Can One Distinguish $\tau-$Neutrinos from Antineutrinos in Neutral-Current Pion Production Processes?

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A potential way to distinguish $\tau-$neutrinos from antineutrinos, below the $\tau-$production threshold, but above the pion production one, is presented. It is based on the different behavior of the neutral current pion production off the nucleon, depending on whether it is induced by neutrinos or antineutrinos. This procedure for distinguishing $\tau-$neutrinos from antineutrinos neither relies on any nuclear model, nor it is affected by any nuclear effect (distortion of the outgoing nucleon waves, etc...). We show that neutrino-antineutrino asymmetries occur both in the totally integrated cross sections and in the pion azimuthal differential distributions. To define the asymmetries for the latter distributions we just rely on Lorentz-invariance. All these asymmetries are independent of the lepton family and can be experimentally measured by using electron or muon neutrinos, due to the lepton family universality of the neutral current neutrino interaction. Nevertheless and to estimate their size, we have also used the chiral model of *hep-ph/0701149* at intermediate energies. Results are really significant since the differences between neutrino and antineutrino induced reactions are always large in all physical channels.

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I. INTRODUCTION

Distinguishing $\tau-$neutrinos from antineutrinos, can be easily done above the $\tau-$production threshold energy, since then the charged–current (CC) channel is open. Conservation of lepton number implies that neutrinos generate a tau lepton, while an antineutrino produces an antilepton. The charge of the tau lepton unambiguously reveals the nature of the incident neutrino beam. Below the $\tau-$threshold, $\tau-$neutrinos or antineutrinos only interact with matter via neutral–current (NC) driven processes, and the outgoing lepton is also a neutrino or antineutrino, and the difference does not manifest itself in this clear way.

Nonetheless, it is out of any doubt the great interest of distinguishing between neutrinos and antineutrinos in different scenarios: CP–violation in ν_μ–ν_τ oscillations [1], supernova–neutrino physics [2]^1, ...

Very recently, Jachowicz and collaborators suggested that in neutrino–nucleus processes, the helicity–related differences between neutrino and antineutrinos induce some asymmetries in the polarization of the ejected nucleons [3]. From this fact, the authors of this reference conclude that these asymmetries represent a potential way to distinguish neutrinos from antineutrinos in NC neutrino scattering on nuclei.

We present in this letter an alternative manner to distinguish $\tau-$neutrinos from antineutrinos, below the $\tau-$production threshold ($\approx$ 3.5 GeV in the Laboratory (LAB) frame for production off the nucleon), but above the pion production one ($\approx$ 0.15 GeV in the LAB frame). The method is based on the different behavior of the NC pion production reaction in the nucleon, depending on whether it is induced by neutrinos or antineutrinos. This procedure for distinguishing $\tau-$neutrinos from antineutrinos neither relies on any nuclear model, nor it is affected by any nuclear effect (distortion of the outgoing nucleon waves, etc...). We show that neutrino-antineutrino asymmetries occur both in the totally integrated cross sections and in the pion azimuthal differential distributions. To define the asymmetries for the latter distributions we just rely on Lorentz-invariance, and experimentally it requires determining the neutrino scattering plane, detecting either a neutral or a positively charged pion and measuring its momentum. Since the outgoing neutrino will not likely be detected, to fix the neutrino scattering plane would also require measuring the outgoing nucleon momentum and some knowledge of the incoming neutrino momentum.

All these asymmetries are independent of the lepton family and can be experimentally measured by using electron or muon neutrinos, due to the lepton family universality of the NC neutrino interaction. Nevertheless and to estimate their size, we have used the chiral model of Ref. [4] to compute them up to neutrino/antineutrino energies of 2 GeV.

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^{1} At typical supernova energies, ν_τ neutrinos do not participate in CC reactions
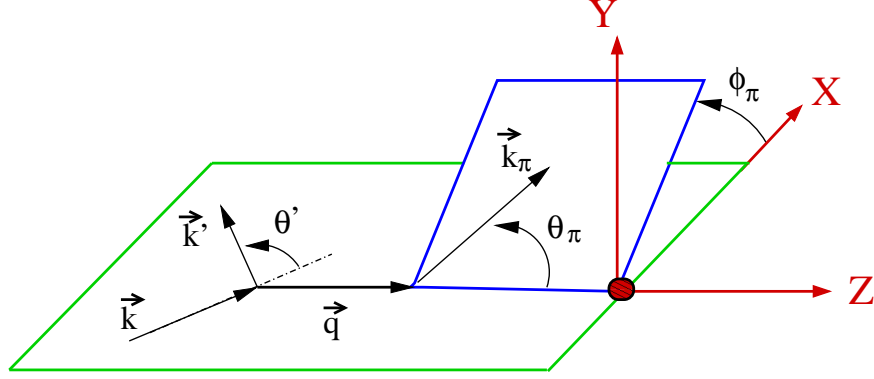
II.   KINEMATICS AND CROSS SECTIONS

FIG. 1: Definition of the different kinematical variables used through this work.

We will focus on the neutrino–pion production reaction off the nucleon driven by neutral currents,

$$\nu_l(k) + N(p) \to \nu_l(k') + N(p') + \pi(k_\pi) \tag{1}$$

though the generalization of the obtained expressions to antineutrino induced reactions is straightforward.

The unpolarized differential cross section, with respect to the outgoing neutrino and pion kinematical variables is given in the LAB frame (kinematics is sketched in Fig 1) by[2]

$$\frac{d^5\sigma_\nu}{d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)} = \frac{|\vec{k}'|}{|\vec{k}|}\frac{G^2}{16\pi^2}\int_0^{+\infty}\frac{dk_\pi k_\pi^2}{E_\pi}L_{\mu\sigma}^{(\nu)}(W_{NC\pi}^{\mu\sigma})^{(\nu)} \tag{2}$$

with $\vec{k}$ and $\vec{k}'$ the LAB neutrino momenta, $E' = |\vec{k}'|$, the energy of the outgoing neutrino, $\vec{k}_\pi$ and E_π, the momentum and energy of the pion in the LAB system, $G = 1.1664 \times 10^{-11}$ MeV^{-2}, the Fermi constant and L and W the leptonic and hadronic tensors, respectively. The leptonic tensor is given by (in our convention, we take $\epsilon_{0123} = +1$ and the metric $g^{\mu\nu} = (+,-,-,-)$):

$$L_{\mu\sigma}^{(\nu)} = (L_s^{(\nu)})_{\mu\sigma} + i(L_a^{(\nu)})_{\mu\sigma} = k'_\mu k_\sigma + k'_\sigma k_\mu - g_{\mu\sigma}k\cdot k' + i\epsilon_{\mu\sigma\alpha\beta}k'^\alpha k^\beta \tag{3}$$

and it is orthogonal to $q^\mu = (k - k')^\mu$ for massless neutrinos, i.e, $L_{\mu\sigma}^{(\nu)}q^\mu = L_{\mu\sigma}^{(\nu)}q^\sigma = 0$.

The hadronic tensor includes all sort of non-leptonic vertices and it reads

$$(W_{NC\pi}^{\mu\sigma})^{(\nu)} = \frac{1}{4M}\overline{\sum_{\text{spins}}}\int\frac{d^3p'}{(2\pi)^3}\frac{1}{2E'_N}\delta^4(p' + k_\pi - q - p)\langle N'\pi|j_{nc}^\mu(0)|N\rangle\langle N'\pi|j_{nc}^\sigma(0)|N\rangle^* \tag{4}$$

with M the nucleon mass and E'_N the energy of the outgoing nucleon. In the sum over initial and final nucleon spins[3], the bar over the sum denotes the average over the initial ones. As for the one particle states they are normalized so that $\langle\vec{p}|\vec{p}'\rangle = (2\pi)^3 2p_0\delta^3(\vec{p} - \vec{p}')$, and finally for the neutral current we take

$$j_{nc}^\mu = \overline{\Psi}_u\gamma^\mu(1 - \frac{8}{3}\sin^2\theta_W - \gamma_5)\Psi_u - \overline{\Psi}_d\gamma^\mu(1 - \frac{4}{3}\sin^2\theta_W - \gamma_5)\Psi_d - \overline{\Psi}_s\gamma^\mu(1 - \frac{4}{3}\sin^2\theta_W - \gamma_5)\Psi_s \tag{5}$$

with Ψ_u, Ψ_d and Ψ_s quark fields, and θ_W the Weinberg angle ($\sin^2\theta_W = 0.231$). Note that with all these definitions, the matrix element $\langle N'\pi|j_{nc}^\mu(0)|N\rangle$ is dimensionless. Both, lepton and hadron tensors are independent of the neutrino

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[2] To obtain Eq. (2) we have neglected the four-momentum carried out by the intermediate Z−boson with respect to its mass (M_Z), and have used the existing relation between the gauge weak coupling constant, $g = e/\sin\theta_W$, and the Fermi constant: $G/\sqrt{2} = g^2/8M_W^2$, with e the electron charge, θ_W the Weinberg angle, $\cos\theta_W = M_W/M_Z$ and M_W the W−boson mass.

[3] Since right-handed neutrinos are sterile, only left-handed neutrinos are considered.

lepton family, and therefore the cross section for the reaction of Eq. (1) is the same for electron, muon or tau incident neutrinos. As the quantity $L_{\mu\sigma}^{(\nu)}(W_{\mathrm{NC}\pi}^{\mu\sigma})^{(\nu)}$ is a Lorentz scalar, to evaluate it we take for convenience $\vec{q}$ in the Z direction (see Fig. 1). Referring now the pion variables to the outgoing πN pair center of mass frame (as it is usual in pion electroproduction) would be readily done by means of a boost in the Z direction. Note that the azimuthal angle ϕ_π is left unchanged by such a boost.

For antineutrino induced reactions we have

$$L_{\mu\sigma}^{(\bar{\nu})} = L_{\sigma\mu}^{(\nu)}, \qquad (W_{\mathrm{NC}\pi}^{\mu\sigma})^{(\bar{\nu})} = (W_{\mathrm{NC}\pi}^{\mu\sigma})^{(\nu)} \tag{6}$$

For the sake of simplicity, from now on, we will omit in the hadronic tensor the explicit reference to ν or $\bar{\nu}$ and the NC π label. By construction, the hadronic tensor accomplishes

$$W^{\mu\sigma} = W_s^{\mu\sigma} + \mathrm{i}W_a^{\mu\sigma} \tag{7}$$

with $W_s^{\mu\sigma}$ ($W_a^{\mu\sigma}$) real symmetric (antisymmetric) tensors. The hadronic tensor is completely determined by up to a total of 19 Lorentz scalar and real, structure functions $W_i(q^2,\, p\cdot q,\, p\cdot k_\pi,\, k_\pi\cdot q)$,

$$W_{s,a}^{\mu\nu} = \left(W_{s,a}^{\mu\nu}\right)^{\mathrm{PC}} + \left(W_{s,a}^{\mu\nu}\right)^{\mathrm{PV}} \tag{8}$$

$$
\begin{aligned}
(W_s^{\mu\nu})^{\mathrm{PC}} &= W_1 g^{\mu\nu} + W_2 p^\mu p^\nu + W_3 q^\mu q^\nu + W_4 k_\pi^\mu k_\pi^\nu + W_5(q^\mu p^\nu + q^\nu p^\mu) + W_6(q^\mu k_\pi^\nu + q^\nu k_\pi^\mu) \\
&\quad + W_7(p^\mu k_\pi^\nu + p^\nu k_\pi^\mu)
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
(W_s^{\mu\nu})^{\mathrm{PV}} &= W_8\left(q^\mu \epsilon_{.\alpha\beta\gamma}^\nu k_\pi^\alpha p^\beta q^\gamma + q^\nu \epsilon_{.\alpha\beta\gamma}^\mu k_\pi^\alpha p^\beta q^\gamma\right) + W_9\left(p^\mu \epsilon_{.\alpha\beta\gamma}^\nu k_\pi^\alpha p^\beta q^\gamma + p^\nu \epsilon_{.\alpha\beta\gamma}^\mu k_\pi^\alpha p^\beta q^\gamma\right) \\
&\quad + W_{10}\left(k_\pi^\mu \epsilon_{.\alpha\beta\gamma}^\nu k_\pi^\alpha p^\beta q^\gamma + k_\pi^\nu \epsilon_{.\alpha\beta\gamma}^\mu k_\pi^\alpha p^\beta q^\gamma\right)
\end{aligned} \tag{10}
$$

$$(W_a^{\mu\nu})^{\mathrm{PV}} = W_{11}(q^\mu p^\nu - q^\nu p^\mu) + W_{12}(q^\mu k_\pi^\nu - q^\nu k_\pi^\mu) + W_{13}(p^\mu k_\pi^\nu - p^\nu k_\pi^\mu) \tag{11}$$

$$
\begin{aligned}
(W_a^{\mu\nu})^{\mathrm{PC}} &= W_{14}\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta + W_{15}\epsilon^{\mu\nu\alpha\beta}p_\alpha k_{\pi\beta} + W_{16}\epsilon^{\mu\nu\alpha\beta}q_\alpha k_{\pi\beta} + W_{17}\left(q^\mu \epsilon_{.\alpha\beta\gamma}^\nu k_\pi^\alpha p^\beta q^\gamma - q^\nu \epsilon_{.\alpha\beta\gamma}^\mu k_\pi^\alpha p^\beta q^\gamma\right) \\
&\quad + W_{18}\left(p^\mu \epsilon_{.\alpha\beta\gamma}^\nu k_\pi^\alpha p^\beta q^\gamma - p^\nu \epsilon_{.\alpha\beta\gamma}^\mu k_\pi^\alpha p^\beta q^\gamma\right) + W_{19}\left(k_\pi^\mu \epsilon_{.\alpha\beta\gamma}^\nu k_\pi^\alpha p^\beta q^\gamma - k_\pi^\nu \epsilon_{.\alpha\beta\gamma}^\mu k_\pi^\alpha p^\beta q^\gamma\right)
\end{aligned} \tag{12}
$$

Though $W^{\mu\nu}$ is not orthogonal to q^μ, the W_3, W_5, W_6, W_8, W_{11}, W_{12} and W_{17} terms do not contribute to the differential cross section since the leptonic tensor is orthogonal to the four vector q^μ for massless neutrinos.

The tensor $(W^{\mu\nu})^{\mathrm{PV}} = (W_s^{\mu\nu})^{\mathrm{PV}} + \mathrm{i}(W_a^{\mu\nu})^{\mathrm{PV}}$ when contracted with the leptonic one, $L_{\mu\nu}^{(\nu)}$, provides a pseudo-scalar quantity, i.e., such contraction is not invariant under a parity transformation. Indeed, under a parity transformation we have,

$$L_{\mu\nu}^{(\nu)} \to (L^{\nu\mu})^{(\nu)}, \qquad (W_{\mu\nu})^{\mathrm{PV}} \to -(W^{\nu\mu})^{\mathrm{PV}} \tag{13}$$

whereas the tensor $(W^{\mu\nu})^{\mathrm{PC}} = (W_s^{\mu\nu})^{\mathrm{PC}} + \mathrm{i}(W_a^{\mu\nu})^{\mathrm{PC}}$ transforms as $(L^{\mu\nu})^{(\nu)}$. This explains the origin of the adopted labels PC and PV, which stand for parity violating and conserving contributions to the fifth differential cross section $d^5\sigma/d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)$, respectively[4]. The triple differential cross section $d^3\sigma/d\Omega(\hat{k}')dE'$ is a scalar, up to the factor $|\vec{k}'|/|\vec{k}|$. Thus all parity-violating contributions must disappear when performing the pion solid angle integration. Note that the coordinate system used to define $d\Omega(\hat{k}_\pi)$ involves the pseudo-vector $\vec{k}\times\vec{k}'$ to set up the $Y-$axis, which induces the non-parity invariant nature of $d^5\sigma/d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)$. In electropion production processes, the leptonic tensor is purely symmetric, and the symmetric part of the hadronic one can not contain terms involving the Levi-Civita tensor, since the electromagnetic interaction preserves parity. Hence in that case $d^5\sigma/d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)$ turns out to be a scalar under parity.

A final remark concerns the time-reversal (T) violation effects apparently encoded in the decomposition of the hadronic tensor in Eqs. (8–12). Under a time reversal transformation, and taking into account the antiunitary

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[4] Similar conclusions can be also reached by just noting that the symmetric part of the leptonic tensor does not contain any Levi-Civita $\epsilon^{\mu\nu\alpha\beta}$ pseudo–tensor, whereas its antisymmetric part is just given by a Levi-Civita tensor. Thus, to get a parity invariant quantity, the symmetric part of the hadronic tensor should not have any Levi-Civita tensor, while all terms of its antisymmetric part should contain a Levi-Civita tensor.

character of the $T-$operator, we have

$$L^{(\nu)}_{\mu\nu} \to (L^{\mu\nu})^{(\nu)}, \qquad (W_{\mu\nu})_{\mathrm{PC}} \to (W^{\mu\nu})^{\mathrm{PC}}, \qquad (W_{\mu\nu})^{\mathrm{PV}} \to -(W^{\mu\nu})^{\mathrm{PV}} \tag{14}$$

and therefore $L^{(\nu)}_{\mu\nu} W^{\mu\nu}$ is not $T-$invariant either, because of the presence of the PV terms in the hadronic tensor[5]. This does not necessarily means that there exists a violation of $T-$invariance in the process [5]. Time reversal is the transformation that changes the direction of time: $t \to -t$. Thus invariance under time reversal means that the description of physical processes does not show any asymmetry if we look backward in time. The invariance under time reversal is equivalent to

$$|M_{i \to f}|^2 = |M_{Tf \to Ti}|^2 \tag{15}$$

where $i \to f$ denotes the transition from the initial state i to the final state f, being $M_{i \to f}$ its corresponding transition amplitude, and Tf and Ti denote the states obtained from f and i, respectively, by reversing the momenta, spins, etc.. When one is dealing with electromagnetic or weak interactions, which may be treated to first order in the interaction Hamiltonian (H_I), the transition matrix operator can be approximated by H_I, being it then hermitian. In these circumstances $|M_{Tf \to Ti}| \approx |M_{Ti \to Tf}|$ and the invariance under time reversal may be written in the following form

$$|M_{i \to f}|^2 = |M_{Tf \to Ti}|^2 \approx |M_{Ti \to Tf}|^2 \tag{16}$$

The above equation implies that the transition $i \to f$ cannot show up correlations that change sign under time reversal ($T-$odd correlations). Thus at first order in the interaction Hamiltonian, the tensor $(W^{\mu\nu})^{\mathrm{PV}}$ should vanish, since it would lead to violations of time reversal invariance. However, here we have strong interacting final states (pion and nucleon) and in fact the transition matrix operator is not hermitian, since it cannot be approximated by H_I. Therefore Eq. (16) does not hold and time reversal invariance does not forbid a non-vanishing $(W^{\mu\nu})^{\mathrm{PV}}$ tensor. In summary, besides genuine time reversal violations [6][6], the existence of strong final state interaction effects[7] allows for the existence of $T-$odd correlations in $(L^{(\nu)})_{\mu\nu} W^{\mu\nu}$, induced by the $(W^{\mu\nu})^{\mathrm{PV}}$ term of the hadronic tensor.

After this discussion, we are in conditions of studying the pion azimuthal angle dependence of the differential cross section. With our election of kinematics ($\vec{k}, \vec{k}'$ in the XZ plane), we find $(L^{(\nu)}_s)_{0y} = (L^{(\nu)}_s)_{xy} = (L^{(\nu)}_s)_{zy} = (L^{(\nu)}_a)_{0x} = (L^{(\nu)}_a)_{0z} = (L^{(\nu)}_a)_{xz} = 0$, and then

$$
\begin{aligned}
\int_0^{+\infty} \frac{dk_\pi k_\pi^2}{E_\pi} (L^{(\nu)}_s)_{\mu\nu} W^{\mu\nu}_s &= \int_0^{+\infty} \frac{dk_\pi k_\pi^2}{E_\pi} \Big\{ (L^{(\nu)}_s)_{00} W^{00}_s + 2(L^{(\nu)}_s)_{0x} W^{0x}_s + 2(L^{(\nu)}_s)_{0z} W^{0z}_s + (L^{(\nu)}_s)_{xx} W^{xx}_s \\
&\quad + (L^{(\nu)}_s)_{yy} W^{yy}_s + (L^{(\nu)}_s)_{zz} W^{zz}_s + 2(L^{(\nu)}_s)_{xz} W^{xz}_s \Big\} \\
&= A_s + B_s \cos\phi_\pi + C_s \cos 2\phi_\pi + D_s \sin\phi_\pi + E_s \sin 2\phi_\pi
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
\int_0^{+\infty} \frac{dk_\pi k_\pi^2}{E_\pi} (L^{(\nu)}_a)_{\mu\nu} W^{\mu\nu}_a &= 2\int_0^{+\infty} \frac{dk_\pi k_\pi^2}{E_\pi} \Big\{ (L^{(\nu)}_a)_{0y} W^{0y}_a + (L^{(\nu)}_a)_{xy} W^{xy}_a + (L^{(\nu)}_a)_{yz} W^{yz}_a \Big\} \\
&= -A_a - B_a \cos\phi_\pi - D_a \sin\phi_\pi
\end{aligned} \tag{18}
$$

where we explicitly show the ϕ_π dependence[8]. The PV term of the hadronic tensor has led to the parity violating $\sin\phi_\pi$ and $\sin 2\phi_\pi$ contributions (all of them proportional to $k_{\pi y}$). They disappear when the pion solid angle integration is

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[5] Note that transformations given in Eqs. (6), (13) and (14) imply that the leptonic tensor, by itself, is invariant under CPT.

[6] One is faced with the problem of eliminating final-state-interaction effects, or at least of having them under control, in order to have a significant test of T violation [5, 6].

[7] For instance, at intermediate energies and for CC driven processes, the $\Delta(1232)$ resonance plays a central role [7]. The inclusion of the resonance width accounts partially for the strong final state interaction effects.

[8] All structure functions, $W_{i=1,\ldots 19}$, depend on the Lorentz scalar $p \cdot k_\pi$ and $k_\pi \cdot q$ factors, which are functions of the angle formed between the $\vec{q}$ and $\vec{k}_\pi$ vectors, and thus they are independent of ϕ_π, when $\vec{q}$ is taken along the $Z-$axis.

performed, as anticipated. Thanks to Eq. (6), we obtain for neutrino and antineutrino reactions

$$(\nu,\nu) \to \frac{d^5\sigma_\nu}{d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)} = \frac{|\vec{k}'|}{|\vec{k}|}\frac{G^2}{16\pi^2}\Big\{A_s + A_a + (B_s + B_a)\cos\phi_\pi + C_s\cos 2\phi_\pi + (D_s + D_a)\sin\phi_\pi$$
$$+ \; E_s\sin 2\phi_\pi\Big\} \tag{19}$$

$$(\bar{\nu},\bar{\nu}) \to \frac{d^5\sigma_{\bar{\nu}}}{d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)} = \frac{|\vec{k}'|}{|\vec{k}|}\frac{G^2}{16\pi^2}\Big\{A_s - A_a + (B_s - B_a)\cos\phi_\pi + C_s\cos 2\phi_\pi + (D_s - D_a)\sin\phi_\pi$$
$$+ \; E_s\sin 2\phi_\pi\Big\} \tag{20}$$

III.   NEUTRINO-ANTINEUTRINO ASYMMETRIES

From Eqs. (19) and (20), we see that neutrino-antineutrino asymmetries may occur both in the totally integrated cross sections and in the pion azimuthal differential distributions. For the neutrino and antineutrino induced total cross sections we have

$$(\nu,\nu) \to \sigma_\nu = \frac{G^2}{16\pi^2|\vec{k}|}\int d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)|\vec{k}'|\Big(A_s + A_a\Big) \tag{21}$$

$$(\bar{\nu},\bar{\nu}) \to \sigma_{\bar{\nu}} = \frac{G^2}{16\pi^2|\vec{k}|}\int d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)|\vec{k}'|\Big(A_s - A_a\Big), \tag{22}$$

while for the azimuthal distributions, relations of the type

$$\frac{d^5\sigma}{d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)}(\phi_\pi) - \frac{d^5\sigma}{d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)}(\phi_\pi + \pi)\Big|_\nu = \{(B_s + B_a)\cos\phi_\pi + (D_s + D_a)\sin\phi_\pi\}\frac{|\vec{k}'|}{|\vec{k}|}\frac{G^2}{8\pi^2} \tag{23}$$

$$\frac{d^5\sigma}{d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)}(\phi_\pi) - \frac{d^5\sigma}{d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)}(\phi_\pi + \pi)\Big|_{\bar{\nu}} = \{(B_s - B_a)\cos\phi_\pi + (D_s - D_a)\sin\phi_\pi\}\frac{|\vec{k}'|}{|\vec{k}|}\frac{G^2}{8\pi^2} \tag{24}$$

enhance neutrino-antineutrino asymmetries. To increase the statistics in these azimuthal asymmetries, it is interesting to study the relations of Eqs. (23) and (24) for integrated cross sections. From the discussion in the Appendix A it follows

$$\frac{1}{2}\left(\frac{d\sigma(\phi_\pi)}{d\phi_\pi} - \frac{d\sigma(\phi_\pi + \pi)}{d\phi_\pi}\right)\Big|_\nu = (\mathcal{B}_s + \mathcal{B}_a)\cos\phi_\pi + (\mathcal{D}_s + \mathcal{D}_a)\sin\phi_\pi \tag{25}$$

$$\frac{1}{2}\left(\frac{d\sigma(\phi_\pi)}{d\phi_\pi} - \frac{d\sigma(\phi_\pi + \pi)}{d\phi_\pi}\right)\Big|_{\bar{\nu}} = (\mathcal{B}_s - \mathcal{B}_a)\cos\phi_\pi + (\mathcal{D}_s - \mathcal{D}_a)\sin\phi_\pi \tag{26}$$

where for each event, ϕ_π is the angle formed by the neutrino and pion scattering planes. Since the outgoing neutrino will not likely be detected, if the incoming neutrino momentum is known, the neutrino scattering plane will be set up by detecting also the outgoing nucleon. Indeed, the neutrino scattering plane is also determined by the incoming neutrino and the transferred momenta, and in the LAB system this latter one is given by the sum of the outgoing nucleon and pion momenta ($\vec{q} = \vec{p}' + \vec{k}_\pi$). Thus the strategy will be detecting in coincidences the outgoing pion and nucleon momenta, use those to determine the neutrino scattering plane and then measure the corresponding ϕ_π angle.

All these relations can be used to distinguish $\tau-$neutrinos from antineutrinos, below the $\tau-$production threshold, but above the pion production one. Thus and using a proton target, the total cross section or the dependence on the angle formed by the neutrino and pion planes in either of the channels,

$$\nu p \to \nu n\pi^+, \quad \nu p \to \nu p\pi^0$$
$$\bar{\nu} p \to \bar{\nu} n\pi^+, \quad \bar{\nu} p \to \bar{\nu} p\pi^0, \tag{27}$$

may be used to determine the nature of the incident $\tau-$neutrino beam below the $\tau-$production threshold.

Moreover, we would like to point out that the $(A_s \pm A_a)$, $(B_s \pm B_a)$ and $(D_s \pm D_a)$ response functions, the integrated cross sections σ_ν, $\sigma_{\bar{\nu}}$ and the partially integrated quantities $(\mathcal{B}_s \pm \mathcal{B}_a)$ and $(\mathcal{D}_s \pm \mathcal{D}_a)$ are independent of the lepton family and can be experimentally measured by using electron or muon neutrinos, due to the lepton family universality of the NC neutrino interaction. Thus one would not have to rely on any theoretical model to determine the asymmetry relations proposed in this section.

A. Neutrino-antineutrino asymmetries from the chiral model of Ref. [4]

The above suggestion for distinguishing neutrino from antineutrino in NC pion production reactions will be of no value if the terms A_a, B_a and D_a are much smaller than A_s, B_s and D_s. To estimate these response functions and address this issue, we have used the model developed in Ref. [4] to study the weak pion production off the nucleon at intermediate energies. In this model, besides the Delta pole mechanism ΔP (weak excitation of the $\Delta(1232)$ resonance and its subsequent decay into $N\pi$), some background terms, required by chiral symmetry, are also considered. These chiral background terms are calculated within a SU(2) non-linear σ model involving pions and nucleons, which implements the pattern of spontaneous chiral symmetry breaking of QCD. The model gives a fair description of the neutrino and antineutrino CC and NC available data. Details can be found in Ref. [4].

In Fig. 2, we show neutrino and antineutrino NC pion production total cross sections predicted by the model of Ref. [4], as a function of the incoming neutrino or antineutrino energy. There, 68% CL bands (external lines), inferred from uncertainties of the model of Ref. [4] are also displayed. Those uncertainties come from re-adjusting the $C_5^A(q^2)$ form–factor that controls the largest term of the $\Delta-$axial contribution. On the other hand, since the main dynamical ingredients of the model are the excitation of the Δ resonance and the non-resonant contributions deduced from the leading SU(2) non-linear σ lagrangian involving pions and nucleons, we will concentrate in the pion-nucleon invariant mass $W \leq 1.4$ GeV region. For larger invariant masses, the chiral expansion will not work, or at least the lowest order used in Ref. [4] will not be sufficient and the effect of heavier resonances will become much more important [8]. For this reason in the figure, we always plot cross sections calculated with an invariant pion-nucleon mass cut of 1.4 GeV, it is to say we compute $\int_{m_\pi+M}^{1.4 \text{ GeV}} dW \, d\sigma/dW$. Such cut is commonly used in the CC pion production experimental analyses at intermediate energies. See for instance [9] where it can also be seen that up to incoming neutrino LAB energies of the order of 1 GeV the implementation of this cut hardly changes the measured cross section. In the figure, we also display low energy data measured in the Argonne National Laboratory (ANL) [10] for the $\nu n \to \nu p \pi^-$ reaction, which do not include any cut in W. As shown in Ref. [4], including or not in the theoretical calculation the cut in W only influences significantly the result for the highest energy data–point, leading to variations of the order of 20%, which are around half the experimental error.

Results of Fig. 2 are really significant since the differences between neutrino and antineutrino induced reactions are always large, about a factor of two, in all physical channels. This confirms that this observable can be used to distinguish neutrino from antineutrino above the pion production threshold.

Next we show results, within this model, for $\mathcal{B}_s \pm \mathcal{B}_a$ and $\mathcal{D}_s \pm \mathcal{D}_a$ defined in Eqs. (25) and (26). In Fig. 3 we plot these response functions for a $p\pi^-$ final state. The rest of channels lead to similar results. Clear neutrino-antineutrino asymmetries can be appreciated in both panels of the figure. In particular $\mathcal{D}_s + \mathcal{D}_a$ and $\mathcal{D}_s - \mathcal{D}_a$ have opposite signs. The difference between the number of events for which the pion comes out above the neutrino plane (N_{abv}) and those where the pion comes out below this plane (N_{blw}) is 4 ($\mathcal{D}_s + \mathcal{D}_a$) [4 ($\mathcal{D}_s - \mathcal{D}_a$)] for a neutrino [antineutrino] induced process. Thus, the sign of this difference would determine whether one has a neutrino or an antineutrino beam. Nevertheless, we should mention that the neutrino-antineutrino asymmetry based in the total cross section discussed above (Fig. 2) would provide a signal with much better statistical significance. This is because the ratio $\frac{N_{\text{abv}}-N_{\text{blw}}}{N_{\text{abv}}+N_{\text{blw}}}$ would be of the order of 10^{-2} or smaller. Asymmetries based on the $\mathcal{B}$ response function will not be as much unfavorable, from the statistical point of view.

These results point out that the neutrino–antineutrino asymmetries based on the total cross sections are certainly much more useful than those based on the the pion azimuthal response functions, for intermediate neutrino energies. The situation could be different for energies larger than those explored with the model of Ref. [4].

Acknowledgments

APPENDIX A: PARTIALLY INTEGRATED AZIMUTHAL DISTRIBUTION

To define an azimuthal pion distribution, by integrating the outgoing neutrino variables and pion polar angle in Eqs. (19) and (20), requires for each event a rotation to guarantee that $\vec{q}$ is in the $Z-$direction and that $\vec{k}$ and $\vec{k}'$ are
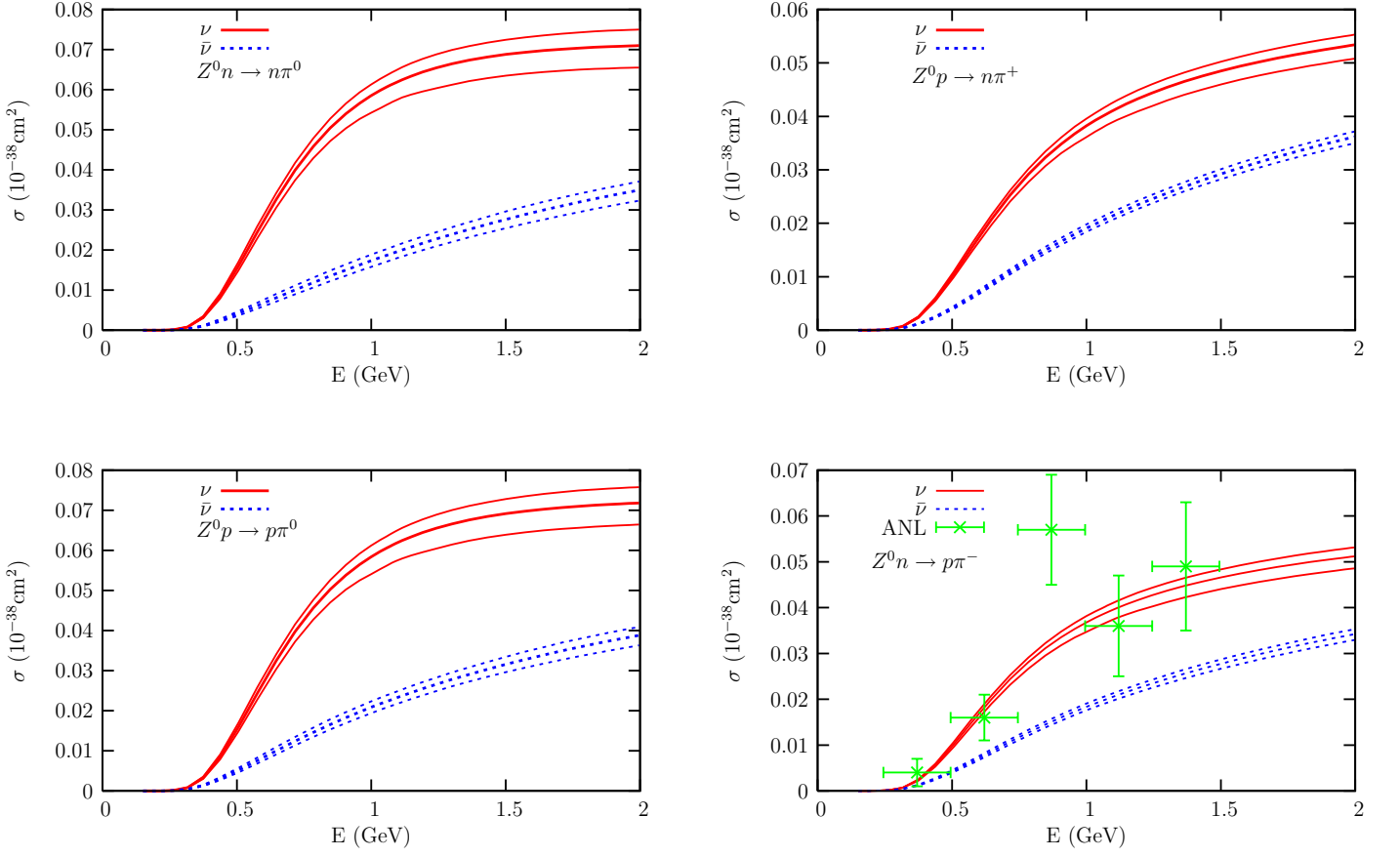
FIG. 2: Neutrino and antineutrino NC pion production total cross sections predicted by the model of Ref. [4], as a function of the incoming neutrino or antineutrino energy. An invariant pion-nucleon mass cut of $W \leq 1.4$ GeV has been implemented. Central lines stand for the result of the model. 68% CL bands (external lines), inferred from uncertainties of the model, are also displayed. Experimental cross sections for neutrino π^- production are taken from Ref. [10].

contained in $XZ-$plane. The totally integrated unpolarized neutrino cross section reads

$$\sigma_\nu = \frac{G^2}{16\pi^2|\vec{k}\,|} \int \frac{d^3k'}{|\vec{k}'|} \frac{d^3k_\pi}{E_\pi} L^{(\nu)}_{\mu\sigma}(k,k') W^{\mu\sigma}(p,q,k_\pi) \tag{A1}$$

To perform the above integrals, we take a fix system of coordinates $\mathcal{XYZ}$, and since $L^{(\nu)}_{\mu\sigma}(k,k') W^{\mu\sigma}(p,q,k_\pi)$ is a scalar under spatial rotations, it can be evaluated as

$$L^{(\nu)}_{\mu\sigma}(Rk, Rk') W^{\mu\sigma}(Rp, Rq, Rk_\pi) \tag{A2}$$

where for each $\vec{k}'$, R is the spatial rotation which brings $\vec{q}$ and $\vec{k} \times \vec{k}'$ into the $\mathcal{Z}-$ and $\mathcal{Y}-$axes, respectively. Such rotation depends on $\vec{k}'$ but it is independent of $\vec{k}_\pi$. Then

$$\begin{aligned}
\sigma_\nu &= \frac{G^2}{16\pi^2|\vec{k}\,|} \int \frac{d^3k'}{|\vec{k}'|} \frac{d^3k_\pi}{E_\pi} L^{(\nu)}_{\mu\sigma}(Rk, Rk') W^{\mu\sigma}(Rp, Rq, Rk_\pi) \\
&= \frac{G^2}{16\pi^2|\vec{k}\,|} \int \frac{d^3k'}{|\vec{k}'|} \frac{d^3k_\pi}{E_\pi} L^{(\nu)}_{\mu\sigma}(Rk, Rk') W^{\mu\sigma}(Rp, Rq, k_\pi)
\end{aligned} \tag{A3}$$

where in the last step, for a fix $\vec{k}'$, we have made a change of variables in the pion integral, $\vec{k}_\pi \to \vec{k}_\pi^R \equiv R\vec{k}_\pi$, and for simplicity we have called the dummy variable $\vec{k}_\pi^R$ again $\vec{k}_\pi$. Finally using spherical coordinates $\vec{k}_\pi =$
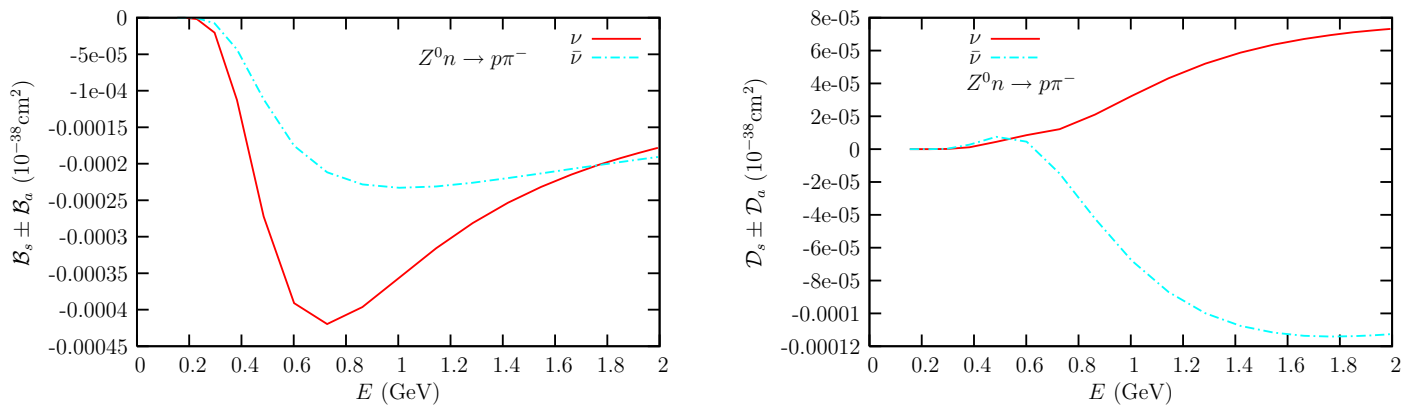
FIG. 3: Predictions of the chiral model of Ref. [4] for the $\mathcal{B}_s \pm \mathcal{B}_a$ and $\mathcal{D}_s \pm \mathcal{D}_a$ (Eqs. (25) and (26)) response functions, as function of the neutrino or antineutrino incoming energy.

$|\vec{k}_\pi|(\sin\theta_\pi \cos\phi_\pi, \sin\theta \sin\phi_\pi, \cos\theta_\pi)$ for the pion integral, we get

$$
\begin{aligned}
\frac{d\sigma_\nu}{d\phi_\pi} &= \frac{G^2}{16\pi^2|\vec{k}\,|} \int \frac{d^3k'}{|\vec{k}'|} \frac{d|\vec{k}_\pi||\vec{k}_\pi|^2 d(\cos\theta_\pi)}{E_\pi} L^{(\nu)}_{\mu\sigma}(Rk, Rk') W^{\mu\sigma}(Rp, Rq, k_\pi) \\
&= \frac{G^2}{16\pi^2|\vec{k}\,|} \int \frac{d^3k'}{|\vec{k}'|} d(\cos\theta_\pi)\Big\{ A_s + A_a + (B_s + B_a)\cos\phi_\pi + C_s\cos 2\phi_\pi + (D_s + D_a)\sin\phi_\pi + E_s \sin 2\phi_\pi \Big\} \\
&= \mathcal{A}_s + \mathcal{A}_a + (\mathcal{B}_s + \mathcal{B}_a)\cos\phi_\pi + \mathcal{C}_s\cos 2\phi_\pi + (\mathcal{D}_s + \mathcal{D}_a)\sin\phi_\pi + \mathcal{E}_s\sin 2\phi_\pi \qquad (A4)
\end{aligned}
$$

since we have $R\vec{q}$ in the $\mathcal{Z}-$direction and $R\vec{k}$ and $R\vec{k}'$ are contained in $\mathcal{XZ}-$plane, and thus the results of Eq. (19) can be used. In the above equations, ϕ_π is the azimuthal angle of the rotated pion momentum in the $\mathcal{XYZ}$ system of coordinates, or equivalently, it is the angle formed by the neutrino and pion planes. The first of these planes is determined by the incoming ($\vec{k}$) and outgoing ($\vec{k}'$) neutrino momenta, while the transferred ($\vec{q}$) and pion ($\vec{k}_\pi$) momenta fix the latter one. Thus, ϕ_π is the angle formed by the vectors $\left((\vec{k}\times\vec{k}')\times\vec{q}\right)$ and $\left(\vec{k}_\pi - (\vec{k}_\pi\cdot\vec{q})\,\vec{q}/|\vec{q}|^2\right)$, and it is therefore invariant under spatial rotations.

The antineutrino cross section can be obtained in a similar way, and thus it follows the neutrino–antineutrino asymmetry relations given in Eqs. (25) and (26) similar to those in Eqs. (23) and (24), but for integrated cross sections.

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