

Quark-mass dependence of baryon resonances

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We study the quark-mass dependence of $J^P = \frac{1}{2}^-$ s-wave and $J^P = \frac{3}{2}^-$ d-wave baryon resonances. Parameter-free results are obtained in terms of the leading order chiral Lagrangian. In the 'heavy' SU(3) limit with $m_\pi = m_K \simeq 500$ MeV the s-wave resonances turn into bound states forming two octets plus a singlet representations of the SU(3) group. Similarly the d-wave resonances turn into bound states forming an octet and a decuplet in this limit. A contrasted result is obtained in the 'light' SU(3) limit with $m_\pi = m_K \simeq 140$ MeV for which no resonances exist.

1. Introduction

The meson-baryon scattering processes are an important test for effective field theories which aim at reproducing QCD at small energies, where the effective degrees of freedom are hadrons rather than quarks and gluons. The task to construct a systematic effective field theory for the meson-baryon scattering processes in the resonance region is closely linked to the fundamental question as to what is the 'nature' of baryon resonances. Here we support the conjecture [1,2,3,4] that baryon resonances not belonging to the large- N_c ground states are generated dynamically by coupled-channel dynamics [5,6,7,8,9,10]. For a comprehensive discussion of this issue we refer to [4]. This conjecture was the basis of the phenomenological model [3], which generated successfully non-strange s- and d-wave resonances by coupled-channel dynamics describing a large body of pion and photon scattering data. Before the event of the quark-model it was already suggested by Wyld [5] and also by Dalitz, Wong and Rajasekaran [6] that a t -channel vector meson exchange model for the s-wave meson-baryon scattering problem has the potential to dynamically generate s-wave baryon resonances upon solving a coupled-channel Schrödinger equation. In a more modern language the t -channel exchange was rediscovered in terms of the Weinberg-

Tomozawa (WT) interaction, the leading term of the chiral Lagrangian that reproduces the first term of the vector meson exchange in an appropriate Taylor expansion [11]. The main difference of the early attempts from computations based on the chiral Lagrangian is the way the coupled-channel scattering equation is regularized and renormalized. The crucial advance over the last years in this field is therefore a significant improvement of the systematics, i.e. how to implement corrections terms into coupled-channel dynamics [12,1,2].

In recent works [1,2,12,13,14], which will be reviewed here, it was shown that chiral dynamics provides a parameter-free prediction for the existence of a wealth of strange and non-strange s- and d-wave wave baryon resonances, once the crossing symmetry constraint is implemented properly. Naively one may expect that chiral dynamics does not make firm predictions for d-wave resonances since the meson baryon-octet interaction in the relevant channels probes a set of counter terms presently unknown. However, this is not necessarily so. Since a d-wave baryon resonance couples to s-wave meson baryon-decuplet states chiral symmetry is quite predictive for such resonances under the assumption that the latter channels are dominant. This is in full analogy to the analysis of the s-wave resonances [5,6,7,8,9,13,15,16] that neglects the effect of the contribution of d-wave meson baryon-decuplet states. The empirical observation that the d-wave resonances $N(1520)$, $N(1700)$ and $\Delta(1700)$ have large branching fractions ($> 50\%$) into the inelastic $N\pi\pi$ channel, even though the elastic πN channel is favored by phase space, supports our assumption.

2. Effective field theory of chiral coupled-channel dynamics

The starting point to describe the meson-baryon scattering process is the chiral SU(3) Lagrangian (see e.g.[17]). A systematic approximation scheme arises due to a successful scale separation justifying the chiral power counting rules [18]. Our effective field theory of the meson-baryon scattering processes is based on the assumption that the scattering amplitudes are perturbative at subthreshold energies with the expansion parameter Q/Λ_χ . The small scale Q is to be identified with any small momentum of the system. The chiral symmetry breaking scale is

$$\Lambda_\chi \simeq 4\pi f \simeq 1.13 \text{ GeV},$$

with the parameter $f \simeq 90$ MeV determined by the pion decay process. Once the available energy is sufficiently high to permit elastic two-body scattering a further typical dimensionless parameter $m_K^2/(8\pi f^2) \sim 1$ arises [12,1,2]. Since this ratio is uniquely linked to two-particle reducible diagrams it is sufficient to sum those diagrams keeping the perturbative expansion of all irreducible diagrams, i.e. the coupled-channel Bethe-Salpeter equation has to be solved. This is the basis of the χ -BS(3) approach developed in [12,1,2,19,20].

At leading order in the chiral expansion one encounters the Weinberg-Tomozawa [11] interaction,

$$\mathcal{L}_{WT} = \frac{i}{8f^2} \text{tr} \left((\bar{B} \gamma^\mu B) \cdot [\Phi, (\partial_\mu \Phi)]_- \right) + \frac{3i}{8f^2} \text{tr} \left((\bar{B}_\nu \gamma^\mu B^\nu) \cdot [\Phi, (\partial_\mu \Phi)]_- \right), \quad (1)$$

where we dropped additional structures that do not contribute to the on-shell scattering process at tree level. The terms in (1) constitute the leading order s-wave interaction of

Goldstone bosons (Φ) with the baryon-octet (B) and baryon-decuplet (B_μ) states. The octet and decuplet fields, Φ, B and B_μ , possess an appropriate matrix structure according to their $SU(3)$ tensor representation.

The scattering process is described by the amplitudes that follow as solutions of the Bethe-Salpeter equation,

$$T(\bar{k}, k; w) = V(\bar{k}, k; w) + \int \frac{d^4 l}{(2\pi)^4} V(\bar{k}, l; w) G(l; w) T(l, k; w),$$

$$G(l; w) = -i D(\frac{1}{2} w - l) S(\frac{1}{2} w + l), \quad (2)$$

where we suppress the coupled-channel structure for simplicity. The meson and baryon propagators, $D(q)$ and $S(p)$, are used in the notation of [3]. We apply the convenient kinematics:

$$w = p + q = \bar{p} + \bar{q}, \quad k = \frac{1}{2}(p - q), \quad \bar{k} = \frac{1}{2}(\bar{p} - \bar{q}), \quad (3)$$

where q, p, \bar{q}, \bar{p} the initial and final meson and baryon 4-momenta. The Bethe-Salpeter scattering equation is recalled for the case of meson baryon-octet scattering. An analogous equation holds for meson baryon-decuplet scattering process (see e.g. [3]). Referring to the detailed discussion given in [2,21,22,23] we assume a systematic on-shell reduction of the Bethe-Salpeter interaction kernel leading to the effective interaction V used in (2). The latter is expanded according to chiral power counting rules. The scattering amplitude $T(\bar{k}, k; w)$ decouples into various sectors characterized by isospin (I) and strangeness (S) quantum numbers. In the case of meson baryon-octet and baryon-decuplet scattering the following channels are relevant

$$(I, S)_{[8 \otimes 8]} = (0, -3), (1, -3), (\frac{1}{2}, -2), (\frac{3}{2}, -2), (0, -1),$$

$$(1, -1), (2, -1), (\frac{1}{2}, 0), (\frac{3}{2}, 0), (0, 1), (1, 1),$$

$$(I, S)_{[8 \otimes 10]} = (\frac{1}{2}, -4), (0, -3), (1, -3), (\frac{1}{2}, -2), (\frac{3}{2}, -2), (0, -1),$$

$$(1, -1), (2, -1), (\frac{1}{2}, 0), (\frac{3}{2}, 0), (\frac{5}{2}, 0), (1, 1), (2, 1). \quad (4)$$

Following the χ -BS(3) approach developed in [2,3] the effective interaction kernel is decomposed into a set of covariant projectors that have well defined total angular momentum, J , and parity, P ,

$$V(\bar{k}, k; w) = \sum_{J,P} V^{(J,P)}(\sqrt{s}) \mathcal{Y}^{(J,P)}(\bar{q}, q, w). \quad (5)$$

The merit of the projectors is that they decouple the Bethe-Salpeter equation (2) into orthogonal sectors labelled by the total angular momentum, J , and parity, P . We insist on the renormalization condition,

$$T^{(I,S)}(\bar{k}, k; w) \Big|_{\sqrt{s}=\mu(I,S)} = V^{(I,S)}(\bar{k}, k; w) \Big|_{\sqrt{s}=\mu(I,S)}, \quad (6)$$

together with the natural choice for the subtraction points,

$$\mu(I, +1) = \mu(I, -3) = \frac{1}{2}(m_\Lambda + m_\Sigma), \quad \mu(I, 0) = m_N,$$

$$\mu(0, -1) = m_\Lambda, \quad \mu(1, -1) = m_\Sigma, \quad \mu(I, -2) = \mu(I, -4) = m_\Xi \quad (7)$$

as explained in detail in [2]. The renormalization condition reflects the crucial assumption our effective field theory is based on, namely that at subthreshold energies the scattering amplitudes can be evaluated in standard chiral perturbation theory. This is achieved by supplementing (2) with (6,7). The subtraction points (7) are the unique choices that protect the s-channel baryon-octet masses manifestly in the p-wave $J = \frac{1}{2}$ scattering amplitudes. The merit of the scheme [12,1,2] lies in the property that for instance the $K \Xi$ and $\bar{K} \Xi$ scattering amplitudes match at $\sqrt{s} \sim m_{\Xi}$ approximately as expected from crossing symmetry. In [2] we suggested to glue s- and u-channel unitarized scattering amplitudes at subthreshold energies. This construction reflects our basic assumption that diagrams showing an s-channel or u-channel unitarity cut need to be summed to all orders at least at energies close to where the diagrams develop their imaginary part. By construction, a glued scattering amplitude satisfies crossing symmetry exactly at energies where the scattering process takes place. At subthreshold energies crossing symmetry is implemented approximatively only, however, to higher and higher accuracy when more chiral correction terms are considered. Insisting on the renormalization condition (6,7) guarantees that subthreshold amplitudes match smoothly and therefore the final 'glued' amplitudes comply with the crossing-symmetry constraint to high accuracy. The natural subtraction points (7) can also be derived if one incorporates photon-baryon inelastic channels. Then additional constraints arise. For instance the reaction $\gamma \Xi \rightarrow \gamma \Xi$, which is subject to a crossing symmetry constraint at threshold, may go via the intermediate states $\bar{K} \Lambda$ or $\bar{K} \Sigma$.

The perturbative nature of subthreshold amplitudes, a crucial assumption of the χ -BS(3) approach proposed in [12,1,2], is not necessarily true in phenomenological coupled-channel schemes in [24,19,20,25,26,16]. Using the subtraction scales as free parameters, as advocated in [25,26,16], may be viewed as promoting the counter terms of chiral order Q^3 to be unnaturally large. If the subtraction scales are chosen far away from their natural values (7) the resulting loop functions are in conflict with chiral power counting rules [12]. Though unnaturally large Q^3 counter terms can not be excluded from first principals one should check such an assumption by studying corrections terms systematically [1,2]. A detailed test of the naturalness of the Q^3 counter terms was performed within the χ -BS(3) scheme [2] demonstrating good convergence in the channels studied without any need for promoting the counter terms of order Q^3 . Possible correction terms in the approach followed in [25,26,16] have so far not been studied systematically for meson-baryon scattering. Moreover, if the scheme advocated in [25,26,16] were applied in all eleven isospin strangeness sectors with $J^P = \frac{1}{2}^-$ a total number of 26 subtraction parameters arise. This should be compared with the only ten counter terms of chiral order Q^3 contributing to the on-shell scattering amplitude at that order [2]. Selecting only the operators that are leading in the large- N_c limit of QCD out of the ten Q^3 operators only four survive [2]. For consistency and before applying the approach of [25,26,16] to all isospin strangeness channels, it would be highly desirable to address the above mismatch of parameters. Our scheme has the advantage that once the parameters describing subleading effects are determined in a subset of sectors one has immediate predictions for all sectors (I, S) . A mismatch of the number of parameters is avoided altogether since the Q^3 counter terms enter the effective interaction kernel directly.

Given the subtraction scales (7) the leading order calculation is parameter free. Of

course chiral correction terms do lead to further so far unknown parameters which need to be adjusted to data. Within the χ -BS(3) approach such correction terms enter the effective interaction kernel V rather than leading to subtraction scales different from (7) as it is assumed in [25,26,16]. In particular the leading correction effects are determined by the counter terms of chiral order Q^2 . The effect of altering the subtraction scales away from their optimal values (7) can be compensated for by incorporating counter terms in the chiral Lagrangian that carry order Q^3 .

3. Baryon resonances from chiral SU(3) symmetry

We begin with a discussion of the s- and d-wave baryon resonance spectrum that arises in the SU(3) limit. The latter is not defined uniquely depending on the magnitude of the current quark masses, $m_u = m_d = m_s$. We consider two scenarios [13,14]. In the 'light' SU(3) limit the current quark masses are chosen such that one obtains $m_\pi = m_K = m_\eta = 140$ MeV. The second case, the 'heavy' SU(3) limit, is characterized by $m_\pi = m_K = m_\eta = 500$ MeV. In the SU(3) limit meson baryon-octet scattering is classified according to,

$$8 \otimes 8 = 27 \oplus \overline{10} \oplus 10 \oplus 8 \oplus 8 \oplus 1. \quad (8)$$

The leading order chiral Lagrangian predicts attraction in the two octet and the singlet channels but repulsion in the 27-plet and decuplet channels. As a consequence in the 'heavy' SU(3) limit the chiral dynamics predicts two degenerate octet bound states together with a non-degenerate singlet state [5,6,7,8,9,13,16,13,14]. In the 'light' SU(3) limit all states disappear. In the $J^P = \frac{3}{2}^-$ sector the Weinberg-Tomozawa interaction is attractive in the octet, decuplet and 27-plet channels, but repulsive in the 35-plet channel,

$$8 \otimes 10 = 35 \oplus 27 \oplus 10 \oplus 8. \quad (9)$$

Therefore one may expect resonances or bound states in the former channels. Indeed, in the 'heavy' SU(3) limit we find $72 = 4 \times (8 + 10)$ bound states in this sector forming an octet and decuplet representation of the SU(3) group. We do not find a 27-plet-bound state reflecting the weaker attraction in that channel. However, if we artificially increase the amount of attraction by about 40 % by lowering the value of f in the Weinberg-Tomozawa term, a clear bound state arises in this channel also. A contrasted result is obtained if we lower the meson masses down to the pion mass arriving at the 'light' SU(3) limit. Then we find neither bound nor resonance octet or decuplet states. This pattern is a clear prediction of chiral coupled-channel dynamics which should be tested with unquenched QCD lattice simulations [27].

We look for poles on the Second Riemann Sheet of our s-wave scattering amplitudes. Positions of the poles determine masses and widths of the resonances while the residues in the different channels define the branching ratios [19,20]. To explore the quark mass dependence of the resonances, we vary the averaged *up* and *down* quark masses, but keep fixed the antikaon mass. A parameter x is introduced in terms of which the pion mass varies as

$$m_\pi^2|_{\text{SU}(3)} = m_\pi^2 + x(m_K^2 - m_\pi^2), \quad x \in [0, 1]. \quad (10)$$

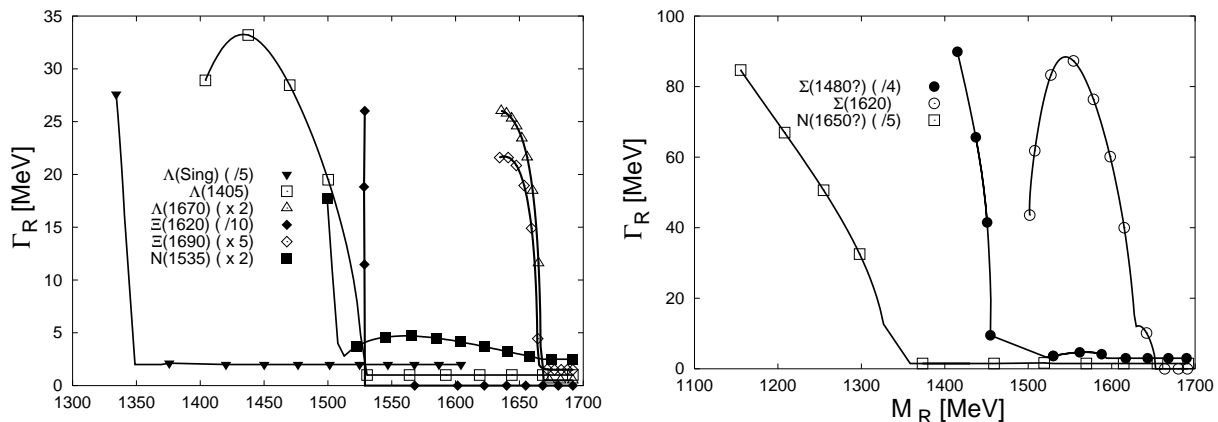


Figure 1. Masses and decay widths of two octets and a singlet of baryon states for several values of the pion mass. For each baryon state we plot eleven points, which correspond to eleven equally spaced values of x (see (10)) ranging from 1 (first point from the right) to 0 (first point from the left). Widths of different baryon resonances have been scaled by factors, as it is detailed in the legend of the plots. To disentangle among different states, for some of them, the widths have been shifted by constants factors. Lines have been plotted just to guide the eye.

A pair $(m_K^2, m_\pi^2|_{\text{SU}(3)})$ determines the η meson mass via the Gell-Mann–Oakes–Renner relation. Given the SU(3) symmetry breaking parameters b_0 , b_D and b_F of [2], the masses of the baryon octet (N(940), $\Lambda(1115)$, $\Sigma(1190)$ and $\Xi(1320)$) are also determined. In the limit $x = 1$ our SU(3) pion is as heavy as the real kaon, while when $x = 0$ the physical world is recovered approximatively. As demonstrated in Fig. 1 we find a remarkable success predicting rather well the bulk of the features of the four stars N(1535), $\Lambda(1405)$ and $\Lambda(1670)$ resonances [13].

We also find a narrow resonance with $(I, S) = (1, -1)$, though it appears in the unphysical 11000 sheet (see [19,13]). In Fig. 1 the trajectory of this pole is labelled with $\Sigma(1620)$, a two star resonance, since it couples strongly to the $\pi\Lambda$, $\pi\Sigma$ and specially to the $\bar{K}N$ channels. There exists also a pole in the 11100 sheet with $M_R = 1466$ MeV and $\Gamma_R = 574$ MeV not shown in Fig. 1. It is above the $\bar{K}N$ threshold and has a large coupling to the $K\Xi$ channel. This very broad pole is precisely the one quoted in [15]. It does not influence the physical scattering at all. However, chiral corrections reduce its width significantly and take its mass closer to the $K\Xi$ threshold indicating that it is the three star $\Sigma(1750)$ resonance [2]. Fig. 1 shows also the fate of the second $(I, S) = (\frac{1}{2}, 0)$ state, which is moved far away from the real axis down to quite low energies. Chiral corrections appear not to take the position of this pole closer to that of the physical N(1650) resonance [2]. A quantitative description may require the inclusion of further inelastic channels, like $\pi\Delta$ and ρN [3]

In the $S = -2$ sector we find two resonances, which can clearly be identified to the $\Xi(1690)$ and $\Xi(1620)$ resonances. Of particular interest is the signal for the $\Xi(1690)$ resonance, where we find a quite small (large) coupling to the $\pi\Xi$ ($\bar{K}\Sigma$) channel, which explains the smallness of the experimental ratio, $\Gamma(\pi\Xi)/\Gamma(\bar{K}\Sigma) < 0.09$ [28] despite of the significant energy difference between the thresholds for the $\pi\Xi$ and $\bar{K}\Sigma$ channels. On the other hand, we also find a third Λ resonance placed also below the $\bar{K}N$ threshold and with a large coupling to the $\pi\Sigma$ channel. This confirms the findings of the recent works

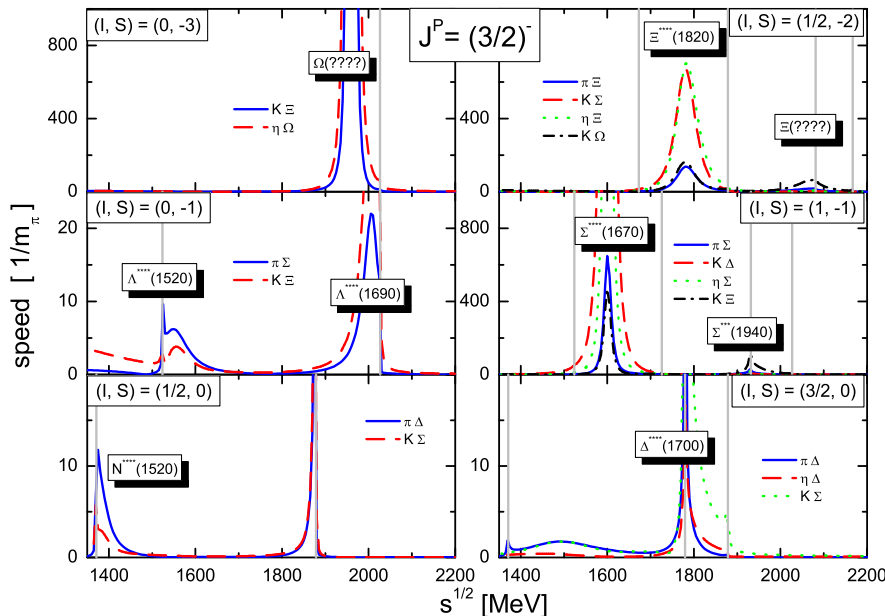


Figure 2. Diagonal speed plots of the $J^P = \frac{3}{2}^-$ sector. The vertical lines show the positions of inelastic meson baryon-decuplet channels. Parameter-free results are obtained in terms of physical masses and $f = 90$ MeV.

[16,15,20], where the dynamics of the $\Lambda(1405)$ states was studied.

In Fig. 2 speed plots of the $J^P = \frac{3}{2}^-$ sector are shown for all channels in which octet and decuplet resonance states are expected. In [14] we generalized the notion of a speed [29] to the case of coupled-channels in way that the latter reveals the coupling strength of a given resonance to any channel, closed or open. If a resonance with not too large decay width sits in the amplitude a clear peak structure emerges in the speed plot even if the resonance structure is masked by a background phase. In the case of s-wave scattering thresholds induce square-root singularities which should not be confused with a resonance signal. It is a remarkable success of the χ -BS(3) approach that it predicts the four star hyperon resonances $\Xi(1820)$, $\Lambda(1520)$, $\Sigma(1670)$ with masses quite close to the empirical values. The nucleon and isobar resonances $N(1520)$ and $\Delta(1700)$ also present in Fig. 2, are predicted with less accuracy. The important result here is the fact that those resonances are generated at all. It should not be expected to obtain already fully realistic results in this leading-order calculation. For instance chiral correction terms are expected to provide a d-wave $\pi \Delta$ -component of the $N(1520)$. We continue with the peak in the (0,-3)-speeds at mass 1950 MeV. Since this is below all thresholds it is in fact a bound state. Such a state has so far not been observed but is associated with a decuplet resonance [30]. Further states belonging to the decuplet are seen in the $(\frac{1}{2}, -2)$ - and $(1, -1)$ -speeds at masses 2100 MeV and 1920 MeV. The latter state can be identified with the three-star $\Xi(1940)$ resonance. Finally we point at the fact that the $(0, -1)$ -speeds show signals of two resonance states consistent with the existence of the four star resonance $\Lambda(1520)$ and $\Lambda(1690)$ even though in the 'heavy' SU(3) limit we observed only one bound state. It appears that the SU(3) symmetry breaking pattern generates the 'missing' state in this particular sector by promoting the weak attraction of the 27-plet contribution in (9).

4. Summary

In this talk we reported on recent progress in the understanding of baryon resonances based on chiral-coupled channel dynamics. The reader was introduced to an effective field theory formulation of chiral coupled-channel dynamics. Leading order results predict the existence of s- and d-wave baryon resonances with a spectrum remarkably close to the empirical pattern without any adjustable parameters. The formation of resonances is a consequence of the chiral SU(3) symmetry of QCD, i.e. in an effective field theory, that was based on the chiral SU(2) symmetry only, no resonances would be formed.

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