Study of the leptonic decays of pseudoscalar B, D and vector B^*, D^* mesons and of the semileptonic $B \to D$ and $B \to D^*$ decays.

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We present results for different observables in weak decays of pseudoscalar and vector mesons with a heavy c or b quark. The calculations are done in a nonrelativistic constituent quark model improved at some instances by heavy quark effective theory constraints. We determine pseudoscalar and vector meson decay constants that within a few per cent satisfy $f_V M_V / f_P M_P = 1$, a result expected in heavy quark symmetry when the heavy quark masses tend to infinity. We also analyze the semileptonic $B \to D$ and $B \to D^*$ decays for which we evaluate the different form factors. Here we impose heavy quark effective theory constraints among form factors that are not satisfied by a direct quark model calculation. The value of the form factors at zero recoil allows us to determine, by comparison with experimental data, the value of the $|V_{cb}|$ Cabbibo-Kobayashi-Maskawa matrix element. From the $B \to D$ semileptonic decay we get $|V_{cb}| = 0.040 \pm 0.006$ in perfect agreement with our previous determination based on the study of the semileptonic $\Lambda_b \to \Lambda_c$ decay and also in excellent agreement with a recent experimental determination by the DELPHI Collaboration. We further make use of the partial conservation of axial current hypothesis to determine the strong coupling constants $g_{B^*B\pi}(0) = 60.5 \pm 1.1$ and $g_{D^*D\pi}(0) = 22.1 \pm 0.4$. The ratio $R = (\ g_{B^*B\pi}(0)\, f_{B^*} \sqrt{M_D}\ )/(\ g_{D^*D\pi}(0)\, f_{D^*} \sqrt{M_B}\ ) = 1.105 \pm 0.005$ agrees with the heavy quark symmetry prediction of 1.

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I. INTRODUCTION

In systems with a heavy quark with mass much larger than the QCD scale (Λ_{QCD}) a new symmetry, known as heavy quark symmetry (HQS) [1, 2, 3, 4], arises. In that limit the dynamics of the light quark degrees of freedom becomes independent of the heavy quark flavor and spin. This is similar to what happens in atomic physics where the electron properties are approximately independent of the spin and mass of the nucleus (for a fixed nuclear charge). HQS can be cast into the language of an effective theory (HQET)[5] that allows a systematic, order by order, evaluation of corrections to the infinity mass limit in inverse powers of the heavy quark masses. HQS and HQET have proved very useful tools to understand bottom and charm physics and they have been extensively used to describe the dynamics of systems containing a heavy c or b quark [6, 7]. For instance, all lattice QCD simulations rely on HQS to describe bottom systems [8].

In a recent publication [9] we have studied the $\Lambda_b^0 \to \Lambda_c^+\, l^- \bar\nu_l$ and $\Xi_b^0 \to \Xi_c^+\, l^- \bar\nu_l$ reactions in a nonrelativistic quark model. The detailed analysis of the different form factors showed how a direct nonrelativistic calculation does not meet HQET constraints and we had to improve our model imposing HQET relations among form factors. Our calculation allowed for a determination of the $|V_{cb}|$ Cabibbo-Kobayashi-Maskawa (CKM) matrix element given by $|V_{cb}| = 0.040 \pm 0.005^{+0.001}_{-0.002}$ in good agreement with a recent determination by the DELPHI Collaboration $|V_{cb}| = 0.0414 \pm 0.0012 \pm 0.0021 \pm 0.0018$ [10]. What we intend to do here is a study of different weak observables of pseudoscalar and vector mesons with a heavy c or b quark in a non relativistic quark model improved at some points with HQET constraints. Weak observables are of great interest as they help to probe the quark structure of hadrons and provide information to measure the CKM matrix elements.

In the case of mesons with a heavy quark HQS leads to many model independent predictions. For instance in the HQS limit the masses of the lowest lying (s-wave) pseudoscalar and vector mesons with a heavy quark are degenerate. Nonrelativistic quark models satisfy this constraint: the reduced mass of the system is just the mass of the light quark and the spin-spin terms, that distinguishes vector from pseudoscalar, are zero if the mass of the heavy quark goes to infinity. HQS also predicts that the masses and leptonic decay constants of pseudoscalars and vector mesons are related via $f_P\, M_P = f_V\, M_V$, relation that is also satisfied in the quark model in the HQS limit. If one looks now at the form factors for the semileptonic $B \to D$ and $B \to D^*$ decays HQS predicts relations among different form factors that are also met by the quark model in the HQS limit. The question is to what extent the deviations from the HQS limit evaluated in the quark model agree with the constraints deduced from HQET. In addition we will make use of these HQET constraints to improve the quark model results and thus come up with reliable predictions.

The paper is organized as follows: in section II we introduce the meson wave functions and interquark potentials we shall use in this work. In section III we analyze the leptonic decays of pseudoscalar and vector B and D mesons, determining the different decay constants. In section IV we study the form factors for the semileptonic $B \to D\, l\, \bar\nu$ and

$B \to D^* l \bar{\nu}$ decays. In section V we evaluate the strong coupling constants $g_{B^*B\pi}$ and $g_{D^*D\pi}$. Finally in section VI we end with the conclusions. The paper also contains three appendices where we collect the expressions for the matrix elements that are needed for the evaluation of different observables.

Apart from lattice QCD and QCD sum rules (QCDSR) calculations with which we shall compare our results and that will be quoted in the following, the different observables analyzed in this work have been studied in the quark model starting with the pioneering work of Ref. [11] within a non relativistic version, to continue with different versions of the relativistic quark model applied to the determination of decay constants [12, 13, 14, 15, 16, 17, 18, 19, 20, 21], form factors and differential decay widths [19, 21, 22, 23, 24, 25, 26, 27], Isgur-Wise functions [21, 28, 29, 30, 31, 32, 33, 34] or strong coupling constants [19, 32, 35, 36, 37, 38] [1].

II. WAVE FUNCTION AND INTERQUARK INTERACTIONS

For a meson M we use the following expression for the wave function

$$\left|M, \lambda\, \vec{P}\right\rangle_{NR} = \int d^3p \sum_{\alpha 1, \alpha_2} \hat{\phi}^{(M,\lambda)}_{\alpha 1, \alpha_2}(\vec{p}) \frac{(-1)^{\frac{1}{2}-s_2}}{(2\pi)^{\frac{3}{2}}\sqrt{2E_{f_1}(\vec{p}_1)2E_{f_2}(\vec{p}_2)}} \left| q,\ \alpha_1\ \vec{p}_1 = \frac{m_{f_1}}{m_{f_1}+m_{f_2}}\vec{P} - \vec{p} \right\rangle \left| \bar{q},\ \alpha_2\ \vec{p}_2 = \frac{m_{f_2}}{m_{f_1}+m_{f_2}}\vec{P} + \vec{p} \right\rangle \tag{1}$$

where $\vec{P}$ stands for the meson three momentum and λ represents the spin projection in the meson center of mass. α_1 and α_2 represent the quantum numbers of spin (s), flavor (f) and color (c)

$$\alpha \equiv (s, f, c) \tag{2}$$

of the quark and the antiquark, while E_{f_1}, $\vec{p}_1$ and E_{f_2}, $\vec{p}_2$ are their respective energies and three-momenta. m_f is the mass of the quark or antiquark with flavor f. The factor $(-1)^{\frac{1}{2}-s_2}$ is included in order that the antiquark spin states have the correct relative phase[2]. The normalization of the quark and antiquark states is

$$\langle\ \alpha'\ \vec{p}\,'\,|\,\alpha\ \vec{p}\rangle = \delta_{\alpha',\ \alpha}\,(2\pi)^3\, 2E\,\delta(\vec{p}\,' - \vec{p}) \tag{3}$$

Furthermore, $\hat{\phi}^{(M,\lambda)}_{\alpha 1, \alpha_2}(\vec{p})$ is the momentum space wave function for the relative motion of the quark-antiquark system. Its normalization is given by

$$\int d^3p \sum_{\alpha_1\,\alpha_2} \left(\hat{\phi}^{(M,\lambda')}_{\alpha_1,\,\alpha_2}(\vec{p})\right)^* \hat{\phi}^{(M,\lambda)}_{\alpha_1,\,\alpha_2}(\vec{p}) = \delta_{\lambda',\,\lambda} \tag{4}$$

and, thus, the normalization of our meson states is

$$_{NR}\left\langle M, \lambda'\, \vec{P}\,'\,|\, M, \lambda\, \vec{P}\right\rangle_{NR} = \delta_{\lambda',\,\lambda}\,(2\pi)^3\,\delta(\vec{P}\,' - \vec{p}) \tag{5}$$

For the particular case of ground state pseudoscalar (P) and vector (V) mesons we can assume the orbital angular momentum to be zero and then we will have

$$\begin{aligned}
\hat{\phi}^{(P)}_{\alpha 1,\,\alpha_2}(\vec{p}) &= \frac{1}{\sqrt{3}}\delta_{c_1,\,c_2}\ \hat{\phi}^{(P)}_{(s_1,\,f_1),\,(s_2,\,f_2)}(\vec{p}) \\
&= \frac{1}{\sqrt{3}}\delta_{c_1,\,c_2}\ (-i)\ \hat{\phi}^{(P)}_{f_1,\,f_2}(|\vec{p}|)\ Y_{00}(\widehat{\vec{p}}\,)\ (1/2, 1/2, 0; s_1, s_2, 0) \\
\hat{\phi}^{(V,\,\lambda)}_{\alpha 1,\,\alpha_2}(\vec{p}) &= \frac{1}{\sqrt{3}}\delta_{c_1,\,c_2}\ \hat{\phi}^{(V,\,\lambda)}_{(s_1,\,f_1),\,(s_2,\,f_2)}(\vec{p}) \\
&= \frac{1}{\sqrt{3}}\delta_{c_1,\,c_2}\ (-1)\ \hat{\phi}^{(V)}_{f_1,\,f_2}(|\vec{p}|)\ Y_{00}(\widehat{\vec{p}}\,)\ (1/2, 1/2, 1; s_1, s_2, \lambda)
\end{aligned} \tag{6}$$

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[1] The list of references is by no means exhaustive.

[2] Note that under charge conjugation ($\mathcal{C}$) quark and antiquark creation operators are related via $\mathcal{C}\, c^\dagger_\alpha(\vec{p})\,\mathcal{C}^\dagger = (-1)^{\frac{1}{2}-s}\, d^\dagger_\alpha(\vec{p})$. This means that the antiquark states with the correct spin relative phase are not $d^\dagger_\alpha(\vec{p})\,|0\rangle = |\,\bar{q},\ \alpha\ \vec{p}\,\rangle$ but are instead given by $(-1)^{\frac{1}{2}-s}\, d^\dagger_\alpha(\vec{p})\,|0\rangle = (-1)^{\frac{1}{2}-s}\,|\,\bar{q},\ \alpha\ \vec{p}\,\rangle$.

where $(j_1, j_2, j_3; m_1, m_2, m_3)$ is a Clebsch-Gordan coefficient, $Y_{00} = 1/\sqrt{4\pi}$ is the $l = m = 0$ spherical harmonic, and $\hat{\phi}^{(M)}_{f_1,\,f_2}(|\,\vec{p}\,|)$ is the Fourier transform of the radial coordinate space wave function. The phases are introduced for later convenience.

To evaluate the coordinate space wave function we shall use several interquark potentials, one suggested by Bhaduri and collaborators [39] (BHAD), and four suggested by Silvestre-Brac and Semay [40, 41] (AL1,AL2,AP1,AP2). The general structure of those potentials in the quark-antiquark sector is

$$V_{ij}^{q\bar{q}}(r) = -\frac{\kappa\left(1 - e^{-r/r_c}\right)}{r} + \lambda r^p - \Lambda + \left\{ a_0 \frac{\kappa}{m_i m_j} \frac{e^{-r/r_0}}{r r_0^2} + \frac{2\pi}{3 m_i m_j} \kappa' \left(1 - e^{-r/r_c}\right) \frac{e^{-r^2/x_0^2}}{\pi^{\frac{3}{2}} x_0^3} \right\} \vec{\sigma}_i \vec{\sigma}_j \tag{7}$$

with $\vec{\sigma}$ the spin Pauli matrices, m_i the constituent quark masses and

$$x_0(m_i, m_j) = A \left(\frac{2 m_i m_j}{m_i + m_j} \right)^{-B} \tag{8}$$

The potentials considered differ in the form factors used for the hyperfine terms, the power of the confining term ($p = 1$, as suggested by lattice QCD calculations [42], or $p = 2/3$ which gives the correct asymptotic Regge trajectories for mesons [43]), or the use of a form factor in the one gluon exchange Coulomb potential. All free parameters in the potentials have been adjusted to reproduce the light (π, ρ, K, K^*, etc.) and heavy-light (D, D^*, B, B^*, etc.) meson spectra. They also lead to precise predictions for the charmed and bottom baryon ($\Lambda_{c,b}$, $\Sigma_{c,b}$, $\Sigma^*_{c,b}$, $\Xi_{c,b}$, $\Xi'_{c,b}$, $\Xi^*_{c,b}$, $\Omega_{c,b}$ and $\Omega^*_{c,b}$) masses [40, 44] and for the semileptonic $\Lambda_b^0 \to \Lambda_c^+ l^- \bar{\nu}_l$ and $\Xi_b^0 \to \Xi_c^+ l^- \bar{\nu}_l$ [9] decays.

We will use the above mentioned interquark interactions to evaluate the different observables. This will provide us with a spread of results that we will consider, and quote, as a theoretical error to the averaged value that will quote as our central result.

III. LEPTONIC DECAY OF PSEUDOSCALAR AND VECTOR B AND D MESONS

In this section we will consider the purely leptonic decay of pseudo scalars (B, D) and vector (B^*, D^*) mesons. The charged weak current operator for a specific pair of quark flavors f_1 and f_2 reads

$$J_\mu^{f_1\, f_2}(0) = J_{V\,\mu}^{f_1\, f_2}(0) - J_{A\,\mu}^{f_1\, f_2}(0) = \sum_{(c_1,\, s_1),\, (c_2,\, s_2)} \delta_{c_1,\, c_2}\ \overline{\Psi}_{\alpha_1}(0) \gamma_\mu (1 - \gamma_5) \Psi_{\alpha_2}(0) \tag{9}$$

with Ψ_{α_1} a quark field of a definite spin, flavor and color. The hadronic matrix elements involved in the processes can be parametrized in terms of a unique pseudoscalar f_P or vector f_V decay constant as

$$\begin{aligned} \langle 0|\, J_\mu^{f_1\, f_2}(0) \left| P, \vec{P} \right\rangle &= \langle 0| - J_{A\,\mu}^{f_1\, f_2}(0) \left| P, \vec{P} \right\rangle = -i P_\mu\, f_P \\ \langle 0|\, J_\mu^{f_1\, f_2}(0) \left| V, \lambda\, \vec{P} \right\rangle &= \langle 0|\, J_{V\,\mu}^{f_1\, f_2}(0) \left| V,\, \lambda\, \vec{P} \right\rangle = \varepsilon_\mu^{(\lambda)}(\,\vec{P}\,) M_V f_V \end{aligned} \tag{10}$$

where the meson states are normalized such that

$$\left\langle M, \lambda'\, \vec{P}\,' \,|\, M, \lambda\, \vec{P} \right\rangle = \delta_{\lambda',\,\lambda}\, (2\pi)^3\, 2E\, \delta(\vec{P}\,' - \vec{p}\,) \tag{11}$$

In the first of Eqs.(10) P_μ is the four-momentum of the meson, while in the second M_V and $\varepsilon_\mu^{(\lambda)}(\,\vec{P}\,)$ are the mass and the polarization vector of the vector meson. In both cases f_1 and f_2 are the flavors of the quark and the antiquark that make up the meson.

Concerns about the experimental determination of the pseudoscalars decay contants have been raised in Ref. [45]. There the effect of radiative decays was analyzed concluding that for B mesons the decay constant determination could be greatly affected by radiative corrections. In the vector sector, and as rightly pointed out in Ref. [46], the vector decay constants are not relevant from a phenomenological point of view since B^* and D^* will decay through the electromagnetic and/or strong interaction. They are nevertheless interesting as a mean to test HQS relations.

For mesons at rest we will obtain

$$\begin{aligned} f_P &= \frac{-i}{M_P}\ \langle 0|\, J_{A\,0}^{f_1\, f_2}(0) \left| P, \vec{0} \right\rangle \\ f_V &= \frac{-1}{M_V}\ \langle 0|\, J_{V\,3}^{f_1\, f_2}(0) \left| V,\, 0\, \vec{0} \right\rangle \end{aligned} \tag{12}$$

with M_P the mass of the pseudoscalar meson. In our model, and due to the different normalization of our meson states, we shall evaluate the decay constants as

$$f_P = -i\sqrt{\frac{2}{M_P}}\,\langle 0|\, J_{A\,0}^{f_1\,f_2}(0)\,\Big|\,P,\vec{0}\,\Big\rangle_{NR}$$
$$f_V = -\sqrt{\frac{2}{M_V}}\,\langle 0|\, J_{V\,3}^{f_1\,f_2}(0)\,\Big|\,V,\,0\,\vec{0}\,\Big\rangle_{NR} \tag{13}$$

The corresponding matrix elements are given in appendix A

The results that we obtain for the different decay constants appear in Tables I and II. Starting with f_D and f_{D_s} our results are larger that the ones obtained in the lattice by the UKQCD Collaboration [54] or the ones evaluated using QCD spectral sum rules (QSSR) [57]. Not only the independent values are larger but also the ratio f_{D_s}/f_D is larger in our case. On the other hand our results are in better agreement with other lattice determinations [55, 56]. They also compare very well with the experimental measurements of f_D and f_{D_s} in Refs. [47, 48, 49, 50, 51, 53] being our f_{D_s}/f_D ratio in very good agreement with the value obtained using recent CLEO Collaboration data [47, 48]. As for f_B and f_{B_s}, we find a very good agreement in the case of f_{B_s} between our results and the ones obtained in the lattice or with the use of QSSR. For f_B our result is smaller and then also our ratio f_{B_s}/f_B is larger.

	f_D [MeV]	f_{D_s} [MeV]	f_{D_s}/f_D
This work	243^{+21}_{-17}	341^{+7}_{-5}	$1.41^{+0.08}_{-0.09}$
Experimental data			
CLEO	$202 \pm 41 \pm 17$ [47]	$280 \pm 19 \pm 28 \pm 34$ [48]	—
ALEPH [49]	—	$285 \pm 19 \pm 40$	—
OPAL [50]	—	$286 \pm 44 \pm 41$	—
BEATRICE [51]	—	$323 \pm 44 \pm 12 \pm 34$	—
E653 [52]	—	$194 \pm 35 \pm 20 \pm 14$	—
BES [53]	$371^{+129}_{-119} \pm 25$	—	—
Lattice data			
UKQCD [54]	$206(4)^{+17}_{-10}$	$229(3)^{+23}_{-12}$	$1.11(1)^{+1}_{-1}$
Fermilab Lattice [55] (Preliminary)	$225^{+11}_{-13} \pm 21$	$263^{+5}_{-9} \pm 24$	—
M. Wingate *et al.* [56]	—	$290 \pm 20 \pm 29 \pm 29 \pm 6$	—
QCD Spectral Sum Rules			
S. Narison [57]	203 ± 23	235 ± 24	1.15 ± 0.04
	f_B [MeV]	f_{B_s} [MeV]	f_{B_s}/f_B
This work	155^{+15}_{-12}	239^{+9}_{-7}	$1.54^{+0.09}_{-0.08}$
Lattice data			
UKQCD [54]	$195(6)^{+24}_{-23}$	$220(6)^{+23}_{-18}$	$1.13(1)^{+1}_{-1}$
M. Wingate *et al.* [56]	—	$260 \pm 7 \pm 26 \pm 8 \pm 5$	—
Lattice world averages	200 ± 30 [58]	230 ± 30 [59]	1.16 ± 0.04 [58]
QCD Spectral Sum Rules			
S. Narison [57]	207 ± 21	240 ± 24	1.16 ± 0.04

TABLE I: Pseudoscalar f_P decay constants for B and D mesons

	$\tilde{f}_{D^*}$	$\tilde{f}_{D_s^*}$	$\tilde{f}_{D^*}/\tilde{f}_{D_s^*}$
This work	$9.1^{+0.9}_{-0.9}$	$6.5^{+0.3}_{-0.4}$	$1.41^{+0.06}_{-0.05}$
UKQCD [54]	$8.6(3)^{+5}_{-9}$	$8.3(2)^{+5}_{-5}$	$1.04(1)^{+2}_{-2}$
	$\tilde{f}_{B^*}$ [MeV]	$\tilde{f}_{B_s^*}$ [MeV]	$\tilde{f}_{B^*}/\tilde{f}_{B_s^*}$
This work	$35.6^{+3.4}_{-3.4}$	$23.0^{+1.0}_{-1.5}$	$1.55^{+0.07}_{-0.06}$
UKQCD [54]	$28(1)^{+3}_{-4}$	$25(1)^{+2}_{-3}$	$1.10(2)^{+2}_{-2}$

TABLE II: $\tilde{f}_V = M_V/f_V$ for B^* and D^* mesons

For the vector meson decay constants we obtain the values

$$\begin{aligned} f_{D^*} &= 223^{+23}_{-19}\,\mathrm{MeV} \qquad & f_{D_s^*} &= 326^{+21}_{-17}\,\mathrm{MeV} \\ f_{B^*} &= 151^{+15}_{-13}\,\mathrm{MeV} \qquad & f_{B_s^*} &= 236^{+14}_{-11}\,\mathrm{MeV} \end{aligned} \tag{14}$$

which are very much the same as the values obtained for the decay constants of their pseudoscalar counterparts. This almost equality of pseudoscalar and vector decay constants is expected in HQS in the limit where the heavy quark masses go to infinity where one would have [4]

$$f_V\, M_V = f_P\, M_P \qquad , \qquad M_V = M_P \tag{15}$$

Our decay constants satisfy the above relation within 2%. On the other hand UKQCD lattice data show deviations as large as 20% for D mesons [54].

In order to compare the values of the vector decay constants with lattice data from Ref. [54] we give in Table II the quantity $\tilde{f}_V = M_V/f_V$. We find good agreement for $\tilde{f}_{D^*}$ and $\tilde{f}_{B_s^*}$ but not so much for the other two. Also our ratios $\tilde{f}_{D^*}/\tilde{f}_{D_s^*}$ and $\tilde{f}_{B^*}/\tilde{f}_{B_s^*}$ are larger than the ones favored by lattice calculations.

On the other hand the ratio

$$\frac{f_{B^*}\sqrt{M_B}}{f_{D^*}\sqrt{M_D}} = 1.138^{+0.011}_{-0.008} \tag{16}$$

is in very good agreement with the expectation in Ref. [60] where they would get $1.05 \sim 1.20$ for that ratio.

IV. SEMILEPTONIC DECAY OF B INTO $Dl\bar{\nu}$ AND $D^*l\bar{\nu}$

In this case the strong matrix elements are parametrized as

$$\begin{aligned} \frac{\left\langle D,\, \vec{P}' \,\middle|\, J^{c\,b}_{\mu}(0) \,\middle|\, B,\, \vec{P}\right\rangle}{\sqrt{M_B M_D}} &= \frac{\left\langle D,\, \vec{P}' \,\middle|\, J^{c\,b}_{V\,\mu}(0) \,\middle|\, B,\, \vec{P}\right\rangle}{\sqrt{M_B M_D}} \\ &= (v+v')_\mu\; h_+(w) + (v-v')_\mu\; h_-(w) \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{\left\langle D^*,\, \lambda\,\vec{P}' \,\middle|\, J^{c\,b}_{\mu}(0) \,\middle|\, B,\, \vec{P}\right\rangle}{\sqrt{M_B M_{D^*}}} &= \varepsilon_{\mu\nu\alpha\beta}\, \left(\varepsilon^{(\lambda)\,\nu}(\,\vec{P}'\,)\right)^*\, v^\alpha\, v'^{\,\beta}\, h_V(w) \\ &- i\, \left(\varepsilon^{(\lambda)}_\mu(\,\vec{P}'\,)\right)^*\, (w+1)\, h_{A_1}(w) \\ &+ i\, \left(\varepsilon^{(\lambda)}\,(\,\vec{P}'\,)\right)^* \cdot v\, \left(v_\mu\, h_{A_2}(w) + v'_\mu\, h_{A_3}(w)\right) \end{aligned} \tag{18}$$

where $v = P/M_B$ and $v' = P'/M_{D,\,D^*}$ are the four velocities of the initial B and final D, D^* mesons, $w = v\cdot v'$ [3] and $\varepsilon_{\mu\nu\alpha\beta}$ is the fully antisymmetric tensor with $\varepsilon_{0123} = +1$.

[3] w is related to the four momentum transferred square q^2 via $q^2 = M_B^2 + M_{D,\,D^*}^2 - 2\,w\,M_B\,M_{D,\,D^*}$

w	β_+	β_-	β_V	β_{A_1}	β_{A_2}	β_{A_3}
1.0	2.6	−5.4	11.9	−1.5	−11.0	2.2
1.1	−0.3	−5.4	8.9	−3.8	−10.3	−0.2
1.2	−3.1	−5.3	6.1	−5.9	−9.8	−2.5
1.3	−5.6	−5.3	3.5	−7.9	−9.3	−4.6
1.4	−8.0	−5.2	1.1	−9.7	−8.8	−6.6
1.5	−10.2	−5.2	−1.1	−11.5	−8.4	−8.5
1.59	−12.1	−5.1				

TABLE III: QCD corrections $\beta_j(w)$ in % as evaluated in Ref. [62]

In the limit of infinite heavy quark masses m_c, $m_b \to \infty$ HQS reduces the six form factors to a unique universal function $\xi(w)$ known as the Isgur-Wise function [4]

$$h_+(w) = h_{A_1}(w) = h_{A_3}(w) = h_V(w) = \xi(w) \tag{19}$$
$$h_-(w) = h_{A_2}(w) = 0 \tag{20}$$

Vector current conservation in the equal mass case implies the normalization

$$\xi(1) = 1 \tag{21}$$

Away from the heavy quark limit those relations are modified by QCD corrections so that one has

$$h_j(w) = \left(\alpha_j + \beta_j(w) + \gamma_j(w) + \mathcal{O}(1/m^2_{c,b})\right)\, \xi(w) \tag{22}$$

The α_j are constants fixed by the behavior of the form factor in the heavy quark limit

$$\begin{aligned} \alpha_+ &= \alpha_{A_1} = \alpha_{A_3} = \alpha_V = 1 \\ \alpha_- &= \alpha_{A_2} = 0 \end{aligned} \tag{23}$$

The different β_j account for perturbative radiative corrections [61] while the γ_j are non perturbative in nature and are proportional to the inverse of the heavy quark masses [62]. At zero recoil ($w = 1$) Luke's theorem [63] imposes the restriction.

$$\gamma_+(1) = \gamma_{A_1}(1) = 0 \tag{24}$$

so that power corrections to $h_+(1)$ and $h_{A_1}(1)$ are of order $\mathcal{O}(1/m^2_{c,b})$. In Tables III and IV we collect the values for the different β_j and γ_j in the full interval of w values allowed in the two decays. These two tables have been taken from Ref. [62].

w	γ_+	γ_-	γ_V	γ_{A_1}	γ_{A_2}	γ_{A_3}
1.0	0.0	−4.1	19.1	0.0	−23.1	−4.1
1.1	2.7	−4.1	20.7	2.9	−21.4	−0.7
1.2	6.2	−4.1	23.1	6.5	−19.8	3.4
1.3	10.5	−4.2	26.3	10.7	−18.3	8.0
1.4	15.3	−4.4	30.0	15.4	−17.0	13.0
1.5	20.6	−4.5	34.3	20.5	−15.8	18.5
1.59	25.7	−4.7				

TABLE IV: Power corrections $\gamma_j(w)$ in % as evaluated in Ref. [62]

A. $B \to D\, l\, \bar{\nu}$ decay

Let us start with the $B \to D\, l\, \bar{\nu}$ case. In the center of mass of the B meson and taking $\vec{P}\,' = -\vec{q} = -|\vec{q}\,|\,\widehat{\vec{k}}$ in the z direction we will have for the form factors $h_+(w)$ and $h_-(w)$[4]

$$h_+(w) = \frac{1}{\sqrt{2M_B 2M_D}} \left(V^0(|\vec{q}\,|) + \frac{V^3(|\vec{q}\,|)}{|\vec{q}\,|} \left(E_D(|\vec{q}\,|) - M_D\right)\right)$$
$$h_-(w) = \frac{1}{\sqrt{2M_B 2M_D}} \left(V^0(|\vec{q}\,|) + \frac{V^3(|\vec{q}\,|)}{|\vec{q}\,|} \left(E_D(|\vec{q}\,|) + M_D\right)\right) \qquad (25)$$

where $E_D(|\vec{q}\,|) = \sqrt{M_D^2 + |\vec{q}\,|^2}$ and $V^\mu(|\vec{q}\,|)$ ($\mu = 0, 3$) is given by

$$V^\mu(|\vec{q}\,|) = \left\langle D, -|\vec{q}\,|\,\vec{k}\ \middle|\ (J_V^{c\,b})^\mu(0)\ \middle|\ B, \vec{0}\right\rangle \qquad (26)$$

In our model $V^\mu(|\vec{q}\,|)$ is evaluated as

$$V^\mu(|\vec{q}\,|) \ = \ \sqrt{2M_B 2E_D(|\vec{q}\,|)}\ \ {}_{NR}\left\langle D, -|\vec{q}\,|\,\widehat{\vec{k}}\ \middle|\ (J_V^{c\,b})^\mu(0)\ \middle|\ B, \vec{0}\right\rangle_{NR} \qquad (27)$$

which expression is given in appendix B.

In the case of equal masses $m_b = m_c$ vector current conservation demands that

$$h_+(1) = 1 \qquad ; \qquad h_-(w) = 0 \qquad (28)$$

In this limit we find that $h_+(1) = 1$ so that our value for $h_+(1)$ complies with vector current conservation. On the other hand $h_-(w) \neq 0$ violating vector current conservation.

In the upper left panel of Fig. 1 we show the values of $h_+(w)$ and $h_-(w)$ for the $B \to D$ transition as obtained from Eqs. (25, 27) with the use of the AL1 interquark potential. The values for $h_-(w)$ are not reliable. Actual calculation shows that they are of the same size as the deviations from zero that one computes in the equal mass case. To improve on this what we shall do instead is to use the form factor $h_+(w)$ and Eq. (22) to extract $\xi(w)$ (we shall call it $\xi_+(w)$) and from there we can re-evaluate $h_-(w)$ with the use of Eq. (22). The results appear in the upper right panel of Fig. 1 where we also show the lattice results for $\xi(w)$ obtained by the UKQCD Collaboration in Ref. [64]. We find good agreement with lattice data. Finally in the lower panel of Fig. 1 we show the different $\xi_+(w)$ obtained with the use of the different interquark potentials. As we see from the figure all $\xi_+(w)$ are very much the same in the whole interval for w.

The slope at the origin of our Isgur-Wise function is given by

$$\rho^2 = -\frac{1}{\xi_+(w)}\,\frac{d\,\xi_+(w)}{dw}\bigg|_{w=1} = 0.35 \pm 0.02 \qquad (29)$$

small compared to the lattice value of $\rho^2 = 0.81^{+17}_{-11}$ extracted from a best fit to data.

1. *Differential decay width*

Neglecting lepton masses the differential decay width for the process $B \to D\, l\, \bar{\nu}$ is given by [65]

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{48\pi^3}\,|V_{cb}|^2\, M_D^3\, (w^2-1)^{3/2}(M_B + M_D)^2\ F_D^2(w) \qquad (30)$$

where $G_F = 1.16637(1) \times 10^{-5}\ \mathrm{GeV}^{-2}$ [79] is the Fermi decay constant, V_{cb} is the CKM matrix element for the $b \to c$ weak transition, and $F_D(w)$ is given by

$$F_D(w) = \left[h_+(w) - \frac{1-r}{1+r}\, h_-(w)\right] \qquad (31)$$

[4] In this case w is related to $|\vec{q}\,|$ via $|\vec{q}\,| = M_D\,\sqrt{w^2-1}$

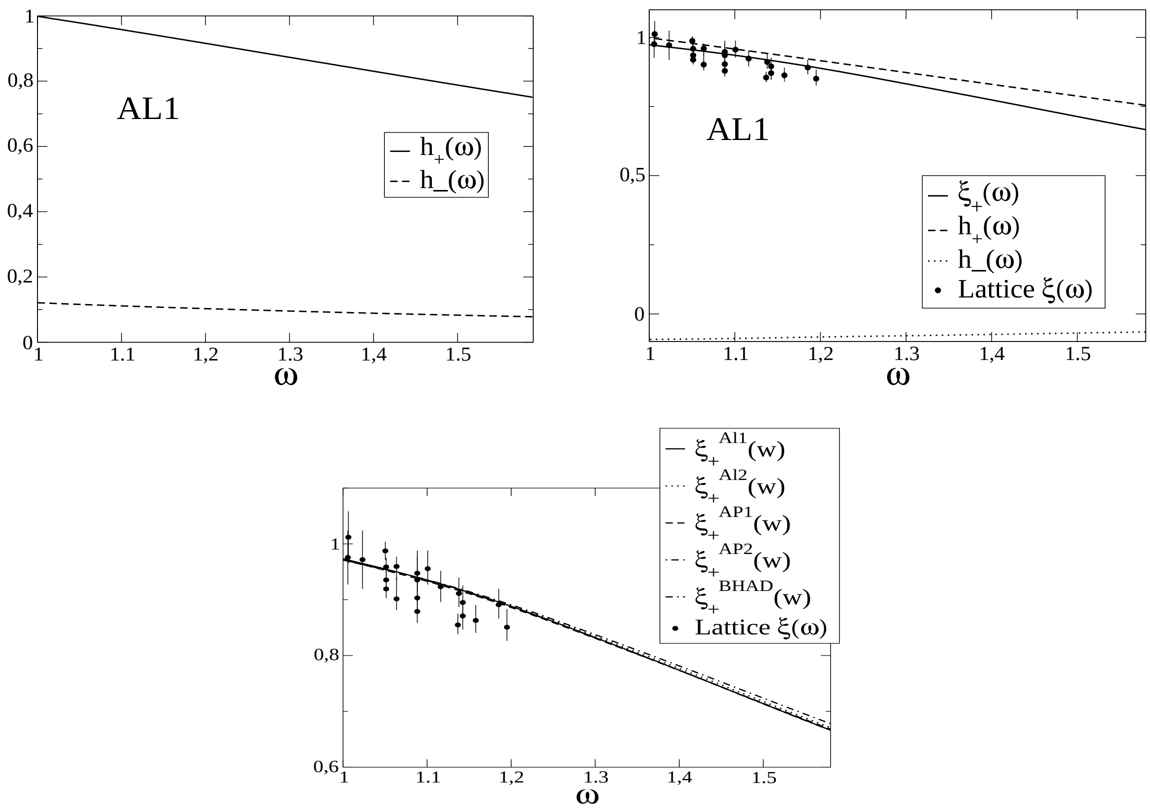

FIG. 1: Upper left panel: $h_+(w)$ and $h_-(w)$ for the $B \to D$ transition as obtained from Eqs. (25, 27) using the AL1 interquark potential. Upper right panel: $h_+(w)$ as before, $\xi_+(w)$ obtained from the values of $h_+(w)$ using Eq. (22), $h_-(w)$ obtained from $\xi_+(w)$ using Eq. (22). We also show the UKQCD lattice data by Bowler *et al.* [64]. Lower panel: Different $\xi_+(w)$ obtained with the above procedure for the different interquark interactions. Lattice data is also shown.

with $r = M_D/M_B$.

In Fig. 2 we show our calculation for $F_D(w)\ |V_{cb}|$ obtained with the AL1 interquark potential and using three different values of $|V_{cb}|$ corresponding to the central and extreme values of the range for $|V_{cb}|$ favored by the Particle Data Group (PDG), $|V_{cb}| = 0.039 \sim 0.044$ [79]. We also show the experimental data for the decays $B^- \to D^0\, l\, \bar{\nu}$ and $\bar{B}^0 \to D^+\, l\, \bar{\nu}$ obtained by the CLEO Collaboration [66], a fit to CLEO data using the form factors of Boyd *et al.* [67], and the experimental data for the decay $\bar{B}^0 \to D^+\, l\, \bar{\nu}$ obtained by the BELLE Collaboration [68]. Our results are larger than experimental data for $w > 1.2$. Our total integrated width will thus be larger than the experimental one for any reasonable value of $|V_{cb}|$. From our data we extract the slope at $w = 1$ given by

$$\rho_D^2 = -\frac{1}{F_D(w)}\frac{dF_D(w)}{dw}\bigg|_{w=1} = 0.38 \pm 0.02 \tag{32}$$

which is small compared to the values extracted from the experimental data: $\rho_D^2 = 0.76 \pm 0.16 \pm 0.08$ [66] and $\rho_D^2 = 0.69 \pm 0.14$ [68] obtained from a linear fit to the data, or $\rho_D^2 = 1.30 \pm 0.27 \pm 0.14$ [66] and $\rho_D^2 = 1.16 \pm 0.25$ [68] obtained using the form factors of Boyd *et al.* [67]. Thus, only our results close to $w = 1$ seem to be reliable. We can use our prediction for $F_D(1)$ to extract the value of $|V_{cb}|$ from the experimental determination of the quantity $|V_{cb}|F_D(1)$. Different values of that quantity appear in Table V

Our result for $F_D(1)$ is given by (we do not show the theoretical error which is or the order of 10^{-4})

$$F_D(1) = 1.04 \tag{33}$$

which is in good agreement with other calculations $F_D(1) = 0.98 \pm 0.07$ [69], $F_D(1) = 1.04$ [11] or $F_D(1) = 1.069 \pm 0.008 \pm 0.002 \pm 0.025$ [70]. From our value for $F_D(1)$ and the experimental values for $|V_{cb}|F_D(1)$ we can obtain $|V_{cb}|$ in the range

$$|V_{cb}| = 0.040 \pm 0.006 \tag{34}$$

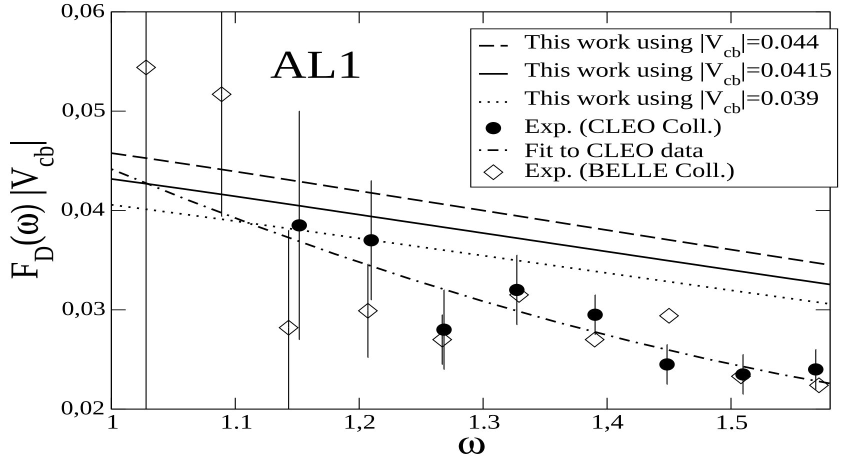

FIG. 2: $F_D(r,w)\ |V_{cb}|$ obtained with the AL1 interquark potential. Solid line: our results using $|V_{cb}| = 0.0415$. Dashed line: our results using $|V_{cb}| = 0.044$. Dotted line: our results using $|V_{cb}| = 0.039$. Circles: experimental data by the CLEO Collaboration [66]. Dashed-dotted line: fit to CLEO data using the form factors of Boyd *et al.* [67]. Diamonds: experimental data by the BELLE Collaboration [68].

	$\|V_{cb}\|\, F_D(1)$
CLEO Collaboration [66]	$0.0416 \pm 0.0047 \pm 0.0037$
BELLE Collaboration [68]	$0.0411 \pm 0.0044 \pm 0.0052$

TABLE V: $|V_{cb}|\, F_D(1)$ values obtained by different experiments.

This result agrees with our recent determination based on the analysis of the $\Lambda_b \to \Lambda_c\, l\, \bar\nu_l$ reaction from where we got $|V_{cb}| = 0.040 \pm 0.005$ [9].

B. $B \to D^*\, l\, \bar\nu$ decay

Working again in the center of mass of the B meson and taking $\vec{P}\,' = -\vec{q} = -|\vec{q}\,|\,\widehat{\vec{k}}$ in the z direction we will have for the form factors $h_V(w)$, $h_{A_1}(w)$, $h_{A_2}(w)$ and $h_{A_3}(w)$ the expressions

$$\begin{aligned}
h_V(w) &= \sqrt{2}\sqrt{\frac{M_{D^*}}{M_B}}\,\frac{V^{(*)}_{-1,\,2}(|\vec{q}\,|)}{|\vec{q}\,|} \\
h_{A_1}(w) &= i\,\frac{\sqrt{2}}{w+1}\,\frac{1}{\sqrt{M_B M_{D^*}}}\,A^{(*)}_{-1,\,1}(|\vec{q}\,|) \\
h_{A_2}(w) &= i\sqrt{\frac{M_{D^*}}{M_B}}\left(-\frac{A^{(*)}_{0,\,0}(|\vec{q}\,|)}{|\vec{q}\,|} + \frac{E_{D^*}(|\vec{q}\,|)\,A^{(*)}_{0,\,3}(|\vec{q}\,|)}{|\vec{q}\,|^2} - \sqrt{2}\,M_{D^*}\frac{A^{(*)}_{-1,\,1}(|\vec{q}\,|)}{|\vec{q}\,|^2}\right) \\
h_{A_3}(w) &= i\frac{M^2_{D^*}}{\sqrt{M_B M_{D^*}}}\left(-\frac{A^{(*)}_{0,\,3}(|\vec{q}\,|)}{|\vec{q}\,|^2} + \frac{\sqrt{2}}{M_{D^*}}\,\frac{E_{D^*}(|\vec{q}\,|)\,A^{(*)}_{-1,\,1}(|\vec{q}\,|)}{|\vec{q}\,|^2}\right)
\end{aligned} \qquad (35)$$

with $E_{D^*}(|\vec{q}\,|) = \sqrt{M^2_{D^*} + |\vec{q}\,|^2}$, and where $V^{(*)}_{\lambda,\,\mu}(|\vec{q}\,|)$ and $A^{(*)}_{\lambda,\,\mu}(|\vec{q}\,|)$ are given by

$$\begin{aligned}
V^{(*)}_{\lambda,\,\mu}(|\vec{q}\,|) &= \left\langle D^*,\, \lambda\ \ -|\vec{q}\,|\,\vec{k}\ \middle|\ (J^{c\,b}_V)_\mu(0)\ \middle|\ B,\, \vec{0}\right\rangle \\
A^{(*)}_{\lambda,\,\mu}(|\vec{q}\,|) &= \left\langle D^*,\, \lambda\ \ -|\vec{q}\,|\,\vec{k}\ \middle|\ (J^{c\,b}_A)_\mu(0)\ \middle|\ B,\, \vec{0}\right\rangle
\end{aligned} \qquad (36)$$

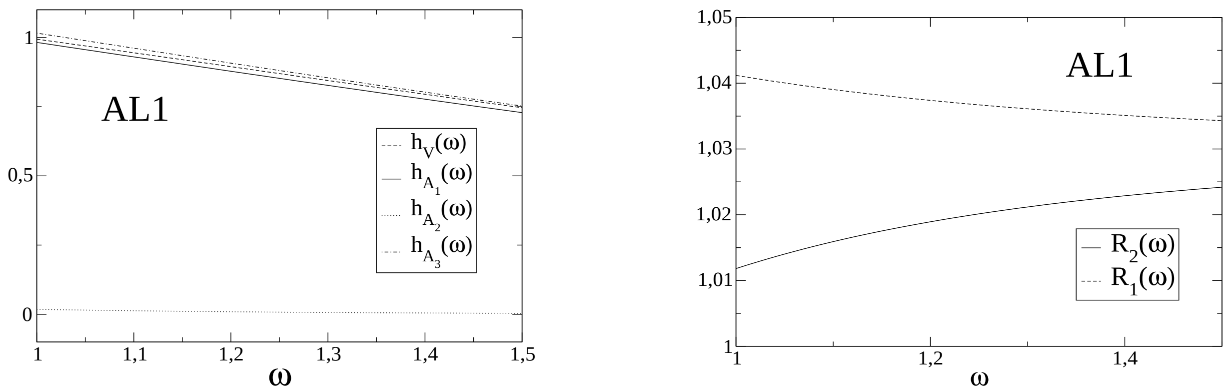

FIG. 3: Left panel: $h_V(w)$, $h_{A_1}(w)$, $h_{A_2}(w)$ and $h_{A_3}(w)$ form factors obtained using Eq. (35). Right panel: $R_1(w)$ and $R_2(w)$ ratios. In both panels the AL1 interquark potential has been used.

In our model $V^{(*)}_{\lambda,\,\mu}(|\vec{q}\,|)$ and $A^{(*)}_{\lambda,\,\mu}(|\vec{q}\,|)$ are evaluated as

$$\begin{aligned} V^{(*)}_{\lambda,\,\mu}(|\vec{q}\,|) &= \sqrt{2M_B 2E_{D^*}(|\vec{q}\,|)}\ \ {}_{NR}\Big\langle D^*,\, \lambda\ -|\vec{q}\,|\,\widehat{\vec{k}}\ \big|\,(J^{c\,b}_V)_\mu(0)\,\big|\ B,\,\vec{0}\Big\rangle_{NR} \\ A^{(*)}_{\lambda,\,\mu}(|\vec{q}\,|) &= \sqrt{2M_B 2E_{D^*}(|\vec{q}\,|)}\ \ {}_{NR}\Big\langle D^*,\, \lambda\ -|\vec{q}\,|\,\widehat{\vec{k}}\ \big|\,(J^{c\,b}_A)_\mu(0)\,\big|\ B,\,\vec{0}\Big\rangle_{NR} \end{aligned} \tag{37}$$

with expressions given in appendix C.

In the left panel of Fig. 3 we show our results for the $h_V(w), h_{A_1}(w), h_{A_2}(w)$ and $h_{A_3}(w)$ form factors, obtained with the AL1 interquark potential and the use of Eq. (35). In the right panel of the same figure we show the ratios

$$\begin{aligned} R_1(w) &= \frac{h_V(w)}{h_{A_1}(w)} \\ R_2(w) &= \frac{h_{A_3}(w) + r\, h_{A_2}(w)}{h_{A_1}(w)} \end{aligned} \tag{38}$$

where now $r = M_{D^*}/M_B$. These ratios are expected to vary very weakly with w. We find indeed that this is so in our case being our values of $R_1(w)$ and $R_2(w)$ within 4% of unity. In Table VI we give now our results for $R_1(1)$ and $R_2(1)$ and compare them to different experimental[5] and theoretical determinations. We find discrepancies of the order of $15 \sim 33\,\%$ for $R_1(1)$ and $13 \sim 46\,\%$ for $R_2(1)$.

One can understand these discrepancies by evaluating the different $\xi(w)$ functions obtained from the form factors with the use of Eq. (22) and the β and γ coefficients of Neubert given in Tables III and IV. The results appear in the upper left panel of Fig. 4. One can infer from the figure that our results for $h_{A_2}(w)$ are not reliable. Also we somehow miss the correct normalization for $h_V(1)$. On the other hand the values of $\xi_{A_1}(w)$ and $\xi_{A_3}(w)$ are equal within 4% and in reasonable agreement with lattice data from Ref. [64].

To improve the nonrelatistic quark model prediction, and similarly to what we did in subsection IV A, we will take $\xi_{A_1}(w)$ as our model determination of the Isgur-Wise function $\xi(w)$ and we will reevaluate the form factors with the use of Eq. (22). What we obtain is now depicted in the upper right panel of Fig. 4. In the lower panel we give the different $\xi_{A_1}(w)$ obtained with the different interquark potentials. They do not show any significant difference.

The slope of the $\xi_{A_1}(w)$ function at the origin is given by

$$\rho^2 = 0.55 \pm 0.02 \tag{39}$$

[5] The experimental results by the CLEO and BABAR Collaborations have been obtained with the assumption that $R_1(w)$ and $R_2(w)$ are constants.

	$R_1(1)$	$R_2(1)$
This work	1.01 ± 0.02	1.04 ± 0.01
CLEO [71]	$1.18 \pm 0.30 \pm 0.12$	$0.71 \pm 0.22 \pm 0.07$
BABAR (Preliminary) [72]	$1.328 \pm 0.055 \pm 0.025 \pm 0.025$	$0.920 \pm 0.044 \pm 0.020 \pm 0.013$
Caprini *et al.* [73]	1.27	0.80
Grinstein *et al.* [74]	1.25	0.81
Close *et al.* [28]	1.15	0.91

TABLE VI: $R_1(1)$ and $R_2(1)$

to be compared to the lattice result $\rho^2 = 0.93^{+47}_{-59}$ [64]. In this case we are within lattice errors, but one can not be very conclusive due to the large value of the latter in this case.

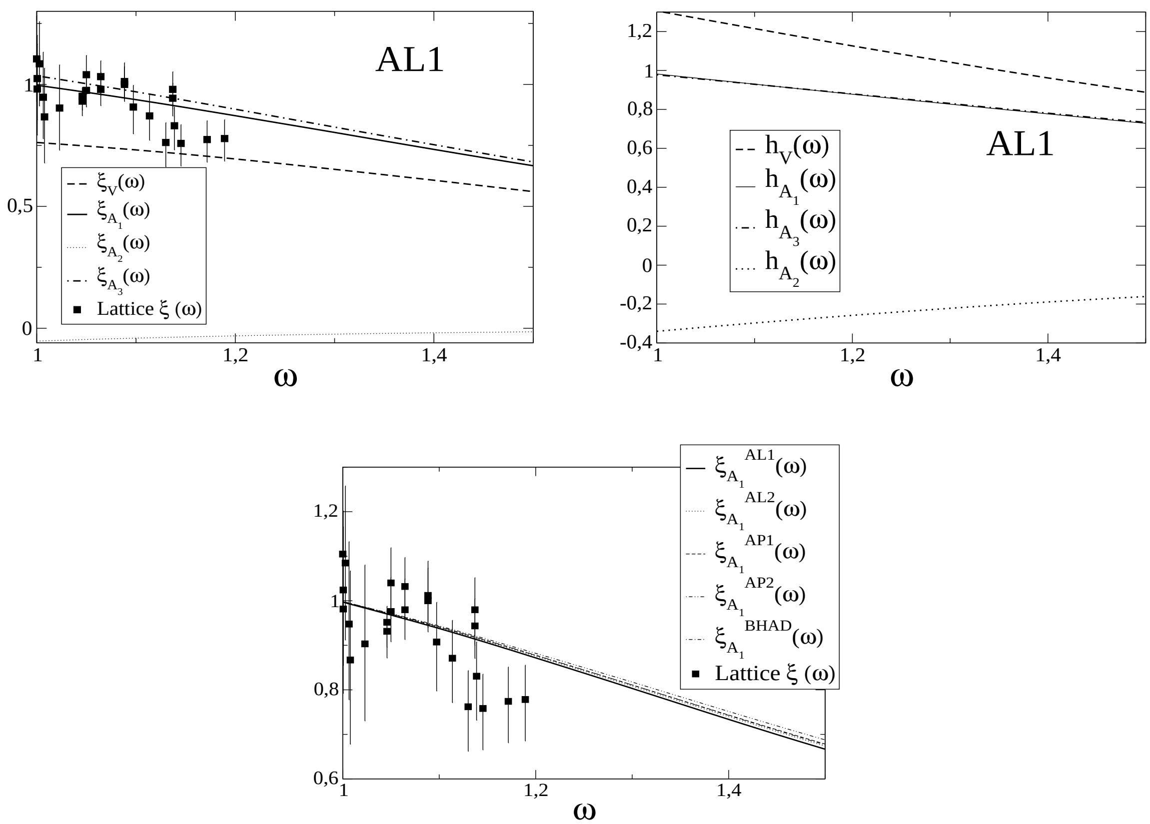

FIG. 4: Upper left panel: different $\xi(w)$ functions obtained from the $h_V(w)$, $h_{A_1}(w)$, $h_{A_2}(w)$ and $h_{A_3}(w)$ form factors using Eq. (22) and the AL1 interquark potential. Lattice data by K. C. Bowler *et al.* from Ref. [64] are also shown. Upper right panel: form factors obtained from $\xi_{A_1}(w)$ with the use of Eq. (22). Lower panel: Different $\xi_{A_1}(w)$ obtained with the different interquark potentials.

Finally in Fig. 5 we give the ratio $\xi_+(w)/\xi_{A_1}(w)$ evaluated with the AL1 interquark potential. We see the differences between the two Isgur-Wise functions are at the level of 3-7%.

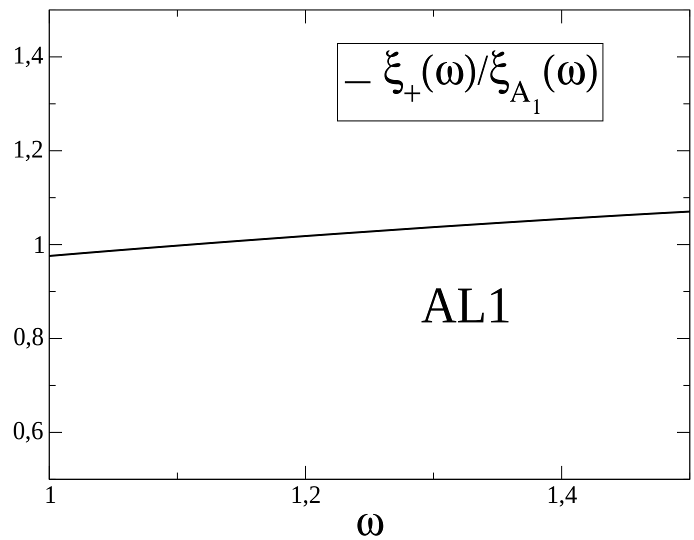

FIG. 5: Ratio of our two Isgur-Wise functions calculated with the AL1 interquark potential.

1. *Differential decay width*

Neglecting lepton masses the differential decay width for the process $B \to D^* \, l \, \bar{\nu}$ is given by [75]

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{48\pi^3} \, |V_{cb}|^2 \, (M_B - M_{D^*})^2 \, M_{D^*}^3 \, \sqrt{(w^2-1)} \, (w+1)^2 \left[1 + \frac{4w}{w+1} \, \frac{1 - 2wr + r^2}{(1-r)^2} \right] \, F_{D^*}^2(w) \tag{40}$$

where $F_{D^*}(w)$ is defined as

$$F_{D^*}(w) = h_{A_1}(w) \sqrt{\frac{\tilde{H}_0^2(w) + \tilde{H}_+^2(w) + \tilde{H}_-^2(w)}{1 + \frac{4w}{w+1} \, \frac{1-2wr+r^2}{(1-r)^2}}} \tag{41}$$

The $\tilde{H}_j(w)$ are helicity form factors given in terms of the $R_1(w)$ and $R_2(w)$ ratios as

$$\begin{aligned} \tilde{H}_0(w) &= 1 + \frac{w-1}{1-r} \, [1 - R_2(w)] \\ \tilde{H}_\pm(w) &= \frac{\sqrt{1 - 2wr + r^2}}{1-r} \left[1 \mp \sqrt{\frac{w-1}{w+1}} \, R_1(w) \right] \end{aligned} \tag{42}$$

Similarly to Fig. 2, in Fig. 6 we show our results for the quantity $F_{D^*}(w) \, |V_{cb}|$ evaluated with the AL1 interquark potential and using the values of $|V_{cb}|$ corresponding to the central and extreme values of the range for $|V_{cb}|$ favored by the PDG. We also show the experimental data by the CLEO Collaboration [71] for the $B^- \to D^{*0} \, l \, \bar{\nu}$ reaction (squares), and for the $\bar{B}^0 \to D^{*+} \, l \, \bar{\nu}$ reaction (circles) together with a best fit, and the experimental data by the BELLE Collaboration [76] for the $\bar{B}^0 \to D^{*+} \, l \, \bar{\nu}$ reaction (diamonds). We find good agreement with CLEO data for small w values. Disagreement starts already at around $w = 1.1$ where our resuls start to go above the experimental data. BELLE data are systematically below our results.

Also our slope at the origin

$$\rho_{D^*}^2 = - \frac{1}{F_{D^*}(w)} \, \frac{dF_{D^*}(w)}{dw} \bigg|_{w=1} = 0.31 \pm 0.02 \tag{43}$$

is smaller than the value obtained by the BELLE Collaboration $\rho_{D^*}^2 = 0.81 \pm 0.12$ [76] using a linear fit to their data. All this means that our total width would be larger than the experimental one for any reasonable value of V_{cb}. On the other hand experimentalists are able to extract the value of $|V_{cb}| \, F_{D^*}(1)$. Different experimental results for that quantity appear in Table VII.

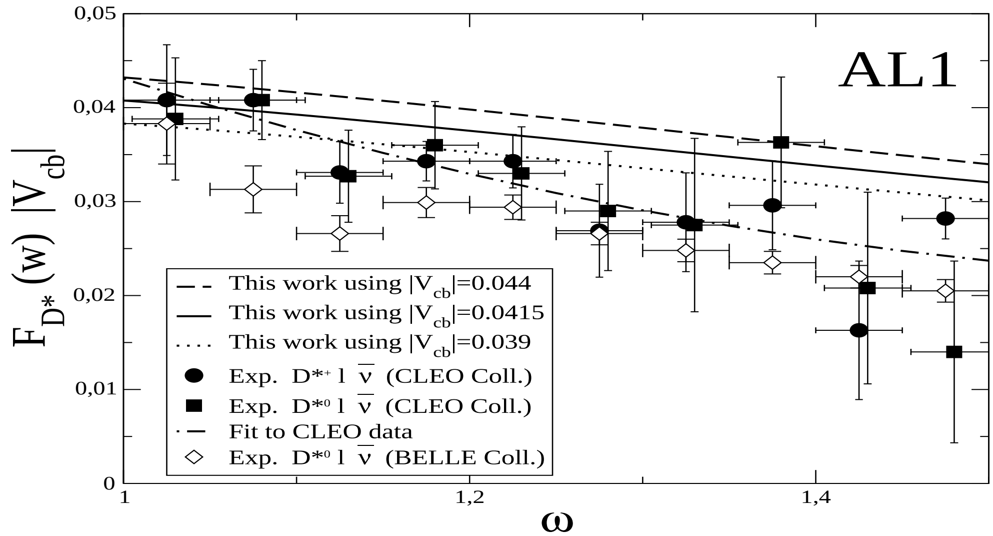

FIG. 6: $F_{D^*}(r,w)\,|V_{cb}|$ obtained with the AL1 interquark potential. Solid line: our results using $|V_{cb}| = 0.0415$. Dashed line: our results using $|V_{cb}| = 0.044$. Dotted line: our results using $|V_{cb}| = 0.039$. Circles and squares: experimental data by the CLEO Collaboration [71]. Dashed-dotted line: fit to CLEO Collaboration data. Diamonds: experimental data by the BELLE Collaboration [76].

		$\lvert V_{cb}\rvert\, F_{D^*}(1)$
(CLEO Coll.)	[71]	$0.0431 \pm 0.0013 \pm 0.0018$
(DELPHI Coll.)	[10]	$0.0392 \pm 0.0018 \pm 0.0023$
(BELLE Coll.)	[76]	$0.0354 \pm 0.0019 \pm 0.0018$
(BABAR Coll)	[77]	$0.0355 \pm 0.0003 \pm 0.0016$

TABLE VII: $|V_{cb}|\, F_{D^*}(1)$ values obtained by different experiments.

Our result for $F_{D^*}(1)$ is given by

$$F_{D^*}(1) = h_{A_1}(1) = 0.983 \pm 0.001 \tag{44}$$

Comparison with the experimental data for $|V_{cb}|\, F_{D^*}(1)$ allows us to extract values for $|V_{cb}|$ in the range

$$|V_{cb}| = 0.0333 \sim 0.0461 \tag{45}$$

One can not be more conclusive due to the dispersion in the experimental data for $|V_{cb}|\, F_{D^*}(1)$. From DELPHI data alone we would obtain $|V_{cb}| = 0.040 \pm 0.003$ in perfect agreement with our determination using the $B \to D$ reaction data. We should also say that our value for $F_{D^*}(1)$ is larger than the lattice determination $F_{D^*}(1) = 0.919^{+0.030}_{-0.035}$ by S. Hashimoto *et al.* [78] normally used by experimentalists to extract their $|V_{cb}|$ values. A new unquenched lattice determination of this quantity by the Fermilab Lattice Collaboration is in progress [59].

V. STRONG COUPLING CONSTANTS $g_{H^*H\pi}$

In this section we will evaluate the strong coupling constants $g_{H^*H\pi}$ where H stands for a B or D meson. To this end we shall make use of the partial conservation of the axial current hypothesis (PCAC) which relates the divergence of the axial current to the pion field as

$$\partial^\mu\, J^{d\,u}_{A\,\mu}(x) = \; f_\pi\, m_\pi^2\, \Phi_{\pi^-}(x) \tag{46}$$

where $f_\pi = 130.7 \pm 0.1 \pm 0.36\,\text{MeV}$ [79] is the pion decay constant, $m_\pi = 139.57\,\text{MeV}$ [79] is the pion mass, and $\Phi_{\pi^-}(x)$ is the charged pion field that destroys a π^- and creates a π^+. Using the LSZ reduction formula one can relate the matrix element of the divergence of the axial current to the pion emission amplitude as

$$\langle H,\ \vec{P}' \,|\, q^\mu\, J^{d\,u}_{A\,\mu}(0)\,|\, H^*, \lambda\, \vec{P}\,\rangle = -i\, f_\pi \frac{m_\pi^2}{q^2 - m_\pi^2}\ \mathcal{A}^{(\lambda)}_{H^* H\pi}(P', P) \tag{47}$$

where $q = P - P'$ and $\mathcal{A}^{(\lambda)}_{H^* H\pi}(P', P)$ is the pion emission amplitude for the process $H^* \to H\pi$ given by[6]

$$\mathcal{A}^{(\lambda)}_{H^* H\pi}(P', P) = -\, g_{H^* H\pi}(q^2)\ \left(q^\mu\, \varepsilon^{(\lambda)}_\mu(\vec{P}\,)\right) \tag{48}$$

The matrix element on the left hand side of Eq.(47) has a pion pole contribution that can be easily evaluated to be

$$\langle H,\ \vec{P}' \,|\, q^\mu\, J^{d\,u}_{A\,\mu}(0)\,|\, H^*, \lambda\, \vec{P}\,\rangle_{pion-pole} = -i\, f_\pi \frac{q^2}{q^2 - m_\pi^2}\ \mathcal{A}^{(\lambda)}_{H^* H\pi}(P', P) \tag{49}$$

so that we can extract a non-pole contribution

$$\begin{aligned}\langle H,\ \vec{P}' \,|\, q^\mu\, J^{d\,u}_{A\,\mu}(0)\,|\, H^*, \lambda\, \vec{P}\,\rangle_{non-pole} &= i\, f_\pi\ \mathcal{A}^{(\lambda)}_{H^* H\pi}(P', P) \\ &= -i\, f_\pi\ g_{H^* H\pi}(q^2)\ \left(q^\mu\, \varepsilon^{(\lambda)}_\mu(\vec{P}\,)\right) \end{aligned}\tag{50}$$

which is the one we shall evaluate within the quark model. For the matrix element on the left hand side of Eq.(50) we can use a parametrization similar to the one used in Eq.(18)

$$\begin{aligned}\langle H,\ \vec{P}' \,|\, q^\mu\, J^{d\,u}_{A\,\mu}(0)\,|\, H^*, \lambda\, \vec{P}\,\rangle_{non-pole} = q^\mu \Big\{ &-i\, \varepsilon^{(\lambda)}_\mu(\vec{P}\,)\ (w+1)\ h_{A_1}(w) \\ &+ i\left(\varepsilon^{(\lambda)}(\vec{P}\,)\cdot v'\right)\left(v'_\mu\, h_{A_2}(w) + v_\mu\, h_{A_3}(w)\right)\Big\}\sqrt{M_H M_{H^*}}\end{aligned} \tag{51}$$

with the result that

$$\begin{aligned} g_{H^* H\pi}(q^2) = \frac{1}{f_\pi}\Big\{ &(w+1)\, h_{A_1}(w) + w\left(\frac{M_{H^*}}{M_H}\, h_{A_2}(w) - h_{A_3}(w)\right) \\ &+ \left(\frac{M_{H^*}}{M_H}\, h_{A_3}(w) - h_{A_2}(w)\right)\Big\}\sqrt{M_H M_{H^*}}\end{aligned} \tag{52}$$

The evaluation of the form factors is done is a similar way as the one described in subsection IV B. The results that we get for $q^2 = 0$ are

$$g_{D^* D\pi}(0) = 22.1 \pm 0.4 \quad , \quad g_{B^* B\pi}(0) = 60.5 \pm 1.1 \tag{53}$$

to be compared to the experimental determination $g_{D^* D\pi}(m_\pi^2) = 17.9 \pm 0.3 \pm 1.9$ by the CLEO Collaboration [80], the lattice results $g_{D^* D\pi}(m_\pi^2) = 18.8 \pm 2.3^{+1.1}_{-2.0}$ [81] and $g_{B^* B\pi}(0) = 47 \pm 5 \pm 8$ [82], or a recent determination using QCDSR for which $g_{D^* D\pi}(m_\pi^2) = 14.0 \pm 1.5$ and $g_{B^* B\pi}(0) = 42.5 \pm 2.6$ [83]. Older QCDSR results give smaller values for both coupling constants. For instance, the calculation within QCDSR on the light cone in Ref. [84] give $g_{D^* D\pi}(m_\pi^2) = 12.5 \pm 1$ and $g_{B^* B\pi}(0) = 29 \pm 3$ [7]. The latter are small compared to lattice data or the experimental determination of $g_{D^* D\pi}(m_\pi^2)$ by the CLEO Collaboration.

From our results we obtain the ratio

$$R = \frac{g_{B^* B\pi}(0)\ f_{B^*}\sqrt{M_D}}{g_{D^* D\pi}(0)\ f_{D^*}\sqrt{M_B}} = 1.105 \pm 0.005 \tag{54}$$

in good agreement with HQS that predicts a value of 1 with $1/m_Q$ corrections appearing in next to leading order [60] [8]. Our result in Eq.(54) is also in agreement with the one obtained combining lattice data for f_{B^*} and f_{D^*} from

[6] Corresponding to the emission of a π^+

[7] Values for both coupling constants obtained prior to 1995 within different approaches can be found in Ref. [84] and references therein

[8] Note the strong coupling constant used in Ref. [60] is given in terms of ours as $(g_{H^* H\pi}\, f_\pi)/(2M_{H^*})$ with $H = B, D$

Ref. [54], for $g_{B^*B\pi}(0)$ from Ref. [81], and the experimental CLEO Collaboration data for $g_{D^*D\pi}(m_\pi^2)$ from Ref. [80]. In this case one gets

$$R|_{Exp.-Latt.} = 1.26 \pm 0.36 \tag{55}$$

where we have added errors in quadratures. A calculation using light cone QCDSR gives [84]

$$R|_{QCDSR} = 0.92 \tag{56}$$

VI. CONCLUDING REMARKS

Our analysis of leptonic decay constants of mesons with a heavy c or b quark shows that in a nonrelativistic calculation the equality $f_V M_V = f_P M_P$ is satisfied within 2%. This equality is expected in HQS in the limit where the heavy quark masses go to infinity. The nonrelativistic result suggests that the HQS infinite mass limit sets in already at the m_c scale, something that is not supported by lattice data for D mesons where one finds deviations as large as 20% from the above equality.

One also finds problems in the semileptonic $B \to D$ and $B \to D^*$ decays. We have seen how the $h_-(w)$ form factor of the $B \to D$ decay and the $h_{A_2}(w)$ form factor of the $B \to D^*$ decay are not reliably calculated in the nonrelativistic quark model where one finds large deviations from the HQET relations of Eq.(22). We have tried to remedy this failure by evaluating the Isgur-Wise function from a form factor whose calculation we trusted in the quark model, $h_+(w)$ for the $B \to D$ decay, and $h_{A_1}(w)$ for the $B \to D^*$ decay. Those Isgur-Wise function were later used together with HQET constraints in Eq.(22) to recalculate all other form factors. The two Isgur-Wise functions thus determined show an overall reasonable agreement with lattice data but in both cases the slope seems to be too small. This deficiency multiply its effects when one goes to larger w values and as a consequence the quantities $F_D(w)\,|V_{cb}|$ and $F_{D^*}(w)\,|V_{cb}|$ go above experimental data for w values larger than 1.2, and for any reasonable value of $|V_{cb}|$. A failure of some kind is expected in a nonrelativistic calculation as w increases. For $w = 1.2$ the three momenta of the final meson amounts to 66% ot its mass and relativistic corrections in the wave function could start to be important. On the other hand we believe our results are sound at zero recoil ($w = 1$). That enables us to obtain $|V_{cb}| = 0.040 \pm 0.006$ from the $B \to D$ decay, in perfect agreement with the value $|V_{cb}| = 0.040 \pm 0.005$ previously obtained by us from the analysis of the $\Lambda_b \to \Lambda_c$ semileptonic decay, and in good agreement with a recent determination $|V_{cb}| = 0.0414 \pm 0.0012 \pm 0.0021 \pm 0.0018$ by the DELPHI Collaboration. The experimental situation concerning the $B \to D^*$ reaction is not so clear as different experiments give values for $F_{D^*}(w)\,|V_{cb}|$ which are hardly compatible. From DELPHI Collaboration data we would get $|V_{cb}| = 0.040 \pm 0.003$ in agreement with the above result.

Finally we have made use of PCAC to evaluate the strong coupling constants $g_{B^*B\pi}$ and $g_{D^*D\pi}$. Our results are larger than experimental data or the results provided by lattice and QCDSR calculations. In this case the final meson is nearly at rest and one would expect a nonrelativistic calculation to perform better. The main difference with the other observables analyzed is that here the two active quarks are the light ones. The discrepancies might hint at a possible sizeable renormalization of the axial coupling for light constituents quarks. On the other hand the value for the ratio R in Eq. (54) agrees with the HQS prediction and with the one evaluated using a combination of lattice and experimental data, being also close to a QCDSR determination.

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APPENDIX A: MATRIX ELEMENTS FOR THE LEPTONIC DECAY OF PSEUDO SCALARS AND VECTOR MESONS

The matrix element needed for the evaluation of the pseudoscalar decay constant is given by

$$\left\langle 0 \left| J_{A\,0}^{f_1 f_2}(0) \right| P, \vec{0} \right\rangle_{NR} = \sqrt{3} \int d^3p \sum_{s_1,s_2} \hat{\phi}^{(P)}_{(s_1,f_1),\,(s_2,f_2)}(\vec{p}) \frac{(-1)^{\frac{1}{2}-s_2}}{(2\pi)^{\frac{3}{2}} \sqrt{2E_{f_1}(\vec{p})2E_{f_2}(\vec{p})}}$$

$$
\begin{aligned}
& \bar{v}_{s_2,f_2}(\vec{p})\,\gamma_0\gamma_5\,u_{s_1,f_1}(-\vec{p}) \\
= {} & \sqrt{3}\int d^3p\sum_{s_1,s_2}\hat{\phi}^{(P)}_{(s_1,f_1),\,(s_2,f_2)}(\vec{p})\frac{(-1)^{\frac{1}{2}-s_2}}{(2\pi)^{\frac{3}{2}}\sqrt{2E_{f_1}(\vec{p})2E_{f_2}(\vec{p})}} \\
& \bar{u}_{-s_2,f_2}(\vec{p})\,\gamma_0\,u_{s_1,f_1}(-\vec{p}) \\
= {} & i\,\frac{\sqrt{3}}{\pi}\int_0^\infty d|\vec{p}|\;\hat{\Phi}^{(P)}_{f_1,\,f_2}(|\vec{p}|)\,|\vec{p}|^2\sqrt{\frac{(E_{f_1}(\vec{p})+m_{f_1})(E_{f_2}(\vec{p})+m_{f_2})}{4E_{f_1}(\vec{p})E_{f_2}(\vec{p})}} \\
& \left(1-\frac{|\vec{p}|^2}{(E_{f_1}(\vec{p})+m_{f_1})(E_{f_2}(\vec{p})+m_{f_2})}\right)
\end{aligned}
\tag{A1}
$$

where we have used the fact that $v_{s,f}(\vec{p}) = \gamma_5\,u_{-s,f}(\vec{p})$.
Similarly for the vector meson case

$$
\begin{aligned}
\left\langle 0\left|J^{f_1\,f_2}_{V\,3}(0)\right|V,\,0\,\vec{0}\right\rangle_{NR} = {} & \sqrt{3}\int d^3p\sum_{s_1,s_2}\hat{\phi}^{(V,\,0)}_{(s_1,f_1),\,(s_2,f_2)}(\vec{p})\frac{(-1)^{\frac{1}{2}-s_2}}{(2\pi)^{\frac{3}{2}}\sqrt{2E_{f_1}(\vec{p})2E_{f_2}(\vec{p})}} \\
& \bar{v}_{s_2,f_2}(\vec{p})\,\gamma_3\,u_{s_1,f_1}(-\vec{p}) \\
= {} & \sqrt{3}\int d^3p\sum_{s_1,s_2}\hat{\phi}^{(V,0)}_{(s_1,f_1),\,(s_2,f_2)}(\vec{p})\frac{(-1)^{\frac{1}{2}-s_2}}{(2\pi)^{\frac{3}{2}}\sqrt{2E_{f_1}(\vec{p})2E_{f_2}(\vec{p})}} \\
& \bar{u}_{-s_2,f_2}(\vec{p})\,\gamma_3\gamma_5\,u_{s_1,f_1}(-\vec{p}) \\
= {} & \frac{-\sqrt{3}}{\pi}\int_0^\infty d|\vec{p}|\;\hat{\Phi}^{(V)}_{f_1,\,f_2}(|\vec{p}|)\,|\vec{p}|^2\sqrt{\frac{(E_{f_1}(\vec{p})+m_{f_1})(E_{f_2}(\vec{p})+m_{f_2})}{4E_{f_1}(\vec{p})E_{f_2}(\vec{p})}} \\
& \left(1+\frac{|\vec{p}|^2}{3(E_{f_1}(\vec{p})+m_{f_1})(E_{f_2}(\vec{p})+m_{f_2})}\right)
\end{aligned}
\tag{A2}
$$

APPENDIX B: EXPRESSION FOR THE $V^\mu(|\vec{q}|)$ MATRIX ELEMENT

The expression for $V^\mu(|\vec{q}|)$ is given by

$$
\begin{aligned}
V^\mu(|\vec{q}|) = {} & \sqrt{2M_B2E_D(|\vec{q}|)}\;{}_{NR}\left\langle D,\,-|\vec{q}|\,\widehat{\vec{k}}\left|(J_V^{c\,b})^\mu(0)\right|B,\,\vec{0}\right\rangle_{NR} \\
= {} & \sqrt{2M_B2E_D(|\vec{q}|)}\int d^3p\sum_{s_1,s_2}\left(\hat{\phi}^{(D)}_{(s_1,c),\,(s_2,f_2)}\Big(\frac{m_{f_2}}{m_c+m_{f_2}}\,|\vec{q}|\widehat{\vec{k}}+\vec{p}\Big)\right)^*\sum_{s_1'}\hat{\phi}^{(B)}_{(s_1',b),\,(s_2,f_2)}(\vec{p}) \\
& \frac{1}{\sqrt{2E_c(|\vec{q}|\widehat{\vec{k}}+\vec{p})2E_b(\vec{p})}}\,\bar{u}_{s_1,c}(-|\vec{q}|\,\widehat{\vec{k}}-\vec{p})\,\gamma^\mu\,u_{s_1',b}(-\vec{p})
\end{aligned}
\tag{B1}
$$

where f_2 represents a light u or d quark.
Going a little further we have

$$
\begin{aligned}
V^0(|\vec{q}|) = {} & \sqrt{2M_B2E_D(|\vec{q}|)}\int d^3p\,\frac{1}{4\pi}\left(\hat{\phi}^{(D)}_{c,\,f_2}\Big(\Big|\frac{m_{f_2}}{m_c+m_{f_2}}\,|\vec{q}|\widehat{\vec{k}}+\vec{p}\Big|\Big)\right)^*\hat{\phi}^{(B)}_{b,\,f_2}(|\vec{p}|) \\
& \sqrt{\frac{\Big(E_c(|\vec{q}|\widehat{\vec{k}}+\vec{p})+m_c\Big)\Big(E_b(\vec{p})+m_b\Big)}{4E_c(|\vec{q}|\widehat{\vec{k}}+\vec{p})\;E_b(\vec{p})}}\left(1+\frac{|\vec{p}|^2+p_z\,|\vec{q}|}{\Big(E_c(|\vec{q}|\widehat{\vec{k}}+\vec{p})+m_c\Big)\Big(E_b(\vec{p})+m_b\Big)}\right)
\end{aligned}
\tag{B2}
$$

In the case of equal masses $m_b = m_c$ and for $|\vec{q}| = 0$ ($w = 1$) we will obtain

$$
V^0(|\vec{q}|)\big|_{|\vec{q}|=0} = 2M_B
\tag{B3}
$$

Similarly

$$V^3(|\vec{q}|) \; = \; -\sqrt{2M_B 2E_D(|\vec{q}|)} \; \int d^3p \; \frac{1}{4\pi} \left(\hat{\phi}^{(D)}_{c,\,f_2}(| \frac{m_{f_2}}{m_c+m_{f_2}} \, |\vec{q}\,|\hat{\vec{k}} + \vec{p}\,|) \right)^* \; \hat{\phi}^{(B)}_{b,\,f_2}(|\,\vec{p}\,|)$$
$$\sqrt{\frac{\left(E_c(|\vec{q}|\hat{\vec{k}}+\vec{p}\,)+m_c\right)\;\left(E_b(\vec{p}\,)+m_b\right)}{4E_c(|\vec{q}|\hat{\vec{k}}+\vec{p}\,)\;E_b(\vec{p}\,)}} \left(\frac{p_z}{E_b(\vec{p}\,)+m_b} + \frac{p_z+|\vec{q}\,|}{E_c(|\vec{q}|\hat{\vec{k}}+\vec{p}\,)+m_c} \right) \tag{B4}$$

APPENDIX C: EXPRESSIONS FOR THE $V^{(*)}_{\lambda,\,\mu}$ AND $A^{(*)}_{\lambda,\,\mu}$ MATRIX ELEMENTS

The expressions for the $V^{(*)}_{\lambda,\,\mu}$ and $A^{(*)}_{\lambda,\,\mu}$ matrix elements are given by

$$\begin{aligned}
V^{(*)}_{\lambda,\,\mu}(|\vec{q}\,|) \; &= \; \sqrt{2M_B 2E_{D^*}(|\vec{q}\,|)} \;{}_{NR}\left\langle D^*,\,\lambda \;\; -|\vec{q}\,|\,\hat{\vec{k}} \,\middle|\, (J^{c\,b}_V)_\mu(0) \,\middle|\, B,\,\vec{0} \right\rangle_{NR} \\
&= \; \sqrt{2M_B 2E_{D^*}(|\vec{q}\,|)} \; \int d^3p \sum_{s_1,s_2} \left(\hat{\phi}^{(D^*,\,\lambda)}_{(s_1,c),\,(s_2,f_2)}(\frac{m_{f_2}}{m_c+m_{f_2}} \, |\vec{q}\,|\hat{\vec{k}}+\vec{p}\,) \right)^* \; \sum_{s_1'} \hat{\phi}^{(B)}_{(s_1',b),\,(s_2,f_2)}(\,\vec{p}\,) \\
&\qquad \frac{1}{\sqrt{2E_c(|\vec{q}\,|\,\hat{\vec{k}}+\vec{p}\,)\,2E_b(\vec{p}\,)}} \, \bar{u}_{s_1,c}(-|\vec{q}\,|\,\hat{\vec{k}}-\vec{p}\,)\,\gamma_\mu\,u_{s_1',b}(-\vec{p}\,) \\
A^{(*)}_{\lambda,\,\mu}(|\vec{q}\,|) \; &= \; \sqrt{2M_B 2E_{D^*}(|\vec{q}\,|)} \;{}_{NR}\left\langle D^*,\,\lambda \;\; -|\vec{q}\,|\,\hat{\vec{k}} \,\middle|\, (J^{c\,b}_A)_\mu(0) \,\middle|\, B,\,\vec{0} \right\rangle_{NR} \\
&= \; \sqrt{2M_B 2E_{D^*}(|\vec{q}\,|)} \; \int d^3p \sum_{s_1,s_2} \left(\hat{\phi}^{(D^*,\,\lambda)}_{(s_1,c),\,(s_2,f_2)}(\frac{m_{f_2}}{m_c+m_{f_2}} \, |\vec{q}\,|\hat{\vec{k}}+\vec{p}\,) \right)^* \; \sum_{s_1'} \hat{\phi}^{(B)}_{(s_1',b),\,(s_2,f_2)}(\,\vec{p}\,) \\
&\qquad \frac{1}{\sqrt{2E_c(|\vec{q}\,|\,\hat{\vec{k}}+\vec{p}\,)\,2E_b(\vec{p}\,)}} \, \bar{u}_{s_1,c}(-|\vec{q}\,|\,\hat{\vec{k}}-\vec{p}\,)\,\gamma_\mu\gamma_5\,u_{s_1',b}(-\vec{p}\,)
\end{aligned} \tag{C1}$$

So that

$$V^{(*)}_{-1,\,2}(|\vec{q}\,|) \; = \; -\frac{1}{\sqrt{2}}\sqrt{2M_B 2E_{D^*}(|\vec{q}\,|)} \; \int d^3p \; \frac{1}{4\pi} \left(\hat{\phi}^{(D^*)}_{c,\,f_2}(| \frac{m_{f_2}}{m_c+m_{f_2}} \, |\vec{q}\,|\hat{\vec{k}}+\vec{p}\,|\,) \right)^* \; \hat{\phi}^{(B)}_{b,\,f_2}(|\,\vec{p}\,|)$$
$$\sqrt{\frac{\left(E_c(|\vec{q}|\hat{\vec{k}}+\vec{p}\,)+m_c\right)\;\left(E_b(\vec{p}\,)+m_b\right)}{4E_c(|\vec{q}|\hat{\vec{k}}+\vec{p}\,)\;E_b(\vec{p}\,)}} \left(\frac{p_z}{E_b(\vec{p}\,)+m_b} - \frac{p_z+|\vec{q}\,|}{E_c(|\vec{q}|\hat{\vec{k}}+\vec{p}\,)+m_c} \right) \tag{C2}$$

$$A^{(*)}_{-1,\,1}(|\vec{q}\,|) \; = \; -\frac{i}{\sqrt{2}}\sqrt{2M_B 2E_{D^*}(|\vec{q}\,|)} \; \int d^3p \; \frac{1}{4\pi} \left(\hat{\phi}^{(D^*)}_{c,\,f_2}(| \frac{m_{f_2}}{m_c+m_{f_2}} \, |\vec{q}\,|\hat{\vec{k}}+\vec{p}\,|\,) \right)^* \; \hat{\phi}^{(B)}_{b,\,f_2}(|\,\vec{p}\,|)$$
$$\sqrt{\frac{\left(E_c(|\vec{q}|\hat{\vec{k}}+\vec{p}\,)+m_c\right)\;\left(E_b(\vec{p}\,)+m_b\right)}{4E_c(|\vec{q}|\hat{\vec{k}}+\vec{p}\,)\;E_b(\vec{p}\,)}} \left(1 + \frac{2p_x^2 - |\,\vec{p}\,|^2 - p_z\,|\vec{q}\,|}{\left(E_c(|\vec{q}|\hat{\vec{k}}+\vec{p}\,)+m_c\right)\;\left(E_b(\vec{p}\,)+m_b\right)} \right) \tag{C3}$$

$$A^{(*)}_{0,\,0}(|\vec{q}\,|) \; = \; -i\sqrt{2M_B 2E_{D^*}(|\vec{q}\,|)} \; \int d^3p \; \frac{1}{4\pi} \left(\hat{\phi}^{(D^*)}_{c,\,f_2}(| \frac{m_{f_2}}{m_c+m_{f_2}} \, |\vec{q}\,|\hat{\vec{k}}+\vec{p}\,|\,) \right)^* \; \hat{\phi}^{(B)}_{b,\,f_2}(|\,\vec{p}\,|)$$

$$\sqrt{\frac{\Big(E_c(|\vec{q}\,|\widehat{\vec{k}}+\vec{p}\,)+m_c\Big)\,\Big(E_b(\vec{p}\,)+m_b\Big)}{4E_c(|\vec{q}\,|\widehat{\vec{k}}+\vec{p}\,)\;E_b(\vec{p}\,)}}\left(\frac{p_z}{E_b(\vec{p}\,)+m_b}+\frac{p_z+|\vec{q}\,|}{E_c(|\vec{q}\,|\widehat{\vec{k}}+\vec{p}\,)+m_c}\right) \tag{C4}$$

$$A^{(*)}_{0,3}(|\vec{q}\,|) \;=\; -i\sqrt{2M_B 2E_{D^*}(|\vec{q}\,|)}\int d^3p\,\frac{1}{4\pi}\left(\hat{\phi}^{(D^*)}_{c,\,f_2}(|\,\frac{m_{f_2}}{m_c+m_{f_2}}\,|\vec{q}\,|\widehat{\vec{k}}+\vec{p}\,|\,)\right)^*\hat{\phi}^{(B)}_{b,\,f_2}(|\,\vec{p}\,|)$$
$$\sqrt{\frac{\Big(E_c(|\vec{q}\,|\widehat{\vec{k}}+\vec{p}\,)+m_c\Big)\,\Big(E_b(\vec{p}\,)+m_b\Big)}{4E_c(|\vec{q}\,|\widehat{\vec{k}}+\vec{p}\,)\;E_b(\vec{p}\,)}}\left(1+\frac{2p_z^2-|\,\vec{p}\,|^2+p_z\,|\vec{q}\,|}{\Big(E_c(|\vec{q}\,|\widehat{\vec{k}}+\vec{p}\,)+m_c\Big)\,\Big(E_b(\vec{p}\,)+m_b\Big)}\right) \tag{C5}$$

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