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Doubly heavy quark baryon spectroscopy and semileptonic decay

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Abstract. Working in the framework of a nonrelativistic quark model we evaluate the spectra and semileptonic decay widths for the ground state of doubly heavy Ξ and Ω baryons. We solve the three-body problem using a variational ansatz made possible by the constraints imposed by heavy quark spin symmetry. In order to check the dependence of our results on the inter-quark interaction we have used five different quark quark potentials which include Coulomb and hyperfine terms coming from one-gluon exchange, plus a confining term. Our results for the spectra are in good agreement with a previous calculation done using a Faddeev approach. For the semileptonic decay our results for the total decay widths are in a good agreement with the ones obtained within a relativistic quark model in the quark-diquark approximation.

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1 Introduction

Even though only recently the mass of a baryon with two heavy quarks has been measured experimentally[1], these systems have been being studied for more than a decade. Working with a system with two heavy quarks one can take advantage of the constraints imposed by heavy quark spin symmetry (HQSS). This symmetry amounts to the decoupling of the heavy quark spins in the infinity heavy quark mass limit. In that limit one can consider the total spin of the two heavy quarks subsystem to be well defined. This result, that we shall assume to be valid for the actual heavy quark masses, will simplify the solution of the three-body problem.

In this contribution we shall present results for masses and total semileptonic decay widths. We have also analyzed other static observables as well as form factors, differential decay widths and angular asymmetries of the weak decays. For a detailed account of the full calculation see Ref. [2]

In Table 1 we summarize the quantum numbers of the baryons considered in this study.

2 The model

Once the centre of mass (CM) motion has been removed, the intrinsic Hamiltonian that describes the inner dynamics of the baryon is given by

$$H^{\rm int} = \sum_{j=1,2} H_j^{sp} + V_{h_1h_2}(\boldsymbol{r}_1 - \boldsymbol{r}_2,\, spin) - \frac{\boldsymbol{\nabla}_1 \cdot \boldsymbol{\nabla}_2}{m_q} + \overline{M}$$

Table 1. Quantum numbers of doubly heavy baryons analyzed in this study. S, J^P are strangeness and the spin parity of the baryon, I is the isospin, and S_h^π is the spin parity of the heavy degrees of freedom. l denotes a light u or d quark .

Baryon	S	J^P	I	S_h^π	Quark content
Ξ_{cc}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	ccl
Ξ_{cc}^*	0	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	ccl
Ω_{cc}	-1	$\frac{1}{2}^+$	0	1^+	ccs
Ω_{cc}^*	-1	$\frac{3}{2}^+$	0	1^+	ccs
Ξ_{bb}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	bbl
Ξ_{bb}^*	0	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	bbl
Ω_{bb}	-1	$\frac{1}{2}^+$	0	1^+	bbs
Ω_{bb}^*	-1	$\frac{3}{2}^+$	0	1^+	bbs
Ξ'_{bc}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	0^+	bcl
Ξ_{bc}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	bcl
Ξ_{bc}^*	0	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	bcl
Ω'_{bc}	-1	$\frac{1}{2}^+$	0	0^+	bcs
Ω_{bc}	-1	$\frac{1}{2}^+$	0	1^+	bcs
Ω_{bc}^*	-1	$\frac{3}{2}^+$	0	1^+	bcs

$$H_j^{sp} = -\frac{\boldsymbol{\nabla}_j^{\,2}}{2\mu_j} + V_{h_jq}(\boldsymbol{r}_j,\, spin),\ j = 1, 2 \qquad (1)$$

where $\boldsymbol{r}_1$, $\boldsymbol{r}_2$ are the relative positions of the h_1, h_2 heavy quarks with respect to the light quark q, $\overline{M} = m_{h_1} + m_{h_2} + m_q$, $\mu_j = \left(1/m_{h_j} + 1/m_q\right)^{-1}$ and $\boldsymbol{\nabla}_j = \partial/\partial \boldsymbol{r}_j$, $j = 1, 2$. V_{h_jq} and $V_{h_1h_2}$ are the heavy-light and heavy-heavy interaction potentials. Note the presence of the Hughes-

Eckart term that results from the separation of the CM motion.

For the quark quark interaction we have considered five different phenomenological potentials, one suggested by Bhaduri and collaborators [3] (BD) and four suggested by B. Silvestre-Brac and C. Semay [4,5] (AL1, AL2, AP1 y AP2). All of them include Coulomb and hyperfine terms coming from one-gluon exchange and a confining term, and differ in the form factor used for the hyperfine term, the use of a form factor in the one gluon exchange Coulomb term or in the power of the confinement term. All free parameters had been adjusted to reproduce the light and heavy-light meson spectra. Details on the potentials can be found in Refs. [3,4,5].

For the interactions considered, the total spin and internal orbital angular momentum commute with the intrinsic Hamiltonian, and thus are well defined. In this work we will study the ground state of baryons with total angular momentum $J = 1/2,\ 3/2$ so we can assume the orbital angular momentum to be 0. This implies that the spatial wave function only depend on r_1, r_2 and $r_{12} = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$. We will also assume that taking the total spin of the heavy degrees of freedom to be well defined, as obtained in the infinite heavy quark mass limit, is a good approximation. That will allow us to write the wave function in a simple way (see Ref. [2] for details).

The spatial part of wave function will be determined using a variational method in which we will assume the following functional form:

$$\Psi^B_{h_1h_2}(r_1, r_2, r_{12}) = N\, F^B(r_{12})\, \phi_{h_1q}(r_1)\, \phi_{h_2q}(r_2) \tag{2}$$

where N is a normalization constant, ϕ_{h_jq} is the S-Wave ground state wave function $\varphi_j(r_j)$ of the single particle Hamiltonian H^{sp}_j corrected at large distances:

$$\phi_{h_jq}(r_j) = (1 + \alpha_j\, r_j)\, \varphi_j(r_j), \quad j = 1, 2 \tag{3}$$

The heavy-heavy Jastrow correlation function F^B will be given as a linear combination of gaussians:

$$F^B(r_{12}) = \sum_{j=1}^{4} a_j e^{-b_j^2 (r_{12}+d_j)^2}, \quad a_1 = 1 \tag{4}$$

where α_i, a_i $i \neq 1$, b_i and d_i are free variational parameters. The values that we get for the variational parameters are compiled in Ref. [2].

Table 2. Doubly heavy Ξ masses in MeV.

	This work	[4]	Exp. [1]	Lattice [7]
Ξ_{cc}	3612^{+17}	3609^{+22}	3519 ± 1	3549 ± 95
Ξ^*_{cc}	3706^{+23}			3641 ± 97
Ξ_{bb}	10197^{+10}_{-17}	10194^{+10}_{-19}		
Ξ^*_{bb}	10236^{+9}_{-17}			
Ξ_{bc}	6919^{+17}_{-7}	6916^{+18}_{-9}		
Ξ'_{bc}	6948^{+17}_{-6}			
Ξ^*_{bc}	6986^{+14}_{-5}			

	This work	[8]	[9]	[10]	[11]	[12]	[13]
Ξ_{cc}	3612^{+17}	3620	3480	3740	3478	3660	3524
Ξ^*_{cc}	3706^{+23}	3727	3610	3860	3610	3740	3548
Ξ_{bb}	10197^{+10}_{-17}	10202	10090	10300	10093	10340	
Ξ^*_{bb}	10236^{+9}_{-17}	10237	10130	10340	10133	10370	
Ξ_{bc}	6919^{+17}_{-7}	6933	6820	7010	6820	7040	
Ξ'_{bc}	6948^{+17}_{-6}	6963	6850	7070	6850	6990	
Ξ^*_{bc}	6986^{+14}_{-5}	6980	6900	7100	6900	7060	

We have also used the wave function obtained in this model to study different doubly $B(1/2^+) \rightarrow B'(1/2^+)$ baryon semileptonic decays involving a $b \rightarrow c$ transition at the quark level. We have worked in the spectator approximation with only one-body currents.
The differential decay width reads

$$\mathrm{d}\Gamma = 8|V_{cb}|^2 m_{B'} G_F^2 \frac{d^3p'}{(2\pi)^3 2E'_{B'}} \frac{d^3k}{(2\pi)^3 2E_{\bar\nu_l}} \frac{d^3k'}{(2\pi)^3 2E'_l} (2\pi)^4 \times \delta^4(p - p' - k - k')\ \mathcal{L}^{\alpha\beta}(k,k') \mathcal{H}_{\alpha\beta}(p,p') \tag{5}$$

where $|V_{cb}|$ is the modulus of the corresponding Cabibbo–Kobayashi–Maskawa matrix element, $m_{B'}$ is the mass of the final baryon, G_F is the Fermi decay constant, p, p', k and k' are the four-momenta of the initial baryon, final baryon, final anti-neutrino and final lepton respectively, and $\mathcal{L}$ and $\mathcal{H}$ are the lepton and hadron tensors.
The lepton tensor is given as

$$\mathcal{L}^{\mu\sigma}(k,k') = k'^{\mu} k^{\sigma} + k'^{\sigma} k^{\mu} - g^{\mu\sigma} k \cdot k' + \mathrm{i}\epsilon^{\mu\sigma\alpha\beta} k'_{\alpha} k_{\beta} \tag{6}$$

where we use the convention $\epsilon^{0123} = -1$, $g^{\mu\mu} = (+,-,-,-)$.
The hadron tensor is given as

$$\mathcal{H}_{\mu\sigma}(p,p') = \frac{1}{2} \sum_{r,r'} \left\langle B', r'\ \boldsymbol{p}' \left| \overline{\Psi}^c(0) \gamma_\mu (I - \gamma_5) \Psi^b(0) \right| B, r\ \boldsymbol{p} \right\rangle \times \left\langle B', r'\ \boldsymbol{p}' \left| \overline{\Psi}^c(0) \gamma_\sigma (I - \gamma_5) \Psi^b(0) \right| B, r\ \boldsymbol{p} \right\rangle^* \tag{7}$$

with $|B, r\ \boldsymbol{p}\rangle$ ($|B', r'\ \boldsymbol{p}'\rangle$) representing the initial (final) baryon with three–momentum $\boldsymbol{p}$ ($\boldsymbol{p}'$) and spin index r (r'). The baryon states are normalized such that

$$\langle r\ \boldsymbol{p} \,|\, r'\ \boldsymbol{p}' \rangle = (2\pi)^3 (E(\boldsymbol{p})/m)\, \delta_{rr'}\, \delta^3(\boldsymbol{p} - \boldsymbol{p}') \tag{8}$$

We compute the widths similarly as we did in Ref. [6] for baryons with a heavy quark.

3 Results and discussion

The mass of the baryon is simply given by the expectation value of the intrinsic Hamiltonian. In table 2 we give our results for doubly heavy Ξ baryons, while in table 3 are the results for the doubly heavy Ω ones. Our central values correspond to the results obtained using the AL1 potential, while the errors quoted take into account the variations found when using the other potentials. That also applies to the quoted results for Ref. [4], obtained with the same interaction potentials but within a Faddeev

Table 3. Doubly heavy Ω masses in MeV.

	This work	[4]	Lattice [7]
Ω_{cc}	3702^{+41}	3711^{+30}_{-2}	3663 ± 97
Ω^*_{cc}	3783^{+22}		3734 ± 98
Ω_{bb}	10260^{+14}_{-34}		
Ω^*_{bb}	10297^{+5}_{-28}		
Ω_{bc}	6986^{+27}_{-17}	7003^{+20}_{-32}	
Ω'_{bc}	7009^{+24}_{-15}		
Ω^*_{bc}	7046^{+11}_{-9}		

	This work	[8]	[9]	[10]	[11]	[12]
Ω_{cc}	3702^{+41}	3778	3590	3760	3590	3740
Ω^*_{cc}	3783^{+22}	3872	3690	3900	3690	3820
Ω_{bb}	10260^{+14}_{-34}	10359	10180	10340	10180	10370
Ω^*_{bb}	10297^{+5}_{-28}	10389	10200	10380	10200	10400
Ω_{bc}	6986^{+27}_{-17}	7088	6910	7050	6910	7090
Ω'_{bc}	7009^{+24}_{-15}	7116	6930	7110	6930	7060
Ω^*_{bc}	7046^{+11}_{-9}	7130	6990	7130	6990	7120

Table 4. Semileptonic decay widths in units of 10^{-14} GeV. We have used $|V_{cb}| = 0.0413$. l stands for $l = e, \mu$

	This work	[14]	[15]	[16]	[17]
$\Gamma(\Xi_{bb} \to \Xi_{bc}\, l\bar{\nu}_l)$	$3.84^{+0.49}_{-0.10}$	3.26		28.5	
$\Gamma(\Xi_{bc} \to \Xi_{cc}\, l\bar{\nu}_l)$	$5.13^{+0.51}_{-0.05}$	4.59	0.79	8.93	4.0
$\Gamma(\Xi_{bb} \to \Xi'_{bc}\, l\bar{\nu}_l)$	$2.12^{+0.26}_{-0.05}$	1.64		4.28	
$\Gamma(\Xi'_{bc} \to \Xi_{cc}\, l\bar{\nu}_l)$	$2.71^{+0.19}_{-0.05}$	1.76		7.76	

	This work	[14]	[16]
$\Gamma(\Omega_{bb} \to \Omega_{bc}\, l\bar{\nu}_l)$	$4.28^{+0.39}_{-0.03}$	3.40	28.8
$\Gamma(\Omega_{bc} \to \Omega_{cc}\, l\bar{\nu}_l)$	$5.17^{+0.39}$	4.95	
$\Gamma(\Omega_{bb} \to \Omega'_{bc}\, l\bar{\nu}_l)$	$2.32^{+0.26}$	1.66	
$\Gamma(\Omega'_{bc} \to \Omega_{cc}\, l\bar{\nu}_l)$	$2.71^{+0.17}$	1.90	

approach. When comparison with this work is possible we find an excellent agreement between the two calculations. Besides we give predictions for states not considered in the study of Ref. [4]. We also compare with other theoretical models. All calculations give similar results that vary within a few percent. From the experimental side only the mass of the Ξ_{cc} has been measured. The experimental value for $M_{\Xi_{cc}}$ obtained by the SELEX Collaboration [1] is 100 MeV smaller than our result. Note nevertheless that no account is given of the systematic error. There are also lattice calculations, by the UKQCD Collaboration [7], of the masses of the doubly charmed Ξ_{cc}, Ξ^*_{cc}, Ω_{cc} and Ω^*_{cc} baryons. Our results are within errors of the lattice determinations.

In table 4 we present our results for the semileptonic decay widths for the different processes under study. Our central values again correspond to the results obtained using the AL1 potential, while the errors show the variations when using the other four potentials. The biggest variations appear for the BD potential, with differences of the order of 7 ∼ 12%. We compare our results with the predictions of different models. For that purpose we need to fix a value for $|V_{cb}|$ for which we take $|V_{cb}| = 0.0413$. Our results are in reasonable agreement with the ones in Ref. [14] where they use a relativistic quark model evaluated in the quark-diquark approximation. For $\Gamma(\Xi_{bc} \to \Xi_{cc})$ we also agree with the value of Ref. [17] obtained using heavy quark effective theory. A much smaller value for the same width is obtained in the relativistic three–quark model calculation of Ref. [15]. In Ref. [16], where they use the Bethe–Salpeter equation applied to a quark-diquark system, they obtain much larger results for all transitions.

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References

1. M. Mattson *et al.* (SELEX) Collaboration, Phys. Rev. Lett. **89** (2002) 112001.
2. E. Hernández, J. Nieves and J.M. Verde-Velasco, hep-ph/0610030, sent to Phys Rev D
3. R. K. Bhaduri, L.E. Cohler, Y. Nogami, Nuovo Cim. A **65** (1981) 376.
4. B. Silvestre-Brac, Few-Body Systems **20** (1996) 1.
5. C. Semay and B. Silvestre-Brac, Z. Phys. C **61** (1994) 271.
6. C. Albertus, E. Hernandez and J. Nieves, Phys. Rev. D **71**, 014012 (2005)
7. J.M. Flynn, F. Mescia, and A.S.B. Tariq (UKQCD Collaboration), JHEP **0307** (2003) 066.
8. D. Ebert, R.N. Faustov, V.O. Galkin, and A.P. Martynenko, Phys. Rev. D **66** (2002) 014008.
9. V.V. Kiselev and A.K. Likhoded, Phys. Usp. **45** (2002) 455 (Usp. Fiz. Nauk **172** (2002) 497). arXiv:hep-ph/0103169.
10. S.-P. Tong, Y.-B. Ding, X.-H. Guo, H.-Y. Jin, X.-Q. Li, P.-N. Shen, and R. Zhang, Phys. Rev. D **62** (2000) 054024.
11. S.S. Gershtein, V.V. Kiselev, A.K. Likhoded, and A.I. Onishchenko, Phys. Rev. D **62** (2000) 054021.
12. R. Roncaglia, D. B. Lichtenberg, and E. Predazzi, Phys. Rev. D **52** (1995) 1722; R. Roncaglia, A. Dzierba, D. B. Lichtenberg, and E. Predazzi, Phys. Rev D **51** (1995) 1248.
13. J. Vijande, H. Garcilazo, A. Valcarce and F. Fernandez, Phys. Rev. D **70**, 054022 (2004)
14. D. Ebert, R.N. Faustov, V.O. Galkin, and A.P. Martynenko, Phys. Rev. D **70** (2004) 014018.
15. A. Faessler, Th. Gutsche, M.A. Ivanov, J. G. Körner, and V.E. Lyubovitdkij, Phys. Lett. B **518** (2001) 55.
16. X.-H. Guo, H.-Y. Jin, and X.-Q. Li, Phys. Rev. D **58** (1998) 114007.
17. M. A. Sanchis-Lozano, Nucl. Phys. B **440** (1995) 251.