Static properties and semileptonic decays of doubly heavy baryons in a nonrelativistic quark model.

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We evaluate static properties and semileptonic decays for the ground state of doubly heavy Ξ, Ξ', Ξ^* and Ω, Ω', Ω^* baryons. Working in the framework of a nonrelativistic quark model, we solve the three–body problem by means of a variational ansatz made possible by heavy quark spin symmetry constraints. To check the dependence of our results on the inter-quark interaction we use five different quark-quark potentials that include a confining term plus Coulomb and hyperfine terms coming from one–gluon exchange. Our results for static properties (masses, charge radii and magnetic moments) are, with a few exceptions for the magnetic moments, in good agreement with a previous Faddeev calculation. Our much simpler wave functions are used to evaluate semileptonic decays of doubly heavy $\Xi, \Xi'(J=1/2)$ and $\Omega, \Omega'(J=1/2)$ baryons. Our results for the decay widths are in good agreement with calculations done within a relativistic quark model in the quark–diquark approximation.

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I. INTRODUCTION

The subject of doubly heavy baryons has been attracting attention for a long time. Magnetic moments of doubly charmed baryons were evaluated back in the 70's by Lichtenberg [1] within a nonrelativistic approach. The infinite heavy quark mass limit was already used in the 90's to relate the spectrum of doubly heavy baryons to the one of mesons with a single heavy quark [2], or to analyze their semileptonic decay [3]. A factor of two error in the hyperfine

Baryon	S	J^P	I	S_h^π	Quark content
Ξ_{cc}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	ccl
Ξ^*_{cc}	0	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	ccl
Ω_{cc}	-1	$\frac{1}{2}^+$	0	1^+	ccs
Ω^*_{cc}	-1	$\frac{3}{2}^+$	0	1^+	ccs
Ξ_{bb}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	bbl
Ξ^*_{bb}	0	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	bbl
Ω_{bb}	-1	$\frac{1}{2}^+$	0	1^+	bbs
Ω^*_{bb}	-1	$\frac{3}{2}^+$	0	1^+	bbs
Ξ'_{bc}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	0^+	bcl
Ξ_{bc}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	bcl
Ξ^*_{bc}	0	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	bcl
Ω'_{bc}	-1	$\frac{1}{2}^+$	0	0^+	bcs
Ω_{bc}	-1	$\frac{1}{2}^+$	0	1^+	bcs
Ω^*_{bc}	-1	$\frac{3}{2}^+$	0	1^+	bcs

TABLE I: Quantum numbers of doubly heavy baryons analyzed in this study. S, J^P are strangeness and the spin parity of the baryon, I is the isospin, and S_h^π is the spin parity of the heavy degrees of freedom. l denotes a light u or d quark .

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splittings of Ref. [2] have been recently noticed by the potential nonrelativistic QCD (pNRQCD) calculation of Ref. [4].

On the experimental side the SELEX Collaboration claimed evidence for the Ξ_{cc}^+ baryon, in the $\Lambda_c^+ K^- \pi^+$ and pD^+K^- decay modes, with a mass of $M_{\Xi_{cc}^+} = 3519 \pm 1$ MeV/c^2 [5]. Those results were challenged by a theoretical analysis [6] which claimed the observed events by the SELEX Collaboration could be explained without the involvement of doubly charmed baryons. Other experimental collaborations like FOCUS [7], *BABAR* [8] and BELLE [9] have found no evidence for doubly charmed baryons so far. At present the Ξ_{cc}^+ has only a one star status and it is not listed in the particle summary table [10].

In hadrons with a heavy quark and working in the infinite heavy quark mass limit the dynamics of the light degrees of freedom becomes independent of the heavy quark flavor and spin. This is known as heavy quark symmetry (HQS) [11, 12, 13, 14]. This symmetry was developed into an effective theory (HQET) [15] that allowed a systematic, order by order, evaluation of corrections in inverse powers of the heavy quark masses. Unfortunately ordinary HQS can not be applied directly to hadrons containing two heavy quarks as the kinetic energy term needed in those systems to regulate infrared divergences breaks heavy flavor symmetry [16]. For those hadrons the symmetry that survives is heavy quark spin symmetry (HQSS) [17], which amounts to the decoupling of the heavy quark spins in the infinite heavy quark mass limit. In that limit one can consider the total spin of the two heavy quark subsystem (S_h) to be well defined. In this work we shall assume this is a good approximation for the actual heavy quark masses. This approximation, which is the only one related to the infinite heavy quark mass limit that we shall use, will certainly simplify the solution of the baryon three–quark problem. Recently the authors of Ref. [4] have developed and effective theory (pNRQCD) suitable to describe baryons with two and three heavy quarks.

Solving the three–body problem is not an easy task and here we shall do it by means of a variational approach. The approach, with obvious changes, was already applied with good results in the study of baryons with one heavy quark [18]. This method, that leads to simple and manageable wave functions, is made possible by the simplifications introduced in the problem by the fact that we can consider S_h to be well defined. We shall consider several simple phenomenological quark–quark interactions [19, 20, 21] which free parameters have been adjusted in the meson sector and are thus free of three–body ambiguities. The use of different interactions will allow us to estimate part of the theoretical uncertainties affecting our calculation. Uncertainties related to the nonrelativistic baryon states that we use are difficult to estimate. We are aware of the limitations of a non-relativistic approach to describe light quark physics. However, for the kind of study performed in this work these are likely not as relevant as in other contexts. We study the semileptonic decays of double heavy baryons, in which the light quark is merely an spectator (heavy-to-heavy transitions). Indeed the lack of a proper relativistic treatment did prevent us to study semileptonic transitions of the type $b \to u$ of a greater phenomenological interest. One should notice however that at least part of the relativistic effects not explicitly taken into account are included in an effective way in the parameters of the interaction which had been fitted to experimental data. We think this explains why the nonrelativistic quark model is so successful phenomenologically even in the presence of light quarks.

Our simple variational calculation reproduces the results for static properties obtained in Ref. [21] by solving more involved Faddeev type equations. Our method has the advantage that we provide explicit and manageable wave functions that can be used to evaluate further observables. Static properties like masses and magnetic moments of doubly heavy baryons have also been studied in other models. Masses have been calculated in the relativistic quark model assuming a light quark heavy diquark structure [22], the potential approach and sum rules of QCD [23], the nonperturbative QCD approach [24], the Bethe–Salpeter equation applied to the light quark heavy diquark [25], the nonrelativistic quark model with harmonic oscillator potential [26] or with the use of QCD derived potentials [27, 28], the relativistic quasi–potential quark model [29], with the use of the Feynman-Hellman theorem and semi-empirical mass formulas within the framework of a nonrelativistic constituent quark model [30], in effective field theories [4, 31], or in lattice nonrelativistic QCD [32]. There are also lattice QCD determinations [33, 34, 35]. Similarly, magnetic moments have been evaluated in a nonrelativistic approach [1], in the relativistic three–quark model [36], the relativistic quark model using different forms of the relativistic kinematics [37], in the skyrmion model [38], in the Dirac equation formalism [39], or using the MIT bag model [40].

We shall further use our manageable wave functions to study semileptonic decays of doubly heavy , $J = 1/2$, baryons. We shall evaluate form factors, decay widths and angular asymmetry parameters. Previous calculations of semileptonic decay widths have been done in different relativistic quark model approaches [41, 42, 43], or with the use of HQET [44].

The paper is organized as follows. In Sect. II we study the Hamiltonian of the system (Subsect. II A) and briefly introduce the different inter-quark interactions used in this work (Subsect. II B). The variational wave functions are discussed in Sect. III. In Sect. IV we present results for the static properties: masses (Subsect. IV A), charge densities and radii (Subsect. IV B), and magnetic moments (Subsect. IV C). Semileptonic decays are analyzed in Sect. V. After the presentation of general formulas, in Subsect. V A we relate the form factors to matrix elements and show how the latter ones are evaluated within our model. In Subsect. V B we present our results for the form factors, differential and total semileptonic decay widths, and angular asymmetry parameters. The findings of this work are summarized

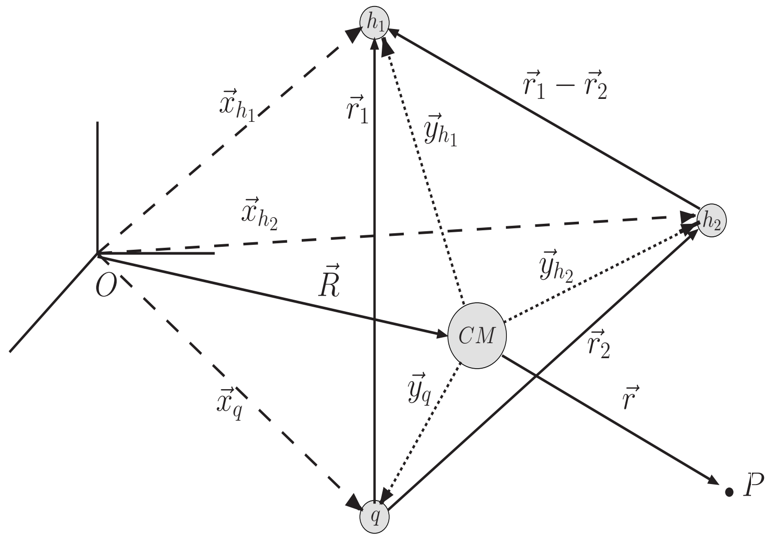

FIG. 1: Definition of different coordinates used throughout this work.

in Sect. VI. The paper also includes three appendices: in appendix A we analyze the infinite heavy quark mass limit of our radial wave functions. In appendix B we relate the form factors for semileptonic decay to the two basic integrals in terms of which all of them can be obtained. Finally, in appendix C we give explicit expressions for those basic integrals.

In Table I we summarize the quantum numbers of the doubly heavy baryons considered in this study[1].

II. THREE BODY PROBLEM

A. Intrinsic Hamiltonian

In the Laboratory (LAB) frame (see Fig. 1), the Hamiltonian (H) of the three quark (h_1, h_2, q, where h_1, $h_2 = c, b$ and $q = l(u, d)$, s) system reads:

$$H = \sum_{j=h_1,\,h_2,\,q} \left(m_j - \frac{\vec{\nabla}^2_{\vec{x}_j}}{2m_j} \right) + V_{h_1h_2} + V_{h_1q} + V_{h_2q} \tag{1}$$

where m_{h_1}, m_{h_2}, m_q are the quark masses and the quark-quark interaction terms V_{jk} depend on the quark spin-flavor quantum numbers and the quark coordinates ($\vec{x}_{h_1}, \vec{x}_{h_2}, \vec{x}_q$ for the h_1, h_2, q quarks respectively). To separate the Center of Mass (CM) free motion, we go to the light quark frame ($\vec{R}, \vec{r}_1, \vec{r}_2$),

$$\begin{aligned} \vec{R} &= \frac{m_{h_1}\vec{x}_{h_1} + m_{h_2}\vec{x}_{h_2} + m_q\vec{x}_q}{m_{h_1} + m_{h_2} + m_q} \\ \vec{r}_1 &= \vec{x}_{h_1} - \vec{x}_q \\ \vec{r}_2 &= \vec{x}_{h_2} - \vec{x}_q \end{aligned} \tag{2}$$

where $\vec{R}$ and $\vec{r}_1$, $\vec{r}_2$ are the CM position in the LAB frame and the relative positions of the h_1, h_2 heavy quarks with respect to the light quark q. The Hamiltonian now reads

$$H = -\frac{\vec{\nabla}^2_{\vec{R}}}{2\overline{\overline{M}}} + H^{\rm int} \tag{3}$$

[1] Note that the definitions of Ξ_{bc} and Ξ'_{bc} are interchanged in some references, with Ξ_{bc} having $S_h = 0$ and Ξ'_{bc} having $S_h = 1$. The same applies to Ω_{bc} and Ω'_{bc}. In tables we always quote the results corresponding to the convention we use.

$$
\begin{aligned}
H^{\rm int} &= \overline{M} + \sum_{j=1,2} H_j^{sp} + V_{h_1h_2}(\vec{r}_1 - \vec{r}_2,\, spin) - \frac{\vec{\nabla}_1 \cdot \vec{\nabla}_2}{m_q} \\
H_j^{sp} &= -\frac{\vec{\nabla}_j^2}{2\mu_j} + V_{h_jq}(\vec{r}_j,\, spin),\ j = 1,2
\end{aligned} \tag{4}
$$

where $\overline{M} = m_{h_1} + m_{h_2} + m_q$, $\mu_j = \left(1/m_{h_j} + 1/m_q\right)^{-1}$ and $\vec{\nabla}_j = \partial/\partial_{\vec{r}_j}$, $j = 1,2$. The intrinsic Hamiltonian $H^{\rm int}$ describes the dynamics of the baryon. Apart from the sum of the quark masses $\overline{M}$, it consists of the sum of two single particle Hamiltonian H_j^{sp}, each of them describing the dynamics of a heavy-light quark system, plus the heavy–heavy interaction term, including the Hughes-Eckart term $(\vec{\nabla}_1 \cdot \vec{\nabla}_2)$. We will use a variational approach to solve it.

B. Quark–Quark Interactions

We have examined five different interactions, one suggested by Bhaduri and collaborators [19] (BD) and four suggested by B. Silvestre-Brac and C. Semay [20, 21] (AL1, AL2, AP1 y AP2). All of them contain a confinement term, plus Coulomb and hyperfine terms coming from one-gluon exchange, and differ from each other in the form factors used for the hyperfine terms, the power of the confining term or the use of a form factor in the one gluon exchange Coulomb potential. All free parameters in the potentials had been adjusted to reproduce the light (π, ρ, K, K^*, etc.) and heavy-light (D, D^*, B, B^*, etc.) meson spectra[2]. All details on the above interactions can be found in Refs. [19, 20, 21].

These interactions were also used in Ref. [21] to obtain, within a Faddeev calculation, the spectrum and static properties of heavy baryons. Our simpler variational method will not only give equally good results for the observables analyzed in [21], but it will also provide us with easy to handle wave functions that can be used to evaluate other observables.

III. VARIATIONAL WAVE FUNCTIONS

For the above mentioned interactions, we have that both the total spin and the internal orbital angular momentum given as

$$
\begin{aligned}
\vec{S} &= \left(\vec{\sigma}_{h_1} + \vec{\sigma}_{h_2} + \vec{\sigma}_q\right)/2 \\
\vec{L} &= \vec{l}_1 + \vec{l}_2, \qquad \text{with } \vec{l}_j = -{\rm i}\ \vec{r}_j \times \vec{\nabla}_j, \quad j = 1,2
\end{aligned} \tag{5}
$$

commute with the intrinsic Hamiltonian and are thus well defined. We are interested in the ground state of baryons with total angular momentum $J = 1/2$, $3/2$ so that we can assume the orbital angular momentum of the baryons to be $L = 0$. This implies that the spatial wave function can only depend on the relative distances r_1, r_2 and $r_{12} = |\vec{r}_1 - \vec{r}_2|$. Furthermore when the heavy quark mass is infinity ($m_h \to \infty$), the total spin of the heavy degrees of freedom, $\vec{S}_{\rm heavy} = \left(\vec{\sigma}_{h_1} + \vec{\sigma}_{h_2}\right)/2$, commutes with the intrinsic Hamiltonian, since the spin–spin terms in the potentials vanish in this limit. We can then assume the spin of the heavy degrees of freedom to be well defined.

With these simplifications we have used the following intrinsic wave functions in our variational approach[3]

- $\Xi_{h_1h_2}$, $\Omega_{h_1h_2}$*−type baryons:*

$$
\begin{aligned}
|\Xi_{h_1h_2}, \Omega_{h_1h_2}; J = \frac{1}{2}, M_J\rangle &= \sum_{M_{S_h} M_{S_q}} (1 \tfrac{1}{2} \tfrac{1}{2} | M_{S_h} M_{S_q} M_J)\ |h_1h_2; 1 M_{S_h}\rangle \otimes |q; \frac{1}{2} M_{S_q}\rangle \\
&\qquad \times \Psi^{\Xi,\,\Omega}_{h_1h_2}(r_1, r_2, r_{12})
\end{aligned} \tag{6}
$$

  where M_J is the third component of the baryon total angular momentum while $|h_1h_2; S_h, M_{S_h}\rangle$ and $|q; \frac{1}{2} M_{S_q}\rangle$ represent spin states of the h_1h_2 subsystem and the light quark respectively. $(j_1 j_2 j | m_1 m_2 m)$ is a Clebsch-Gordan

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[2] To get the quark–quark interaction starting from a quark-antiquark one the usual $V_{ij}^{qq} = V_{ij}^{q\bar{q}}/2$ prescription, coming from a $\vec{\lambda}_i \vec{\lambda}_j$ color dependence ($\vec{\lambda}$ are the Gell-Mann matrices) of the whole potential, has been assumed.

[3] We omit the antisymmetric color wave function and the plane wave for the center of mass motion which are common to all cases.

coefficient. For $h_1=h_2$ we need $\Psi^{\Xi,\,\Omega}_{h_1h_1}(r_1,r_2,r_{12})=\Psi^{\Xi,\,\Omega}_{h_1h_1}(r_2,r_1,r_{12})$ to guarantee a complete symmetry of the wave function under the exchange of the two heavy quarks.

- $\Xi^*_{h_1h_2},\,\Omega^*_{h_1h_2}$*−type baryons:*

$$|\Xi^*_{h_1h_2},\,\Omega^*_{h_1h_2};J=\frac{3}{2},M_J\rangle \;=\; \sum_{M_{S_h}M_{S_q}}(1\frac{1}{2}\frac{3}{2}|M_{S_h}M_{S_q}M_J)\;|h_1h_2;1M_{S_h}\rangle\otimes|q;\frac{1}{2}M_{S_q}\rangle$$
$$\times\Psi^{\Xi^*,\,\Omega^*}_{h_1h_2}(r_1,r_2,r_{12}) \tag{7}$$

Similarly to the case before for $h_1=h_2$ we need $\Psi^{\Xi^*,\,\Omega^*}_{h_1h_1}(r_1,r_2,r_{12})=\Psi^{\Xi^*,\,\Omega^*}_{h_1h_1}(r_2,r_1,r_{12})$.

- $\Xi'_{h_1h_2},\,\Omega'_{h_1h_2}$*−type baryons:*

$$|\Xi'_{h_1h_2},\,\Omega'_{h_1h_2};J=\frac{1}{2},M_J\rangle \;=\; |h_1h_2;00\rangle\otimes|q;\frac{1}{2}M_J\rangle$$
$$\times\Psi^{\Xi',\,\Omega'}_{h_1h_2}(r_1,r_2,r_{12}) \tag{8}$$

In this case $h_1\neq h_2$ and we do not need the orbital part to have a definite symmetry under the exchange of the two quarks.

The spatial wave functions $\Psi(r_1,r_2,r_{12})$ in the above expressions will be determined by the variational principle: $\delta\langle B|H^{\rm int}|B\rangle=0$. For simplicity, we shall assume a Jastrow–type functional form[4]:

$$\Psi^B_{h_1h_2}(r_1,r_2,r_{12}) \;=\; N\,F^B(r_{12})\,\phi_{h_1q}(r_1)\,\phi_{h_2q}(r_2) \tag{9}$$

where N is a constant, which is determined from normalization

$$1=\int d^3r_1\int d^3r_2\;\left|\Psi^B_{h_1h_2}(r_1,r_2,r_{12})\right|^2=8\pi^2\int_0^{+\infty}dr_1\;r_1^2\;\int_0^{+\infty}dr_2\;r_2^2\int_{-1}^{+1}d\mu\;\left|\Psi^B_{h_1h_2}(r_1,r_2,r_{12})\right|^2 \tag{10}$$

with μ being the cosine of the angle between the vectors $\vec r_1$ and $\vec r_2$ ($r_{12}=(\,r_1^2+r_2^2-2r_1r_2\mu)^{1/2}$).

The functions ϕ_{h_1q} and ϕ_{h_2q} will be taken as the $S-$wave ground states $\varphi_j(r_j)$ of the single particle Hamiltonians H^{sp}_j of Eq. (4) modified at large distances.

$$\phi_{h_jq}(r_j) \;=\; (1+\alpha_j\,r_j)\,\varphi_j(r_j),\quad j=1,2 \tag{11}$$

The heavy–heavy correlation function F^B will be given by a linear combination of gaussians[5]

$$F^B(r_{12}) \;=\; \sum_{j=1}^{4}a_je^{-b_j^2(r_{12}+d_j)^2},\quad a_1=1 \tag{12}$$

The value of one of the a_j parameters can be absorbed into the normalization constant N, so that we fix $a_1=1$. The rest of the variational parameters are determined by the variational condition $\delta\langle B|H^{\rm int}|B\rangle=0$.

Although it is not self-evident from the functional form assumed, our variational wave functions are consistent with the infinite heavy quark mass limit, reducing in that limit to the product of the internal wave function for the heavy diquark times the wave function for the relative motion of the light quark with respect to a pointlike heavy diquark. All this is discussed in appendix A.

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[4] A similar form lead to very good results in the case of baryons with a single heavy quark [18].

[5] Note that F^B should vanish at large distances because of the confinement potential. The confinement potential is also responsible for the non–vanishing values of the parameters $\alpha_j,\ j=1,2$ in Eq. (11).

		AL1	AL2	AP1	AP2	BD
Ξ_{cc}	VAR	3612	3619	3629	3630	3639
	FAD [21]	3609	3616	3625	3628	3633
Ξ^*_{cc}	VAR	3706	3715	3722	3729	3722
Ξ_{bb}	VAR	10197	10180	10207	10179	10202
	FAD [21]	10194	10175	10204	10176	10197
Ξ^*_{bb}	VAR	10236	10219	10245	10219	10235
Ξ_{bc}	VAR	6919	6912	6933	6917	6936
	FAD [21]	6916	6913	6928	6907	6934
Ξ'_{bc}	VAR	6948	6942	6957	6944	6965
Ξ^*_{bc}	VAR	6986	6981	7000	6987	6993

		AL1	AL2	AP1	AP2	BD
Ω_{cc}	VAR	3702	3718	3711	3710	3743
	FAD [21]	3711	3718	3710	3709	3741
Ω^*_{cc}	VAR	3783	3802	3800	3802	3805
Ω_{bb}	VAR	10260	10249	10259	10226	10274
	FAD [21]	10267	10246	10258	10224	10271
Ω^*_{bb}	VAR	10297	10287	10301	10269	10302
Ω_{bc}	VAR	6986	6986	6990	6969	7013
	FAD [21]	7003	6996	6996	6971	7023
Ω'_{bc}	VAR	7009	7010	7011	6994	7033
Ω^*_{bc}	VAR	7046	7047	7055	7037	7057

TABLE II: Doubly heavy Ξ and Ω baryons masses in MeV. VAR stands for the results of our variational calculation. FAD stands for the results obtained in Ref. [21] using the same interquark interactions but within a Faddeev approach.

IV. STATIC PROPERTIES

A. Masses

The mass of the baryon is simply given by the expectation value of the intrinsic Hamiltonian. Our results (VAR) appear in Table II where we compare them with the ones obtained in Ref. [21] with the use of the same inter-quark interactions but within a Faddeev approach (FAD). For that purpose we have eliminated from the latter a small three-body force contribution of the type $V_{123} = \text{constant}/m_{h_1}m_{h_2}m_q$ that was also included in the evaluation of Ref. [21]. We will show their full results in the following tables. Whenever comparison is possible we find an excellent agreement between the two calculations. In some cases the variational masses are even lower than the Faddeev ones. Besides we give predictions for states that were not considered in the study of Ref. [21].

In Tables III and IV we compare our results with other theoretical calculations[6]. Our central values correspond to the results obtained with the AL1 potential, while the errors quoted take into account the variation when using different potentials. The same presentation is used for the results obtained in Ref. [21] for which we now show their full values including the contribution of the three-body force. All calculations give similar results that vary within a few per cent. From the experimental point of view the SELEX Collaboration [5] has recently measured the value of $M_{\Xi_{cc}}$. This experimental value is 100 MeV smaller that our result. On account of what has been said in the introduction, one should take this experimental value with due caution. Note also that in Ref. [5] the systematic error is not given. There are also different lattice determinations for baryons with two equal heavy quarks [33, 34, 35]. Our results agree within errors with the lattice data for baryons with two c quarks, while they are roughly 100 MeV below lattice results for doubly b-quark baryons. The best overall agreement with lattice data available so far is achieved in the calculation of Ref. [30] where they use the Feynman-Hellmann theorem and semiempirical mass formulas in the framework of a nonrelativistic quark model but without the use of an explicit Hamiltonian. Within full dynamical calculations, ours and the relativistic calculations in Refs. [22, 25, 29] have the best overall agreement with lattice

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[6] Note in Ref. [30] the Ξ_{bc}, Ξ'_{bc} and Ω_{bc}, Ω'_{bc} baryons are defined such that the total spin of the light q quark and the heavy c quark are well defined, being 0 for Ξ_{bc}, Ω_{bc} and 1 for Ξ'_{bc}, Ω'_{bc}. They are thus linear combinations of our states. The different spin functions are related by

$$\begin{aligned}\left(|qc;0\rangle \otimes |b;\tfrac{1}{2}\rangle\right)^{J=1/2} &= \frac{1}{2}\left(|bc;0\rangle \otimes |q;\tfrac{1}{2}\rangle\right)^{J=1/2} - \frac{\sqrt{3}}{2}\left(|bc;1\rangle \otimes |q;\tfrac{1}{2}\rangle\right)^{J=1/2}\\ \left(|qc;1\rangle \otimes |b;\tfrac{1}{2}\rangle\right)^{J=1/2} &= -\frac{\sqrt{3}}{2}\left(|bc;0\rangle \otimes |q;\tfrac{1}{2}\rangle\right)^{J=1/2} - \frac{1}{2}\left(|bc;1\rangle \otimes |q;\tfrac{1}{2}\rangle\right)^{J=1/2}\end{aligned} \tag{13}$$

In order to extract their predictions for the Ξ_{bc}, Ξ'_{bc} and Ω_{bc}, Ω'_{bc} baryons with total spin of the two heavy quarks well defined, we have assumed that the above relations, but with coefficients square, are also valid for the masses. Note this may be incorrect as we are neglecting a possible non negligible interference contribution.

data.

	This work	[21]	[22]	[23]	[24]	[25]	[26]	[27]	[28]	[29]	[30]	[31]	[32]
Ξ_{cc}	3612^{+17}	3607^{+24}	3620	3480	3690	3740	3646	3524	3478	3660	3660 ± 70	3610	3588 ± 72
Ξ_{cc}^*	3706^{+23}		3727	3610		3860	3733	3548	3610	3810	3740 ± 80	3680	
Ξ_{bb}	10197^{+10}_{-17}	10194^{+10}_{-19}	10202	10090	10160	10300			10093	10230	10340 ± 100		
Ξ_{bb}^*	10236^{+9}_{-17}		10237	10130		10340			10133	10280	10370 ± 100		
Ξ_{bc}	6919^{+17}_{-7}	6915^{+17}_{-9}	6933	6820	6960	7010			6820	6950	$6965 \pm 90^\dagger$		6840 ± 236
Ξ_{bc}'	6948^{+17}_{-6}		6963	6850		7070			6850	7000	$7065 \pm 90^\dagger$		
Ξ_{bc}^*	6986^{+14}_{-5}		6980	6900		7100			6900	7020	7060 ± 90		

	This work	Exp. [5]	Latt. [33]	Latt. [34]	Latt. [35]
Ξ_{cc}	3612^{+17}	3519 ± 1		3605 ± 23	3549 ± 95
Ξ_{cc}^*	3706^{+23}			3685 ± 23	3641 ± 97
Ξ_{bb}	10197^{+10}_{-17}		10314 ± 47		
Ξ_{bb}^*	10236^{+9}_{-17}		10333 ± 55		

TABLE III: First panel: doubly heavy Ξ masses in MeV as obtained in different models. Our central values, and the ones of Ref. [21], have been evaluated with the AL1 potential. Second panel: we compare our results with the experimental value for $M_{\Xi_{cc}}$ measured by the SELEX Collaboration [5] (Note the cautions that appear on this experimental mass in the introduction), and lattice results from Refs. [33, 34, 35]. Entries with † should be taken with due caution (see footnote 6).

	This work	[21]	[22]	[23]	[24]	[25]	[26]	[28]	[29]	[30]	[31]	[32]
Ω_{cc}	3702^{+41}	3710^{+29}_{-2}	3778	3590	3860	3760	3749	3590	3760	3740 ± 70	3710	3698 ± 65
Ω_{cc}^*	3783^{+22}		3872	3690		3900	3826	3690	3890	3820 ± 80	3760	
Ω_{bb}	10260^{+14}_{-34}	10267^{+4}_{-43}	10359	10180	10340	10340		10180	10320	10370 ± 100		
Ω_{bb}^*	10297^{+5}_{-28}		10389	10200		10380		10200	10360	10400 ± 100		
Ω_{bc}	6986^{+27}_{-17}	7003^{+20}_{-32}	7088	6910	7130	7050		6910	7050	$7045 \pm 90^\dagger$		6954 ± 225
Ω_{bc}'	7009^{+24}_{-15}		7116	6930		7110		6930	7090	$7105 \pm 90^\dagger$		
Ω_{bc}^*	7046^{+11}_{-9}		7130	6990		7130		6990	7110	7120 ± 90		

	This work	Latt. [33]	Latt. [34]	Latt. [35]
Ω_{cc}	3702^{+41}		$3733 \pm 9^{+7}_{-38}$	3663 ± 97
Ω_{cc}^*	3783^{+22}		$3801 \pm 9^{+3}_{-34}$	3734 ± 98
Ω_{bb}	10260^{+14}_{-34}	$10365 \pm 40^{-11}_{+12}{}^{+16}_{-0}$		
Ω_{bb}^*	10297^{+5}_{-28}	$10383 \pm 39^{-8}_{+8}{}^{+12}_{-0}$		

TABLE IV: Same as Table III for doubly heavy Ω baryons.

There are also independent determinations of mass splittings in lattice QCD [33, 34, 35], nonrelativistic lattice QCD [32] and pNRQCD [4]. In Table V we compare those results to the ones obtained in the present calculation and in other models. Our central results evaluated with the AL1 potential are larger than the ones obtained in lattice QCD [33, 34, 35] and lattice nonrelativistic QCD [32]. The agreement is better when we use the BD potential of Ref. [19] for which we always get the lowest results. Similar results are obtained by the relativistic calculation of Ref. [22], whereas for the relativistic calculations in Refs. [23, 25, 29] and the nonrelativistic one in Ref. [28] the agreement with lattice QCD and nonrelativistic lattice QCD data worsens. As for the calculation in Ref. [30] we do not quote their results due to the large theoretical errors involved.

B. Charge densities and radii

The baryon charge density at the point P (coordinate vector $\vec{r}$ in the CM frame, see Fig. 1) is given by:

$$\begin{aligned} \rho_e^B(\vec{r}) &= \int d^3r_1 d^3r_2 \left|\Psi^B_{h_1 h_2}(r_1, r_2, r_{12})\right|^2 \left\{ e_{h_1}\delta^3(\vec{r} - \vec{y}_{h_1}) + e_{h_2}\delta^3(\vec{r} - \vec{y}_{h_2}) + e_q \delta^3(\vec{r} - \vec{y}_q) \right\} \\ &\equiv \rho_e^B(\vec{r})\big|_{h_1} + \rho_e^B(\vec{r})\big|_{h_2} + \rho_e^B(\vec{r})\big|_q \end{aligned} \tag{14}$$

	This work	[22]	[23]	[25]	[28]	[29]	[4]	[32]	Latt. [33]	Latt. [34]	Latt. [35]
$M_{\Xi^*_{cc}} - M_{\Xi_{cc}}$	94^{+5}_{-11}	107	130	120	132	150	120 ± 40	70 ± 13		80 ± 11	87 ± 19
$M_{\Xi^*_{bb}} - M_{\Xi_{bb}}$	39^{+1}_{-6}	35	40	40	40	50	34 ± 4	20 ± 7	20 ± 6		
$M_{\Xi^*_{bc}} - M_{\Xi_{bc}}$	67^{+3}_{-10}	47	80	90	80	70		43 ± 11			
$M_{\Xi'_{bc}} - M_{\Xi_{bc}}$	29^{+1}_{-5}	30	30	60	30	50		9 ± 7			
$M_{\Omega^*_{cc}} - M_{\Omega_{cc}}$	81^{+11}_{-19}	94	100	140	100	130		63 ± 9		68 ± 7	67 ± 16
$M_{\Omega^*_{bb}} - M_{\Omega_{bb}}$	37^{+6}_{-9}	30	20	40	20	40		19 ± 5	20 ± 5		
$M_{\Omega^*_{bc}} - M_{\Omega_{bc}}$	60^{+8}_{-16}	42	80	80	80	60		39 ± 8			
$M_{\Omega'_{bc}} - M_{\Omega_{bc}}$	23^{+2}_{-3}	28	20	60	20	40		9 ± 6			

TABLE V: Mass splittings in MeV for doubly heavy Ξ and Ω baryons. Our central values have been obtained with the AL1 potential. Entries with a † should be taken with due caution (see footnote 6).

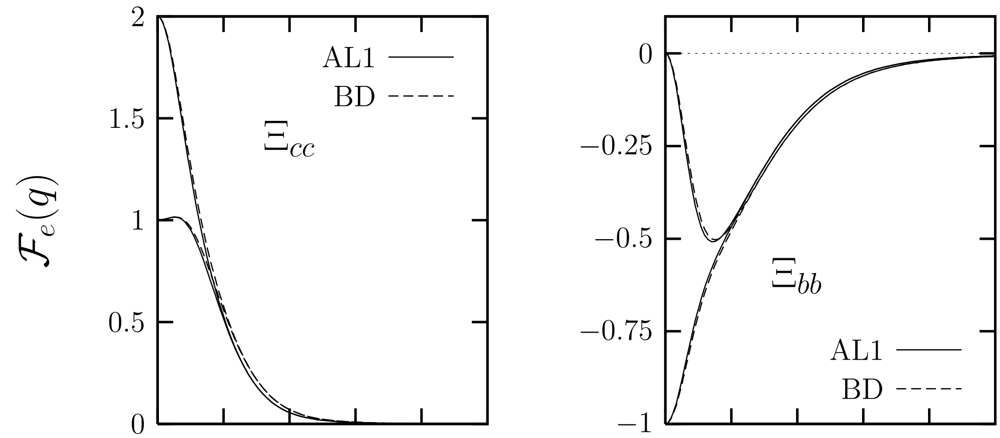

FIG. 2: Charge form factor of the Ξ_{cc} and Ξ_{bb} baryons evaluated with the AL1 [20, 21] (solid line) and BD [19] (dashed line) potentials. We show the two possible charge states. Similar results are obtained for Ξ^*_{cc} and Ξ^*_{bb}.

where $e_{h_1,h_2,q}$ are the quark charges in proton charge units e, and from Fig. 1 we have[7] $\vec{y}_{h_1} = \vec{y}_q + \vec{r}_1$, $\vec{y}_{h_2} = \vec{y}_q + \vec{r}_2$ and $\vec{y}_q = -\left(m_{h_1}\vec{r}_1 + m_{h_2}\vec{r}_2\right)/\overline{M}$. Since our $L = 0$ wave functions only depend on scalars (r_1, r_2 and r_{12}) the charge density is spherically symmetric ($\rho^B_e(\vec{r}\,) = \rho^B_e(|\vec{r}\,|)$).

The charge form factor is defined as usual

$$\mathcal{F}^B_e(\vec{q}\,) = \int d^3r\; e^{\mathrm{i}\vec{q}\cdot\vec{r}} \rho^B_e(r) \tag{15}$$

and it only depends on $|\vec{q}\,|$. Its value at $\vec{q} = \vec{0}$ gives the baryon charge in units of the proton charge.

The charge mean square radii are defined

$$\langle r^2 \rangle^B_e = \int d^3r\; r^2 \rho^B_e(r) = 4\pi \int_0^{+\infty} dr\; r^4 \rho^B_e(r) \tag{16}$$

In Figs. 2, 3 and 4 we show the charge form factors for Ξ_{cc}, Ξ_{bb}, Ω_{cc}, Ω_{bb} and Ξ_{bc}, Ω_{bc} baryons. In each case similar results are obtained for the star and prime excitations. We show the calculations with both the AL1 potential of Refs. [20, 21] and the BD potential of Ref. [19]. The differences between the two calculations are minor in most cases.

In Table VI we show the charge mean square radii. With the exceptions of the Ξ^0_{bc} and Ω^0_{bc}, we find good agreement with the results obtained in Ref. [21] within a Faddeev calculation. The possible presence of a $S_h = 0$ contribution in the wave functions of Ref. [21] could be the possible explanation for this discrepancy. We also compare with the results obtained, for a few states, in Ref. [37] with the use a relativistic quark model in the instant form. The agreement is bad in this case.

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[7] There exists the obvious restriction $m_{h_1}\vec{y}_{h_1} + m_{h_2}\vec{y}_{h_2} + m_q\vec{y}_q = \vec{0}$.

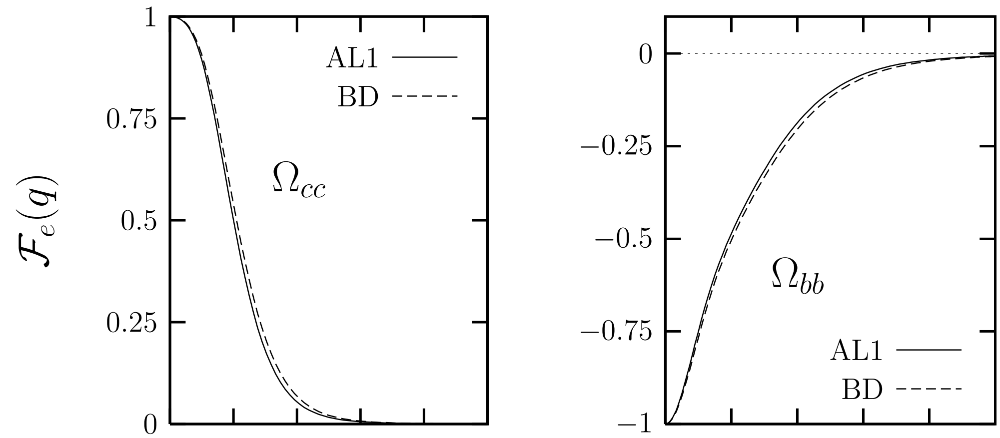

FIG. 3: Charge form factor of the Ω_{cc} and Ω_{bb} baryons evaluated with the AL1 (solid line) and BD (dashed line) potentials. Similar results are obtained for Ω_{cc}^* and Ω_{bb}^*.

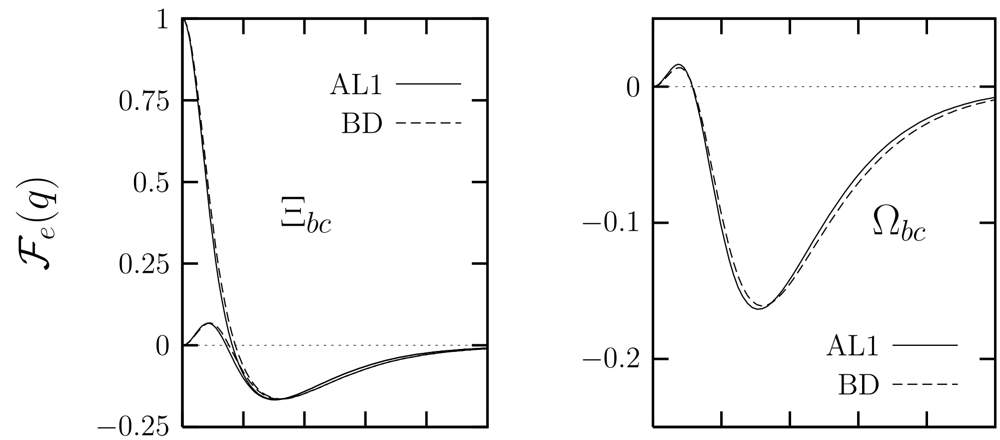

FIG. 4: Charge form factor of the Ξ_{bc} and Ω_{bc} baryons evaluated with the AL1 (solid line) and BD (dashed line) potentials. For Ξ_{bc} baryons we show the two possible charge states. Similar results are obtained for Ξ_{bc}^*, Ξ_{bc}' and Ω_{bc}^*, Ω_{bc}'

C. Magnetic moments

The orbital part of the magnetic moment is defined in terms of the velocities $\vec{v}$ of the quarks, with respect to the position of the CM, and it reads

$$\begin{aligned}\mu^B &= \int d^3r_1 d^3r_2 \left(\Psi^B_{h_1h_2}(r_1,r_2,r_{12})\right)^* \left\{\frac{e_{h_1}}{2m_{h_1}}(\vec{y}_{h_1}\times m_{h_1}\vec{v}_{h_1})_z\right.\\ &+ \left.\frac{e_{h_2}}{2m_{h_2}}(\vec{y}_{h_2}\times m_{h_2}\vec{v}_{h_2})_z + \frac{e_q}{2m_q}(\vec{y}_q\times m_q\vec{v}_{y_q})_z\right\}\Psi^B_{h_1h_2}(r_1,r_2,r_{12})\end{aligned} \tag{17}$$

with[8] $m_{h_1}\vec{v}_{h_1} = -i\vec{\nabla}_1$, $m_{h_2}\vec{v}_{h_2} = -i\vec{\nabla}_2$ and $m_q\vec{v}_q = i\left(\vec{\nabla}_1+\vec{\nabla}_2\right)$. Since our orbital wave function has $L=0$, the orbital magnetic moment vanishes. The magnetic moment of the baryon is then entirely given by the spin contribution.

$$\langle B;\, J,\, M_J=J|\ \frac{e_{h_1}}{2m_{h_1}}(\vec{\sigma}_{h_1})_z + \frac{e_{h_2}}{2m_{h_2}}(\vec{\sigma}_{h_2})_z + \frac{e_q}{2m_q}(\vec{\sigma}_q)_z\ |B;\, J,\, M_J=J\rangle \tag{18}$$

Those matrix elements are trivially evaluated with the results

$$\Xi_{h1h_2},\ \Omega_{h1h_2}\ \longrightarrow\ \frac{2}{3}\left(\frac{e_{h_1}}{2m_{h_1}} + \frac{e_{h_2}}{2m_{h_2}} - \frac{1}{2}\ \frac{e_q}{2m_q}\right)$$

[8] Note that the classical kinetic energy has a term on $\vec{v}_{h_1}\cdot\vec{v}_{h_2}$ and then the operator $m_{h_1}\vec{v}_{h_1}$ is not proportional to $-i\overrightarrow{\nabla}_{y_{h_1}}$, but it is rather given by $m_{h_1}\vec{v}_{h_1} = (\overline{M}-m_{h_1})/\overline{M}\cdot(-i\overrightarrow{\nabla}_{y_{h_1}}) - m_{h_2}/\overline{M}\cdot(-i\overrightarrow{\nabla}_{y_{h_2}}) = (-i\overrightarrow{\nabla}_1)$. Similarly $m_{h_2}\vec{v}_{h_2} = (-i\overrightarrow{\nabla}_2)$.

	This work	[21]	[37]
Ξ_{cc}^{+}	$-0.030^{+0.003}_{-0.016}$	$-0.038^{+0.004}_{-0.016}$	0.1
Ξ_{cc}^{++}	$0.298^{+0.034}_{-0.028}$	$0.315^{+0.035}_{-0.030}$	1.3
Ξ_{cc}^{*+}	$-0.042^{+0.007}_{-0.019}$		
Ξ_{cc}^{*++}	$0.341^{+0.041}_{-0.042}$		
Ξ_{bb}^{0}	$0.221^{+0.033}_{-0.025}$	$0.242^{+0.035}_{-0.027}$	
Ξ_{bb}^{-}	$-0.133^{+0.014}_{-0.016}$	$-0.143^{+0.006}_{-0.018}$	
Ξ_{bb}^{*-}	$-0.142^{+0.018}_{-0.018}$		
Ξ_{bb}^{*0}	$0.238^{+0.035}_{-0.031}$		
Ξ_{bc}^{0}	$-0.057^{+0.006}_{-0.013}$	$-0.072^{+0.008}_{-0.017}$	
Ξ_{bc}^{+}	$0.279^{+0.026}_{-0.031}$	$0.306^{+0.035}_{-0.011}$	
$\Xi_{bc}^{\prime 0}$	$-0.065^{+0.010}_{-0.015}$		
$\Xi_{bc}^{\prime +}$	$0.283^{+0.036}_{-0.025}$		
Ξ_{bc}^{*0}	$-0.065^{+0.010}_{-0.018}$		
Ξ_{bc}^{*+}	$0.305^{+0.031}_{-0.039}$		

	This work	[21]	[37]
Ω_{cc}^{+}	$0.013^{+0.001}_{-0.002}$	$0.009_{-0.003}$	0.2
Ω_{cc}^{*+}	$0.009^{+0.001}_{-0.002}$		
Ω_{bb}^{-}	$-0.086^{+0.008}_{-0.001}$	$-0.090^{+0.007}_{-0.002}$	
Ω_{bb}^{*-}	$-0.092^{+0.011}_{-0.001}$		
Ω_{bc}^{0}	$-0.016_{+0.003}$	$-0.025^{+0.002}_{-0.003}$	
$\Omega_{bc}^{\prime 0}$	$-0.019^{+0.003}_{-0.003}$		
Ω_{bc}^{*0}	$-0.021^{+0.004}_{-0.002}$		

TABLE VI: Charge mean square radii in fm^2 for doubly heavy Ξ and Ω baryons. Our central values, and the ones of Ref. [21], have been evaluated with the AL1 potential.

$$\begin{aligned} \Xi^*_{h1h_2},\ \Omega^*_{h1h_2} &\longrightarrow \frac{e_{h_1}}{2m_{h_1}}+\frac{e_{h_2}}{2m_{h_2}}+\frac{e_q}{2m_q} \\ \Xi'_{h1h_2},\ \Omega'_{h1h_2} &\longrightarrow \frac{e_q}{2m_q} \end{aligned} \tag{19}$$

In Table VII we give our numerical results. Our central values, as the ones obtained in Ref. [21] within a Faddeev approach, have been evaluated with the use of the AL1 potential. When compared to the values obtained in Ref. [21] we find very good agreement with just a few exceptions (Ξ^0_{bc}, Ξ^+_{bc}, Ω^0_{bc}). The discrepancy for the latter baryons may come from a possible non negligible $S_h = 0$ contribution to their wave functions in the calculation of Ref. [21]. In our case we have fixed $S_h = 1$ which we think is a very good approximation since in the limit of infinite heavy quark masses the spin of the heavy quark degrees of freedom is well defined.

We also compare our results to the ones obtained in Refs.[1, 36, 37, 38, 39, 40] using different approaches. The differences between different calculations are in some cases large. Being $L = 0$ a good approximation the magnetic moments are essentially determined by the spin contribution of the quarks. With $m_b \gg m_u$, m_d, m_s, the contribution from the b quarks is negligible compared to the one of the light quark. This is also true to a lesser extent for the c quark.

V. SEMILEPTONIC DECAY

In this section we shall use the wave functions obtained with the variational method to study different doubly $B(1/2^+) \to B'(1/2^+)$ baryon semileptonic decays involving a $b \to c$ transition at the quark level.

The differential decay width reads

$$d\Gamma = 8|V_{cb}|^2 m_{B'} G_F^2 \frac{d^3p'}{(2\pi)^3 2E'_{B'}} \frac{d^3k}{(2\pi)^3 2E_{\bar\nu_l}} \frac{d^3k'}{(2\pi)^3 2E'_l} (2\pi)^4 \delta^4(p-p'-k-k')\ \mathcal{L}^{\alpha\beta}(k,k')\mathcal{H}_{\alpha\beta}(p,p') \tag{20}$$

where $|V_{cb}|$ is the modulus of the corresponding Cabibbo–Kobayashi–Maskawa matrix element, $m_{B'}$ is the mass of the final baryon, $G_F = 1.16637(1)\times 10^{-11}\,\mathrm{MeV}^{-2}$[10] is the Fermi decay constant, p, p', k and k' are the four-momenta of the initial baryon, final baryon, final anti-neutrino and final lepton respectively, and $\mathcal{L}$ and $\mathcal{H}$ are the lepton and hadron tensors.

The lepton tensor is given as

$$\mathcal{L}^{\mu\sigma}(k,k') \ = \ k'^{\mu}k^{\sigma} + k'^{\sigma}k^{\mu} - g^{\mu\sigma}k\cdot k' + \mathrm{i}\epsilon^{\mu\sigma\alpha\beta}k'_{\alpha}k_{\beta} \tag{21}$$

	This work	[21]	[1]	[36]	[37]	[38]	[39]	[40]
Ξ_{cc}^{+}	$0.785^{+0.050}_{-0.030}$	$0.784^{+0.050}_{-0.029}$	0.806	0.72	0.72	$0.89 \sim 0.98$	$0.778 \sim 0.790$	0.86
Ξ_{cc}^{++}	$-0.208^{+0.035}_{-0.086}$	$-0.206^{+0.034}_{-0.086}$	-0.124	0.13	-0.10	-0.47	$-0.172 \sim -0.154$	0.17
Ξ_{cc}^{*+}	$-0.311^{+0.052}_{-0.130}$		-0.186			$-1.17 \sim -0.98$		0.20
Ξ_{cc}^{*++}	$2.67^{+0.27}_{-0.15}$		2.60			$3.16 \sim 3.18$		2.54
Ξ_{bb}^{0}	$-0.742^{+0.044}_{-0.091}$	$-0.742^{+0.044}_{-0.092}$		-0.53			$-0.726 \sim -0.705$	0.61
Ξ_{bb}^{-}	$0.251^{+0.045}_{-0.021}$	$0.251^{+0.046}_{-0.021}$		0.18			$0.226 \sim 0.236$	0.14
Ξ_{bb}^{*0}	$1.87^{+0.27}_{-0.13}$							1.37
Ξ_{bb}^{*-}	$-1.11^{+0.06}_{-0.14}$							-0.95
Ξ_{bc}^{0}	$0.518^{+0.048}_{-0.020}$	$0.058^{+0.059}_{-0.054}$		0.42				
Ξ_{bc}^{+}	$-0.475^{+0.040}_{-0.088}$	$-0.198^{+0.057}_{-0.056}$		-0.12				
$\Xi_{bc}^{\prime 0}$	$-0.993^{+0.065}_{-0.137}$			-0.76			$-0.385 \sim -0.366$	
$\Xi_{bc}^{\prime +}$	$1.99^{+0.27}_{-0.13}$			1.52			$1.50 \sim 1.54$	
Ξ_{bc}^{*0}	$-0.712^{+0.059}_{-0.133}$							-0.39
Ξ_{bc}^{*+}	$2.27^{+0.27}_{-0.14}$							2.04

	This work	[21]	[1]	[36]	[37]	[38]	[39]	[40]
Ω_{cc}^{+}	$0.635^{+0.012}_{-0.015}$	$0.635^{+0.011}_{-0.015}$	0.688	0.67	0.72	$0.59 \sim 0.64$	$0.657 \sim 0.663$	0.84
Ω_{cc}^{*+}	$0.139^{+0.009}_{-0.017}$		0.167			$-0.20 \sim 0.03$		0.39
Ω_{bb}^{-}	$0.101^{+0.007}_{-0.007}$	$0.101^{+0.007}_{-0.006}$		0.04			$0.105 \sim 0.108$	0.084
Ω_{bb}^{*-}	$-0.662^{+0.022}_{-0.024}$							-1.28
Ω_{bc}^{0}	$0.368^{+0.010}_{-0.011}$	$0.009^{+0.038}_{-0.029}$		0.45				
$\Omega_{bc}^{\prime 0}$	$-0.542^{+0.021}_{-0.024}$			-0.61			$-0.130 \sim -0.125$	
Ω_{bc}^{*0}	$-0.261^{+0.015}_{-0.021}$							-0.22

TABLE VII: Magnetic moments, in nuclear magnetons ($|e|/2m_p$, with m_p the proton mass), of doubly heavy Ξ and Ω baryons. Our central values, and the ones of Ref. [21], have been evaluated with the AL1 potential.

where we use the convention $\epsilon^{0123} = -1$, $g^{\mu\mu} = (+,-,-,-)$.

The hadron tensor is given as

$$\mathcal{H}_{\mu\sigma}(p,p') \;=\; \frac{1}{2}\sum_{r,r'} \left\langle B', r' \; \vec{p}\,' \left| \overline{\Psi}^c(0)\gamma_\mu(I-\gamma_5)\Psi^b(0) \right| B, r \; \vec{p} \right\rangle \; \left\langle B', r' \; \vec{p}\,' \left| \overline{\Psi}^c(0)\gamma_\sigma(I-\gamma_5)\Psi^b(0) \right| B, r \; \vec{p} \right\rangle^* \tag{22}$$

with $|B, r\;\vec{p}\rangle$ ($|B', r'\;\vec{p}\,'\rangle$) representing the initial (final) baryon with three–momentum $\vec{p}\,(\vec{p}\,')$ and spin third component r (r'). The baryon states are normalized such that $\langle r\;\vec{p}\,|\,r'\;\vec{p}\,'\rangle = (2\pi)^3 (E(\vec{p}\,)/m)\,\delta_{rr'}\,\delta^3(\vec{p}-\vec{p}\,')$. The hadron matrix elements can be parametrized in terms of six form factors as

$$\begin{aligned}\left\langle B', r' \; \vec{p}\,' \left| \overline{\Psi}^c(0)\gamma_\mu(I-\gamma_5)\Psi^b(0) \right| B, r \; \vec{p} \right\rangle \;=\; & \bar{u}^{B'}_{r'}(\vec{p}\,')\Big\{\gamma_\mu\left(F_1(w)-\gamma_5 G_1(w)\right) + v_\mu\left(F_2(w)-\gamma_5 G_2(w)\right) \\ & + v'_\mu\left(F_3(w)-\gamma_5 G_3(w)\right)\Big\} u^B_r(\vec{p}\,)\end{aligned} \tag{23}$$

where $u^{B,B'}$ are dimensionless Dirac spinors, normalized as $\bar{u}u = 1$, and $v_\mu = p_\mu/m_B$ ($v'_\mu = p'_\mu/m_{B'}$) is the four velocity of the initial B (final B') baryon. The form factors are functions of the velocity transfer $w = v\cdot v'$ or equivalently of the four momentum transfer ($q = p - p'$) square $q^2 = m_B^2 + m_{B'}^2 - 2m_B m_{B'} w$. In the decay w ranges from $w = 1$, corresponding to zero recoil of the final baryon, to a maximum value given by $w = w_{\max} = (m_B^2 + m_{B'}^2)/(2m_B m_{B'})$ which depends on the transition.

Neglecting lepton masses, we have for the differential decay rates from transversely (Γ_T) and longitudinally (Γ_L) polarized W's (the total width is $\Gamma = \Gamma_L + \Gamma_T$) [45]

$$
\begin{aligned}
\frac{\mathrm{d}\Gamma_T}{\mathrm{d}w} &= \frac{G_F^2|V_{cb}|^2}{12\pi^3} m_{B'}^3 \sqrt{w^2-1}\, q^2 \Big\{(w-1)|F_1(w)|^2 + (w+1)|G_1(w)|^2\Big\} \\
\frac{\mathrm{d}\Gamma_L}{\mathrm{d}w} &= \frac{G_F^2|V_{cb}|^2}{24\pi^3} m_{B'}^3 \sqrt{w^2-1} \Big\{(w-1)|\mathcal{F}^V(w)|^2 + (w+1)|\mathcal{F}^A(w)|^2\Big\} \\
\mathcal{F}^{V,A}(w) &= \Big[(m_B \pm m_{B'})F_1^{V,A}(w) + (1\pm w)\Big(m_{B'}F_2^{V,A}(w) + m_B F_3^{V,A}(w)\Big)\Big], \\
& \qquad F_j^V \equiv F_j(w),\;\; F_j^A \equiv G_j(w),\;\; j=1,2,3
\end{aligned}
\tag{24}
$$

One can also evaluate the polar angle distribution [45]:

$$
\frac{\mathrm{d}^2\Gamma}{\mathrm{d}w\,\mathrm{d}\cos\theta} = \frac{3}{8}\left(\frac{\mathrm{d}\Gamma_T}{\mathrm{d}w} + 2\frac{\mathrm{d}\Gamma_L}{\mathrm{d}w}\right)\Big\{1 + 2\alpha'\cos\theta + \alpha''\cos^2\theta\Big\} \tag{25}
$$

where θ is the angle between $\vec{k}'$ and $\vec{p}'$ measured in the off–shell W rest frame, and α' and α'' are asymmetry parameters given by

$$
\begin{aligned}
\alpha' &= -\frac{G_F^2|V_{cb}|^2}{6\pi^3} m_{B'}^3 \frac{q^2\,(w^2-1)\,F_1(w)G_1(w)}{\mathrm{d}\Gamma_T/\mathrm{d}w + 2\,\mathrm{d}\Gamma_L/\mathrm{d}w} \\
\alpha'' &= \frac{\mathrm{d}\Gamma_T/\mathrm{d}w - 2\,\mathrm{d}\Gamma_L/\mathrm{d}w}{\mathrm{d}\Gamma_T/\mathrm{d}w + 2\,\mathrm{d}\Gamma_L/\mathrm{d}w}
\end{aligned}
\tag{26}
$$

These asymmetry parameters are functions of the velocity transfer w and on averaging over w the numerators and denominators are integrated separately and thus we have

$$
\langle\alpha'\rangle = -\frac{G_F^2|V_{cb}|^2}{6\pi^3}\frac{m_{B'}^3}{\Gamma_T}\frac{\int_1^{w_{\max}} q^2\,(w^2-1)\,F_1(w)G_1(w)\mathrm{d}w}{1+2R_{L/T}}, \qquad \langle\alpha''\rangle = \frac{1-2R_{L/T}}{1+2R_{L/T}}, \qquad R_{L/T} = \frac{\Gamma_L}{\Gamma_T} \tag{27}
$$

A. Form factors

To obtain the form factors we have to evaluate the matrix elements

$$
\Big\langle B', r'\; \vec{p}'\,\Big|\,\overline{\Psi}^c(0)\gamma_\mu(I-\gamma_5)\Psi^b(0)\Big|\,B, r\;\vec{p}\Big\rangle \tag{28}
$$

which in our model are given by

$$
\sqrt{\frac{E_B(\vec{p})}{m_B}}\sqrt{\frac{E_{B'}(\vec{p}\,')}{m_{B'}}}\;{}_{NR}\Big\langle B', r'\; \vec{p}'\,\Big|\,\overline{\Psi}^c(0)\gamma_\mu(I-\gamma_5)\Psi^b(0)\Big|\,B, r\;\vec{p}\Big\rangle_{NR} \tag{29}
$$

where the suffix "NR" denotes our nonrelativistic states and the factors $\sqrt{E/m}$ take into account the different normalization. We shall work in the initial baryon rest frame so that $\vec{p} = \vec{0}$, $\vec{p}\,' = -\vec{q}$, and take $\vec{q}$ in the positive z direction. Furthermore we shall use the spectator approximation. Having all this in mind we have in momentum space

$$
\begin{aligned}
&\sqrt{\frac{E_{B'}(-\vec{q})}{m_{B'}}}\;{}_{NR}\Big\langle B', r'\; -\vec{q}\,\Big|\,\overline{\Psi}^c(0)\gamma_\mu(I-\gamma_5)\Psi^b(0)\Big|\,B, r\;\vec{0}\Big\rangle_{NR} \\
&\quad = \sqrt{2}\sqrt{\frac{E_{B'}(-\vec{q})}{m_{B'}}}\sum_{s_1}\sum_{s_2}\left(\frac{1}{2}\frac{1}{2}S_h\Big|s_1, s_2-s_1, s_2\right)\left(S_h\frac{1}{2}\frac{1}{2}\Big|s_2, r-s_2, r\right) \\
&\qquad\qquad \times\left(\frac{1}{2}\frac{1}{2}S_h'\Big|r'-r+s_1, s_2-s_1, r'-r+s_2\right)\left(S_h'\frac{1}{2}\frac{1}{2}\Big|r'-r+s_2, r-s_2, r'\right)
\end{aligned}
$$

$$\times \int d^3q_1\, d^3q_2 \left(\Phi^{B'}_{c\,h_2}(\vec{q}_1 - \frac{m_{h_2}+m_q}{\overline{M}\,'}\,\vec{q},\, \vec{q}_2 + \frac{m_{h_2}}{\overline{M}\,'}\,\vec{q})\right)^* \Phi^{B}_{b\,h_2}(\vec{q}_1,\vec{q}_2)$$
$$\times\sqrt{\frac{m_b}{E_b(\vec{q}_1)}}\sqrt{\frac{m_c}{E_c(\vec{q}_1-\vec{q})}}\;\bar{u}^c_{r'-r+s_1}(\vec{q}_1-\vec{q})\gamma_\mu(I-\gamma_5)\,u^b_{s_1}(\vec{q}_1) \tag{30}$$

where $\Phi^B_{b\,h_2}(\vec{q}_1,\vec{q}_2)$ ($\Phi^{B'}_{c\,h_2}(\vec{q}_1,\vec{q}_2)$) is the Fourier transform of the coordinate space wave function $\Psi^B_{b\,h_2}(r_1,r_2,r_{12})$ ($\Psi^{B'}_{c\,h_2}(r_1,r_2,r_{12})$) with $\vec{q}_1$, $\vec{q}_2$ being the conjugate momenta to the space variables $\vec{r}_1$, $\vec{r}_2$. The factor of two comes from the fact that: i) for $bc-$baryon decays, the charm quark resulting from the $b\to c$ transition could be either the particle 1 or the particle 2 in the final cc baryon, while ii) for $bb-$baryon decays, there exist two equal contributions resulting for the decay of each of the two bottom quarks of the initial baryon.

The actual calculation is done in coordinate space where we have

$$\sqrt{\frac{E_{B'}(-\vec{q})}{m_{B'}}}\;{}_{NR}\left\langle B', r' \;\; -\vec{q}\,\middle|\,\overline{\Psi}^c(0)\gamma_\mu(I-\gamma_5)\Psi^b(0)\,\middle|\,B, r\;\vec{0}\right\rangle_{NR}$$
$$= \sqrt{2}\sqrt{\frac{E_{B'}(-\vec{q})}{m_{B'}}}\sum_{s_1}\sum_{s_2}\left(\frac{1}{2}\frac{1}{2}S_h\middle|s_1,s_2-s_1,s_2\right)\left(S_h\frac{1}{2}\frac{1}{2}\middle|s_2,r-s_2,r\right)$$
$$\times\left(\frac{1}{2}\frac{1}{2}S'_h\middle|r'-r+s_1,s_2-s_1,r'-r+s_2\right)\left(S'_h\frac{1}{2}\frac{1}{2}\middle|r'-r+s_2,r-s_2,r'\right)$$
$$\times\int d^3r_1\,d^3r_2\,\Psi^{B'}_{c\,h_2}(r_1,r_2,r_{12})\;e^{i\frac{m_{h_2}}{\overline{M}\,'}\,\vec{q}\cdot\vec{r}_2}\;e^{-i\frac{m_{h_2}+m_q}{\overline{M}\,'}\,\vec{q}\cdot\vec{r}_1}$$
$$\times\sqrt{\frac{m_b}{E_b(\vec{l}\,)}}\sqrt{\frac{m_c}{E_c(\vec{l}-\vec{q})}}\;\bar{u}^c_{r'-r+s_1}(\vec{l}-\vec{q})\gamma_\mu(I-\gamma_5)\,u^b_{s_1}(\vec{l}\,)\;\Psi^B_{b\,h_2}(r_1,r_2,r_{12}) \tag{31}$$

where $\vec{l}=-i\,\vec{\nabla}_1$ represents an internal momentum which is much smaller than the heavy quark masses m_b, m_c. On the other hand $|\vec{q}|$ can be large[9]. Thus, to evaluate the above expression we have made use of an expansion in $\vec{l}$, introduced in Ref. [46], where second order terms in $\vec{l}$ are neglected, while all orders in $|\vec{q}|$ are kept. For instance $E_c(\vec{l}-\vec{q})$ is approximated by $E_c(\vec{l}-\vec{q})\approx E_c(\vec{q})\times(1-\vec{l}\cdot\vec{q}/E^2_c(\vec{q}))$ with $E_c(\vec{q})=\sqrt{m_c^2+\vec{q}^{\,2}}$.

The three vector and three axial form factors can be extracted from the set of equations[10]

$$\begin{aligned}
\left\langle B', 1/2\;\; -\vec{q}\,\middle|\,\overline{\Psi}^c(0)\gamma_1\Psi^b(0)\,\middle|\,B,-1/2\;\vec{0}\right\rangle &= \sqrt{\frac{E_{B'}(-\vec{q})+m_{B'}}{2m_{B'}}}\,\frac{|\vec{q}|}{E_{B'}(-\vec{q})+m_{B'}}\,F_1(|\vec{q}|)\\
\left\langle B', 1/2\;\; -\vec{q}\,\middle|\,\overline{\Psi}^c(0)\gamma_3\Psi^b(0)\,\middle|\,B,1/2\;\vec{0}\right\rangle &= \sqrt{\frac{E_{B'}(-\vec{q})+m_{B'}}{2m_{B'}}}\,|\vec{q}|\left(\frac{F_1(|\vec{q}|)}{E_{B'}(-\vec{q})+m_{B'}}+\frac{F_3(|\vec{q}|)}{m_{B'}}\right)\\
\left\langle B', 1/2\;\; -\vec{q}\,\middle|\,\overline{\Psi}^c(0)\gamma_0\Psi^b(0)\,\middle|\,B,1/2\;\vec{0}\right\rangle &= \sqrt{\frac{E_{B'}(-\vec{q})+m_{B'}}{2m_{B'}}}\left(F_1(|\vec{q}|)+F_2(|\vec{q}|)+\frac{E_{B'}(-\vec{q})}{m_{B'}}\,F_3(|\vec{q}|)\right)
\end{aligned}\tag{32}$$

for the vector form factors and

$$\begin{aligned}
\left\langle B', 1/2\;\; -\vec{q}\,\middle|\,\overline{\Psi}^c(0)\gamma_1\gamma_5\Psi^b(0)\,\middle|\,B,-1/2\;\vec{0}\right\rangle &= \sqrt{\frac{E_{B'}(-\vec{q})+m_{B'}}{2m_{B'}}}\,(-G_1(|\vec{q}|))\\
\left\langle B', 1/2\;\; -\vec{q}\,\middle|\,\overline{\Psi}^c(0)\gamma_3\gamma_5\Psi^b(0)\,\middle|\,B,1/2\;\vec{0}\right\rangle &= \sqrt{\frac{E_{B'}(-\vec{q})+m_{B'}}{2m_{B'}}}\left(-G_1(|\vec{q}|)+\frac{|\vec{q}|^2\,G_3(|\vec{q}|)}{m_{B'}(E_{B'}(-\vec{q})+m_{B'})}\right)
\end{aligned}$$

---

[9] At $q^2=0$ one has $|\vec{q}|=(m_B^2-m_{B'}^2)/2m_B$ which is $\approx m_B/3$ for the transitions under study.

[10] Remember $\vec{q}$ is in the z direction. Notice also that, for $\vec{p}=\vec{0}$, w is just a function of $|\vec{q}|$.

$$\left\langle B', 1/2\ \ -\vec{q}\left|\,\overline{\Psi}^c(0)\gamma_0\gamma_5\Psi^b(0)\right| B, 1/2\ \vec{0}\right\rangle \ = \ \sqrt{\frac{E_{B'}(-\vec{q}\,)+m_{B'}}{2m_{B'}}}\ \frac{|\vec{q}\,|}{E_{B'}(-\vec{q}\,)+m_{B'}}\Bigg(-G_1(|\vec{q}\,|)+G_2(|\vec{q}\,|) + \frac{E_{B'}(-\vec{q}\,)}{m_{B'}}\,G_3(|\vec{q}\,|)\Bigg) \tag{33}$$

for the axial ones. All the left hand side terms can be evaluated using Eq.(31) with the approximation mentioned above.

For each transition there are only two different coordinate space integrals from which all different matrix elements can be evaluated. Those integrals are

$$\begin{aligned}
\mathcal{I}^{B'B}(|\vec{q}\,|) &= \int d^3r_1\, d^3r_2\, e^{i\frac{m_{h_2}}{\overline{M}'}\,\vec{q}\cdot\vec{r}_2}\, e^{-i\frac{m_{h_2}+m_q}{\overline{M}'}\,\vec{q}\cdot\vec{r}_1}\ \left[\Psi^{B'}_{c\,h_2}(r_1,r_2,r_{12})\right]^*\ \Psi^{B}_{b\,h_2}(r_1,r_2,r_{12}) \\
\mathcal{K}^{B'B}(|\vec{q}\,|) &= \frac{1}{|\vec{q}\,|^2}\int d^3r_1\, d^3r_2\, e^{i\frac{m_{h_2}}{\overline{M}'}\,\vec{q}\cdot\vec{r}_2}\, e^{-i\frac{m_{h_2}+m_q}{\overline{M}'}\,\vec{q}\cdot\vec{r}_1}\ \left[\Psi^{B'}_{c\,h_2}(r_1,r_2,r_{12})\right]^*\ \vec{l}\cdot\vec{q}\ \Psi^{B}_{b\,h_2}(r_1,r_2,r_{12})
\end{aligned} \tag{34}$$

In appendix B we relate the form factors to the integrals $\mathcal{I}^{B'B}(|\vec{q}\,|)$ and $\mathcal{K}^{B'B}(|\vec{q}\,|)$ for the different $S_h,\ S'_h$ combinations, while in appendix C we give the actual expressions we use to evaluate those integrals.

1. *Current conservation*

In the limit $m_b = m_c$ and for $B' = B$ (and thus $S_h = S'_h$) vector current conservation provides a relation among the vector F_2 and F_3 form factors, namely

$$F_2(w) = F_3(w) \tag{35}$$

On the other hand, if the two quarks in the current had the same flavour, vector current conservation would fix the normalization of the zeroth component of the vector current matrix element at $w = 1$, since this just counts the number of heavy quarks, two, in that case. For quarks with distinct flavours, but still in the limit $m_b = m_c$, the forward vector matrix element has an extra Clebsch-Gordan factor of $1/\sqrt{2}$, and thus we have

$$F_1(1) + F_2(1) + F_3(1) = \sqrt{2} \tag{36}$$

In this limiting situation the integrals $\mathcal{I}^{BB}(|\vec{q}\,|)$ and $\mathcal{K}^{BB}(|\vec{q}\,|)$ are related by[11]

$$\mathcal{K}^{BB}(|\vec{q}\,|) = \frac{m_{h_2}+m_q}{2\overline{M}}\ \mathcal{I}^{BB}(|\vec{q}\,|) \tag{37}$$

Besides one has that $\mathcal{I}^{BB}(0) = 1$.

Using now the relations in Eq.(B4) in appendix B we see that our model satisfies the constraint in Eq.(36) exactly. On the other hand we have a small violation of vector current conservation due to binding effects. For instance, and again using the relations in Eq.(B4), we obtain for $w = 1$

$$F_2(1) = F_3(1) + \sqrt{2}(1 - \frac{m_B}{\overline{M}}) \tag{38}$$

which shows that current conservation is violated by a term proportional to the binding energy of the baryon divided by the sum of the masses of its constituents. This violation disappears in the infinite heavy quark mass limit. This deficiency is avoided in most calculations by the neglect of binding effects between the heavy diquark and the light quark[12]. Improvements on vector current conservation would require at minimum the introduction of two–body currents [47], going thus beyond the spectator approximation, that we have not considered in this analysis.

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[11] One just has to integrate by parts in the $\mathcal{K}^{BB}(|\vec{q}\,|)$ expression

[12] In Refs. [41, 43] the currents are constructed at the diquark level. At the baryon level, vector current is conserved because light quark-heavy diquark binding effects are neglected. The calculation in Ref. [44] uses the infinite heavy quark mass limit thus canceling binding effects.

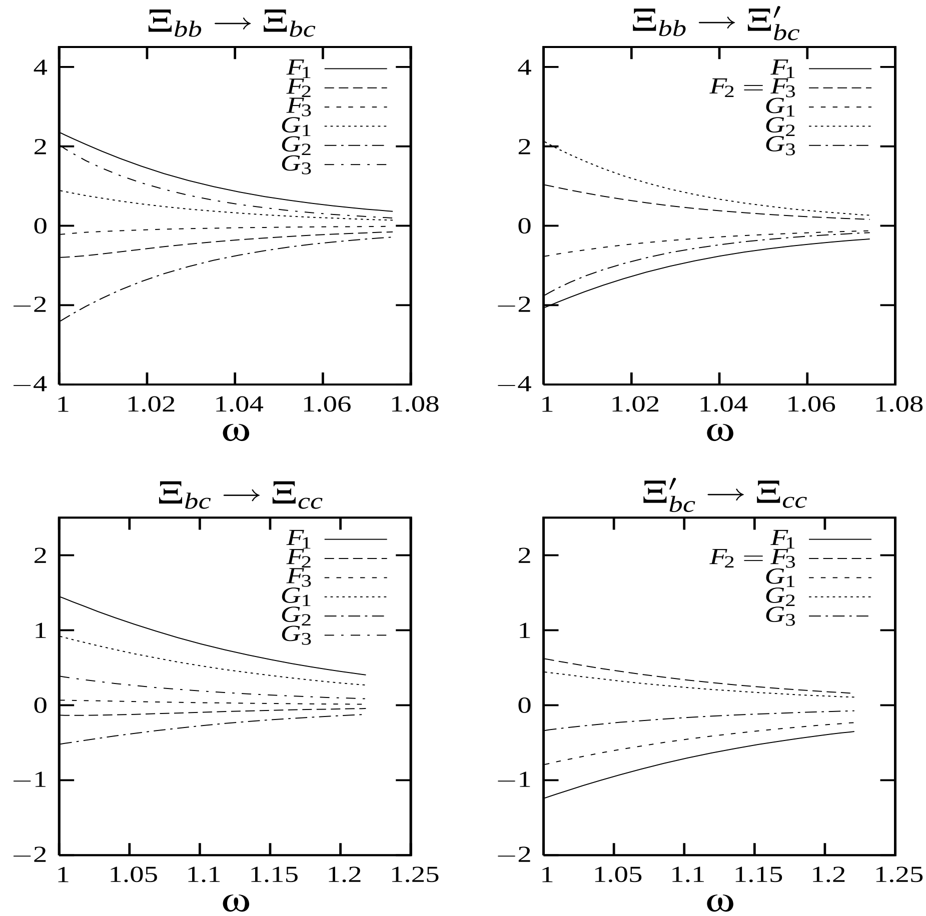

FIG. 5: Vector F_1, F_2, F_3 and axial G_1, G_2, G_3 form factors for doubly $\Xi(J=1/2)$ baryons decays evaluated with the AL1 potential. The results for $\Omega(J=1/2)$ baryons (not shown) are very much the same.

Note also that, for transitions that do not conserve the spin of the heavy quark subsystem S_h (i.e. $\Xi'^{0}_{bc} \to \Xi^{+}_{cc} l \bar{\nu}_l$) we have in the $m_b = m_c$ limit and at zero recoil that

$$F_1(1) + F_2(1) + F_3(1) = 0 \tag{39}$$

due to the orthogonality of the initial and final baryon wave–functions.

B. Results

In Fig. 5 we show the form factors for Ξ decays evaluated with the AL1 potential. Variations when using a different potential are at the level a few per cent at most. The results for doubly heavy Ω decays are almost identical to the corresponding ones for doubly heavy Ξ decays. The fact that we have two heavy quarks and that the light one acts as a spectator makes the results almost independent of the light quark mass.

In Fig. 6 we show now our results for the differential $d\Gamma_T/dw$, $d\Gamma_L/dw$ and $d\Gamma/dw$ decay widths evaluated with the AL1 and BD potentials. The differences between the results obtained with the two inter-quark interactions could reach 30% for some transitions and for some regions of w. As a consequence of the apparent SU(3) symmetry in the form factors we also find that the results for doubly heavy Ξ and Ω decays (not shown) are very close to each other. This apparent SU(3) symmetry goes over to the integrated decay widths and asymmetry parameters.

In Table VIII we give our results for the semileptonic decay width (transverse Γ_T, longitudinal Γ_L and total Γ) for the different processes under study. Our central values have been evaluated using the AL1 potential while the errors show the variations when changing the interaction. The biggest variations appear for the BD potential for which one obtains results which are larger by $7 \sim 12\%$. In Table IX we compare our results to the ones calculated in different models. For that purpose we need a value for $|V_{cb}|$ for which we take $|V_{cb}| = 0.0413$. Our results are in

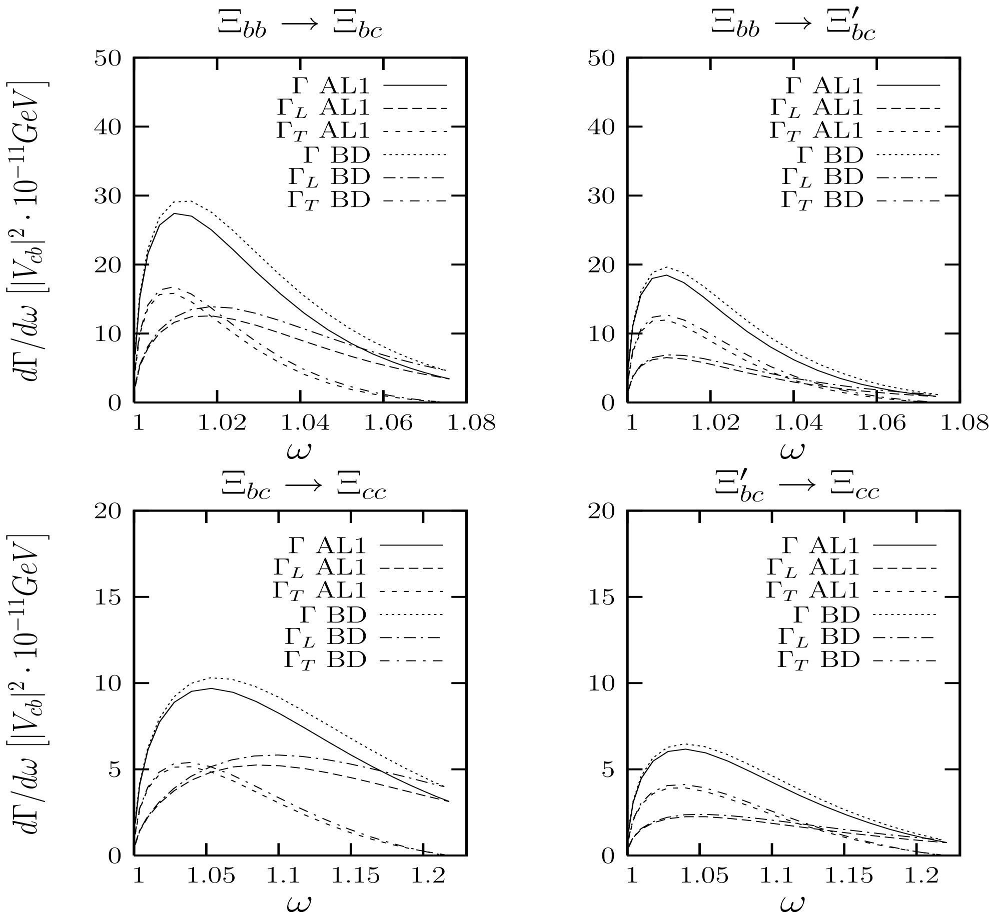

FIG. 6: $d\Gamma/dw$, $d\Gamma_L/dw$ and $d\Gamma_T/dw$ semileptonic decay widths in units of $|V_{cb}|^2 \cdot 10^{-11}$ GeV, for doubly $\Xi(J = 1/2)$ baryons decays. Solid line, long–dashed line and short–dashed line: $d\Gamma/dw$, $d\Gamma_L/dw$ and $d\Gamma_T/dw$ evaluated with the AL1 potential; dotted line, long–dashed dotted line and short–dashed dotted line: $d\Gamma/dw$, $d\Gamma_L/dw$ and $d\Gamma_T/dw$ evaluated with the BD potential. The results for Ω decay (not shown) are very similar.

reasonable agreement with the ones in Ref. [41] where they use a relativistic quark model evaluated in the quark-diquark approximation[13]. The result for $\Gamma(\Xi_{bc} \to \Xi_{cc})$ obtained in Ref. [44] using HQET is a factor of 4 larger than ours[14]. We can not directly compare our result for the latter transition with the one published in Ref. [42], the reason being that in this reference the Ξ_{bc} state is defined differently, with the c and the light quark coupled to total spin 1. Had they defined the state with the bc pair coupled to spin 1 they would had obtained roughly a factor of 4 larger result [50], in reasonable agreement with our calculation or the one by Ebert *et al.*[15]. Finally in Ref. [43], where they use the Bethe–Salpeter equation applied to a quark-diquark system, they obtain much larger results for all transitions.

In Table X we compile our results for the average angular asymmetries α' and α'', as well as the $R_{L/T}$ ratio, introduced in Eq.(27). The central values have been obtained with the AL1 potential. Being all quantities ratios the variation when changing the inter-quark interaction are in most cases small.

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[13] Note we have divided by a factor 2 the results originally published in Ref. [41] as there, and as we initially did, the authors forgot a symmetry factor $1/\sqrt{2}$ for the case of diquarks with two equal quarks [48].

[14] Note we have multiplied by a factor 2 the result originally published in Ref. [44] as the author overlooked a normalization factor $\sqrt{2}$ [49].

[15] Note the actual physical states would be an admixture of what we call $\Xi_{bc}\,\Xi'_{bc}$, and similarly in the Ω case. A measurement of the decay width could give information on the admixtures.

	Γ_T	Γ_L	Γ
$\Xi_{bb} \to \Xi_{bc}\, l\bar\nu_l$	$0.49^{+0.5}_{-0.01}$	$0.64^{+0.10}_{-0.02}$	$1.13^{+0.15}_{-0.03}$
$\Xi_{bc} \to \Xi_{cc}\, l\bar\nu_l$	$0.58^{+0.04}_{-0.01}$	$0.93^{+0.11}_{-0.01}$	$1.51^{+0.15}_{-0.02}$
$\Xi_{bb} \to \Xi'_{bc}\, l\bar\nu_l$	$0.37^{+0.04}_{-0.01}$	$0.26^{+0.04}_{-0.01}$	$0.62^{+0.08}_{-0.02}$
$\Xi'_{bc} \to \Xi_{cc}\, l\bar\nu_l$	$0.45^{+0.03}_{-0.01}$	$0.35^{+0.03}_{-0.01}$	$0.80^{+0.06}_{-0.02}$

	Γ_T	Γ_L	Γ
$\Omega_{bb} \to \Omega_{bc}\, l\bar\nu_l$	$0.53^{+0.04}_{-0.01}$	$0.73^{+0.08}_{-0.01}$	$1.26^{+0.12}_{-0.01}$
$\Omega_{bc} \to \Omega_{cc}\, l\bar\nu_l$	$0.58^{+0.03}$	$0.94^{+0.09}$	$1.52^{+0.12}$
$\Omega_{bb} \to \Omega'_{bc}\, l\bar\nu_l$	$0.40^{+0.04}$	$0.29^{+0.04}$	$0.68^{+0.08}$
$\Omega'_{bc} \to \Omega_{cc}\, l\bar\nu_l$	$0.45^{+0.03}$	$0.35^{+0.03}$	$0.80^{+0.05}$

TABLE VIII: Semileptonic decay widths in units of $|V_{cb}|^2 \cdot 10^{-11}$ GeV. Γ_T and Γ_L stand for the transverse and longitudinal contributions to the width Γ. The central values have been obtained with the AL1 potential. l stands for a light charged lepton, $l = e,\ \mu$

	This work	[41]†	[42]‡	[43]	[44]§
$\Gamma(\Xi_{bb} \to \Xi_{bc}\, l\bar\nu_l)$	$1.92^{+0.25}_{-0.05}$	1.63		28.5	
$\Gamma(\Xi_{bc} \to \Xi_{cc}\, l\bar\nu_l)$	$2.57^{+0.26}_{-0.03}$	2.30	0.79	8.93	8.0
$\Gamma(\Xi_{bb} \to \Xi'_{bc}\, l\bar\nu_l)$	$1.06^{+0.13}_{-0.03}$	0.82		4.28	
$\Gamma(\Xi'_{bc} \to \Xi_{cc}\, l\bar\nu_l)$	$1.36^{+0.10}_{-0.03}$	0.88		7.76	

	This work	[41]†
$\Gamma(\Omega_{bb} \to \Omega_{bc}\, l\bar\nu_l)$	$2.14^{+0.20}_{-0.02}$	1.70
$\Gamma(\Omega_{bc} \to \Omega_{cc}\, l\bar\nu_l)$	$2.59^{+0.20}$	2.48
$\Gamma(\Omega_{bb} \to \Omega'_{bc}\, l\bar\nu_l)$	$1.16^{+0.13}$	0.83
$\Gamma(\Omega'_{bc} \to \Omega_{cc}\, l\bar\nu_l)$	$1.36^{+0.09}$	0.95

TABLE IX: Semileptonic decay widths in units of 10^{-14} GeV. We have used a value $|V_{cb}| = 0.0413$. l stands for a light charged lepton, $l = e,\ \mu$. For results with †, ‡ and § see text for details.

VI. SUMMARY

We have evaluated static properties and semileptonic decays for the ground state of doubly heavy Ξ and Ω baryons. The calculations have been done in the framework of a nonrelativistic quark model with the use of five different inter-quark interactions. The use of different quark-quark potentials allows us to obtain an estimation of the theoretical uncertainties related to the quark-quark interaction. In order to build our wave functions we have made use of the constraints imposed by the infinite heavy quark mass limit. In this limit the spin–spin interactions vanish and the total spin of the two heavy quarks is well defined. With this approximation we have used a simple variational approach, with Jastrow type orbital wave functions, to solve the involved three-body problem.

Among the static properties, our results for the masses are in very good agreement with previous results obtained with the same inter-quark interactions but within a more complicated Faddeev approach [21]. In some cases we even get lower, and thus better, masses[16]. We have calculated mass densities and charge densities (charge form factors) finding that the corresponding mean square radii are again in good agreement with the Faddeev calculation of Ref. [21]. We have also evaluated magnetic moments. Being the total orbital angular momentum of the baryon

	$\langle\alpha'\rangle$	$\langle\alpha''\rangle$	$R_{L/T}$
$\Xi_{bb} \to \Xi_{bc}\, l\bar\nu_l$	$-0.13^{+0.01}$	$-0.45_{-0.02}$	$1.33^{+0.06}_{-0.01}$
$\Xi_{bc} \to \Xi_{cc}\, l\bar\nu_l$	$-0.12^{+0.01}$	$-0.53_{-0.01}$	$1.62^{+0.07}$
$\Xi_{bb} \to \Xi'_{bc}\, l\bar\nu_l$	-0.19	$-0.17_{-0.01}$	$0.71^{+0.01}_{-0.01}$
$\Xi'_{bc} \to \Xi_{cc}\, l\bar\nu_l$	-0.19	$-0.23_{-0.01}$	$0.79^{+0.02}$

	$\langle\alpha'\rangle$	$\langle\alpha''\rangle$	$R_{L/T}$
$\Omega_{bb} \to \Omega_{bc}\, l\bar\nu_l$	$-0.13^{+0.01}$	$-0.47_{-0.01}$	$1.37^{+0.06}$
$\Omega_{bc} \to \Omega_{cc}\, l\bar\nu_l$	$-0.12^{+0.01}$	$-0.53_{-0.01}$	$1.63^{+0.06}$
$\Omega_{bb} \to \Omega'_{bc}\, l\bar\nu_l$	$-0.19_{-0.01}$	$-0.18_{-0.01}$	$0.72^{+0.02}$
$\Omega'_{bc} \to \Omega_{cc}\, l\bar\nu_l$	-0.19	$-0.23_{-0.01}$	$0.79^{+0.02}$

TABLE X: Averaged values of the asymmetry parameters α' and α'' evaluated as indicated in Eq.(27). We also show the ratio $R_{L/T} = \Gamma_L/\Gamma_T$. The central values have been obtained with the AL1 potential. l stands for a light charged lepton, $l = e,\ \mu$

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[16] For a given Hamiltonian a variational mass is an upper bound of the true ground state mass.

$L = 0$, the magnetic moments come from the spin contributions alone. With the exception of Ξ^0_{bc}, Ξ^+_{bc} and Ω^0_{bc} we agree perfectly with the Faddeev calculation in Ref. [21]. For the magnetic moments of Ξ^0_{bc}, Ξ^+_{bc} and Ω^0_{bc} the discrepancies between the two calculation are very large. The origin might be attributed to the presence of a non–negligible $S_h = 0$ component in the wave functions of Ref. [21]. In our case we have $S_h = 1$ which we think is a good approximation based on the infinite heavy quark mass limit. This assertion seems to be corroborated by the results obtained in the relativistic calculation of Ref. [36], at least for the Ξ^0_{bc} and Ω^0_{bc} cases.

We have used our wave functions to study the semileptonic decay of doubly $\Xi(J = 1/2)$ and $\Omega(J = 1/2)$ baryons. We have worked in the spectator approximation with one–body currents alone. In the $m_b = m_c$ case and for $B = B'$ baryons we have checked that our model satisfies baryon number conservation. On the other hand we have a small vector current violation by an amount given by the binding energy over the mass of the baryon, violation that disappears in the infinite heavy quark mass limit. Small vector current violations due to binding effects seems to be present in most calculations to date. With this model we have evaluated form factors, asymmetry parameters, differential decay widths and total decay widths. Our results for the latter are in reasonable agreement with the ones obtained in Ref. [41] using a relativistic quark model in the quark–diquark approximation, while they are much smaller than the ones obtained in Ref. [43] by means of the Bethe–Salpeter equation applied to a quark-diquark system.

For the weak form factors the results exhibit an apparent SU(3) symmetry when going from Ξ to Ω baryons. This is due to the fact that we have two heavy quarks and the light one acts as a spectator in the weak transition. This apparent symmetry appears also in the decay widths and asymmetry parameters. On the other hand SU(3) violating effects are clearly visible in some static quantities like the charge form factors and radii, and the magnetic moments, that depend strongly on the light quark charge and/or mass.

Acknowledgments

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APPENDIX A: INFINITE HEAVY QUARK MASS LIMIT OF OUR VARIATIONAL WAVE FUNCTIONS

In the infinite heavy quark mass limit the wave function of the system should look like the one for a "meson" composed of a light quark and a heavy diquark. The two heavy quarks bind into a $\bar{3}$ color source diquark that to the light degrees of freedom appears to be pointlike [3]. In our model the pointlike nature of the heavy diquark comes about through the one-gluon exchange Coulomb potential which binds the two heavy quarks into a distance[17]

$$r_{h_1h_2} \propto \frac{1}{\mu_{h_1h_2}};\ \mu_{h_1h_2} = \frac{m_{h_1}m_{h_2}}{m_{h_1}+m_{h_2}} \tag{A1}$$

that tends to zero if both quark masses go to infinity.

For our wave functions we can define the probability distribution P_{h1h2} for the two heavy quarks to be found at a distance $r_{h_1h_2}$

$$P_{h1h2}\left(r_{h_1h_2}\right) = \int d^3r_1 \int d^3r_2\ \delta(r_{12} - r_{h_1h_2}) \left|\Psi^B_{h_1h_2}(r_2, r_2, r_{12})\right|^2 \tag{A2}$$

In Fig. 7, we show the P_{h1h2} probability distributions for the Ξ_{cc}, Ξ_{bc}, Ξ_{bb} and Ω_{cc}, Ω_{bc}, Ω_{bb} baryons evaluated for the AL1 potential. We see how the maximum moves, as expected, to lower $r_{h_1h_2}$ values as the quark masses increase.

On the other hand the relative wave function for the light quark with respect to the heavy quark subsystem should tend to the one of a light quark relative to a pointlike diquark. This limit is not evident in the coordinates we work as one has first to separate the diquark internal orbital wave function from the total one. To see that this limiting

[17] The relation can only be approximate due to confinement and the interaction with the light quark.

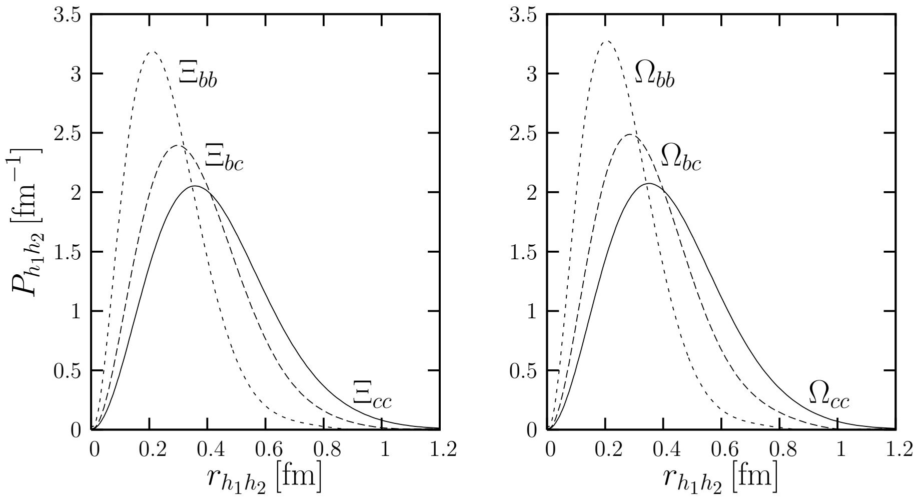

FIG. 7: $P_{h_1h_2}$ probabilities (see Eq.(A2)) for the Ξ_{cc}, Ξ_{bc}, Ξ_{bb} and Ω_{cc}, Ω_{bc}, Ω_{bb} baryons evaluated with the AL1 potential.

situation is in fact reached within our variational ansatz let us do the following: introduce the new set of coordinates

$$\begin{aligned} \vec{R} &= \frac{m_{h_1}\vec{x}_{h_1}+m_{h_2}\vec{x}_{h_2}+m_q\vec{x}_q}{m_{h_1}+m_{h_2}+m_q} \\ \vec{r}_{12} &= \vec{x}_{h_1}-\vec{x}_{h_2} \\ \vec{r}_q &= \frac{m_{h_1}\vec{x}_{h_1}+m_{h_2}\vec{x}_{h_2}}{m_{h_1}+m_{h_2}}-\vec{x}_q = \frac{m_{h_1}\vec{r}_1+m_{h_2}\vec{r}_2}{m_{h_1}+m_{h_2}} \end{aligned} \tag{A3}$$

in terms of which the Hamiltonian now reads

$$\begin{aligned} H &= -\frac{\vec{\nabla}^2_{\vec{R}}}{2\overline{M}}+H^{\rm int} \\ H^{\rm int} &= \overline{M}+H_{h1h_2}+H_{qh_1h_2} \end{aligned} \tag{A4}$$

where

$$\begin{aligned} H_{h1h_2} &= -\frac{\vec{\nabla}^2_{12}}{2\mu_{h_1h_2}}+V_{h_1h_2}(\vec{r}_{12},\,spin) \\ H_{qh_1h_2} &= -\frac{1}{2}\left(\frac{1}{m_{h_1}+m_{h_2}}+\frac{1}{m_q}\right)\vec{\nabla}^2_q+V_{h_1q}(\vec{r}_q+\frac{m_{h_2}}{m_{h_1}+m_{h_2}}\vec{r}_{12},\,spin) \\ &\qquad +V_{h_2q}(\vec{r}_q-\frac{m_{h_1}}{m_{h_1}+m_{h_2}}\vec{r}_{12},\,spin) \end{aligned} \tag{A5}$$

with $\vec{\nabla}_{12}=\partial/\partial_{\vec{r}_{12}}$ and $\vec{\nabla}_q=\partial/\partial_{\vec{r}_q}$. Defining now

$$H^0_{qh_1h_2} = -\frac{1}{2}\left(\frac{1}{m_{h_1}+m_{h_2}}+\frac{1}{m_q}\right)\vec{\nabla}^2_q+V_{h_1q}(\vec{r}_q,\,spin)+V_{h_2q}(\vec{r}_q,\,spin) \tag{A6}$$

one would have

$$H^{\rm int}=H_{h_1h_2}+H^0_{qh_1h_2}+(H_{qh_1h_2}-H^0_{qh_1h_2}) \tag{A7}$$

$H_{h_1h_2}$ is the Hamiltonian for the relative movement of the two heavy quarks while $H^0_{qh_1h_2}$ is the Hamiltonian for the relative movement of the light quark with respect to a pointlike heavy diquark where the two heavy quarks are located in their center of mass. Both movements are coupled through the term $(H_{qh_1h_2}-H^0_{qh_1h_2})$. If the heavy quark masses get arbitrarily large the average r_{12} value tends to zero and thus one can neglect the effect of $H_{qh_1h_2}-H^0_{qh_1h_2}$. In that limiting situation the light and heavy quark degrees of freedom decouple completely and the internal Hamiltonian reduces, as it should, to the sum of the Hamiltonians $H_{h_1h_2}$, corresponding to the relative movement of the two heavy quarks, and $H^0_{qh_1h_2}$, corresponding to the relative movement of the light quark with respect to the pointlike heavy diquark subsystem.

As to the wave function it should reduce in that limit to the product $\Phi_{h_1h_2}(r_{12})\cdot\Phi^0_{qh_1h_2}(r_q)$, being $\Phi_{h_1h_2}(r_{12})$, $\Phi^0_{qh_1h_2}(r_q)$ the ground state wave functions for $H_{h_1h_2}$, $H^0_{qh_1h_2}$ respectively. To check that we reach that limiting situation we have evaluated the projection $\mathcal{P}$ of our variational wave functions onto $\Phi_{h_1h_2}(r_{12})\cdot\Phi^0_{qh_1h_2}(r_q)$ evaluated with the actual heavy quark masses. This projection is given by[18]

$$\mathcal{P}=\int d^3r_1\int d^3r_2\left(\Psi^B_{h_1h_2}(r_1,r_2,r_{12})\right)^*\Phi_{h_1h_2}(r_{12})\;\Phi^0_{qh_1h_2}(r_q) \tag{A8}$$

and the values for $|\mathcal{P}|^2$ that we obtain for the Ξ_{cc}, Ξ_{bc}, Ξ_{bb} and Ω_{cc}, Ω_{bc}, Ω_{bb} baryons using the AL1 potential are given in Table XI. We see how $|\mathcal{P}|^2$ increases with increasing heavy quark masses indicating that for very high heavy quark masses the total wave function tends to the one of a light quark relative to a pointlike diquark times the diquark internal wave function. In summary, our variational wave functions respect the infinite heavy quark mass limit.

	Ξ_{cc}	Ξ_{bc}	Ξ_{bb}	Ω_{cc}	Ω_{bc}	Ω_{bb}
$\vert\mathcal{P}\vert^2$	0.974	0.975	0.991	0.959	0.966	0.984

TABLE XI: Absolute value square of the $\mathcal{P}$ projection coefficient defined in Eq. (A8)

We note by passing that the product $\Phi_{h_1h_2}(r_{12})\cdot\Phi^0_{qh_1h_2}(r_q)$ corrected by a correlation function in the variable $\vec r_{12}\cdot\vec r_q$ would have been a good variational orbital wave function. For instance, a calculation using the AL1 potential and an orbital wave function given just by that product $\Phi_{h_1h_2}(r_{12})\cdot\Phi^0_{qh_1h_2}(r_q)$ gives for the expectation value of $H^{\rm int}$ for the Ξ_{cc}, Ξ_{bc}, Ξ_{bb} baryons

$$\langle H^{\rm int}\rangle\big|^{AL1}_{\Xi_{cc}}=3640\,{\rm MeV};\;\;\langle H^{\rm int}\rangle\big|^{AL1}_{\Xi_{bc}}=6943\,{\rm MeV};\;\;\langle H^{\rm int}\rangle\big|^{AL1}_{\Xi_{bb}}=10198\,{\rm MeV} \tag{A9}$$

which are respectively 28 MeV, 24 MeV, and 1 MeV larger, and therefore worse (see footnote 13), than our best values in Table II. The correlation function would clearly be needed, being its role more important for the "*cc*" and "*bc*" systems. The improvement in going from a "*cc*" to a "*bc* system" is not as good as one would naively expect, the reason being that, considering $\vec r_{12}$ to be a small quantity, $H_{qh_1h_2}-H^0_{qh_1h_2}$ is first order in $\vec r_{12}$ for the "*bc*" system while, for symmetry reasons, it is necessarily of second order for the "*cc*" and "*bb*" ones. In any case one sees how the expectation values obtained with this simple ansatz get closer to our best values in Table II as the heavy quark masses increase.

APPENDIX B: FORM FACTORS IN TERMS OF THE $\mathcal{I}^{B'B}(|\vec q|)$ AND $\mathcal{K}^{B'B}(|\vec q|)$ INTEGRALS

In this appendix we relate the vector F_1, F_2, F_3 and axial G_1, G_2, G_3 form factors, that we evaluate in the center of mass of the decaying baryon, to the integrals $\mathcal{I}^{B'B}(|\vec q|)$, $\mathcal{K}^{B'B}(|\vec q|)$ defined in Eq.(34). To simplify the expressions it is convenient to introduce

$$\begin{aligned}\widehat F_j(|\vec q|)&=\sqrt{\frac{E_{B'}(-\vec q)+m_{B'}}{2E_{B'}(-\vec q)}}\sqrt{\frac{2E_c(\vec q)}{E_c(\vec q)+m_c}}\;F_j(|\vec q|)\quad,\;j=1,2,3\\ \widehat G_j(|\vec q|)&=\sqrt{\frac{E_{B'}(-\vec q)+m_{B'}}{2E_{B'}(-\vec q)}}\sqrt{\frac{2E_c(\vec q)}{E_c(\vec q)+m_c}}\;G_j(|\vec q|)\quad,\;j=1,2,3\end{aligned} \tag{B1}$$

- Cases $S_h=1$, $S'_h=0$ or $S_h=0$, $S'_h=1$:

$$\frac{\widehat F_1(|\vec q|)}{E_{B'}(-\vec q)+m_{B'}}=-\sqrt{\frac{2}{3}}\left(\frac{\mathcal{I}^{B'B}(|\vec q|)}{E_c(\vec q)+m_c}-\frac{\mathcal{K}^{B'B}(|\vec q|)}{2}\left(\frac{m_c}{E^2_c(\vec q)}-\frac{1}{m_b}\right)\right)$$

[18] Note r_q can be expressed in terms of r_1,r_2,r_{12} and that $d^3r_1d^3r_2=d^3r_{12}d^3r_q$

$$
\begin{aligned}
\frac{\widehat{F}_1(|\vec{q}\,|)}{E_{B'}(-\vec{q}\,)+m_{B'}}+\frac{\widehat{F}_3(|\vec{q}\,|)}{m_{B'}} &= 0\\
\widehat{F}_1(|\vec{q}\,|)+\widehat{F}_2(|\vec{q}\,|)+\frac{E_{B'}(-\vec{q}\,)}{m_{B'}}\,\widehat{F}_3(|\vec{q}\,|) &= 0
\end{aligned}
\tag{B2}
$$

where $E_c(\vec{q}\,)=\sqrt{m_c^2+\vec{q}^{\,2}}$. From the above expressions we have that $F_2=F_3$.

$$
\begin{aligned}
-\,\widehat{G}_1(|\vec{q}\,|) &= \sqrt{\frac{2}{3}}\left(\mathcal{I}^{B'B}(|\vec{q}\,|)+\frac{|\vec{q}\,|^2\,\mathcal{K}^{B'B}(|\vec{q}\,|)}{2\,(E_c(\vec{q}\,)+m_c)}\left(\frac{m_c}{E_c^2(\vec{q}\,)}+\frac{1}{m_b}\right)\right)\\
\left(-\widehat{G}_1(|\vec{q}\,|)+\frac{|\vec{q}\,|^2\,\widehat{G}_3(|\vec{q}\,|)}{m_{B'}(E_{B'}(-\vec{q}\,)+m_{B'})}\right) &= \sqrt{\frac{2}{3}}\left(\mathcal{I}^{B'B}(|\vec{q}\,|)+\frac{|\vec{q}\,|^2\,\mathcal{K}^{B'B}(|\vec{q}\,|)}{2\,(E_c(\vec{q}\,)+m_c)}\left(\frac{m_c}{E_c^2(\vec{q}\,)}-\frac{1}{m_b}\right)\right)\\
\frac{-\widehat{G}_1(|\vec{q}\,|)+\widehat{G}_2(|\vec{q}\,|)+\frac{E_{B'}(-\vec{q}\,)}{m_{B'}}\,\widehat{G}_3(|\vec{q}\,|)}{E_{B'}(-\vec{q}\,)+m_{B'}} &= \sqrt{\frac{2}{3}}\left(\frac{\mathcal{I}^{B'B}(|\vec{q}\,|)}{E_c(\vec{q}\,)+m_c}-\frac{\mathcal{K}^{B'B}(|\vec{q}\,|)}{2}\left(\frac{m_c}{E_c^2(\vec{q}\,)}+\frac{1}{m_b}\right)\right)
\end{aligned}
\tag{B3}
$$

- Case $S_h=1$, $S_h'=1$

$$
\begin{aligned}
\frac{\widehat{F}_1(|\vec{q}\,|)}{E_{B'}(-\vec{q}\,)+m_{B'}} &= \frac{4}{3\sqrt{2}}\left(\frac{\mathcal{I}^{B'B}(|\vec{q}\,|)}{E_c(\vec{q}\,)+m_c}-\frac{\mathcal{K}^{B'B}(|\vec{q}\,|)}{2}\left(\frac{m_c}{E_c^2(\vec{q}\,)}-\frac{1}{m_b}\right)\right)\\
\frac{\widehat{F}_1(|\vec{q}\,|)}{E_{B'}(-\vec{q}\,)+m_{B'}}+\frac{\widehat{F}_3(|\vec{q}\,|)}{m_{B'}} &= \sqrt{2}\left(\frac{\mathcal{I}^{B'B}(|\vec{q}\,|)}{E_c(\vec{q}\,)+m_c}-\frac{\mathcal{K}^{B'B}(|\vec{q}\,|)}{2}\left(\frac{m_c}{E_c^2(\vec{q}\,)}+\frac{1}{m_b}\right)\right)\\
\widehat{F}_1(|\vec{q}\,|)+\widehat{F}_2(|\vec{q}\,|)+\frac{E_{B'}(-\vec{q}\,)}{m_{B'}}\,\widehat{F}_3(|\vec{q}\,|) &= \sqrt{2}\left(\mathcal{I}^{B'B}(|\vec{q}\,|)+\frac{|\vec{q}\,|^2\,\mathcal{K}^{B'B}(|\vec{q}\,|)}{2\,(E_c(\vec{q}\,)+m_c)}\left(\frac{m_c}{E_c^2(\vec{q}\,)}-\frac{1}{m_b}\right)\right)
\end{aligned}
\tag{B4}
$$

$$
\begin{aligned}
\widehat{G}_1(|\vec{q}\,|) &= \frac{4}{3\sqrt{2}}\left(\mathcal{I}^{B'B}(|\vec{q}\,|)+\frac{|\vec{q}\,|^2\,\mathcal{K}^{B'B}(|\vec{q}\,|)}{2\,(E_c(\vec{q}\,)+m_c)}\left(\frac{m_c}{E_c^2(\vec{q}\,)}+\frac{1}{m_b}\right)\right)\\
\left(-\widehat{G}_1(|\vec{q}\,|)+\frac{|\vec{q}\,|^2\,\widehat{G}_3(|\vec{q}\,|)}{m_{B'}(E_{B'}(-\vec{q}\,)+m_{B'})}\right) &= -\frac{4}{3\sqrt{2}}\left(\mathcal{I}^{B'B}(|\vec{q}\,|)+\frac{|\vec{q}\,|^2\,\mathcal{K}^{B'B}(|\vec{q}\,|)}{2\,(E_c(\vec{q}\,)+m_c)}\left(\frac{m_c}{E_c^2(\vec{q}\,)}-\frac{1}{m_b}\right)\right)\\
\frac{-\widehat{G}_1(|\vec{q}\,|)+\widehat{G}_2(|\vec{q}\,|)+\frac{E_{B'}(-\vec{q}\,)}{m_{B'}}\,\widehat{G}_3(|\vec{q}\,|)}{E_{B'}(-\vec{q}\,)+m_{B'}} &= -\frac{4}{3\sqrt{2}}\left(\frac{\mathcal{I}^{B'B}(|\vec{q}\,|)}{E_c(\vec{q}\,)+m_c}-\frac{\mathcal{K}^{B'B}(|\vec{q}\,|)}{2}\left(\frac{m_c}{E_c^2(\vec{q}\,)}+\frac{1}{m_b}\right)\right)
\end{aligned}
\tag{B5}
$$

APPENDIX C: $\mathcal{I}^{B'B}(|\vec{q}\,|)$ AND $\mathcal{K}^{B'B}(|\vec{q}\,|)$ INTEGRALS

To evaluate $\mathcal{I}^{B'B}(|\vec{q}\,|)$ and $\mathcal{K}^{B'B}(|\vec{q}\,|)$ we use a partial wave expansion of the orbital wave functions:

$$
\begin{aligned}
\Psi^{B}_{b\,h_2}(r_1,r_2,r_{12}) &= \sum_{l=0}^{\infty} f_l^{B}(r_1,r_2)P_l(\mu)\\
\Psi^{B'}_{c\,h_2}(r_1,r_2,r_{12}) &= \sum_{l=0}^{\infty} f_l^{B'}(r_1,r_2)P_l(\mu)
\end{aligned}
\tag{C1}
$$

where μ is the cosine of the angle made by $\vec{r}_1$ and $\vec{r}_2$ and $P_l(\mu)$ is a Legendre polynomial of rank l. The radial functions $f_l^B(r_1, r_2)$, $f_l^{B'}(r_1, r_2)$, are evaluated as

$$\begin{aligned} f_l^B(r_1,r_2) &= \frac{2l+1}{2}\int_{-1}^{+1} d\mu\; P_l(\mu)\,\Psi^B_{b\,h_2}(r_1,r_2,r_{12}) \\ f_l^{B'}(r_1,r_2) &= \frac{2l+1}{2}\int_{-1}^{+1} d\mu\; P_l(\mu)\,\Psi^{B'}_{c\,h_2}(r_1,r_2,r_{12}) \end{aligned} \tag{C2}$$

In terms of those we have

- $\mathcal{I}^{B'B}(|\vec{q}\,|)$

$$\begin{aligned} \mathcal{I}^{B'B}(|\vec{q}\,|) &= 16\pi^2 \sum_l \sum_{l'} \sum_{l''} (ll'l''\,|\,000)^2 \int_0^\infty d\,r_1\, r_1^2\; j_{l''}(\frac{m_{h_2}+m_q}{\overline{M}'}|\vec{q}\,|r_1) \int_0^\infty d\,r_2\, r_2^2\; j_{l''}(\frac{m_{h_2}}{\overline{M}'}|\vec{q}\,|r_2) \\ &\quad \times f_{l'}^{B'}(r_1,r_2)\, f_l^B(r_1,r_2) \end{aligned} \tag{C3}$$

  with $j's$ being spherical Bessel functions.

- $\mathcal{K}^{B'B}(|\vec{q}\,|)$

$$\begin{aligned} \mathcal{K}^{B'B}(|\vec{q}\,|) &= -\frac{16\pi^2}{\sqrt{3}\,|\vec{q}\,|} \sum_l \sum_{l'} \sum_{l''} \sum_{l'''} \sum_L (-1)^{(l''+l'''+1)/2} \sqrt{(2L+1)(2l''+1)(2l'''+1)} \\ &\quad \times (ll'l''\,|\,000)\; (l''l'''1\,|\,000)\; (l'Ll'''\,|\,000)\; W(l''l'1L : ll''') \\ &\quad \times \int_0^\infty d\,r_1\, r_1^2\; j_{l'''}(\frac{m_{h_2}+m_q}{\overline{M}'}|\vec{q}\,|r_1) \int_0^\infty d\,r_2\, r_2^2\; j_{l''}(\frac{m_{h_2}}{\overline{M}'}|\vec{q}\,|r_2)\; f_{l'}^{B'}(r_1,r_2)\,\Omega_L\, f_l^B(r_1,r_2) \end{aligned} \tag{C4}$$

  with $W(l''l'1L; l, l''')$ being a Racah coefficient and Ω_L the differential operator[19]

$$\begin{aligned} \Omega_{L=l+1} &= -\sqrt{\frac{l+1}{2l+1}}\left(\frac{\partial}{\partial r_1} - \frac{l}{r_1}\right) \\ \Omega_{L=l-1} &= \sqrt{\frac{l}{2l+1}}\left(\frac{\partial}{\partial r_1} + \frac{l+1}{r_1}\right) \end{aligned} \tag{C5}$$

For the actual evaluation we restrict the l, l' values to l, $l' = 0, \cdots, 6$

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