

A Heavy Quark Symmetry Approach to Baryons

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We evaluate different properties of baryons with a heavy c or b quark. The use of Heavy Quark Symmetry (HQS) provides with an important simplification of the non relativistic three body problem which can be solved by means of a simple variational approach. This scheme is able to reproduce previous results obtained with more involved Faddeev calculations. The resulting wave functions are parametrized in a simple manner, and can be used to calculate further observables.

1. INTRODUCTION

Since the discovery of Λ_b [1], [2] and most of the charmed baryons of the SU(3) multiplet on the second level of the SU(4) 20-plet [3], a great deal of theoretical work has been devoted to their study (See for instance Refs. [4]-[6]).

In this context HQS has proved to be a useful tool to understand bottom and charmed physics, being one of the basis of lattice simulations of bottom systems. HQS is an approximate SU(N_F) symmetry of QCD, being N_F the number of heavy flavours. This symmetry appears in systems containing heavy quarks, with masses much larger than any other energy scale ($q = \Lambda_{QCD}, m_u, m_d, m_s, \dots$) controlling the dynamics of the remaining degrees of freedom. For baryons containing a heavy quark, and up to corrections of order $\mathcal{O}(\frac{q}{m_h})^2$, HQS guarantees that the heavy baryon light degrees of freedom quantum numbers are always well defined.

However, HQS has not been systematically used within the context of non relativistic constituent quark models (NRCQM). The model we present here solves the non relativistic three body problem, for the ground state of baryons with a heavy c or b quark, making full use of the simplifications of HQS [7]. Thanks to HQS, the method proposed provides us with simple wave functions, while the results obtained for the spectrum and other

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²Here q stands for a typical energy scale relevant for the light degrees of freedom while m_h is the mass of the heavy quark

observables compare quite well with more sophisticated Faddeev calculations done in Ref. [8].

2. THE MODEL

Once the centre of mass (CM) motion has been removed, the intrinsic hamiltonian that describes the dynamics of the baryon is given by

$$H^{int} = \sum_{i=q,q'} h_i^{sp} + V_{qq'}(\vec{r}_1 - \vec{r}_2, spin) - \frac{\vec{\nabla}_1 \cdot \vec{\nabla}_2}{m_Q} + \sum_{i=Q,q,q'} m_i \quad (1)$$

$$h_i^{sp} = -\frac{\vec{\nabla}_i^2}{2\mu_i} + V_{Qi}(\vec{r}_i, spin), \quad i = q, q' \quad (2)$$

where \vec{r}_i is the position of the i -th light quark (q, q') with respect to the heavy one (Q), μ_i accounts for the reduced mass of the heavy and the i -th light quark system, $V_{Q(q,q')}$ and $V_{qq'}$ are the light-heavy and light-light interaction potentials, and $spin$ stands for possible spin dependence of the potentials. Note the presence of the Hughes-Eckart term $\vec{\nabla}_1 \cdot \vec{\nabla}_2/m_Q$ that results from the separation of the CM motion.

The phenomenological potentials used in this work are the one proposed in Ref. [9] and the set of potentials introduced in Ref. [8]. We have also considered a potential derived in the context of the SU(2) linear sigma model in Ref. [10] and that contains a pattern of spontaneous chiral symmetry breaking.

For the interactions considered, both the total spin and the total orbital angular momentum with respect to the heavy quark commute with the intrinsic hamiltonian. Assuming now that the ground state of the baryons are in s -wave, $L = 0$, the spatial wave function can only depend on the relative distances r_1 , r_2 and $r_{12} = |\vec{r}_1 - \vec{r}_2|$. If we consider the case in which the heavy quark mass goes to infinity ($m_Q \rightarrow \infty$), the total spin of the light degrees of freedom also commutes with the hamiltonian, since the terms of the type $\vec{\sigma}_Q \cdot \vec{\sigma}_i/m_Q m_i$, $i = q, q'$ vanish. In that limiting case the total spin of the light degrees of freedom is well defined and one can easily write the wave function for the system (see Ref. [7] for details).

Even in this limit, solving the three body problem is a nontrivial task. To do so we adopt a variational approach with a family of spatial wave functions of the type

$$\Psi_{qq'}^{BQ}(r_1, r_2, r_{12}) = N F^{BQ}(r_{12}) \phi_q^Q(r_1) \phi_{q'}^Q(r_2) \quad (3)$$

where N is a normalization constant, ϕ_i^Q is the s -wave ground state solution (ψ_i^Q) of the single particle hamiltonian (h_i^{sp}) corrected at large distances in the form

$$\phi_i^Q(r_i) = (1 + \alpha_i r_i) \psi_i^Q(r_i), \quad i = q, q' \quad (4)$$

and finally F^{BQ} is a Jastrow correlation function in the relative distance of the two light quarks for which we take

$$F^{BQ}(r_{12}) = f^{BQ}(r_{12}) \sum_{j=1}^4 a_j e^{-b_j^2(r_{12}+d_j)^2}, \quad a_1 = 1 \quad (5)$$

$$f^{BQ}(r_{12}) = \begin{cases} 1 - e^{-cr_{12}} & \text{if } V_{qq'}^B(r_{12} = 0) \gg 0 \\ 1 & (c \rightarrow +\infty) \text{ if } V_{qq'}^B(r_{12} = 0) \leq 0 \end{cases} \quad (6)$$

Table 1

Variational results of charmed and bottom baryons masses (in MeV). We also show the Faddeev results of Ref. [8], the lattice results of Ref. [4] and, when available, the experimental masses [3]. s^π stands for the spin-parity of the light degrees of freedom.

B	s^π	$Q = c$				$Q = b$			
		$M_{exp.}$	$M_{Latt.}$	M_{Var}	$M_{Fad.}$	$M_{exp.}$	$M_{Latt.}$	M_{Var}	$M_{Fad.}$
Λ_Q	0^+	2285 ± 1	2270 ± 50	2295	2296	5624 ± 9	5640 ± 60	5643	5643
Σ_Q	1^+	2452 ± 1	2460 ± 80	2469	2466		5770 ± 70	5851	5849
Σ_Q^*	1^+	2518 ± 2	2440 ± 70	2548			5780 ± 70	5882	
Ξ_Q	0^+	2469 ± 3	2410 ± 50	2474	2473		5760 ± 60	5808	5808
Ξ_Q'	1^+	2576 ± 2	2570 ± 80	2578			5900 ± 70	5946	
Ξ_Q^*	1^+	2646 ± 2	2550 ± 80	2655			5900 ± 80	5975	
Ω_Q	1^+	2698 ± 3	2680 ± 70	2681	2678		5990 ± 70	6033	6035
Ω_Q^*	1^+		2660 ± 80	2755			6000 ± 70	6063	

Table 2

Mass mean square radii in fm^2 for charmed and bottom baryons.

B	$Q = c$		$Q = b$	
	$\langle r^2 \rangle_{Var}$	$\langle r^2 \rangle_{Fad.}$	$\langle r^2 \rangle_{Var}$	$\langle r^2 \rangle_{Fad.}$
Λ_Q	0.106	0.104	0.045	0.045
Σ_Q	0.123	0.121	0.057	0.054
Σ_Q^*	0.135		0.060	
Ξ_Q	0.049	0.048	0.049	0.048
Ξ_Q'	0.119		0.060	
Ξ_Q^*	0.123		0.059	
Ω_Q	0.108	0.108	0.057	0.054
Ω_Q^*	0.120		0.059	

being α_i , $a_{i \neq 1}$, b_i and d_i are free variational parameters.

3. RESULTS AND CONCLUSIONS

In this work we have considered the $\Lambda_{b,c}$, $\Sigma_{b,c}$, $\Xi_{b,c}$ and $\Omega_{b,c}$ baryons, and also the $\Sigma_{b,c}^*$, $\Xi_{b,c}^*$, $\Xi'_{b,c}$ and $\Omega_{b,c}^*$ baryons which were not evaluated in Ref. [8].

Our variational results for charm and bottom masses for the AL1 potential of Ref. [8] can be found in Table 1. The results are in good agreement with previous Faddeev calculations done in Ref. [8]³. They also agree with lattice results of Ref. [4] and with the experimental masses.

Using the wave functions obtained with this method, we have calculated mass and charge form factors (See Figs. 2-5 of Ref. [7]), from which one can obtain mass and charge mean square radii. Our results for the latter are shown in Tables 2 and 3. Again we find very good agreement with the results obtained in Ref. [8]. As a further test of the wave functions, we have also calculated the so called ‘‘wave function at the origin’’ (See

³Our model does not take into account three body terms considered in Ref. [8]. Thus, we have subtracted their effect from the Faddeev results

Table 3

Charge mean square radii in fm^2 for charmed and bottom baryons. We only show results for baryons with the lesser positive charge.

B	$Q = c$		$Q = b$	
	$\langle r^2 \rangle_{Var}$	$\langle r^2 \rangle_{Fad.}$	$\langle r^2 \rangle_{Var}$	$\langle r^2 \rangle_{Fad.}$
Λ_Q	0.131	0.129	0.127	0.128
Σ_Q	-0.261	-0.256	-0.332	-0.318
Σ_Q^*	-0.283		-0.349	
Ξ_Q	-0.163	-0.161	-0.213	-0.212
Ξ_Q'	-0.192		-0.267	
Ξ_Q^*	-0.198		-0.266	
Ω_Q	-0.124	-0.124	-0.189	-0.183
Ω_Q^*	-0.138		-0.196	

Ref. [7]), for which we have good agreement in all cases, except for the Ξ baryons, with the values obtained in Ref. [8]. The absolute value of this quantity is claimed to be dependent of the numerical procedure used. Results obtained with the other interquark interactions can be found in Ref. [7].

In this contribution we have outlined the variational scheme developed in Ref. [7] to describe baryons with a heavy c or b quark. This method for solving the three body problem has been possible thanks to the simplifications introduced by the use of HQS. We have evaluated different properties of the baryons using several interquark interactions. Our results are in good agreement with previous, more involved, Faddeev calculations done with the same interquark potentials. They also compare well with experimental data and lattice results. Our wave functions are much more simpler and manageable than those obtained from the Faddeev calculation and we have already used them to study the semileptonic decay of Λ_b and Ξ_b baryons [11], [12]

REFERENCES

1. C. Albajar *et al.*, Phys. Lett. B 273 (1991) 540.
2. G. Bari *et al.*, Nuovo Cimento A 10 (1991) 1787.
3. K. Hagiwara *et al.*, Phys. Rev. D 66 (2002) 010001.
4. K. C. Bowler *et al.*, Phys. Rev. D 57 (1998) 6948.
5. N. Isgur and M. B. Wise, Phys. Lett. B 232 (1989) 113.
6. M. Neubert, Phys. Rep. 245 (1994) 259.
7. C. Albertus, J. E. Amaro, E. Hernández, J. Nieves, Nucl. Phys. A 740 (2004) 333.
8. B. Silvestre-Brac, Few-Body Systems 20 (1996) 1.
9. R. K. Bhaduri, L. E. Cohler, Y. Nogami, Nuovo Cimento, A 65, 58 (1981).
10. F. Fernández, A. Valcarce, U. Straub, A. Faessler, J. Phys. G 19 (1993) 2013.
11. C. Albertus, E. Hernández, J. Nieves, Proceedings of the 6th International Conference on Hyperons, Charm and Beauty Hadrons (Beach 2004), Nucl. Phys. B Proc. Sup. *in print* (hep-th/0408065).
12. C. Albertus, E. Hernández, J. Nieves, Phys. Rev. D *in print* (nucl-th/0412006).