt{3}}\int d^3Q_1\;d^3Q_2\;\phi^{S_l=1}_{u,s,c}(\vec{Q}_1,\vec{Q}_2)\left(\phi^{S_l=0}_{d,s,c}(\vec{Q}_1-\frac{m_s+m_c}{\overline{M}_{\Xi_c^0}}\,|\vec{q}\,|\,\vec{k},\;\vec{Q}_2+\frac{m_s}{\overline{M}_{\Xi_c^0}}\,|\vec{q}\,|\,\vec{k})\right)^*$$
$$\times\sqrt{\frac{\Big(E_d(|\vec{Q}_1-|\vec{q}\,|\,\vec{k}|)+m_d\Big)\Big(E_u(|\vec{Q}_1|)+m_u\Big)}{2E_d(|\vec{Q}_1-|\vec{q}\,|\,\vec{k}|)2E_u(|\vec{Q}_1|)}}\;\left(\frac{Q_1^z}{E_u(|\vec{Q}_1|)+m_u}+\frac{Q_1^z-|\vec{q}\,|}{E_d(|\vec{Q}_1-|\vec{q}\,|\,\vec{k}|)+m_d}\right)\qquad\text{(A5)}$$

$$A^{1/2,1/2}_{\Xi_c^{*\,+}\Xi_c^0,\,3}=-\frac{\sqrt{2}}{\sqrt{3}}\int d^3Q_1\;d^3Q_2\;\phi^{S_l=1}_{u,s,c}(\vec{Q}_1,\vec{Q}_2)\left(\phi^{S_l=0}_{d,s,c}(\vec{Q}_1-\frac{m_s+m_c}{\overline{M}_{\Xi_c^0}}\,|\vec{q}\,|\,\vec{k},\;\vec{Q}_2+\frac{m_s}{\overline{M}_{\Xi_c^0}}\,|\vec{q}\,|\,\vec{k})\right)^*$$
$$\times\sqrt{\frac{\Big(E_d(|\vec{Q}_1-|\vec{q}\,|\,\vec{k}|)+m_d\Big)\Big(E_u(|\vec{Q}_1|)+m_u\Big)}{2E_d(|\vec{Q}_1-|\vec{q}\,|\,\vec{k}|)2E_u(|\vec{Q}_1|)}}\;\left(1+\frac{2(Q_1^z)^2-|\vec{Q}_1|^2-Q_1^z|\vec{q}\,|}{\Big(E_d(|\vec{Q}_1-|\vec{q}\,|\,\vec{k}|)+m_d\Big)\Big(E_u(|\vec{Q}_1|)+m_u\Big)}\right)\qquad\text{(A6)}$$

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