Large N_c Weinberg-Tomozawa interaction and negative parity s-wave baryon resonances

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It is shown that in the 70 and 700 SU(6) irreducible spaces, the SU(6) extension of the Weinberg-Tomozawa (WT) s-wave meson-baryon interaction incorporating vector mesons (*hep-ph/0505233*) scales as $\mathcal{O}(N_c^0)$, instead of the well known $\mathcal{O}(N_c^{-1})$ behavior for its SU(3) counterpart. However, the WT interaction behaves as order $\mathcal{O}(N_c^{-1})$ within the 56 and 1134 meson-baryon spaces. Explicit expressions for the WT couplings (eigenvalues) in the irreducible SU($2N_F$) spaces, for arbitrary N_F and N_c, are given. This extended interaction is used as a kernel of the Bethe-Salpeter equation, to study the large N_c scaling of masses and widths of the lowest–lying negative parity s-wave baryon resonances. Analytical expressions are found in the $N_c \to \infty$ limit, from which it can be deduced that resonance widths and excitation energies ($M_R - M$) behave as order $\mathcal{O}(N_c^0)$, in agreement with model independent arguments, and moreover they fall in the 70-plet, as expected in constituent quark models for an orbital excitation. For the 56 and 1134 spaces, excitation energies and widths grow $\mathcal{O}(N_c^{1/2})$ indicating that such resonances do not survive in the large N_c limit. The relation of this latter N_c behavior with the existence of exotic components in these resonances is discussed. The interaction comes out repulsive in the 700.

I. INTRODUCTION

Quantum Chromodynamics (QCD), the theory of the strong interactions, is a non-abelian gauge theory based on the gauge group SU(N_c), with the number of colors $N_c = 3$. Several authors have pointed out that many features of QCD can be understood by studying the $1/N_c$ expansion of the theory and that, even at the Leading Order (LO) $N_c \to \infty$, non-trivial and realistic features can be inferred [1, 2, 3].

The question of what is the true nature of baryon resonances has attracted considerable attention in recent modern constructions of effective field theories describing meson-baryon scattering. The pattern of Spontaneous Chiral Symmetry Breaking (SCSB) of QCD, together with an appropriate non-perturbative scheme, turns out to be a crucial ingredient to better understand the main features of the resonances. On the other hand, one might wonder what is the behavior of these hadron states in the large N_c limit of QCD.

To incorporate SCSB, we work in a recently developed framework to describe meson-baryon, both in s- and d-waves ([4] and [5], respectively), scattering and resonances. It is based on the solution of the Bethe Salpeter Equation (BSE) with a kernel determined by the flavor SU(3) chiral counting rules and a particular Renormalization Scheme (RS). The claim of the authors of [6] is that, in the SU(3) limit, this RS restores crossing symmetry for a given total Center of Mass (CM) energy ($\sqrt{s}$) below the unitarity threshold. At LO, in the chiral expansion of the kernel, all parameters are determined, and the obtained results are in a remarkable agreement with data [4, 5, 7]. Extensions of the formalism to the meson-meson sector [8] and the study of charm baryon resonances [9] also lead to excellent results.

One of the findings of Ref. [4] is the existence of two SU(3) octets plus a singlet of $J^P = \frac{1}{2}^-$ s-wave baryon resonances ($N(1535)$, $N(1650)$, $\Lambda(1405)$, $\Xi(1690)$, $\cdots$), which are dynamically generated. In this work, we aim at describing the large N_c−dependence of their masses and widths. It is well known that in the $N_c \to \infty$ limit, the spin 3/2 baryon decuplet (Δ, Σ^*, Ξ^*, Ω) is degenerate to the nucleon octet. Therefore, for consistency [10], such degrees of freedom have to be considered, which will force us to work with a larger spin-flavor symmetry group (SU(6)). Spin-flavor symmetry in the meson sector is not a direct consequence of large N_c. However, vector mesons ($K^*, \rho, \omega, \bar{K}^*, \phi$) do exist, they will couple to baryons and presumably will influence the properties of the resonances. Lacking better theoretical founded models to take into account vector mesons, we study here the spin-flavor symmetric scenario, as reasonable first step.

This paper is organized as follows. In the next section, we briefly sketch the chiral unitary model of Ref. [4], and its LO N_c limit is discussed in Sect. III. The baryon decuplet and vector meson nonet effects are considered in Sect. IV. First in Subsect. IV A , we use the chiral Bethe Salpeter approach to SU(6) meson-baryon scattering developed in Ref. [11]. In Subsect. IV B, we extend the latter model for arbitrary N_c and present the final and more robust results of this work. Finally, in Sect. V, we present our main conclusions. There are four appendices where some useful formulae of interest for Sect. II and Subsect. IV B are compiled.

II. CHIRAL BETHE SALPETER APPROACH TO SU(3) MESON-BARYON SCATTERING ($\chi-$BS(3))

The leading term of the s-wave chiral meson-baryon Lagrangian is the well known Weinberg-Tomozawa (WT) interaction [15]. Since the pioneering works of the group of Weise [16], and using the WT Lagrangian as the input of the BSE[1], several approaches to s-wave baryon resonances in different strangeness and isospin sectors have been carried out [6, 17, 18, 19]. From the theoretical point of view, the used RS constitutes indeed the main difference among all of these works (see Ref. [7] for details).

In order to find resonances in this approach, the coupled channel BSE is solved, with an interaction kernel expanded in chiral perturbation theory as formulated in [20]. The involved hadrons are the Goldstone pseudoscalar meson $(K, \pi, \eta, \bar{K})$ and the lowest $\frac{1}{2}^{+}$ baryon (N, Σ, Λ, Ξ) octets. The solution for the coupled channel s-wave meson-baryon scattering amplitude, $T(\sqrt{s})$ in the so called *on-shell* scheme [6, 21] where the offshellness of the BSE is ignored, can be expressed in terms of a renormalized matrix of loop functions, $J(\sqrt{s})$, and an effective on-shell interaction kernel, $V(\sqrt{s})$, as follows[2]

$$T(\sqrt{s}) = \frac{1}{1 - V(\sqrt{s})\, J(\sqrt{s})}\, V(\sqrt{s})\,. \tag{1}$$

Thanks to the conservation of Isospin (I) and Strangeness (S), the problem decouples into different (I, S) sectors. In each sector, there are several coupled channels, N^{IS}. For instance, in the $(I, S) = (0, -1)$ sector $N^{IS} = 4$ and the corresponding coupled channels are $\pi\Sigma$, $\eta\Lambda$, $\bar{K}N$ and $K\Xi$. Thus for a given (I, S) sector, all objects in Eq. (1) are square matrices of dimension N^{IS} in the coupled channel space. The effective on-shell interaction kernel V is expanded in chiral perturbation theory. The chiral LO interaction kernel $V(\sqrt{s})$, as determined by the WT interaction reads

$$V_{ab}^{IS}(\sqrt{s}) = D_{ab}^{IS} \frac{2\sqrt{s} - M_a - M_b}{4\, f^2}\,, \tag{2}$$

where M_a (M_b) is the baryon mass of the initial (final) channel. The D's matrices can be found in the literature [17, 19] or deduced from Eq. (23) of Subsect. IV A (see [11] for some more details). The eigenvalues (λ's) of the D^{IS} matrices are 2,0,−3,−3 for both $IS = (1/2, 0)$ and $IS = (1/2, -2)$, and 2,−3,−3,−6 and 2,0,0,−3,−3 for $IS = (0, -1)$ and $IS = (1, -1)$, respectively. Those eigenvalues follow a pattern inferred from the SU(3) group representation reduction

$$8 \otimes 8 = 27 \oplus 10 \oplus 10^* \oplus 8_a \oplus 8_b \oplus 1 \tag{3}$$

being

$$\lambda_{8_a} = \lambda_{8_b} \equiv \lambda_8 = -3, \qquad \lambda_1 = -6, \qquad \lambda_{10} = \lambda_{10^*} = 0, \qquad \lambda_{27} = 2, \tag{4}$$

the eigenvalues associated to octets, singlet, decuplet and antidecuplet, and 27–plet SU(3) representations, respectively [4].

On the other hand, the diagonal loop functions, $J^{IS}(\sqrt{s})$, can be found in the Appendix A. The loop function logarithmically diverges and one subtraction is needed to make it finite. Such a freedom is fixed by the renormalization condition [6]

$$T^{IS}(\sqrt{s} = \mu) = V^{IS}(\mu)\,, \qquad \mu = \mu(I, S) \tag{5}$$

with the choice

$$\begin{aligned} \mu(1/2, -2) &= m_\Xi, \quad \mu(0, -1) = m_\Lambda, \\ \mu(1, -1) &= m_\Sigma, \quad \mu(1/2, 0) = m_N \end{aligned} \tag{6}$$

The renormalization condition of Eq. (5) is implemented by imposing that the renormalized loop functions $J_a^{IS}(\sqrt{s})$, $\forall a = 1, \cdots, N^{IS}$, vanish at the appropriate points $\sqrt{s} = \mu(I, S)$. In this way, all the constants $\mathcal{J}_a^{IS}(s = (m_a + M_a)^2), a = 1, \cdots, N^{IS}$ in Eq. (A2) turn out to be completely determined in terms of the involved baryon and meson masses. Furthermore, taking the LO of the chiral expansion for the interaction kernel, $V(\sqrt{s})$, as

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[1] In some of the works, the authors use the Lippmann-Schwinger equation instead of the relativistic BSE.
[2] The T matrix defined in Eq. (1), coincides with the t matrix defined in Eq. (33) of the first entry of Ref. [19].

determined by the WT interaction, there are no free parameters besides the meson (m's) and baryon (M's) masses and the pion weak decay constant in the chiral limit ($f \simeq 90$ MeV). At this chiral LO, the framework leads already to excellent results for physical s-wave meson-baryon scattering [4] (an extension of the model to d-wave scattering works also quite nicely [5]). Besides, the framework allows also to study the dependence of the scattering process on the quark masses, which made possible to unravel the SU(3) structure of the lowest lying s-wave baryon resonances. The findings of Ref. [4] indicate that two full SU(3) octets plus an additional singlet of $\frac{1}{2}^-$ resonances are dynamically generated. Some of them are the four stars $N(1535)$, $N(1650)$, $\Lambda(1405)$, $\Lambda(1670)$ or the three stars $\Xi(1690)$ resonances. All these resonances appear in the sectors $(I,S) = (\frac{1}{2}, 0), (0, -1), (1, -1)$ and $(\frac{1}{2}, -2)$. Similar conclusions, within a different RS, can be drawn from the work of Ref. [22], though there only the strangeness -1 sector is studied in detail.

III. LARGE N_c LIMIT OF THE $\chi-$BS(3)

The $N_c \to \infty$ limit of the LO $\chi-$BS(3) model is particularly simple, since as discussed above, the model has no free parameters besides the hadron masses and the pion weak decay constant in the chiral limit. The $N_c \to \infty$ behavior of those quantities is well established (see f.i. Ref. [3]), and neglecting $1/N_c^\epsilon$ terms ($\epsilon > 0$), one finds

$$f(N_c) \sim f_0 \times \sqrt{\frac{N_c}{3}} \tag{7}$$

$$M_a(N_c) \sim M_0 \frac{N_c}{3} + b_1 \frac{\sqrt{3}}{2} \left(1 - \frac{3N_s^{(a)}}{N_c}\right) \tag{8}$$

$$m_a(N_c) \sim m_a \tag{9}$$

with $M_0 \approx 1097$ MeV from the coefficient a_0 in Eq. (7.4) of Ref. [3], $b_1 \approx -257$ MeV and $f_0 \approx 90$ MeV. Note that the number of strange quarks, N_s, could be a fraction of the total number of quarks (N_c) of the colorless baryon.

Resonances manifest as poles in the fourth quadrant of the second Riemann sheet of the $T-$matrix. Positions of the poles,

$$s_R = M_R^2 - iM_R\Gamma_R, \tag{10}$$

determine masses (M_R) and widths (Γ_R) of the resonances while the residues for the different channels define the corresponding branching ratios. As mentioned above, an exhaustive study of the $J^P = \frac{1}{2}^-$ s-wave baryon resonance properties for the $S = 0, -1$ and -2 channels was performed in Ref. [4]. In what follows, we neglect the meson masses, which become truly massless Goldstone bosons, and the b_1 term contribution to the baryon masses, since they do not affect the LO $N_c \to \infty$ properties of those resonances[3] . In this way SU(3) flavor symmetry is also restored, and one has two degenerate octets and one singlet of resonances. Indeed, for $N_c = 3$ we essentially recover the "light" SU(3) limit introduced in Ref. [4]. Within this framework, our RS leads to the conditions $\mathcal{J}_a^{IS}(s = M^2) = 0, a = 1, \cdots, N^{IS}$, where M is the N_c LO SU(3) baryon mass

$$M_a \sim M = M_0 \times N_c/3, \qquad \forall a \tag{11}$$

For each IS channel, the position of the poles is determined by

$$\beta(s)\Big|_{s=s_R \equiv M_R^2 - iM_R\Gamma_R} = \lambda_i, \quad i = 1, 8, 10, 10^*, 27 \tag{12}$$

with $M_R > M$ and $\Gamma_R > 0$. Besides, λ_i are the eigenvalues of the real and symmetric matrix D^{IS} (Eq. (4)) and the dimensionless function $\beta(s)$ (see Eqs. (1)–(2)) reads

$$\beta(s) = \frac{2f^2}{J_{II}^{N_c \gg 1}(\sqrt{s})(\sqrt{s} - M)} \tag{13}$$

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[3] The N_c dependence of the correction induced by finite meson masses can be estimated by shifting the baryon mass by an amount of order N_c^0.

with $J_{II}^{N_c \gg 1}$ the loop function of Eq. (A1) with $M_a = M, m_a = 0$, but defined in the second Riemann sheet. In the fourth quadrant, it reads [19]

$$J_{II}^{N_c \gg 1}(\sqrt{s}) = \frac{(\sqrt{s} + M)^2}{2\sqrt{s}\,(4\pi)^2} \left(\frac{s - M^2}{s}\right) \left\{ \log|R(s)| + i\mathrm{Arg}(R(s)) - 3i\pi \right\} \tag{14}$$

with $R(s) = (s - M^2)/M^2$ and $\mathrm{Arg}(R(s))$ should be taken in the interval $[0, 2\pi[$.

The equation (12) has solutions only for negative eigenvalues, λ_8 and λ_1. Thus, at LO of the N_c expansion only those s-wave $\frac{1}{2}^-$ resonant states ($N(1535)$, $N(1650)$, $\Lambda(1405)$, $\Lambda(1670)$, $\Sigma(1620)$, $\Xi(1620)$, $\Xi(1690)$, $\Lambda(1390)$[4]) belonging to the two octets and singlet SU(3) representations are dynamically generated from Goldstone meson ($K, \pi, \eta, \bar{K}$) and the lowest $J^P = \frac{1}{2}^+$ baryon (N, Σ, Λ, Ξ) octets re-scattering. Reciprocally, LO N_c results disfavor the existence of dynamically generated decuplet, antidecuplet and 27-plet states. Though, this will change after the inclusion of baryon decuplet and vector meson nonet effects in the next section. For its nowadays interest, we remark that LO N_c $\chi-$BS(3) model strongly disfavors that the $S = +1$ isoscalar Θ^+ resonance, which would be the isospin singlet state of the antidecuplet representation, could be described just in terms of dynamical KN resonant re-scattering[5]. Taking into account also K^* and Δ degrees of freedom, within a larger spin-flavor symmetry scheme, might permit the existence of the *so called* pentaquarks [11].

Octet and singlet resonance masses and widths from Eq. (12) are depicted in Fig. 1. Several comments are in order:

- Since M increases as N_c, the shift $M_R - M$ and the resonance width, Γ_R, increase with N_c slower than $\sqrt{N_c}$.

- The ratio $\Gamma_R/(M_R - M)$ approaches to zero as N_c increases, both for singlet and octet resonances.

- The approximate formula

$$\frac{\Gamma_R}{M} = -\frac{\pi\delta}{\log(2\delta)}, \quad \text{with} \quad \delta \equiv \frac{M_R - M}{M} \tag{15}$$

works notably well in the large N_c limit. Indeed, in the limit $N_c \to \infty$ one easily finds

$$\delta^2 \log \delta = \frac{24\pi^2 f_0^2}{N_c \lambda_i M_0^2} \tag{16}$$

$$\frac{\Gamma_R}{M} = -\lambda_i \frac{N_c \delta^3 M_0^2}{24\pi f_0^2}, \quad i = 8, 1 \tag{17}$$

which suggest a large N_c behavior of the type

$$\delta \sim \frac{1}{\sqrt{N_c \log N_c}} \tag{18}$$

$$\frac{\Gamma_R}{M} \sim \frac{1}{\sqrt{N_c \log^3 N_c}} \tag{19}$$

- The presence of logarithms of N_c in the mass and width of the resonances is against standard large N_c counting rules [2]. In the present approach it comes out from the baryon mass in the loop function. Such logarithms are almost certainly an artifact of the implementation of the effective theory, and are expected to dissappear using a more appropriate treatment along the lines of Heavy Baryon Chiral Perturbation Theory [12], or the Infrared Regularization of Ellis and Tang [13], and Becher and Leutwyler [14]. So the BSE approach as used in this work should reliably predict the power-like part of the N_c dependence but not necessarily logarithmic corrections.

The original work of Witten [2] pointed out that the excited baryons have both natural widths and excitation energies of order $\mathcal{O}(N_c^0)$. More recently, some questions have been raised about the general validity of that result and some arguments in favor of the existence of narrow (widths of $\mathcal{O}(1/N_c)$) excited baryons at large N_c have been given [24].

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[4] The $\Lambda(1390)$ state corresponds to the SU(3) singlet representation [4, 19, 22, 23]. In this list of resonances, there is a Σ state missing. Perhaps, it could be the $\Sigma(1750)$ resonance.
[5] Moreover, we should remind here that the WT chiral meson-baryon Lagrangian predicts a vanishing on shell interaction kernel $V(\sqrt{s})$, for isoscalar KN scattering.
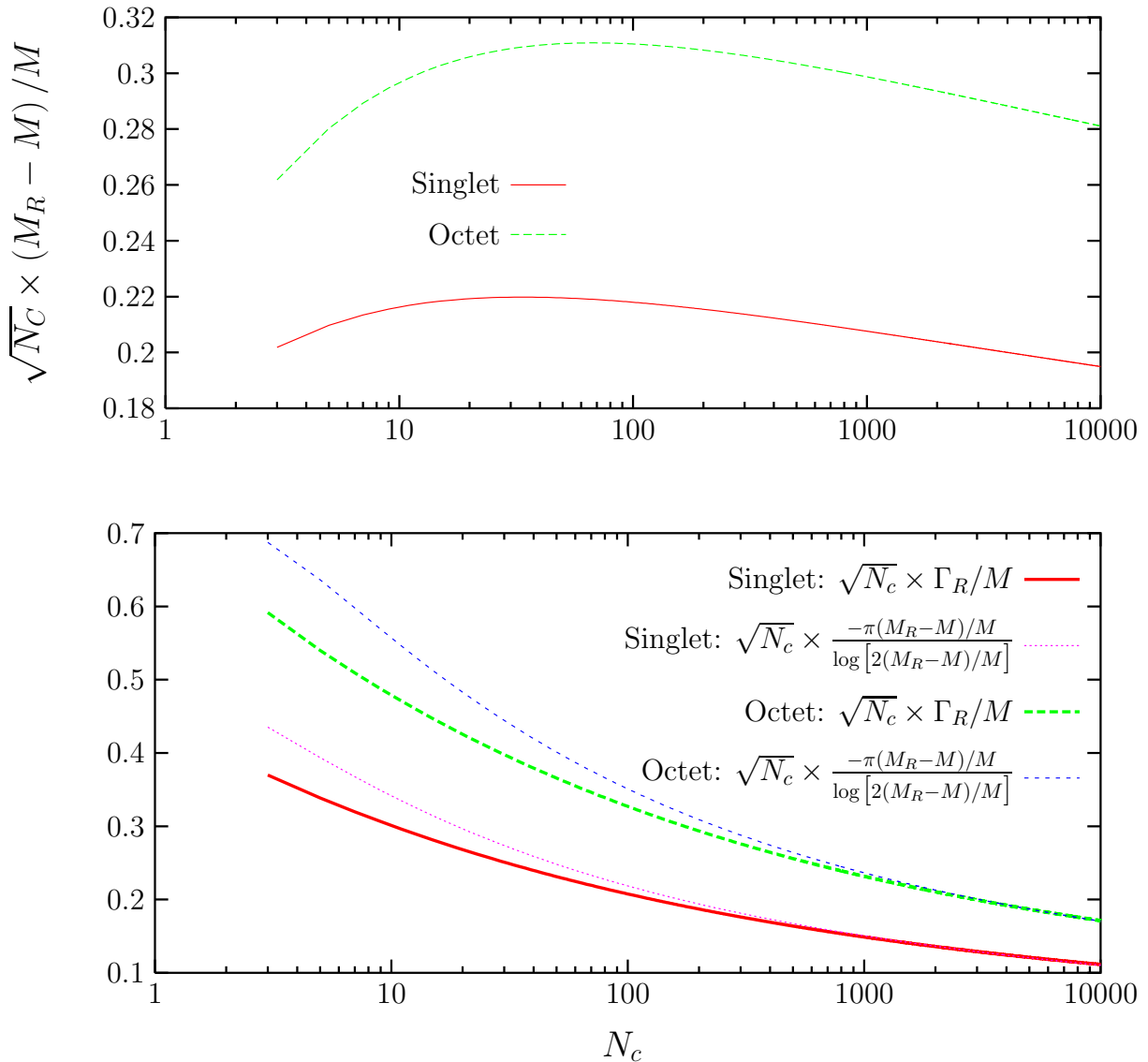
FIG. 1: Singlet and octet resonance masses (M_R) and widths (Γ_R) as a function of N_c from the naive large N_c limit of the $\chi-$BS(3).

Nevertheless, it seems that a general large N_c QCD analysis does not predict such narrow states [25], which has been also corroborated by other authors [26]. On the other hand, resonances are unstable particles, and one may question the validity of a Hamiltonian formalism[6], since it must be assumed that the resonant states exist for a sufficiently long time in order to be described as eigenstates of a Hamiltonian. Chiral soliton models, such as the Skyrme model, improve on that point and in those models, resonances show up as poles in meson–baryon scattering amplitudes [29]. Recently, it has been proved that both schemes are compatible in some sense, and give rise to a set of multiplets of degenerate states, for which any complete spin-flavor multiplet within one picture fills the quantum numbers of complete multiplets in the other picture [30].

The results of this section do not support the existence of narrow states either, but lead to widths and excitation energies $(M_R - M)$ which do not behave as order $\mathcal{O}(N_c^0)$, but instead grow, in the $N_c \to \infty$ limit, as $\sqrt{N_c}$ (modulo subleading logarithmic corrections not under control in the present BSE treatment). It might point out to a serious deficiency of the present analysis. Indeed, as we will show below, the results presented in this section are not reliable.

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[6] In such scenario resonances are described as single–quark orbital excitations about a closed-shell core [27, 28].

At least, there are two aspects which should be revised. First, as mentioned in the introduction baryon decuplet degrees of freedom should be included. Second, baryons carry the quantum numbers of N_c quarks (in order to form an SU(N_c) color singlet from color–fundamental irreps), and therefore the baryon SU(3) irreps might depend on N_c, which could induce an N_c dependence of the eigenvalues (λ's). As we will see, the extension of spin-flavor symmetry to the meson sector will also be essential.

IV. BARYON DECUPLET AND VECTOR MESON NONET EFFECTS

Let us start revising the chiral Bethe Salpeter approach to SU(6) meson-baryon scattering ($\chi-$BS(6)) developed in Ref.[11]. For ground state baryons, there exists an exact spin–flavor symmetry in the large N_c limit [3]. This is to say that the light quark–light quark interaction is approximately spin independent as well as SU(3) independent. This corresponds to treating the six states of a light quark (u, d or s with spin up, $\uparrow$, or down, $\downarrow$) as equivalent, and leads us to the invariance group SU(6). Since the pure SU(3) transformations commute with the pure SU(2) (spin) transformations within SU(6), it follows that a SU(6) multiplet can be decomposed into SU(3) multiplets each of definite total spin. With the inclusion of the spin there are 216 three quark states, and the SU(6) group representation reduction (denoting the SU(6) multiplets by their dimensionality and a SU(3) multiplet μ of spin J by μ_{2J+1}) reads

$$6 \otimes 6 \otimes 6 = 56 \oplus 70 \oplus 70 \oplus 20 = \underbrace{8_2 \oplus 10_4}_{56} \oplus \underbrace{1_4 \oplus 8_2}_{20} \oplus$$

$$\oplus \ 2 \times \left\{ \underbrace{10_2 \oplus 8_4 \oplus 8_2 \oplus 1_2}_{70} \right\} \tag{20}$$

It is natural to assign the lowest–lying baryons to the 56–plet of SU(6), since it can accommodate an octet of spin–1/2 baryons and a decuplet of spin–3/2 baryons, which are exactly the SU(3)–spin combinations of the low–lying baryon states ((N, Σ, Λ, Ξ) and (Δ, Σ^*, Ξ^*, Ω)). Furthermore, the 56–plet of SU(6) is totally symmetric, which allows the baryon to be made of three quarks in s-wave. Color degrees of freedom take care of the Fermi's statistics.

In the meson sector, assuming that the lowest lying states are obtained from s-wave quark–antiquark interactions and taking into account the group reduction

$$6 \otimes 6^* = 35 \oplus 1 = \underbrace{8_1 \oplus 8_3 \oplus 1_3}_{35} \oplus \underbrace{1_1}_{1}, \tag{21}$$

the octet of pseudoscalar ($K, \pi, \eta, \bar{K}$) and the nonet of vector ($K^*, \rho, \omega, \bar{K}^*, \phi$) mesons are commonly placed in the 35 representation of SU(6). A ninth 0^- meson (η') must go in the 1 of SU(6). The nonet of vector mesons and the octet of Goldstone bosons are clearly not degenerated. As mentioned in the introduction, spin-flavor symmetry in the meson sector is not a direct consequence of large N_c. However, vector mesons do exist, they will couple to baryons and presumably will influence the properties of the resonances. Since the splitting between the pseudo-scalar and vector mesons is of order N_c^0 as the meson masses themselves, and having neglected these latter ones with respect to the baryon masses, it is not unreasonable to assume a spin-flavor symmetry in large N_c in the meson sector, as well. Lacking better theoretical founded models to take into account vector mesons, studying the spin-flavor symmetric scenario seems a reasonable first step. Moreover an underlying static chiral $U(6) \times U(6)$ symmetry has been advocated by Caldi and Pagels [31] in which vector mesons would be "dormant" Goldstone bosons acquiring mass thorough relativistic corrections. This scheme solves a number of theoretical problems in the classification of mesons and also makes predictions which are in remarkable agreement with the experiment.

Thus for consistency, the spin–3/2 decuplet baryon and the vector meson nonet degrees of freedom have to be added to the resonance analysis carried out in the previous section. As a consequence, for a given sector (JIS), there now appear some more coupled channels than when the involved hadrons were only the Goldstone pseudoscalar meson and the lowest $J^P = \frac{1}{2}^+$ baryon octets. For instance, in the $(JIS) = (1/2, 0, -1)$ sector, besides the $\pi\Sigma$, $\eta\Lambda$, $\bar{K}N$ and $K\Xi$ channels, we also consider now the $K^*\Xi$, $K^*\Xi^*$, $\rho\Sigma$, $\rho\Sigma^*$, $\omega\Lambda$, $\bar{K}^*N$ and $\phi\Lambda$ ones.

We will limit ourselves to s-wave meson–baryon resonances[7] and we will make use of the SU(6) extension of the Weinberg-Tomozawa (WT) meson-baryon chiral Lagrangian recently carried out in Ref. [11]. Chiral Symmetry

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[7] We are aware of possible d-wave mixings, which within the framework outlined in Ref. [11] will be examined elsewhere.

(CS) at leading order (WT Lagrangian) is much more predictive than SU(3) symmetry[8] and determines the on-shell interaction kernel, $V(\sqrt{s})$, for (8_1)meson–(8_2)baryon s-wave scattering in Eq. (1) in terms of a unique parameter (f), besides the hadron masses (see Eq. (2)). From an SU(6) point of view, one should work with s-wave meson–baryon states, constructed out of the SU(6) 35 (mesons) and 56 (baryons) multiplets. The SU(6) decomposition yields

$$35 \otimes 56 = 56 \oplus 70 \oplus 700 \oplus 1134, \tag{22}$$

and thus one has four (Wigner-Eckart irreducible matrix elements of the SU(6) invariant Hamiltonian) free functions of the meson–baryon Mandelstam variable s. It is clear that not all SU(3) invariant interactions in the (8_1)meson–(8_2)baryon sector can be extended to a SU(6) invariant interaction. Remarkably, the WT interaction turns out to be consistent with SU(6) and, moreover, the extension is unique. In other words, there is a choice of the four couplings for the $35 \otimes 56$ interaction that, when restricted to the $8_1 \otimes 8_2$ sector, reproduces the WT on-shell interaction kernel $V(\sqrt{s})$ of Eq. (2), and such choice is unique [11]. Indeed, the *potential* of Eq. (2) can be recovered, in the SU(3) limit, by taking

$$
\begin{aligned}
\langle \mathcal{M}'\mathcal{B}'; JIY|V|\mathcal{M}\mathcal{B}; JIY \rangle &= \sum_{\phi_{\mathrm{SU(6)}}} \bar{\lambda}_{\phi_{\mathrm{SU(6)}}} \frac{\sqrt{s}-M}{2\,f^2} \mathcal{P}^{\phi_{\mathrm{SU(6)}}, JIY}_{\mathcal{M}\mathcal{B}, \mathcal{M}'\mathcal{B}'}, \\
\mathcal{P}^{\phi_{\mathrm{SU(6)}}, JIY}_{\mathcal{M}\mathcal{B}, \mathcal{M}'\mathcal{B}'} &= \sum_{\mu_{\mathrm{SU(3)}}, \alpha} \begin{pmatrix} 35 & 56 & \phi_{\mathrm{SU(6)}} \\ \mu_M J_M & \mu_B J_B & \mu_{\mathrm{SU(3)}} J\alpha \end{pmatrix} \begin{pmatrix} \mu_M & \mu_B & \mu_{\mathrm{SU(3)}} \\ I_M Y_M & I_B Y_B & IY \end{pmatrix} \\
&\times \begin{pmatrix} \mu'_{M'} & \mu'_{B'} & \mu_{\mathrm{SU(3)}} \\ I'_{M'} Y'_{M'} & I'_{B'} Y'_{B'} & IY \end{pmatrix} \begin{pmatrix} 35 & 56 & \phi_{\mathrm{SU(6)}} \\ \mu'_{M'} J'_{M'} & \mu'_{B'} J'_{B'} & \mu_{\mathrm{SU(3)}} J\alpha \end{pmatrix}.
\end{aligned}
\tag{23}
$$

with

$$\bar{\lambda}_{56} = -12, \qquad \bar{\lambda}_{70} = -18, \qquad \bar{\lambda}_{700} = 6, \qquad \bar{\lambda}_{1134} = -2 \tag{24}$$

and M now being the common octet and decuplet baryon mass. Besides, Y stands for the hypercharge (strangeness plus baryon number), and we use the notation $\mathcal{M} \equiv [(\mu_M)_{2J_M+1}, I_M, Y_M]$ for mesons and similarly for baryons ($\mathcal{B}$). Thus, $\mu_M = 8, 1$ and $\mu_B = 8, 10$ are the meson and baryon SU(3) multiplets, respectively, and $J_{M,B}, I_{M,B}, Y_{M,B}$ are the spin, isospin and hypercharge quantum numbers of the involved hadrons. Finally in Eq. (23), SU(3) isoscalar factors [32], and the SU(6)–multiplet coupling factors [33] are also used. For more details see Ref. [11].

A. Naive large N_c limit of the $\chi-$BS(6)

In this subsection we use Eq. (23) as the interaction kernel to solve the BSE in the large N_c limit. Thus, we improve on the analysis of Sect. III by including baryon decuplet and vector meson nonet effects. We assume that the ground state baryons fall in the 56–plet of SU(6) (only possible if N_c is odd) for large N_c, however we still ignore the fact that the spin-flavor irreps might depend on N_c.

Since V is a SU(6) scalar operator, one readily realizes that the resonance equation reads now,

$$\beta(s)\Big|_{s=s_R \equiv M_R^2 - iM_R\Gamma_R} = \bar{\lambda}_{\phi_{SU(6)}}, \quad \phi_{SU(6)} = 56, 70, 700, 1134 \tag{25}$$

with $M_R > M$ and $\Gamma_R > 0$ and thus, the approximated relation of Eq. (15) is accomplished, as well. This equation has solutions only for negative eigenvalues, $\bar{\lambda}_{70}$, $\bar{\lambda}_{56}$ and $\bar{\lambda}_{1134}$. Note that the 70 of SU(6) leads to the most attractive s-wave meson–baryon interaction. This is also the scenario commonly adopted in most large N_c works, where the first negative parity baryon excited states are considered as members of the 70 multiplet (see f.i. Ref [28]). Beyond the large N_c LO, there appear terms in the meson–baryon Hamiltonian which explicitly break down the spin–flavor symmetry and thus, one expects SU(6) configuration mixings. Hence the $N(1535)$, $N(1650)$, $\Lambda(1405)$, $\Lambda(1670)$, $\Sigma(1620)$, $\Xi(1620)$, $\Xi(1690)$, $\Lambda(1390)$ s-wave $\frac{1}{2}^{-}$ resonances will be constructed out of the SU(6) 70, 56 and 1134

---

[8] From the SU(3) decomposition of Eq. (3), one easily deduces [11] that SU(3) symmetry describes the Goldstone pseudoscalar meson and the lowest $J^P = \frac{1}{2}^{+}$ baryon octets s-wave scattering in terms of seven undetermined functions (Wigner–Eckart matrix elements) of the meson–baryon Mandelstam variable s.

resonant states. From the spin–flavor content of the SU(6) representations, we expect the SU(3) singlet resonance[9] to be a linear combination of the 70 and 1134 states, while for the SU(3) octet ones, the SU(6) 56 resonant states will have to be considered as well[10].

Thus, the properties of these resonances ($N(1535)$, $N(1650)$, $\Lambda(1405)$,...) are modified by their coupling to the baryon decuplet and vector meson nonet states. Assuming that these states belong to the 70 [28][11], 56 and 1134 multiplets, we find that the relations of Eqs. (16)–(17) are correct, just by replacing $\lambda_{1,8}$ by $\bar{\lambda}_{70,56,1134}$.

On the other hand, the relations of Eqs. (18)–(19) still hold, with no modifications. As a conclusion, considering baryon decuplet and vector meson nonet effects would lead to some quantitative changes on resonance masses and widths at relatively low values of N_c, and would affect to the rate how the $N_c \to \infty$ relations of Eqs. (18)–(19) are reached. Therefore, widths and excitation energies ($M_R - M$) would not behave as order $\mathcal{O}(N_c^0)$, but they would still grow as $\sqrt{N_c}$. Thus, the inclusion of baryon decuplet and vector meson nonet degrees of freedom, treated as in this subsection, does not modify this behavior, possibly incorrect.

In the analysis presented up to here, it has been ignored the fact that, since baryons for arbitrary N_c contain N_c valence quarks, the corresponding baryon SU(6) representations also grow in size with N_c [10]. As we will see in the next subsection, this will provide an explicit N_c dependence for the eigenvalues $\bar{\lambda}_{\phi_{SU(6)}}$. This further N_c dependence will allow us first to show that, in some SU(6) irreducible spaces, the SU(6) extension of the WT s-wave meson-baryon interaction, sketched in this subsection, scales as $\mathcal{O}(N_c^0)$, instead of the well known $\mathcal{O}(N_c^{-1})$ behavior for its SU(3) counterpart, and second to recover the Witten's scaling rules for both widths and excitation energies of the resonant states.

B. Extension of the $\chi-$BS(6) Model for Arbitrary N_c

1. SU(6) representations and WT Lagrangian for Arbitrary N_c

Mesons at arbitrary N_c still carry the quantum numbers of a single $q\bar{q}$, and hence their SU($2N_F$) spin-flavor irreps are unchanged when N_c is changed. Thus, the octet of pseudoscalar ($K, \pi, \eta, \bar{K}$) and the nonet of vector ($K^*, \rho, \omega, \bar{K}^*, \phi$) mesons are placed in the 35 representation of SU(6). Baryons, on the other hand, carry the quantum numbers of N_c quarks (in order to form an SU(N_c) color singlet from color–fundamental irreps), and therefore the baryon SU($2N_F$) spin-flavor irreps grow in size with N_c. We wish to identify these large N_c representations with their $N_c = 3$ counterparts. As it is done in Ref. [35], to keep our notation simple and aid in the extrapolation to three colors case, we use quotes to denote the generalized SU($2N_F$) representations familiar from three colors. The ground-state spin-flavor multiplet is taken to be completely symmetric N_c-tableau representation, which is the analog to the SU(6) 56 for three flavors,

$$\underbrace{\boxed{}\ldots\boxed{}}_{N_c \text{ boxes}} \qquad (26)$$

Notationally, we denote such arbitrary-N_c generalization as "56" and its dimension is $\begin{pmatrix} N_c + 5 \\ 5 \end{pmatrix}$. The SU(6) decomposition of Eq. (22) now, for arbitrary N_c, reads

$$35 \otimes \text{"56"} \equiv \; \text{[Young tableau]} \; \otimes \; \overbrace{\text{[Young tableau]}}^{N_c}$$

---

[9] The resonance $\Lambda(1390)$ will have a large SU(3) singlet component.

[10] The 56-plet should be included, since there is only one 8_2 multiplet in the 70 of SU(6).

[11] The available analysis of the negative parity 70–plet baryon masses within the $1/N_c$ expansion suffer from a serious deficiency. Those studies do not consider the $\Lambda(1390)$ state, which existence has been firmly established from a theoretical point of view [4, 19, 22], and also there are some indications supporting its existence in the $K^-p \to \pi^0\pi^0\Sigma^0$ reaction data, as it has been recently pointed out in Ref. [23]. Traditionally, large N_c studies construct the isospin singlet states of the 8_2 and 1_2 SU(3)$_{2J+1}$ representations, entering in the 70 SU(6) multiplet, as linear combinations of the $\Lambda(1405)$ and $\Lambda(1670)$ resonances. It is clear, that the $\Lambda(1390)$ state should be considered, and presumably it will have a large 1_2 component.

$$= \text{“56”} \oplus \text{“70”} \oplus \text{“700”} \oplus \text{“1134”}, \tag{27}$$

where the dimensions of the "70", "700" and "1134" irreps are $\frac{5(N_c-1)}{(N_c+5)}\begin{pmatrix} N_c+5 \\ 5 \end{pmatrix}$, $\frac{5(N_c+7)}{(N_c+1)}\begin{pmatrix} N_c+5 \\ 5 \end{pmatrix}$ and $\frac{24N_c(N_c+6)}{(N_c+5)(N_c+1)}\begin{pmatrix} N_c+5 \\ 5 \end{pmatrix}$, respectively. Thus, we find a first remarkable result: assuming SU(6) spin-flavor symmetry, the s-wave 35–meson "56"–baryon scattering for an arbitrary value of N_c, can still be described in terms of four (Wigner-Eckart irreducible matrix elements of the SU(6) invariant Hamiltonian) undetermined functions of the meson–baryon Mandelstam variable s. This is also the case for any number of flavors $N_F \geq 2$.

Next step is to make use of the underlying CS to further constrain these four undetermined functions. For SU(3) flavor symmetry and $N_c = 3$, the latter functions, at LO in the chiral expansion, are determined by the WT Lagrangian. It is not just SU(3) symmetric but also chiral ($\mathrm{SU}_L(3) \otimes \mathrm{SU}_R(3)$) invariant. Symbolically,

$$\mathcal{L}_{\mathrm{WT}} = \mathrm{Tr}([M^\dagger, M]B^\dagger B) \tag{28}$$

This structure, dictated by CS, is more suitably analyzed in the t-channel. The meson, M, and baryon, B, fields fall in the representation SU(3) 8 which is also the adjoint representation. The commutator $[M^\dagger, M]$ indicates a t-channel coupling to the 8_a (antisymmetric) representation, thus

$$\mathcal{L}_{\mathrm{WT,SU(3)}} = \left((M^\dagger \otimes M)_{8_a} \otimes (B^\dagger \otimes B)_8\right)_1 \tag{29}$$

The unique SU(6) extension is then

$$\mathcal{L}_{\mathrm{WT,SU(6)}} = \left((M^\dagger \otimes M)_{35_a} \otimes (B^\dagger \otimes B)_{35}\right)_1 \tag{30}$$

since the 35 is the adjoint representation of SU(6). The t-channel decompositions $35 \otimes 35 = 1 \oplus 35_s \oplus 35_a \oplus 189 \oplus 280 \oplus 280^* \oplus 405$ and $56 \otimes 56^* = 1 \oplus 35 \oplus 405 \oplus 2695$ indicate that the coupling in Eq. (30) exists and is indeed unique [11], all coupling constants being reduced to a single independent one, namely, that of the WT Lagrangian (pion weak decay constant, besides the hadron masses). To extend this result to arbitrary N_c, we should first consider

where the dimension of the tableau with $2n$ boxes in the first row is $(2n+5)\begin{pmatrix} n+5 \\ 5 \end{pmatrix}\begin{pmatrix} n+4 \\ 4 \end{pmatrix}/(n+5)$, accomplishing

$$\sum_{n=0}^{N_c} \frac{2n+5}{n+5}\begin{pmatrix} n+5 \\ 5 \end{pmatrix}\begin{pmatrix} n+4 \\ 4 \end{pmatrix} = \begin{pmatrix} N_c+5 \\ 5 \end{pmatrix}^2 \tag{32}$$

to verify the equality between the dimensions of both sides of Eq. (31). We see that the SU(6) 35 (adjoint representation) appears in the decomposition into irreps of "56"⊗"56*" (Eq. (31)), and thus we find that the SU(6) extension of the WT Lagrangian (Eq. (30)) can still be done for arbitrary N_c.

Let us denote the contravariant and covariant spin-flavor quark and antiquark components

$$q^i = \begin{pmatrix} u\uparrow \\ d\uparrow \\ s\uparrow \\ u\downarrow \\ d\downarrow \\ s\downarrow \end{pmatrix}, \qquad \bar{q}_i = \left(\bar{u}\downarrow, -\bar{d}\downarrow, -\bar{s}\downarrow, -\bar{u}\uparrow, \bar{d}\uparrow, \bar{s}\uparrow \right) \tag{33}$$

where q^i ($\bar{q}_i$) annihilates[12] a quark (antiquark) with the spin-flavor i. For instance $\bar{u}\downarrow$ annihilates an antiquark with flavor $\bar{u}$ and $S_z = -1/2$. Mesons fall in the adjoint representation and we represent the annihilation operators of mesons in the 35 of SU(6) by means of a traceless tensor M^i_j, which under SU(6) transformations behaves like

$$q^i \bar{q}_j - \frac{1}{2N_F} q^m \bar{q}_m \delta^i_j, \quad i,j = 1, \ldots 2N_F \tag{34}$$

with N_F the number of flavors, three in this work. We represent the annihilation operators of baryons in the "56" of SU(6), for arbitrary N_c, by means of a completely symmetric tensor $B^{i_1 i_2 \ldots i_{N_c}}$, which under SU(6) transformations behaves like $q^{i_1} q^{i_2} \ldots q^{i_{N_c}}$. We treat q^i as boson fields, since the color wave function, not explicitly shown, is fully antisymmetric. The corresponding Wick's contractions of these fields read

$$\underline{M^k_l M^{\dagger i}_j} = \delta^k_j \delta^i_l - \frac{1}{2N_F} \delta^i_j \delta^k_l$$

$$\underline{B^{j_1 j_2 \ldots j_{N_c}} B^\dagger_{i_1 i_2 \ldots i_{N_c}}} = \sum_{P \in S_{N_c}} \delta^{P(j_1)}_{i_1} \delta^{P(j_2)}_{i_2} \ldots \delta^{P(j_{N_c})}_{i_{N_c}} \tag{35}$$

where S_{N_c} is the group of permutations of N_c objects and we use a notation such that the N_c-tuple $P(i_1) P(i_2) \ldots P(i_{N_c})$ is equal to $P(i_1 i_2 \ldots i_{N_c})$.

From the discussion above, we find that Eq. (30) is still the unique SU(6) extension of the WT s-wave meson-baryon interaction for arbitrary N_c. Thus, we find that the group structure, $\mathcal{G}^{\mathrm{SU}(N_c)}_{\mathrm{WT,SU}(6)}$, of the SU(6) extension of the WT, up to constant factors, takes the form[13]

$$\mathcal{G}^{\mathrm{SU}(N_c)}_{\mathrm{WT,SU}(6)} = \frac{2}{(N_c - 1)!} : \left(M^i_j M^{\dagger j}_k - M^{\dagger i}_j M^j_k \right) B^\dagger_{i i_2 \ldots i_{N_c}} B^{k i_2 \ldots i_{N_c}} : \tag{36}$$

where $: \ldots :$ denotes the normal product and the factor $2/(N_c - 1)!$ has been introduced for convenience. To obtain the full form of the Hamiltonian, one should specify some constant factors

$$\mathcal{H}^{\mathrm{SU}(N_c)}_{\mathrm{WT,SU}(6)} \propto \mathcal{G}^{\mathrm{SU}(N_c)}_{\mathrm{WT,SU}(6)}, \tag{37}$$

which depend on kinematics and possibly also on the number of colors. These factors will be discussed in Subsect. IV B 2.

2. Explicit form of $\mathcal{H}^{\mathrm{SU}(N_c)}_{\mathrm{WT,SU}(6)}$

First and for $N_c = 3$, we will write a s-wave meson-baryon Lagrangian invariant under SU(3)$\times$ SU(2) transformations and involving only the Goldstone boson and the nucleon octets. Starting from the lowest order in the chiral expansion [20][14]

$$\mathcal{L}_1 = \mathrm{Tr} \left\{ \bar{\Psi}_B \left(\mathrm{i} \overslash{\nabla} - M \right) \Psi_B \right\} \tag{38}$$

---

[12] We use a convention such that $\begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix}$ is a standard basis of SU(2), that is $\bar{d} = |1/2, 1/2\rangle$ and $\bar{u} = |1/2, -1/2\rangle$. Thus, $\bar{u}, \bar{d}, \bar{s}$ is a standard basis of the 3^* representation of SU(3) with de Swart's convention [32].

[13] Here and in most of the Subsect. IV B, though we give explicitly expressions for the $N_F = 3$ case, the formulae are easily extended for an arbitrary number of flavors.

[14] We have omitted the pieces proportional to the couplings $\mathcal{D}$ and $\mathcal{F}$ ($\mathcal{F} + \mathcal{D} = g_A = 1.25$) because they do not lead to the WT interaction Lagrangian.

where M is the common mass of the baryon octet due to SCSB for massless quarks and "Tr" stands for the trace in SU(3). In addition,

$$
\begin{aligned}
\nabla^\mu \Psi_B &= \partial^\mu \Psi_B + [A_3^\mu, \Psi_B] \\
A_3^\mu &= \frac{1}{2}\left(u_3^\dagger \partial^\mu u_3 + u_3 \partial^\mu u_3^\dagger\right) = \frac{1}{4f^2}[\Phi_3, \partial^\mu \Phi_3] + \mathcal{O}((\Phi_3)^4) \\
U_3 = u_3^2 &= e^{i\sqrt{2}\Phi_3/f}
\end{aligned}
\tag{39}
$$

The SU(3) matrices for the meson and the baryon octets are written in terms of the meson and baryon Dirac fields respectively and are given by[15]

$$
\Phi_3 = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix},
\tag{40}
$$

and

$$
\Psi_B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}.
\tag{41}
$$

respectively. Performing a non-relativistic reduction[16] of Eq. (38), we find

$$
\mathcal{L}_{1\,\text{norel}} = \text{Tr}\left\{ B_3^\dagger \left(i\nabla_0 - M + \frac{1}{2M}(\vec{\sigma} \cdot \vec{\nabla})^2 \right) B_3 \right\}
\tag{42}
$$

where now the B_3-fields (large components of the Ψ_B ones) do not contain antiparticle degrees of freedom, that is they are bispinors which account for the spin degrees of freedom of the non-relativistic baryons. The above Lagrangian is not invariant under SU(3)$\times$ SU(2) transformations, yet. This is because of a spin-orbit type interaction generated by the Pauli matrices. Such a term does not contribute to s-wave and neglecting it, we get

$$
\mathcal{L}_{\text{SU(3)}\times\text{SU(2)}} = \text{Tr}\left\{ B_3^\dagger \left(i\nabla_0 - M + \frac{1}{2M}\vec{\nabla}^2 \right) B_3 \right\}
\tag{43}
$$

which is now SU(3)$\times$SU(2) invariant. Neglecting non s-wave contributions and including explicit baryon mass breaking terms, the interaction part of the above Lagrangian leads to the chiral LO amplitude of Eq. (2).

The extension of the Lagrangian of Eq. (43) to describe also baryon decuplet and vector meson nonet degrees of freedom is now straightforward and it reads

$$
\mathcal{L}_{\text{SU(6)}} = \text{Tr}\left\{ B_6^\dagger \left(i\nabla_0 - M + \frac{1}{2M}\vec{\nabla}^2 \right) B_6 \right\}
\tag{44}
$$

where "Tr" stands now for the trace in SU(6). In addition,

$$
\begin{aligned}
\nabla^\mu B_6 &= \partial^\mu B_6 + A_6^\mu * B_6 \\
A_6^\mu &= \frac{1}{2}\left(u_6^\dagger \partial^\mu u_6 + u_6 \partial^\mu u_6^\dagger\right) = \frac{1}{4f_6^2}[\Phi_6, \partial^\mu \Phi_6] + \mathcal{O}((\Phi_6)^4) \\
U_6 = u_6^2 &= e^{i\sqrt{2}\Phi_6/f_6}
\end{aligned}
\tag{45}
$$

M is now the common mass of the 56 baryon representation and $f_6 = f/\sqrt{2}$, as shown in Appendix B. Besides, B_6 and Φ_6 are the baryon and meson fields which now belong to the 56 and 35 irreps of SU(6), respectively and the

---

[15] For the purpose of our work we do not consider any mixing between octet and singlet SU(3) representations

[16] In order to find an SU(6) invariant Lagrangian, it is natural to perform an non-relativistic reduction, since the no-go Coleman-Mandula theorem [36] forbids an exact hybrid symmetry mixing a compact internal flavor symmetry with the non-compact Poincare symmetry of spin angular momentum. Furthermore, in the large N_c limit, a non-relativistic treatment of baryons is totally justified.

meaning of $A_6^\mu * B_6$ will be specified later (see Eq. (49))[17]. Obviously, we need to check that the restriction of the above Lagrangian to the $8_1 \otimes 8_2$ sector reproduces that given in Eq. (43). This is explicitly shown in Appendix B, for three flavors, though the extension to N_F flavors is straightforward.

In the above equations, Φ_6 is a dimension six matrix made of full meson fields, which depend on the space-time coordinates. The annihilation part of the meson matrix $[\Phi_6]_j^i$ is determined by the operators M_j^i (see Eq. (34)). On the other hand, for SU(6) and arbitrary N_c, we will work with baryon fields $\mathcal{B}^{i_1 i_2 \ldots i_{N_c}}$ such that their Fock space structure is determined by the operators $B^{i_1 i_2 \ldots i_{N_c}}$ introduced in Subsect. IV B 1. Thus, we have[18]

$$\frac{1}{N_c!} \mathcal{B}^\dagger_{i_1 i_2 \ldots i_{N_c}} \mathcal{B}^{i_1 i_2 \ldots i_{N_c}} = \sum_{\lambda \in \text{``56''}} b^\dagger_\lambda b_\lambda, \qquad i_1, i_2, \ldots, i_{N_c} \in \{1, \ldots, 2N_F\} \tag{47}$$

with b_λ a "56" baryon field. In terms of these baryon fields the extension of the Lagrangian of Eq. (44) for an arbitrary number of colors reads

$$\mathcal{L}^{\text{SU}(N_c)}_{\text{SU}(6)} = \frac{1}{N_c!} \mathcal{B}^\dagger_{i_1 i_2 \ldots i_{N_c}} \left(i\nabla_0 - M + \frac{1}{2M} \vec{\nabla}^2 \right) \mathcal{B}^{i_1 i_2 \ldots i_{N_c}}, \qquad i_1, i_2, \ldots, i_{N_c} \in \{1, \ldots, 2N_F\} \tag{48}$$

where the covariant derivative acts on the baryon fields $\mathcal{B}$ as usual

$$(\nabla^\mu \mathcal{B})^{i_1 i_2 \ldots i_{N_c}} = (\partial^\mu \mathcal{B} + A_6^\mu * \mathcal{B})^{i_1 i_2 \ldots i_{N_c}} = \partial^\mu \mathcal{B}^{i_1 i_2 \ldots i_{N_c}} + [A_6^\mu]_k^{i_1} \mathcal{B}^{k i_2 \ldots i_{N_c}} + \ldots + [A_6^\mu]_k^{i_{N_c}} \mathcal{B}^{i_1 i_2 \ldots i_{N_c-1} k} \tag{49}$$

and therefore, we find thanks to the symmetry of the baryonic tensor $\mathcal{B}^{i_1 i_2 \ldots i_{N_c}}$

$$\mathcal{B}^\dagger_{i_1 i_2 \ldots i_{N_c}} (\nabla^\mu \mathcal{B})^{i_1 i_2 \ldots i_{N_c}} = \mathcal{B}^\dagger_{i_1 i_2 \ldots i_{N_c}} \left(\partial^\mu \mathcal{B}^{i_1 i_2 \ldots i_{N_c}} + N_c [A_6^\mu]_k^{i_1} \mathcal{B}^{k i_2 \ldots i_{N_c}} \right) \tag{50}$$

From Eqs. (48) and (50) we get[19]

$$\mathcal{L}^{\text{SU}(N_c)}_{\text{SU}(6)} = \mathcal{L}^{\text{SU}(N_c)}_{\text{kin, SU}(6)} + \mathcal{L}^{\text{SU}(N_c)}_{\text{WT, SU}(6)} \tag{51}$$

$$\mathcal{L}^{\text{SU}(N_c)}_{\text{kin, SU}(6)} = \sum_{\lambda \in \text{``56''}} b^\dagger_\lambda \left(i\partial_0 - M + \frac{1}{2M} \vec{\partial}^2 \right) b_\lambda \tag{52}$$

$$\mathcal{L}^{\text{SU}(N_c)}_{\text{WT, SU}(6)} = \frac{iN_c}{2f^2} [\Phi_6, \partial_0 \Phi_6]_k^j \frac{1}{N_c!} \mathcal{B}^\dagger_{j i_2 \ldots i_{N_c}} \mathcal{B}^{k i_2 \ldots i_{N_c}} \tag{53}$$

which corresponds to the decomposition of $\mathcal{L}^{\text{SU}(N_c)}_{\text{SU}(6)}$ into a baryon kinetic and an s-wave meson-baryon interaction (WT) terms. The WT Hamiltonian[20] $\left(\mathcal{H}^{\text{SU}(N_c)}_{\text{WT,SU}(6)} = -\mathcal{L}^{\text{SU}(N_c)}_{\text{WT,SU}(6)} \right)$, acting on meson-baryon Fock states $|r\rangle$, takes the form

$$\mathcal{H}^{\text{SU}(N_c)}_{\text{WT,SU}(6)} |r\rangle = \frac{\sqrt{s} - M}{2f^2} \times \mathcal{G}^{\text{SU}(N_c)}_{\text{WT,SU}(6)} |r\rangle \tag{54}$$

with $\mathcal{G}^{\text{SU}(N_c)}_{\text{WT,SU}(6)}$ defined in Eq. (36). From the results of Appendix C, we conclude that $\mathcal{H}^{\text{SU}(N_c)}_{\text{WT,SU}(6)}$ is diagonal in the spaces associated to the "56", "70", "700" and "1134" representation of SU(6) and with eigenvalues:

$$\bar{\lambda}_{\text{``56''}} = -4N_F, \qquad \bar{\lambda}_{\text{``70''}} = -2(N_c + 2N_F), \qquad \bar{\lambda}_{\text{``700''}} = 2N_c, \qquad \bar{\lambda}_{\text{``1134''}} = -2 \tag{55}$$

Note that for the case $N_F = 3$ and $N_c = 3$, we nicely recover $\bar{\lambda}_{56} = -12$, $\bar{\lambda}_{70} = -18$, $\bar{\lambda}_{700} = 6$ and $\bar{\lambda}_{1134} = -2$ (Eq. (24)). Remarkably, we see that in the "70" and "700" irreducible spaces, the SU(6) extension of the WT s-wave meson-baryon interaction scales as $\mathcal{O}(N_c^0)$, instead of the well known $\mathcal{O}(N_c^{-1})$ behavior for its SU(3) counterpart.

---

[17] For the SU(3) case $A_3^\mu * B_3$ reduces to the usual commutator. For SU(6), it will not be a commutator since while A_6^μ are dimension six traceless matrices, the B_6 baryon field is a fully symmetric tensor with N_c indices.

[18] With this convention $\mathcal{B}^{123 \ldots N_c} = \mathcal{B}^{213 \ldots N_c} = \ldots$ or $\mathcal{B}^{111 \ldots 1}/\sqrt{N_c!}$ are baryon fields with the usual normalization. This is because

$$B^{123 \ldots N_c} B^\dagger_{123 \ldots N_c} = 1, \qquad \text{while}$$

$$B^{111 \ldots 1} B^\dagger_{111 \ldots 1} = N_c! \tag{46}$$

Thus, for instance for $N_c = 3$, $\mathcal{B}^{111}/\sqrt{3!} = \Delta^{++}(S_z = +3/2)$.

[19] We have replaced $\vec{\nabla}^2$ by $\vec{\partial}^2$ since the difference between both operators does not contribute to s-wave.

[20] This is an abuse of notation, we really mean the on-shell scattering amplitude, at LO in the chiral expansion, which is used as the kernel, V, for the on-shell BSE.

3.   Large N_c SU(6) versus SU(3) WT interaction

It would seem that the additional N_c factor obtained here, as compared to the standard SU(3) calculation, comes only from a proper treatment of the baryon, namely, to use the correct N_c-dependent "56" representation instead of the 56. This is only partially true: another crucial ingredient has been the introduction of vector mesons in the scheme. Note that if one only considers pseudoscalar mesons, the interaction being s-wave, the various baryonic spin sectors will never mix and if one starts with nucleons, the "decuplet" states will not be seen. In order to further analyzed this point, let us write the meson field in the form

$$\Phi_6 = \frac{1}{2}\pi_\alpha\lambda_\alpha + \frac{1}{2}\rho_{\alpha i}\lambda_\alpha\sigma_i + \frac{1}{\sqrt{2N_F}}\omega_i\sigma_i := \sqrt{2}\phi_A t_A\,. \tag{56}$$

Here σ_i and λ_α are Pauli and (su(N_F) algebra) Gell-Mann matrices with $i = 1, 2, 3$ and $\alpha = 1, \ldots, N_F^2 - 1$, and π, ρ, ω are the hermitian meson fields corresponding to the pseudoscalar octect and the vector nonet. The matrices t_A are the SU($2N_F$) group generators in the fundamental representation, namely, $(\lambda_\alpha \otimes 1)/\sqrt{8}$, $(1 \otimes \sigma_i)/\sqrt{4N_F}$, and $(\lambda_\alpha \otimes \sigma_i)/\sqrt{8}$, and ϕ_A the associated meson fields with $A = 1, \ldots, (2N_F)^2 - 1$. For the matrix element $M_A B \to M_{A'} B'$ one then finds

$$
\begin{aligned}
\frac{1}{4}\langle M_{A'} B'|\mathcal{G}_{\mathrm{WT,SU(6)}}^{\mathrm{SU(N_c)}}|M_A B\rangle &= \frac{1}{(N_c - 1)!}([t_A, t_{A'}])^j{}_k \langle B'|B_{ji_2\ldots i_{N_c}}^\dagger B^{ki_2\ldots i_{N_c}}|B\rangle \\
&= ([t_A, t_{A'}])^j{}_k \langle B'|q_j^\dagger q^k|B\rangle \\
&= \langle B'|[G_A, G_{A'}]|B\rangle
\end{aligned}
\tag{57}
$$

where

$$G_A = (t_A)^j{}_k\, q_j^\dagger q^k \tag{58}$$

are the SU($2N_F$) generators on the baryon sector,

$$G_A = T_\alpha,\, S_i\,, G_{\alpha i}\,. \tag{59}$$

As we have discussed above, the matrix element (57) is generically of $\mathcal{O}(N_c)$. However, if A and A' are pseudoscalars, the baryonic matrix element couples to purely flavor generators. As a consequence, in the physically relevant case of B and B' being baryonic states with finite flavor (i.e., isospin and hypercharge of $\mathcal{O}(N_c^0)$), the matrix element turns out to be $\mathcal{O}(N_c^0)$ instead of $\mathcal{O}(N_c)$. A similar statement holds for $\omega B \to \omega' B'$ provided B and B' have finite spin, since S_i is the relevant operator in this case. For matrix elements of the type $\pi B \to \rho B'$, the commutation relation

$$[T_\alpha, G_{\beta i}] = if_{\alpha\beta\gamma}G_{\gamma i} \tag{60}$$

($f_{\alpha\beta\gamma}$ being the flavor structure constants) indicates that the driving operator is of the type $G_{\alpha i}$, which is $\mathcal{O}(N_c)$ even for finite spin-flavor baryons. (Note that the Casimir operator $T_A T_A$ has a common large value $\mathcal{O}(N_c^2)$, to wit, $N_c(N_c + 2N_F)(2N_F - 1)/(4N_F)$, for all states in the same irreducible representation "56".) For $\rho B \to \rho' B'$ the driving operator is

$$[G_{\alpha i}, G_{\beta j}] = \frac{i}{4}\delta_{ij}f_{\alpha\beta\gamma}T_\gamma + \frac{i}{2}\epsilon_{ijk}\left(\frac{1}{N_F}\delta_{\alpha\beta}S_k + d_{\alpha\beta\gamma}G_{\gamma k}\right) \tag{61}$$

so generically the matrix element between finite baryons will be large for $N_F \geq 3$.

As illustration, for two flavors and odd N_c, we can consider the "nucleon" state with spin and isospin 1/2

$$|N_{a\sigma}\rangle \propto \epsilon^{a_2 a_3}\epsilon^{\sigma_2\sigma_3}\cdots B_{a\sigma,a_2\sigma_2,\ldots}^\dagger|0\rangle\,, \qquad a, \sigma \in \{1, 2\} \tag{62}$$

consisting of a single quark carrying the spin and isospin of the baryon plus $(N_c - 1)/2$ pairs of quarks coupled to spin and isospin zero. An easy computation gives for the "nucleon" matrix elements corresponding to the generators T_α, S_i and $G_{\alpha i}$

$$\frac{1}{2}\tau_\alpha\,, \quad \frac{1}{2}\sigma_i\,, \quad \frac{1}{12}(N_c + 2)\tau_\alpha\sigma_i\,, \tag{63}$$

respectively, consistently with our previous remarks. Thus the extension to include vector mesons is indeed essential to activate the generic large N_c dependences found above.
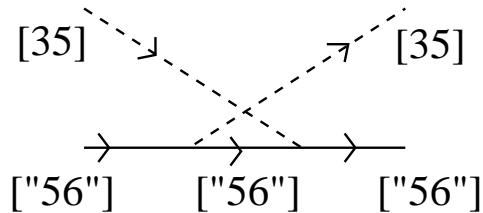
FIG. 2: Diagrammatic representation of the crossed nucleon pole-type term.

4. Crossed nucleon-pole terms

The $8_1 \otimes (8_2 \oplus 10_4)$ crossed nucleon pole-type terms, included among those depicted in Fig. 2, are believed to scale as $g_A^2/(Mf^2)$ and therefore behave as $\mathcal{O}(N_c^0)$ in the large N_c limit [10]. As we have just seen, the standard WT term scales as $\mathcal{O}(N_c^{-1})$, from its $1/f^2$ dependence, and therefore in the large N_c limit, it is a sub-leading correction to the crossed nucleon pole-type term. We have shown that this picture changes when the effects induced by vector mesons and the N_c dependence of the "56" irrep are considered, being incorrect within the "70" and "700" meson-baryon spaces. Furthermore, one might wonder whether the N_c−behavior of the crossed nucleon pole-type Hamiltonian, $\mathcal{H}_{\mathrm{CN,SU(6)}}^{\mathrm{SU(N_c)}}$, depends on the SU(6) representation, as it happens in the case of the WT interaction. To answer such a question, it would be useful to have a SU(6) symmetric model, for arbitrary N_c, for this interaction term. Because of the p-wave nature and the spin dependence of the MBB coupling, this might not be possible, and at least we have not been able to come up with a consistent model. Likely, the spaces that diagonalize this interaction do not form SU(6) irreps. This is because in general spin-flavor symmetry is not exact for excited baryons even in the large N_c [28]. However, phenomenologically for $N_c = 3$, the spin–flavor symmetry breaking term is small and comparable in magnitude to that of the $1/N_c$ corrections [28]. Even assuming that $\mathcal{H}_{\mathrm{CN,SU(6)}}^{\mathrm{SU(N_c)}}$ scales as $\mathcal{O}(N_c^0)$ in the "70" irrep space, the WT term provides the large N_c dominant contribution in this space, since both $\mathcal{H}_{\mathrm{CN,SU(6)}}^{\mathrm{SU(N_c)}}$ and $\mathcal{H}_{\mathrm{WT,SU(6)}}^{\mathrm{SU(N_c)}}$ would follow the same N_c scaling law and the latter one is dominant for $N_c = 3$.

5. Resonance Masses and Widths from $\mathcal{H}_{\mathrm{WT,SU(6)}}^{\mathrm{SU(N_c)}}$

The resonance equation reads,

$$\beta(s)\Big|_{s=s_R \equiv M_R^2 - iM_R\Gamma_R} = \bar{\lambda}_{\phi_{SU(6)}}, \quad \phi_{SU(6)} = \text{"56", "70", "700", "1134"}, \qquad M_R > M, \Gamma_R > 0 \tag{64}$$

There are solutions only for negative eigenvalues, $\bar{\lambda}_{\text{"70"}}$, $\bar{\lambda}_{\text{"56"}}$ and $\bar{\lambda}_{\text{"1134"}}$, and as before, the "70" irrep of SU(6) leads to the most attractive s-wave meson–baryon interaction, and it becomes the only non-vanishing WT contribution in the strict limit $N_c \to \infty$.

The approximated relations of Eqs. (16) and (17), having in mind that $\bar{\lambda}_{\text{"70"}} \sim N_c$, lead to new scaling relations

$$M_R - M, \Gamma_R = \mathcal{O}(N_c^0) \tag{65}$$

for the "70"–plet. From the above N_c−behavior one deduces that widths and excitation resonance energies, behave now as order one, as predicted by Witten almost 30 years ago. For the "56" and "1134"-plets, the scenario has not been modified, and we are still in the same situation as in Subsect. IV A, with widths and excitation energies growing as $\sqrt{N_c}$. That is, resonances would disappear, since they become wider and heavier as N_c increases. The different N_c behaviors exhibited by resonance masses and widths, deduced from the WT Lagrangian, in each irreducible space can be appreciated in Fig. 3.

The crossed nucleon pole-type contribution (CNPC) might change this picture. As discussed at the end of the previous Subsect., we believe that the CNPC will never be dominant in the "70" irrep space. However, if we focus on the "56" and "1134" resonance plets, the WT interaction could be subleading in the large N_c limit, if the crossed nucleon pole force would scale as $\mathcal{O}(N_c^0)$ in those spaces. If this latter interaction were repulsive, the "56" and "1134" plets of resonances would dissapear at sufficiently large values of N_c, while if it were attractive, widths and excitation resonance energies would behave as order $\mathcal{O}(N_c^0)$. If the CNPC scales as $\mathcal{O}(N_c^{-1})$ or lower in any of those spaces, the corresponding plet of resonances will either never be formed (if the combined WT contribution plus CNPC
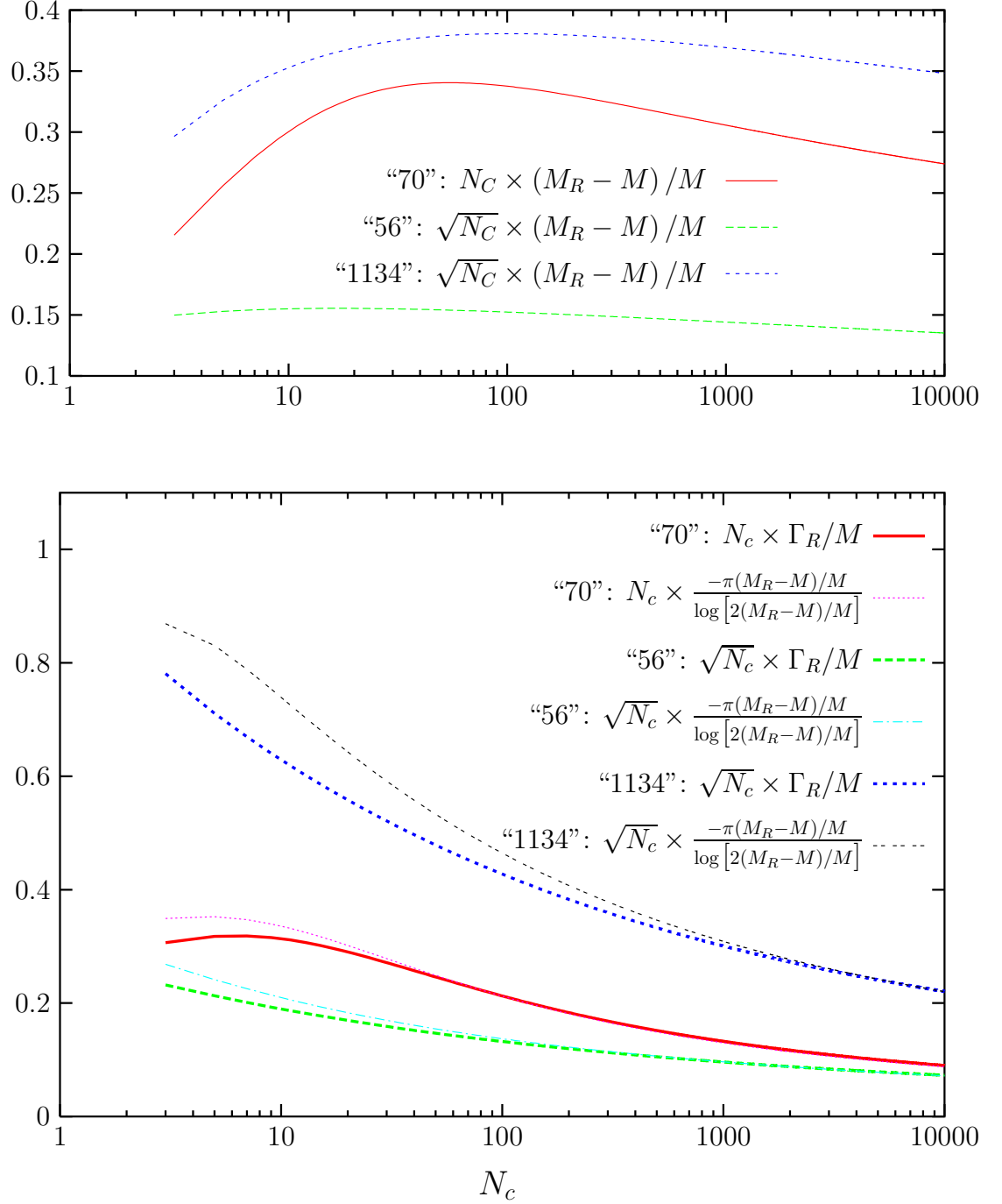
FIG. 3: SU(6) "70", "56" and "1134" resonance masses (M_R) and widths (Γ_R) obtained from $\mathcal{H}_{\mathrm{WT,SU(6)}}^{\mathrm{SU(N_c)}}$, as a function of N_c.

is repulsive) or they will dissappear (become wider and heavier) at sufficiently large values of N_c. For illustrative purposes in the appendix D we develop a toy model for the CNPC, somehow unrealistic since it neglects the spin dependence of the couplings. However, this model shows that is feasible to have situations in which the CNPC N_c behavior depends on the particular irrep space and that the WT term provides the large N_c dominant contribution in the "70", "700" and "1134" irreps spaces.

Nevertheless, there will be also d-wave mixings or new s-wave meson-baryon couplings[21] which might also modify the whole picture.

V.   CONCLUDING REMARKS

One of the interesting results of this paper is the Lagrangian in (48), which accounts for the $SU(2N_F)$ symmetric version of the WT interaction, for arbitrary N_c and N_F, as well as its particular case (44) for $N_c = N_F = 3$. As we have noted above, due to the action of the covariant derivative (49–50), it follows that generically (that its, prior to projection to particular sectors) such extended WT amplitude scales as $\mathcal{O}(N_c^0)$, instead of $\mathcal{O}(N_c^{-1})$, characteristic of the standard WT $SU(N_F)$ symmetric amplitude. Two factors combine to achieve this result. First, in a large N_c world, the flavor representation of the lightest baryon depends on N_c, and the standard commutator $[A_3, B_3]$ becomes a covariant derivative, which acts on each baryon index in turn. Less technically, and using a graphic quark model picture for the baryon, in the WT interaction, the meson-meson pair may couple to any of the N_c quarks of the baryon, allowing a further N_c factor in the amplitude. This mechanism is also at work in the p-wave pseudoscalar-baryon coupling and gives the standard large N_c scaling $g_A = \mathcal{O}(N_c)$. However, in the standard $SU(N_F)$ case, the pseudoscalar-pseudoscalar amplitude depends on the flavor generator baryonic matrix element, which is $\mathcal{O}(N_c)$ for generic baryons but $\mathcal{O}(N_c^0)$ for the relevant baryons, namely, those with finite flavor and spin. The second essential factor is thus the inclusion of vector mesons. They coupled to spin-flavor generators which are $\mathcal{O}(N_c)$ even for baryons with finite flavor and spin. As a consequence, in the "70" and "700" $SU(6)$ irreducible spaces, the $SU(6)$ extension of the WT s-wave meson-baryon interaction scales as $\mathcal{O}(N_c^0)$, instead of the well known $\mathcal{O}(N_c^{-1})$ behavior for its $SU(3)$ counterpart. However, the WT interaction behaves as order $\mathcal{O}(N_c^{-1})$ within the "56" and "1134" meson-baryon spaces.

From constituent quark model considerations, it is accepted that the excited baryon states that correspond to the first radial and orbital excitations fit well into respectively a positive parity 56^+ and negative parity 70^- irreps [37]. From the study carried out in this work, we confirm the existence of a narrow $70-$plet of negative parity resonances, which masses depart from the lowest–lying 56 multiplet baryon mass by the typical amount of a meson mass. The non existence, in the large N_c limit, of negative parity 56^- resonances can be understood if the crossed nucleon pole force is repulsive or if it is attractive, it should decrease at least as $\mathcal{O}(N_c^{-1})$ in this irrep. Thus, one of the two $\frac{1}{2}^-$ $SU(3)$ octets of s-wave baryon resonances found in Ref. [4] would dissappear in the large N_c limit, since there exists only one 8_2 multiplet included into the 70 representation of $SU(6)$. However, the $SU(3)$ singlet spin-parity $\frac{1}{2}^-$ resonance will become presumably narrow, in the large N_c limit, thanks to its 70 component.

On the other hand, the WT interaction predicts for the 1134-plet that both excitation energies and widths grow with an approximate $\sqrt{N_c}$ rate. This presumably implies that these states do not appear in the large N_c-QCD spectrum, which most likely reflects the existence of exotic, f.i. $qqqq\bar{q}$, components[22] for $N_c = 3$. Note that exotic components are certainly included in the $SU(3)$ antidecuplet belonging to the 1134 $SU(6)$ representation.

Finally, as we have noted previously, in the present approach the power-like $1/N_c$ expansion comes with subleading logarithmic corrections (see for instance Eqs. (18-19)), which are believed to be spurious. It remains to be studied in deep, how the logarithmic corrections depend on details of the RS prescription, the baryon wavefunction renormalization, etc. This subject is worth studying and clearly it would be highly desirable to consider this issue for future research, however, is beyond the scope of the present work.

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[21] Hamiltonians of the form (symbolically) $\left((M^\dagger \otimes M)_{35_s} \otimes (B^\dagger \otimes B)_{35}\right)_1$ or $\left((M^\dagger \otimes M)_1 \otimes (B^\dagger \otimes B)_1\right)_1$.

[22] A similar analysis has been carried out in Ref. [38] in the the meson–meson context. There, it was found that while the ρ and K^* meson widths have the $q\bar{q}$ expected behavior ($\mathcal{O}(1/N_c)$), the s-wave σ and κ poles show a totally different behavior, since their widths grow with N_c, in conflict with a $q\bar{q}$ interpretation, and leaving room for sizeable tetra-quark or glue ball components.

APPENDIX A: SOME DETAILS ON $\chi-$BS(3)

For a given isospin–strangeness sector (for simplicity we will omit the IS upper indices), the element in the position aa $(a = 1, ..N^{IS})$ of the diagonal matrix loop function $J(\sqrt{s})$ reads [19]

$$J_a(\sqrt{s}) = \frac{(\sqrt{s} + M_a)^2 - m_a^2}{2\sqrt{s}} \mathcal{J}_a(s) \tag{A1}$$

where $M_a(m_a)$ is the baryon (meson) mass in the channel a and

$$\mathcal{J}_a(s) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_a^2} \frac{1}{(P - q)^2 - M_a^2} = \bar{\mathcal{J}}_a(s) + \mathcal{J}_a(s = (m_a + M_a)^2) \tag{A2}$$

with $P^2 = s$, $\mathcal{J}_a(s = (m_a + M_a)^2)$ a divergent quantity and the finite function $\bar{\mathcal{J}}_a(s)$ given by

$$\bar{\mathcal{J}}_a(s) = \frac{1}{(4\pi)^2} \left\{ \left[\frac{M_a^2 - m_a^2}{s} - \frac{M_a - m_a}{M_a + m_a} \right] \ln \frac{M_a}{m_a} + L_a(s) \right\} \tag{A3}$$

and for real s and above threshold, $s > (m_a + M_a)^2$, we have

$$L_a(s) \equiv L_a(s + i\epsilon) = \frac{\lambda^{1/2}(s, m_a^2, M_a^2)}{s} \left\{ \log \left[\frac{1 + \sqrt{\frac{s - s_+}{s - s_-}}}{1 - \sqrt{\frac{s - s_+}{s - s_-}}} \right] - i\pi \right\} \tag{A4}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$, the pseudothreshold and threshold variables are $s_{\mp} = (M_a \mp m_a)^2$ respectively, and the logarithm is taken to be real. Note that $L_a(s_+) = 0$. The definition of the $L_a(s)$ in the whole complex plane and the definition of its different Riemann sheets can be found in Ref. [19].

APPENDIX B: RESTRICTION OF $\mathcal{L}_{\text{SU}(6)}$ TO THE $8_1 \otimes 8_2$ SECTOR

In this appendix we check that the restriction of the SU(6) Lagrangian of Eq. (44) to the $8_1 \otimes 8_2$ sector reproduces that given in Eq. (43), which provides the standard WT amplitudes of Eq. (2). We will do it for three flavors, though the extension to N_F flavors is straightforward. We will start studying the meson part of the Lagrangian.

The operator M_j^i (see Eq. (34)) is essentially the annihilation part of the meson matrix $[\Phi_6]_j^i$. The projection $(\equiv (\Phi_6)_3)$ of Φ_6 to the 8_1 octet is

$$[(\Phi_6)_3]_b^a = \frac{1}{\sqrt{2}} \sum_{\sigma=1,2} [\Phi_6]_{b\sigma}^{a\sigma} = \frac{1}{\sqrt{2}} [\text{Tr}_{\text{SU}(2)}(\Phi_6)]_b^a, \quad a, b = 1, 2, 3 \tag{B1}$$

that is, in the above equation a, b account for the quark and antiquark flavors, while for quark (antiquark), $\sigma = 1$ corresponds to $S_z = 1/2 \, (-1/2)$ and $\sigma = 2$ corresponds to $S_z = -1/2 \, (1/2)$. Thus, for instance $[(\Phi_6)_3]_1^1 = ([\Phi_6]_1^1 + [\Phi_6]_4^4) / \sqrt{2} = \pi^0/\sqrt{2} + \eta/\sqrt{6}$, as one can deduce from $(M_1^1 + M_4^4)/\sqrt{2}$. Reciprocally, the contribution $(\equiv (\Phi_3)_6)$ of the 8_1 octet to Φ_6 is

$$[(\Phi_3)_6]_{b\sigma'}^{a\sigma} = \frac{1}{\sqrt{2}} [\Phi_3]_b^a \times \delta_{\sigma'}^{\sigma} \tag{B2}$$

and thus we have $(\Phi_3)_6 = (\Phi_3 \otimes 1_{\text{SU}(2)}) / \sqrt{2}$. We see that the consistency relations $((\Phi_3)_6)_3 = \Phi_3$ and $\text{Tr}[(\Phi_3')_6 \cdot (\Phi_3)_6] = \text{Tr}[\Phi_3' \cdot \Phi_3]$ are trivially satisfied. On the other hand, when $\Phi_6 = (\Phi_3)_6$ we must require $U_6 = U_3 \otimes 1_{\text{SU}(2)}$, then

$$f_6 = \frac{f}{\sqrt{2}} \tag{B3}$$

Besides, it is also satisfied that

$$(A_3^\mu)_6 = A_3^\mu \otimes 1_{\text{SU}(2)} \tag{B4}$$

For SU(3) and $N_c = 3$, the B_3 field is normalized such that

$$\text{Tr}(B_3^\dagger B_3) = \sum_\lambda b_\lambda^\dagger b_\lambda = p^\dagger p + n^\dagger n + \Lambda^\dagger \Lambda + \Sigma^{0\dagger}\Sigma^0 + \Sigma^{+\dagger}\Sigma^+ + \Sigma^{-\dagger}\Sigma^- + \Xi^{0\dagger}\Xi^0 + \Xi^{-\dagger}\Xi^- \tag{B5}$$

with b_λ a 8_2 baryon field. The normalization above is consistent with that adopted for the SU(6) fields in Eq. (47). On the other hand, we will denote the B_3 field indices as $[B_3]^a{}_b{}^\sigma$, $a, b = 1, 2, 3$ and $\sigma = \pm$ accounts for the spin third component of the baryon ($\pm 1/2$), while for B_6 we will use $\mathcal{B}^{ijk}$, $i, j, k = 1, \ldots 6$. The projection ($\equiv (B_6)_3$) of B_6 to the 8_2 octet is (up to a global sign)

$$[(B_6)_3]^a{}_{.\ b}{}^\sigma = \frac{1}{2\sqrt{3}} \mathcal{B}^{a\sigma\ c\sigma' d\sigma''} \epsilon_{bcd}\, \epsilon_{\sigma'\sigma''}, \quad a, b, c, d = 1, 2, 3, \quad \sigma, \sigma', \sigma'' = \pm. \tag{B6}$$

with $\epsilon_{++} = \epsilon_{--} = 0$ and $\epsilon_{+-} = -\epsilon_{-+} = 1$. This definition satisfies $[(B_6)_3]^a{}_{.\ a}{}^\sigma = 0$, as required to ensure that those states belong to the octet irrep of SU(3), and the normalization can be easily tested by checking, for instance, that

$$[(B_6)_3^\dagger]^{1+}_3 \, [(B_6)_3]^{1+}_3 = 1, \qquad [(B_6)_3^\dagger]^{1+}_1 \, [(B_6)_3]^{1+}_1 = 2/3 \tag{B7}$$

which correctly accounts for the normalization of a diagonal, $p(S_z = +1/2)$, and of a non-diagonal, $\left\{ \sqrt{3}\Sigma^0(S_z = +1/2) + \Lambda(S_z = +1/2) \right\}/\sqrt{6}$, elements of the B_3 matrix. Reciprocally, the contribution ($\equiv (B_3)_6$) of the 8_2 octet to B_6 is

$$(B_3)_6 \to (\mathcal{B}_3)_6^{a\sigma\ b\sigma' c\sigma''} = \frac{1}{\sqrt{3}} \left([B_3]^a{}_{.\ d}{}^\sigma \epsilon^{dbc}\epsilon^{\sigma'\sigma''} + [B_3]^b{}_{.\ d}{}^{\sigma'} \epsilon^{dac}\epsilon^{\sigma\sigma''} + [B_3]^c{}_{.\ d}{}^{\sigma''} \epsilon^{dab}\epsilon^{\sigma\sigma'} \right) \tag{B8}$$

With these definitions it is straightforward to prove the consistency relations $((B_3)_6)_3 = B_3$, and[23]

$$(B_3^\dagger)_6 \cdot (B_3)_6 = [B_3^\dagger]^a{}_{.\ b\sigma}[B_3]^b{}_{.\ a}{}^\sigma = \text{Tr}(B_3^\dagger B_3), \tag{B12}$$

$$\left[(A_3^\mu)_6 * (B_3)_6 \right]^{a\sigma\ b\sigma' c\sigma''} = \frac{1}{\sqrt{3}} \left([A_3^\mu, B_3]^a{}_{.\ d}{}^\sigma \epsilon^{dbc}\epsilon^{\sigma'\sigma''} + [A_3^\mu, B_3]^b{}_{.\ d}{}^{\sigma'} \epsilon^{dac}\epsilon^{\sigma\sigma''} + [A_3^\mu, B_3]^c{}_{.\ d}{}^{\sigma''} \epsilon^{dab}\epsilon^{\sigma\sigma'} \right)$$

$$= (A_3^\mu * B_3)_6^{a\sigma\ b\sigma' c\sigma''}, \qquad \text{with}\ \ A_3^\mu * B_3 = [A_3^\mu, B_3] \tag{B13}$$

where $B_6^\dagger \cdot B_6 \equiv \frac{1}{3!}\mathcal{B}^\dagger_{i_1 i_2 i_3}\mathcal{B}^{i_1 i_2 i_3}$. From the above equations, it follows

$$(B_3^\dagger)_6 \cdot ((A_3^\mu)_6 * (B_3)_6) = (B_3^\dagger)_6 \cdot (A_3^\mu * B_3)_6 = \text{Tr}(B_3^\dagger \cdot (A_3^\mu * B_3)) = \text{Tr}(B_3^\dagger[A_3^\mu, B_3]) \tag{B14}$$

This latter equation together Eq. (B12), shows that the restriction of the Lagrangian in Eq. (44) to the $8_1 \otimes 8_2$ sector reproduces that given in Eq. (43).

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[23] To deduce Eq. (B13) we have made use of

$$\left[(A_3^\mu)_6 * (B_3)_6 \right]^{a\sigma\ b\sigma' c\sigma''} = [(A_3^\mu)_6]^{i_1}_k [(\mathcal{B}_3)_6]^{k i_2 i_3} + [(A_3^\mu)_6]^{i_2}_k [(\mathcal{B}_3)_6]^{i_1 k i_3} + [(A_3^\mu)_6]^{i_3}_k [(\mathcal{B}_3)_6]^{i_1 i_2 k}$$

$$= \frac{1}{\sqrt{3}} \left([A_3^\mu]^a_e [B_3]^e{}_{.\ d}{}^\sigma \epsilon^{dbc}\epsilon^{\sigma'\sigma''} + [A_3^\mu]^a_e [B_3]^b{}_{.\ d}{}^{\sigma'} \epsilon^{dec}\epsilon^{\sigma\sigma''} + [A_3^\mu]^a_e [B_3]^c{}_{.\ d}{}^{\sigma''} \epsilon^{deb}\epsilon^{\sigma\sigma'} + \begin{smallmatrix} a & \leftrightarrow & b \\ \sigma & \leftrightarrow & \sigma' \end{smallmatrix} + \begin{smallmatrix} a & \leftrightarrow & c \\ \sigma & \leftrightarrow & \sigma'' \end{smallmatrix} \right)$$

$$= \frac{1}{\sqrt{3}} \left([A_3^\mu]^a_e [B_3]^e{}_{.\ d}{}^\sigma \epsilon^{dbc}\epsilon^{\sigma'\sigma''} + 2[A_3^\mu]^a_e [B_3]^b{}_{.\ d}{}^{\sigma'} \epsilon^{dec}\epsilon^{\sigma\sigma''} + \begin{smallmatrix} a & \leftrightarrow & b \\ \sigma & \leftrightarrow & \sigma' \end{smallmatrix} + \begin{smallmatrix} a & \leftrightarrow & c \\ \sigma & \leftrightarrow & \sigma'' \end{smallmatrix} \right)$$

$$= \frac{1}{\sqrt{3}} \left([A_3^\mu]^a_e [B_3]^e{}_{.\ d}{}^\sigma \epsilon^{dbc}\epsilon^{\sigma'\sigma''} + 2[A_3^\mu]^b_e [B_3]^a{}_{.\ d}{}^\sigma \epsilon^{dec}\epsilon^{\sigma'\sigma''} + \begin{smallmatrix} a & \leftrightarrow & b \\ \sigma & \leftrightarrow & \sigma' \end{smallmatrix} + \begin{smallmatrix} a & \leftrightarrow & c \\ \sigma & \leftrightarrow & \sigma'' \end{smallmatrix} \right) \tag{B9}$$

where we associate to the SU(6) indices i_1, i_2 and i_3, the SU(3)$\times$ SU(2) $a\sigma$, $b\sigma'$ and $c\sigma''$ ones, respectively. Because of the antisymmetric tensor $\epsilon^{\sigma'\sigma''}$, and the underlying symmetry under the interchange ($i_1 \leftrightarrow i_2$), one should only keep the antisymmetric contribution, when the SU(3) indices b and c are interchanged, of the second term in Eq. (B9), thus we find

$$\left[(A_3^\mu)_6 * (B_3)_6 \right]^{a\sigma\ b\sigma' c\sigma''} = \frac{1}{\sqrt{3}} \left([A_3^\mu]^a_e [B_3]^e{}_{.\ d}{}^\sigma \epsilon^{dbc}\epsilon^{\sigma'\sigma''} + [A_3^\mu]^b_e [B_3]^a{}_{.\ d}{}^\sigma \epsilon^{dec}\epsilon^{\sigma'\sigma''} - [A_3^\mu]^c_e [B_3]^a{}_{.\ d}{}^\sigma \epsilon^{deb}\epsilon^{\sigma'\sigma''} + \begin{smallmatrix} a & \leftrightarrow & b \\ \sigma & \leftrightarrow & \sigma' \end{smallmatrix} + \begin{smallmatrix} a & \leftrightarrow & c \\ \sigma & \leftrightarrow & \sigma'' \end{smallmatrix} \right)$$

$$= \frac{1}{\sqrt{3}} \left([A_3^\mu, B_3]^a{}_{.\ d}{}^\sigma \epsilon^{dbc}\epsilon^{\sigma'\sigma''} + \begin{smallmatrix} a & \leftrightarrow & b \\ \sigma & \leftrightarrow & \sigma' \end{smallmatrix} + \begin{smallmatrix} a & \leftrightarrow & c \\ \sigma & \leftrightarrow & \sigma'' \end{smallmatrix} \right) \tag{B10}$$

thanks to the relation

$$[A_3^\mu]^b_e\, \epsilon^{dec} - [A_3^\mu]^c_e\, \epsilon^{deb} = -[A_3^\mu]^d_e\, \epsilon^{ebc} \tag{B11}$$

APPENDIX C: EIGENVALUES OF $\mathcal{G}_{\mathrm{WT,SU(6)}}^{\mathrm{SU(N_c)}}$

Since the 35-meson–"56"-baryon hamiltonian $\mathcal{H}_{\mathrm{WT,SU(6)}}^{\mathrm{SU(N_c)}}$ (or equivalently $\mathcal{G}_{\mathrm{WT,SU(6)}}^{\mathrm{SU(N_c)}}$) is a SU(6) scalar, its eigenvectors follow from the SU(6) reduction given in Eq. (27).

A meson-baryon state belonging to the "56" representation is of the form

$$\sum_{P \in S(N_c)} M_{P(i_1)}^{\dagger m} B_{m P(i_2)...P(i_{N_c})}^{\dagger} |0\rangle \tag{C1}$$

where $|0\rangle$ is the ground state in the Fock space (state containing zero hadrons). All of these states are eigenstates of $\mathcal{G}_{\mathrm{WT,SU(6)}}^{\mathrm{SU(N_c)}}$, with eigenvalue proportional to $\bar{\lambda}_{\text{"56"}}$, which might depend on N_c. One of these $\begin{pmatrix} N_c + 5 \\ 5 \end{pmatrix}$ states is

$$|1\rangle = M_1^{\dagger m} B_{m11...1}^{\dagger} |0\rangle \tag{C2}$$

One finds,

$$\begin{aligned} \mathcal{G}_{\mathrm{WT,SU(6)}}^{\mathrm{SU(N_c)}} |1\rangle &= \frac{2}{(N_c-1)!} \left(M_k^{\dagger j} \underline{M_j^i} M_1^{\dagger m} - M_j^{\dagger i} \underline{M_k^j} M_1^{\dagger m} \right) B_{ii_2...i_{N_c}}^{\dagger} \underline{B^{ki_2...i_{N_c}}} B_{m1...1}^{\dagger} |0\rangle \\ &= (-4 N_F) |1\rangle \end{aligned} \tag{C3}$$

where we have made use of the Wick's contractions given in Eq. (35).

Therefore $\bar{\lambda}_{\text{"56"}} = -4 N_F$. Analogously and using the states

$$\begin{aligned} |2\rangle &= \left(M_2^{\dagger i} B_{i11...1}^{\dagger} - M_1^{\dagger i} B_{i21...1}^{\dagger} \right) |0\rangle \\ |3\rangle &= M_1^{\dagger 2} B_{111...1}^{\dagger} |0\rangle \\ |4\rangle &= \left(M_2^{\dagger 3} B_{111...1}^{\dagger} - M_1^{\dagger 3} B_{211...1}^{\dagger} \right) |0\rangle \end{aligned} \tag{C4}$$

for the "70", "700" and "1134" eigenspaces respectively, we find the eigenvalues of $\mathcal{G}_{\mathrm{WT,SU(6)}}^{\mathrm{SU(N_c)}}$ in these spaces. Thus, we finally conclude

$$\left[\bar{\lambda}_{\text{"56"}}, \bar{\lambda}_{\text{"70"}}, \bar{\lambda}_{\text{"700"}}, \bar{\lambda}_{\text{"1134"}} \right] = \left[-4 N_F, -2(N_c + 2 N_F), 2 N_c, -2 \right] \tag{C5}$$

APPENDIX D: A TOY MODEL FOR THE N_c DEPENDENCE OF THE $\mathcal{H}_{CN}$ SU(6) EIGENVALUES

For illustrative purposes, in what follows we develop a simple model, where we ignore the spin dependence of the couplings. Symbolically, the crossed nucleon pole-type hamiltonian, $\mathcal{H}_{CN}$, might take the form (see diagram of Fig. 2)

$$\mathcal{H}_{CN} = \left((M^{\dagger} \otimes B)_{\text{"56"}} \otimes (M \otimes B^{\dagger})_{\text{"56*"}} \right)_1 \tag{D1}$$

Since

$$(M^{\dagger} \otimes B)_{\text{"56"}}^{i_1 i_2 ... i_{N_c}} = \frac{1}{N_c!} \sum_{P \in S_{N_c}} B^{j P(i_2) P(i_3)...P(i_{N_c})} M_j^{\dagger P(i_1)} \tag{D2}$$

the group structure, $\mathcal{G}_{\mathrm{CN,SU(6)}}^{\mathrm{SU(N_c)}}$, of the crossed nucleon pole-type hamiltonian, up to constant factors and within this simplified model, would take the form

$$\begin{aligned} \mathcal{G}_{\mathrm{CN,SU(6)}}^{\mathrm{SU(N_c)}} &= \frac{1}{(N_c-1)!} \frac{1}{N_c!} : \sum_{P \in S_{N_c}} B^{j P(i_2) P(i_3)...P(i_{N_c})} M_j^{\dagger P(i_1)} B_{l i_2 i_3 ... i_{N_c}}^{\dagger} M_{i_1}^l : \\ &= : \frac{1}{N_c!} B^{j i_2 i_3 ... i_{N_c}} M_j^{\dagger k} B_{l i_2 i_3 ... i_{N_c}}^{\dagger} M_k^l + \frac{N_c - 1}{N_c!} B^{j k i_3 ... i_{N_c}} M_j^{\dagger r} B_{l r i_3 ... i_{N_c}}^{\dagger} M_k^l : \end{aligned} \tag{D3}$$

From the eigenvalues of $\mathcal{G}_{\mathrm{CN,SU(6)}}^{\mathrm{SU(N_c)}}$, we obtain the following proportionality relations for those of $\mathcal{H}_{\mathrm{CN,SU(6)}}^{\mathrm{SU(N_c)}}$ in this model[24]

$$\left[\lambda_{\text{``}56\text{''}}^{\mathrm{CN}}, \lambda_{\text{``}70\text{''}}^{\mathrm{CN}}, \lambda_{\text{``}700\text{''}}^{\mathrm{CN}}, \lambda_{\text{``}1134\text{''}}^{\mathrm{CN}}\right] \propto \left[N_c + 2N_F - 1 - \frac{N_c}{2N_F} - \frac{2N_F}{N_c}, -1 - \frac{2N_F}{N_c}, 1, -\frac{1}{N_c}\right]. \tag{D4}$$

We see that in the large N_c limit both, $\lambda_{\text{``}70\text{''}}^{\mathrm{CN}}/\lambda_{\text{``}56\text{''}}^{\mathrm{CN}}$ and $\lambda_{\text{``}700\text{''}}^{\mathrm{CN}}/\lambda_{\text{``}56\text{''}}^{\mathrm{CN}}$, behave as $\mathcal{O}(1/N_c)$, while $\lambda_{\text{``}1134\text{''}}^{\mathrm{CN}}/\lambda_{\text{``}56\text{''}}^{\mathrm{CN}}$ is suppressed by N_c^{-2}. Thus in this model, we find that the crossed nucleon pole-type contribution depends on the SU(6) representation, being the "56" the dominant one. Assuming that $\lambda_{\text{``}56\text{''}}^{\mathrm{CN}}$ scales as $\mathcal{O}(N_c)$, we find that the WT term provides the large N_c dominant contribution in the "70", "700" and "1134" irreducible representation spaces. Indeed within this toy model, the crossed nucleon pole-type contributions in those spaces are suppressed by $1/N_c$ with respect the WT ones.

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