

Neutrino Induced Coherent Pion Production off Nuclei and PCAC

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We review the Rein–Sehgal model and criticize its use for low energy neutrino induced coherent pion production. We have studied the validity of the main approximations implicit in that model, trying to compare with physical observables when that is possible and with microscopical calculations. Next, we have tried to elaborate a new improved model by removing the more problematic approximations, while keeping the model still reasonably simple. Last, we have discussed the limitations intrinsic to any approach based on the partial conservation of the axial current hypothesis. In particular, we have shown the inability of such models to determine the angular distribution of the outgoing pion with respect to the direction of the incoming neutrino, except for the $q^2 = 0$ kinematical point.

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I. INTRODUCTION

The theoretical modelling of neutrino-induced coherent pion production in nuclei in the low (~ 1 GeV) energy region is a quite complicated task. The reasons are twofold. First, our limited knowledge of the weak pion production on nucleons at these energies, (see for instance Refs. [1, 2, 3, 4, 5, 6, 7, 8, 9]), that relies upon conflicting and low statistics bubble chamber experimental data [10, 11]. Second, the need of a quantum mechanical treatment of the multiple scattering involving the strong interaction of pions and nucleons that can hardly be accommodated in typical MonteCarlo approaches. Many different models have been proposed. Some of them are based on the assumption that coherent pion production is dominated by the divergence of the axial current and the use of the Partial Conservation of the Axial Current (PCAC) hypothesis [12, 13, 14, 15, 16, 17, 18, 19]. Other works develop a microscopical model for both pion production and distortion assuming that the pion nuclear interaction is dominated by the $\Delta(1232)$ resonance [20, 21, 22, 23, 24, 25]. Thus, they can only be applied at relatively low neutrino energies. Despite the large effort dedicated to this process the situation is hardly satisfactory and large discrepancies are found among different theoretical predictions, both for total and differential cross sections.

Besides, the understanding of these processes is very important for the analysis of neutrino oscillation experiments. For instance charged current (CC) coherent pion production is one of the candidate processes responsible for the deficit found by the K2K collaboration in the forward scattering events, which seriously limited the prediction accuracy of the neutrino energy spectrum at the far detector [26]. Similarly, neutral current (NC) π^0 production is one of the largest background sources in muon neutrino experiments, in particular for electron neutrino appearance measurements [27, 28], because it can mimic electron events when one of the π^0 decay photons is not detected. A significant part of the π^0 's in the forward direction could come from coherent production.

Due to its success in the description of CC coherent pion production data at high energies and also its simplicity and easy implementation, the Rein-Sehgal (RS) model of Ref. [12] has been widely used by experimental collaborations in their analyses [26, 28, 29], even at quite low neutrino energies clearly beyond the scope of the original paper. We should remark that some of the RS model basic approximations are better suited for high energies and heavy nuclei, as this implies more forward peaked cross sections. In fact, the minimum neutrino energy discussed in Ref. [12] was 2 GeV whereas the model has been used for the analysis of the coherent process at neutrino energies down to the pion production threshold.

When compared to other approaches, which are more reliable for low energy neutrinos, the differences clearly show up in the total production rate (see e.g. Fig. 2 of Ref. [29]) and also in the pion angular or energy distributions (see Fig. 8 of Ref. [25]¹). Some of the discrepancies, like the wider angular distributions of the pions predicted by the

¹ The curves labeled as MiniBooNE Coll. correspond to the MiniBooNE implementation of the RS model

RS model, can be easily traced back to the approximations used in its derivation and they could lead to important consequences in the determination of the size of the pion yield and/or the ratio of coherent to non-coherent pion production for low energy incident neutrinos.

Given the deficiencies of the RS model for low neutrino energies, it seems clear the need of the use of alternative approaches. However, up to our knowledge, the currently available models are of limited value for that purpose. On the one hand, the PCAC based calculations share many of the limitations of the RS model that will be discussed below. On the other hand, the microscopical models based on the Δ dominance are expected to be reliable only for low energy neutrinos ($\lesssim 1$ GeV).

Some important steps have been given in the direction of developing a model for pion production on the nucleon beyond the Δ resonance, e.g. Ref. [30], but we are still in need of more experimental data (cross sections) that could better constrain the little known transition form factors. This is a long term project that will require a collaborative effort of both experimental and theoretical groups.

In the meantime, and lacking a microscopical approach for the whole energy region of interest, it looks worthwhile to try to modify the RS model to extend its applicability towards lower neutrino energies. Some recent works in this line include Refs. [18, 31] which already correct some of the known RS problems. In this work, we will further explore this possibility. First, we will carefully study the validity of the main approximations implicit in the RS model, trying to compare with physical observables when that is possible and with microscopical calculations. Second, we will try to elaborate a new improved model by removing the more problematic approximations, while keeping the model still reasonably simple. Last, we will consider the limitations intrinsic to any PCAC based approach.

The paper is organized as follows. After some basic formalism in Sect. II, we review the RS model in Sect. III. In Sect. IV we present our improved model. In Sect. V we confront an extension of both models with low energy pion-nucleus scattering data. In Sect. VI, we show the results of both models for low energy neutrino-induced coherent pion production and we also compare them with a microscopical calculation that we consider reliable in that energy region. Finally, we summarize the paper in Sect. VII.

II. NC NEUTRINO COHERENT PION PRODUCTION STRUCTURE FUNCTIONS

We will focus on NC π^0 coherent production off a nucleus in its ground state \mathcal{N}_{gs} ,

$$\nu_l(k) + \mathcal{N}_{gs} \rightarrow \nu_l(k') + \mathcal{N}_{gs} + \pi^0(k_\pi). \quad (1)$$

The modifications required for the CC case are straightforward. The process starts with a weak pion (π^0) production followed by the strong distortion of the pion in its way out of the nucleus. The nucleus is left in its ground state unlike the case for incoherent production where the nucleus is either broken or left in some excited state.

Defining the four momentum transfer $q = k - k'$ and taking \vec{q} and $\vec{k} \times \vec{k}'$ along the positive z - and y -axis respectively, one can write the differential cross section with respect to the outgoing neutrino variables, the Mandelstam variable $t = (q - k_\pi)^2$ and the pion azimuthal angle, $\phi_{k_\pi q}$, in the LAB system as:

$$\frac{d\sigma}{dE' d\Omega(\vec{k}') dt d\phi_{k_\pi q}} = \frac{|\vec{k}'|}{|\vec{k}|} \frac{G^2}{16\pi^2} L_{\mu\sigma} \mathcal{H}^{\mu\sigma}, \quad (2)$$

with E' the energy of the final neutrino and G the Fermi constant. The leptonic tensor is given by:

$$L_{\mu\sigma} = k'_\mu k_\sigma + k'_\sigma k_\mu + \frac{q^2}{2} g_{\mu\sigma} + i\epsilon_{\mu\sigma\alpha\beta} k'^\alpha k^\beta, \quad (3)$$

and it is orthogonal to the transferred four momentum q^μ . In our convention, we take $\epsilon_{0123} = +1$ and the metric $g^{\mu\nu} = (+, -, -, -)$. The hadronic part, \mathcal{H} , is determined by the matrix element of the NC between the initial and final hadronic states, and it includes all the nuclear effects. Introducing the variable $y = q^0/E$, with E the incident neutrino energy, we can write

$$\frac{d\sigma}{dq^2 dy dt d\phi_{k_\pi q}} = \frac{G^2}{16\pi^2} E \kappa \left(-\frac{q^2}{|\vec{q}|^2} \right) \left(u^2 \frac{d\sigma_L}{dt d\phi_{k_\pi q}} + v^2 \frac{d\sigma_R}{dt d\phi_{k_\pi q}} + 2uv \frac{d\sigma_S}{dt d\phi_{k_\pi q}} + \frac{d\mathcal{A}}{dt d\phi_{k_\pi q}} \right), \quad (4)$$

where

$$\kappa = q^0 + \frac{q^2}{2M}, \quad u, v = \frac{E + E' \pm |\vec{q}|}{2E}, \quad (5)$$

and

$$\begin{aligned}
\frac{d\sigma_S}{dt d\phi_{k_\pi q}} &= -\frac{1}{q^2} \frac{\pi}{\kappa} (|\vec{q}|^2 \mathcal{H}_{00} + q^0 |\vec{q}| (\mathcal{H}_{0z} + \mathcal{H}_{z0}) + (q^0)^2 \mathcal{H}_{zz}) , \\
\frac{d\sigma_L}{dt d\phi_{k_\pi q}} &= \frac{\pi}{2\kappa} (\mathcal{H}_{xx} + \mathcal{H}_{yy} + i(\mathcal{H}_{xy} - \mathcal{H}_{yx})) , \\
\frac{d\sigma_R}{dt d\phi_{k_\pi q}} &= \frac{\pi}{2\kappa} (\mathcal{H}_{xx} + \mathcal{H}_{yy} - i(\mathcal{H}_{xy} - \mathcal{H}_{yx})) , \\
\frac{d\mathcal{A}}{dt d\phi_{k_\pi q}} &= \frac{\pi}{\kappa} \left(uv (\mathcal{H}_{xx} - \mathcal{H}_{yy}) + \frac{E + E'}{E} \sqrt{\frac{|\vec{q}|^2}{-q^2}} \sqrt{uv} \left((\mathcal{H}_{0x} + \mathcal{H}_{x0}) + \frac{q^0}{|\vec{q}|} (\mathcal{H}_{zx} + \mathcal{H}_{xz}) \right) \right. \\
&\quad \left. + i \frac{|\vec{q}|}{E} \sqrt{\frac{|\vec{q}|^2}{-q^2}} \sqrt{uv} \left((\mathcal{H}_{0y} - \mathcal{H}_{y0}) + \frac{q^0}{|\vec{q}|} (\mathcal{H}_{zy} - \mathcal{H}_{yz}) \right) \right) . \tag{6}
\end{aligned}$$

Note that

$$\frac{d\sigma_S}{dt d\phi_{k_\pi q}} = \frac{1}{2\pi} \frac{d\sigma_S}{dt} , \quad \frac{d\sigma_R}{dt d\phi_{k_\pi q}} = \frac{1}{2\pi} \frac{d\sigma_R}{dt} , \quad \frac{d\sigma_L}{dt d\phi_{k_\pi q}} = \frac{1}{2\pi} \frac{d\sigma_L}{dt} , \tag{7}$$

as neither \mathcal{H}_{00} , \mathcal{H}_{zz} nor the combinations $\mathcal{H}_{0z} + \mathcal{H}_{z0}$, $\mathcal{H}_{xx} + \mathcal{H}_{yy}$, $\mathcal{H}_{xy} - \mathcal{H}_{yx}$ can depend on $\phi_{k_\pi q}$. The only quantity that depends on $\phi_{k_\pi q}$ is $\frac{d\mathcal{A}}{dt d\phi_{k_\pi q}}$. The latter is not a proper differential cross section as it can take on positive as well as negative values. Besides, it cancels upon integration on $\phi_{k_\pi q}$ (see below). Note also that for $q^2 = 0$ only the σ_S term contributes.

The tensor $H^{\mu\nu}$,

$$H^{\mu\nu} = \int d\phi_{k_\pi q} \mathcal{H}^{\mu\nu} , \tag{8}$$

only depends on q^μ , $p^\mu = (\mathcal{M}, \vec{0})$ (the four vector of the initial nuclear state, with \mathcal{M} the nucleus mass) and t . Due to the tensorial character of H and the fact that, being the transferred momentum \vec{q} aligned with the z -axis, it is invariant under rotations around the z -axis, one can prove

$$H^{xx} = H^{yy} , \quad H^{xz} = H^{zx} = H^{yz} = H^{zy} = H^{0x} = H^{0y} = H^{x0} = H^{y0} = 0 , \quad H^{xy} = -H^{yx} , \tag{9}$$

Thus, integrating in $\phi_{k_\pi q}$ one obtains for the $\frac{d\sigma}{dq^2 dy dt}$ differential cross section [32]

$$\frac{d\sigma}{dq^2 dy dt} = \frac{G^2}{16\pi^2} E \kappa \left(-\frac{q^2}{|\vec{q}|^2} \right) \left(u^2 \frac{d\sigma_L}{dt} + v^2 \frac{d\sigma_R}{dt} + 2uv \frac{d\sigma_S}{dt} \right) , \tag{10}$$

where

$$\begin{aligned}
\frac{d\sigma_S}{dt} &= -\frac{1}{q^2} \frac{\pi}{\kappa} (|\vec{q}|^2 H_{00} + q^0 |\vec{q}| (H_{0z} + H_{z0}) + (q^0)^2 H_{zz}) , \\
\frac{d\sigma_L}{dt} &= \frac{\pi}{\kappa} (H_{xx} + i H_{xy}) , \\
\frac{d\sigma_R}{dt} &= \frac{\pi}{\kappa} (H_{xx} - i H_{xy}) . \tag{11}
\end{aligned}$$

As stated above, at $q^2 = 0$ only σ_S contributes, and given that in this case $q^0 = |\vec{q}|$, one finds that the cross section goes as

$$(|\vec{q}|^2 H_{00} + q^0 |\vec{q}| (H_{0z} + H_{z0}) + (q^0)^2 H_{zz}) = q^\mu q^\nu H_{\mu\nu} . \tag{12}$$

In other words, in the $q^2 = 0$ limit, the lepton tensor of Eq. (3), turns out to be proportional to $q^\mu q^\nu$ and thus, one is left to compute the matrix element of the divergence of the hadronic current. Since the vector NC is conserved, only the divergence of the axial part contributes to the cross section. PCAC can then be used to express the divergence of the axial current in terms of the pion field operator $\Phi(x)$,

$$\partial_\mu A_{NC}^\mu(x) = 2f_\pi m_\pi^2 \Phi(x) , \tag{13}$$

with $f_\pi \approx 92.4$ MeV, the pion decay constant. Treating the nucleus as an elementary particle, one can write

$$\langle \mathcal{N}_{gs} \pi^0(k_\pi) | q_\mu A_{NC}^\mu | \mathcal{N}_{gs} \rangle_{q^2=0} = -2i f_\pi T(\mathcal{N}_{gs} \pi^0(k_\pi) \leftarrow \pi^0(q) \mathcal{N}_{gs}) \Big|_{q^2=0}, \quad (14)$$

where $T(f \leftarrow i)$ is the transition amplitude between the initial hadron state plus a pion of four momentum q^μ , and the final hadronic state. Using this relation, one obtains

$$q^2 \frac{d\sigma_S}{dt} \Big|_{q^2=0} = -4 \frac{E_\pi}{\kappa} f_\pi^2 \frac{d\sigma(\pi^0 \mathcal{N}_{gs} \rightarrow \pi^0 \mathcal{N}_{gs})}{dt} \Big|_{q^2=0}, \quad (15)$$

and then, neglecting the nucleus recoil ($q^0 = E_\pi$), one can further write (Adler's PCAC formula [33])

$$\frac{d\sigma}{dq^2 dy dt} \Big|_{q^2=0} = \frac{G^2 f_\pi^2 E u v}{2\pi^2 |\vec{q}|} \frac{d\sigma(\pi^0 \mathcal{N}_{gs} \rightarrow \pi^0 \mathcal{N}_{gs})}{dt} \Big|_{q^2=0, E_\pi=q^0}. \quad (16)$$

In the $q^2 = 0$ limit that we are using

$$\frac{E u v}{|\vec{q}|} = \frac{1-y}{y}, \quad (17)$$

and thus

$$\frac{d\sigma}{dx dy dt} \Big|_{q^2=0} = \frac{G^2 M E}{\pi^2} f_\pi^2 (1-y) \frac{d\sigma(\pi^0 \mathcal{N}_{gs} \rightarrow \pi^0 \mathcal{N}_{gs})}{dt} \Big|_{q^2=0, E_\pi=q^0}, \quad (18)$$

with $x = -q^2/2Mq^0$ and M the nucleon mass. This latter form, Eq. (18), was adopted in the original RS model [12]. Recently, Berger and Sehgal [18] have proposed to use Eq. (16) instead, for finite q^2 . To our understanding, both choices are somehow arbitrary. As shown in the derivation, there are several factors of the type $q^0/|\vec{q}|$, which do not affect the $q^2 = 0$ calculation, and that might lead to different corrections far from that limit.

An important remark is in order here. Adler's PCAC formula relates the neutrino induced cross section to the off-shell ($0 = q^2 \neq m_\pi^2 = k_\pi^2$) elastic pion-nucleus one (see Eqs. (15), (16), and (18)). It is tempting to neglect off-shell effects and approximate the latter cross section by the experimental one, or by any realistic model for it. This can be only strictly correct in the case of a pointlike nucleus. This was firstly pointed out by J.S. Bell shortly after Adler proposed his PCAC formula. Because of absorption and inelastic collisions, physical pions do not penetrate into the interior of heavy nuclei, and thus the pion-nucleus elastic cross section is only sensitive to the nuclear surface (roughly it scales as $A^{2/3}$, with A the nucleus mass number). But neutrinos penetrate to all parts of nuclei, being then sensitive to the whole nuclear volume; for them cross sections scale as A [34]. Thus Bell expected, that although the on-shell $\sigma_{\pi^0 \mathcal{N}}$ cross section is surface-like, the off-shell ($q^2 = 0$) one turned out to be volume-like. That clearly hints at a non-trivial off-shell behaviour for $\sigma_{\pi^0 \mathcal{N}}$. Similar arguments were raised and discussed by Bernabeu, Ericson and Jarlskog in the context of muon capture in nuclei [35].

With all these caveats, there are cases where approximating the off-shell pion-nucleus cross section by the on-shell one in Eqs. (15), (16), and (18) could be accurate and easily obtained in any good model for pion nuclear scattering. For instance, i) long wavelength limit: the pion wavelength is larger than the nucleus and it probes the whole nuclear volume (this in the case of the nuclear beta decay), or ii) short wavelength limit: here the pion interaction is weak and there is little multiple scattering. Practically we have a collection of independent scatterers (nucleons).

This problem has not been taken into account in the recent works of Refs. [18, 19], where it is proposed to use the experimental elastic pion-nucleus cross section in the Adler's PCAC formula at energies where the process is dominated by the weak excitation of the $\Delta(1232)$ resonance and its subsequent decay into a πN pair. For resonant energies, the pion wavelength is such that it renders doubtful the assumption of a pointlike nucleus, while the pion-nucleon interaction is sufficiently strong to expect that surface effects due to the distortion of the incoming pion waves, included in the experimental elastic pion-nucleus cross section, might induce inaccuracies in the computation of the weak process.

In order to remove such an effect from the physical $\sigma_{\pi^0 \mathcal{N}}$ one should rely on a distortion model, for instance the one described below in Subsect. IV B. The original model of Ref. [12] deals with this *volume-surface* problem when $\langle x \rangle$, the average path length of the pions, is calculated by choosing the path length traversed by pions uniformly produced in the nucleus instead of the path length of pions scattered on the nucleus.

III. REVIEW OF THE REIN–SEHGAL MODEL

As discussed above, the Rein–Sehgal model [12] uses the Adler’s PCAC formula [33] and approximates the coherent π^0 production differential cross section in the laboratory frame by means of Eq. (18). Besides the neglect of the off-shell ($q^2 \neq m_\pi^2$) effects in the right hand side of Eq. (18), this expression already involves a few approximations, that will not be discussed in this work. For instance, the final nucleus recoil energy is also neglected, which allows to approximate q^0 by the pion energy E_π .

In Ref. [12], the expression of Eq. (18) for neutrino coherent pion production is continued to non-forward lepton directions ($q^2 \neq 0$) by attaching a propagator term $1/(1 - q^2/m_A^2)^2$ with $m_A \approx 1$ GeV. This does not mean that the q^2 dependence is merely given by the added propagator term, and in fact it is much more pronounced than that induced by this factor. This is because large (and negative) q^2 values are suppressed by the elastic pion–nucleus differential cross section that strongly favours $t = 0$. Within the $q^0 = E_\pi$ approximation, a zero value for t would imply $q^2 = m_\pi^2$, a kinematical point that cannot be reached since q^2 is space-like. Thus, the lowest values accessible for t must come from the $q^2 \approx 0$ region.

Once $q^2 \neq 0$ there are other finite contributions stemming from σ_L , σ_R and additional pieces of σ_S fully disregarded in the RS model. These contributions come from both the axial and the vector part of the NC. These further approximations have been, somehow, justified because of the strongly forward peaked character of the process that implies that only quite small values of q^2 will be relevant. This assumption will be discussed later in more detail. In particular, as the contribution of the vector current is neglected, the model predicts equal neutrino and antineutrino cross sections.

Next, in Ref. [12], the pion–nucleus cross section, with the caveats mentioned above, is expressed in terms of the pion–nucleon one and then one obtains

$$\frac{d\sigma}{dx dy dt} = \frac{G^2 ME}{\pi^2} f_\pi^2 (1 - y) \frac{1}{(1 - q^2/1 \text{ GeV}^2)^2} \left(|F_A(t)|^2 F_{\text{abs}} \frac{d\sigma(\pi^0 N \rightarrow \pi^0 N)}{dt} \Big|_{E_\pi=q^0, t=0} \right), \quad (19)$$

where $F_A(t)$ is the nuclear form factor which can be calculated as $F_A(t) = \int d^3\vec{r} e^{i(\vec{q} - \vec{k}_\pi) \cdot \vec{r}} \{\rho_p(\vec{r}) + \rho_n(\vec{r})\}$, with the density $\rho_{p(n)}$ normalized to the number of protons (neutrons). Finally, according to [12], F_{abs} is a t -independent attenuation factor that takes into account effects of the outgoing pion absorption in the nucleus².

At this point, some new approximations have been implemented to estimate the pion–nucleus cross section. First, the pion–nucleon cross section is evaluated at $t = 0$, namely, in the forward direction. This can be justified if the nuclear form factor is sufficiently forward peaked. The larger the pion energy and the heavier the nucleus, the better this approximation becomes. In the original paper [12], Eq. (19) was employed for a medium size nucleus, aluminum, and neutrino energies above 2 GeV, for which the relevant pion energies are quite high. However, as we will show below and it was already pointed out in [25], for neutrino energies below 1 GeV and lighter nuclei, like carbon or oxygen, the nuclear form factor is not enough forward peaked to render the finite t -dependence of the pion–nucleon cross section negligible, and even in the forward direction the t value is not close enough to zero. Second, the distortion factor F_{abs} is an oversimplification since in any realistic scattering model this factor should depend on t .

IV. IMPROVEMENTS ON THE RS MODEL

There exist sophisticated microscopical calculations of the neutrino coherent π^0 production off nuclei based on the dominance of the $\Delta(1232)$ resonance [20, 21, 22, 23, 24, 25]. However, these are difficult to extend for the higher neutrino energies present in the current experiments and to implement in Monte Carlo algorithms. Actually, one of the virtues of the RS model is its simplicity, that allows its use for different nuclei and pion energies. This is one of the reasons why the RS model has been widely used.

We propose here a minimal extension of the model in which we implement two main corrections. Firstly, we will take into account the t -dependence of the pion–nucleon cross section when computing the pion–nucleus scattering, and secondly we will work out a more realistic description of the outgoing pion distortion³.

² As defined in [12], F_{abs} only removes from the flux pions that undergo inelastic collisions but no true pion absorption is actually accounted for by means of this factor [25].

³ The pion–nucleus cross section can not be factorized, in addition to the πN elementary cross section, as the product of a nuclear form factor (defined as the Fourier transform of the density) and a distortion factor. This is in contrast with purely electroweak processes not involving strong final state interactions.

A. t -dependence of the pion-nucleon cross section

The $t = 0$ approximation in the right hand side of Eq. (19) is unnecessary and can be easily removed, and one obtains

$$\left(\frac{d\sigma}{dx dy dt}\right)_{q^2=0} = \frac{G^2 M E}{\pi^2} f_\pi^2 (1-y) |F_A(t)|^2 F_{\text{abs}} \frac{d\sigma_{nsf}(\pi^0 N \rightarrow \pi^0 N)}{dt} \Big|_{E_\pi=q^0}, \quad (20)$$

where σ_{nsf} is the non spin-flip part of the pion nucleon cross section because spin-flip processes do not contribute to the elastic pion nucleus cross section. Given that for $t = 0$ the spin-flip cross section is identically zero this constraint was unnecessary in Ref. [12]. The non spin-flip cross section can be evaluated in terms of the phase shifts ($\delta_{I,\ell}^J$) and inelasticities ($\eta_{I,\ell}^J$), that we take from Ref. [36] (J, ℓ and I stand respectively for total angular momentum, orbital angular momentum and isospin of the pion-nucleon pair). For an incoming neutral pion of laboratory energy E_π , the non spin-flip cross section reads

$$\begin{aligned} \frac{d\sigma_{nsf}}{dt} &= \frac{\pi}{9|\vec{k}_\pi|_{c.m.}^2} \left| 2a^{\frac{3}{2}} + a^{\frac{1}{2}} \right|^2, \\ a^I(s, t) &= \sum_\ell \left[(\ell + 1) f_{I,\ell}^{\ell+1/2} + \ell f_{I,\ell}^{\ell-1/2} \right] P_\ell(\cos \theta), \\ f_{I,\ell}^J(s) &= \frac{\eta_{I,\ell}^J e^{2i\delta_{I,\ell}^J} - 1}{2i|\vec{k}_\pi|_{c.m.}}, \quad \cos \theta = 1 + \frac{t}{2|\vec{k}_\pi|_{c.m.}^2}, \end{aligned} \quad (21)$$

with $s = m_\pi^2 + M^2 + 2ME_\pi$, $|\vec{k}_\pi|_{c.m.}^2 = (s - (M + m_\pi)^2)(s - (M - m_\pi)^2)/4s$, the pion momentum squared and P_ℓ the Legendre polynomials. For the sake of completeness we also give here the expression for the inelastic pion-nucleon cross section, σ_{inel}

$$\sigma_{inel}(s) = \frac{4\pi}{|\vec{k}_\pi|_{c.m.}^2} \text{Im} \left(\frac{2}{3} a^{\frac{3}{2}}(s, t=0) + \frac{1}{3} a^{\frac{1}{2}}(s, t=0) \right) - 4\pi \sum_\ell \left[(\ell + 1) |a_\ell^{\ell+1/2}|^2 + \ell |a_\ell^{\ell-1/2}|^2 \right], \quad (22)$$

with $a_\ell^J(s) = (2f_{3/2,\ell}^J(s) + f_{1/2,\ell}^J(s))/3$. To discuss the importance of the t dependence of the cross section we plot in Fig. 1 the ratio $W(t)/W(0)$ for carbon and iron at two different pion energies. The function $W(t)$ is defined as

$$W(t) = |F_A(t)|^2 \frac{d\sigma_{nsf}(\pi^0 N \rightarrow \pi^0 N)}{dt}, \quad (23)$$

and for the RS model $\frac{d\sigma_{nsf}(\pi^0 N \rightarrow \pi^0 N)}{dt}$ is taken at $t = 0$. To allow for a better comparison with the RS model we have used the same nuclear form factor as in Ref. [12], given by $F_A(t) = A e^{bt/6}$, with $b = R_0^2 A^{2/3}$ and A the number of nucleons. As in Ref. [12] we take $R_0 = 1$ fm. As can be appreciated in the figure, the extra dependence of the non-spin flip pion-nucleon cross section on t , neglected in Ref. [12], produces drastic changes in a light nucleus like ^{12}C for low energy pions. We have chosen $E_\pi = 250$ MeV because it corresponds approximately to the maximum of the π^0 yield in the MiniBooNE experiment [28]. The corrected distribution is much narrower and leads to a significantly smaller area. Effects become less important for increasing pion energies and for heavier nuclei thanks to the more pronounced behavior of the nuclear form-factor. In the original work of Ref. [12], aluminum and neutrino energies, at least of 2 GeV, were considered. For 2 GeV neutrinos, the pion spectrum will peak around 1 GeV, and for such pion energies the corrections to the RS formula turn out to be moderately small. But as seen, they can be relevant for the low energy pions produced in low energy neutrino processes.

B. Pion distortion

Next, we pay attention to the distortion of the outgoing pion. We use an eikonal approximation, in the spirit of the original RS model, and taking \vec{q} in the z direction, we replace

$$|F_A(t)|^2 F_{\text{abs}} \equiv \left| \int d^3\vec{r} e^{i(\vec{q} - \vec{k}_\pi) \cdot \vec{r}} \rho(\vec{r}) \right|^2 F_{\text{abs}} \quad (24)$$

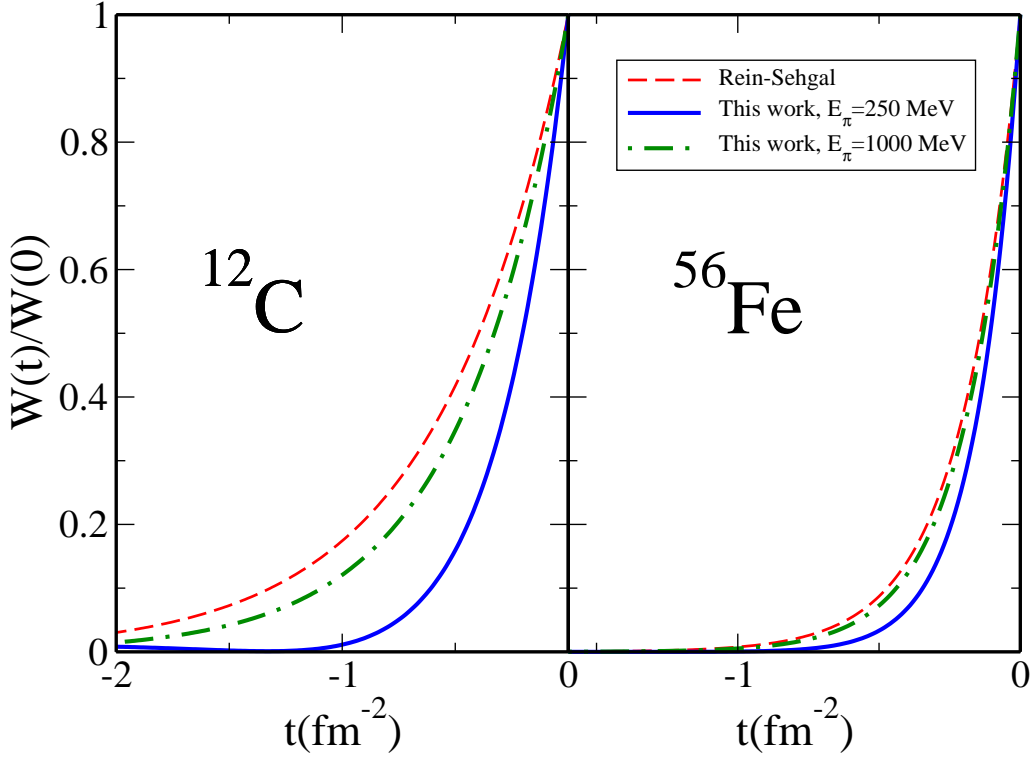


FIG. 1: $W(t)/W(0)$ ratio for carbon and iron at two pion energies.

by

$$|F_{\mathcal{A}}^{distor}(|\vec{q}|, |\vec{k}_{\pi}|, \vec{q} \cdot \vec{k}_{\pi})|^2 \equiv \left| \int d^3\vec{r} e^{i(\vec{q}-\vec{k}_{\pi}) \cdot \vec{r}} \rho(\vec{r}) \Gamma(b, z) \right|^2 \quad (25)$$

$$= \left| 2\pi \int_0^{+\infty} db b J_0(b p_T) \int_{-\infty}^{+\infty} dz \rho(\sqrt{b^2 + z^2}) \Gamma(b, z) e^{ip_z z} \right|^2, \quad (26)$$

with $\rho(\vec{r})$ the nuclear density, $b = \sqrt{x^2 + y^2}$ the impact parameter, $p_z = |\vec{q}| - \vec{q} \cdot \vec{k}_{\pi} / |\vec{q}|$, $p_T = \sqrt{\vec{k}_{\pi}^2 - (\vec{q} \cdot \vec{k}_{\pi})^2 / \vec{q}^2}$ and J_0 the Bessel function. The eikonal distortion factor $\Gamma(b, z)$ is defined as

$$\Gamma(b, z) = \exp\left(-\frac{1}{2}\sigma_{inel} \int_z^{+\infty} dz' \rho(\sqrt{b^2 + z'^2})\right). \quad (27)$$

If one takes $\Gamma(b, z)$ independent of b and z , $F_{\mathcal{A}}^{distor}(|\vec{q}|, |\vec{k}_{\pi}|, \vec{q} \cdot \vec{k}_{\pi})$ depends only on $p_z^2 + p_T^2 = -t$ and it reduces to the structure originally proposed in Ref. [12], where all t dependence is ascribed to the nuclear form factor $F_{\mathcal{A}}(t)$. Indeed, in the original RS model some volumetric average for $\Gamma(b, z)$ is assumed. Neglecting the t -dependence, inherited from the impact parameter dependence of the distortion of the pion waves leads to flatter angular distributions as we will show.

V. PION NUCLEUS SCATTERING

In this section we will confront with data the predictions of the two previous approaches when extended to elastic pion nucleus scattering. By studying low energy pion-nucleus scattering we magnify the effect of the distortion factor and in this way we expect to gain some insight into its possible deficiencies when applied to low energy neutrino-induced coherent pion production.

Following Ref. [12], one would write for the pion nucleus elastic cross section

$$\frac{d\sigma(\pi^0 \mathcal{N}_{gs} \rightarrow \pi^0 \mathcal{N}_{gs})}{dt} = |F_{\mathcal{A}}(t)|^2 F_{\text{abs}}^{\pi} \frac{d\sigma(\pi^0 N \rightarrow \pi^0 N)}{dt} \Big|_{t=0}, \quad (28)$$

where we have replaced the original absorption factor F_{abs} , appropriate for neutrino induced pion production, by F_{abs}^π that also takes into account the initial pion distortion. We have done this following Ref. [12]. Namely, the original absorption factor F_{abs} given in Ref. [12] reads:

$$F_{\text{abs}} = e^{-\langle x \rangle / \lambda}, \quad (29)$$

where $\langle x \rangle$ is the average path length traversed by a π^0 produced homogeneously in the nuclear volume by the neutrino, and $\lambda^{-1} = \sigma_{\text{inel}} \rho$ with σ_{inel} the inelastic π^0 -nucleon cross section. The average $\langle x \rangle$ is found to be $3R/4$ assuming a hard sphere density of radius R for the nucleus. Thus, taking $R = R_0 A^{1/3}$, one finds

$$F_{\text{abs}} = e^{-\frac{9A^{1/3}}{16\pi R_0^2} \sigma_{\text{inel}}}, \quad (30)$$

To properly compare with pion nucleus data one should also consider the distortion of the incoming pion. In this case, after averaging over the impact parameter one obtains $\langle x \rangle = 4R/3$ and thus we replace F_{abs} by F_{abs}^π where

$$F_{\text{abs}}^\pi = e^{-\frac{4A^{1/3}}{\pi R_0^2} \sigma_{\text{inel}}}. \quad (31)$$

Using the original F_{abs} instead of F_{abs}^π amounts to a change of scale but does not modify the angular shape of the cross section. Finally, we obtain σ_{inel} and $\left. \frac{d\sigma(\pi^0 N \rightarrow \pi^0 N)}{dt} \right|_{t=0}$ from the SAID partial wave analysis [36]; detailed expressions were given before. Before discussing the results, we would like to point out that the calculation is for π^0 -nucleus scattering while data correspond to charged pions. However, we have only selected data from isoscalar nuclei and except for Coulomb effects (relevant at small angles) the cross section should be the same.

We would like to stress the fact that the authors of Ref. [12] did not actually make a model for low and medium energy pion-nucleus scattering. The extension to those energies that we are showing here should certainly not be attributed to the RS model.

What we see in Fig. 2 (solid lines), for all nuclei and pion energies shown, is that both the size and the angular dependence predicted by this model derived from the RS approach strongly disagree with data. We have explored the available data for other nuclei and pion energies and the outcome is similar. Within this model, the size of the elastic cross section could be largely affected by even small variations of R_0 (see Eq. (31)), and moreover it is not very clear why one should use σ_{inel} in the absorption factor, as we discuss below.

In the case of coherent pion production induced by neutrinos one expects distortion effects, accounted for by F_{abs} , to be less relevant and besides one should bear in mind that experimental analyses do often adjust the size of the cross section with a free parameter. However, those neutrino analyses always rely on a theoretical model for the outgoing pion angular distribution. As shown in the figure, the RS derived model leads for low pion energies to π -nucleus differential cross sections much flatter than experiment for all nuclei and one would expect this will also be the case for coherent pion production induced by low energy neutrinos. From this latter perspective, we tentatively conclude that the RS model for neutrino-induced coherent pion production, widely used in the literature, might induce important uncertainties in the analysis of the neutrino oscillation experiments for low energy incident neutrinos.

We discuss now the results that one obtains with the modifications to the RS model introduced in section IV. The pion-nucleus scattering would now read

$$\frac{d\sigma(\pi^0 \mathcal{N}_{gs} \rightarrow \pi^0 \mathcal{N}_{gs})}{dt} = |F_{\mathcal{A}}^{\text{distor}-\pi}(\vec{q}, \vec{k}_\pi)|^2 \frac{d\sigma_{\text{nsf}}(\pi^0 N \rightarrow \pi^0 N)}{dt}, \quad (32)$$

where

$$|F_{\mathcal{A}}^{\text{distor}-\pi}(\vec{q}, \vec{k}_\pi)|^2 \equiv \left| \int d^3\vec{r} e^{i(\vec{q}-\vec{k}_\pi) \cdot \vec{r}} \rho(\vec{r}) \Gamma^\pi(b) \right|^2 \quad (33)$$

$$\Gamma^\pi(b) = \exp\left(-\frac{1}{2} \sigma_{\text{inel}} \int_{-\infty}^{+\infty} dz' \rho(\sqrt{b^2 + z'^2})\right). \quad (34)$$

The profile Γ^π does not depend on z since both the incoming and outgoing pions should now be distorted.

We show our predictions (Eq. (32)) in Fig. 2 with dashed-lines. As can be seen in the plots the two simple corrections (t -dependence of the pion-nucleon cross section and a more realistic description of the pion distortion) lead to a better description of the angular dependence, improving enormously previous RS based model results.

Finally, we would like to point out that it is not clear why one should use σ_{inel} in the distortion profile. Indeed, within the eikonal approximation one should use the imaginary part of a realistic pion-nucleus optical potential [22].

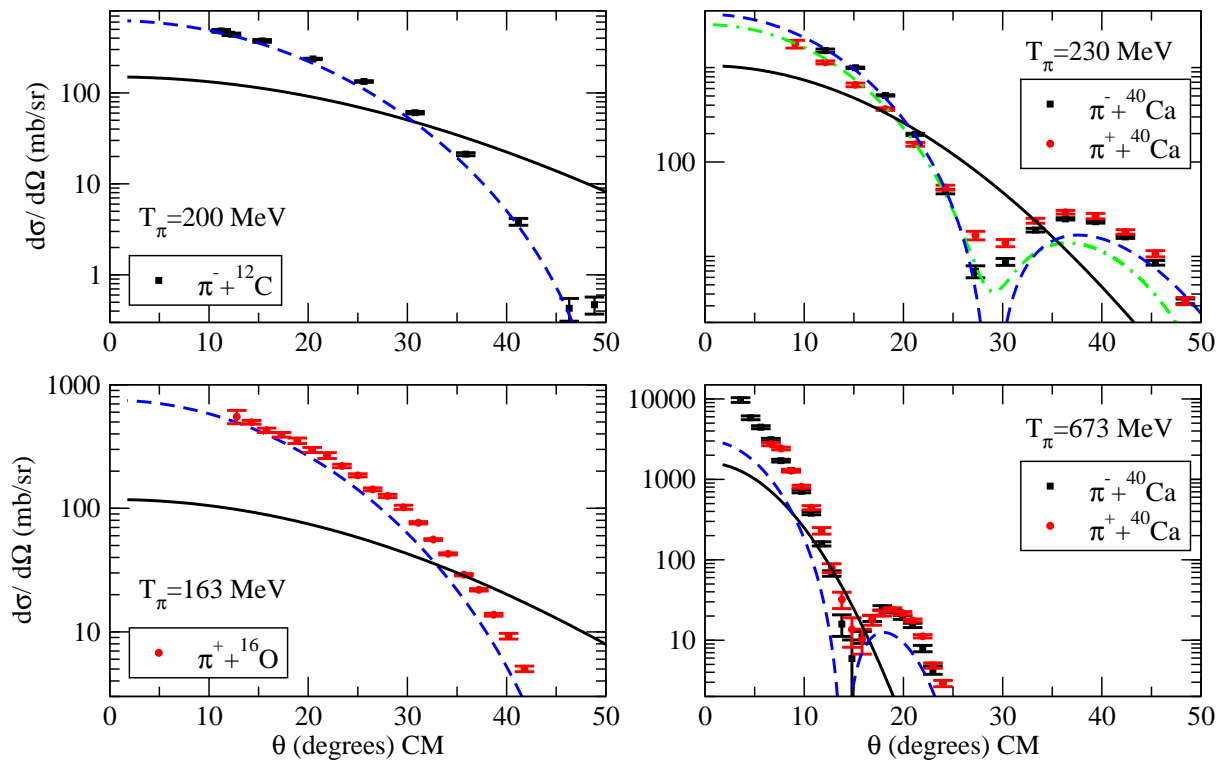


FIG. 2: Pion nucleus elastic cross section for several nuclei and energies. Solid line: RS based model. Dashed line: Improved model of Eq. (32). For comparison in the right top panel, we also display results (dashed dotted green line) obtained from the solution of the Klein–Gordon equation with a microscopic potential calculated within the Δ -hole model [37], as employed in Refs. [23, 24, 25]. To compute our results, we have used modified harmonic oscillator [$\rho(r) = \rho_0(1 + a(r/R)^2) \exp(-(r/R)^2)$] densities for carbon and oxygen and a two parameter Fermi distribution [$\rho(r) = \rho_0/(1 + \exp((r - c)/a_0))$] for calcium. Density parameters: $R = 1.692$ fm, $a = 1.082$ and $R = 1.833$ fm, $a = 1.544$ for ^{12}C and ^{16}O ; $c = 3.51$ fm and $a_0 = 0.563$ fm for ^{40}Ca . Data are from Refs. [38] (^{12}C), [39] (^{16}O), [40] (^{40}Ca at $T_\pi = 230$ MeV) and [41] (^{40}Ca at $T_\pi = 673$ MeV).

This would eliminate those pions that are absorbed, undergo quasielastic processes or suffer inelastic reactions, taking into account Pauli blocking, Fermi motion and other many-body effects to evaluate the corresponding reaction probabilities. Such kind of sophisticated optical potential is only available for low energy pions, up to the region of the $\Delta(1232)$ resonance (see for instance Refs. [37, 42, 43]), and it has been used in the microscopical approaches of Refs. [22, 23, 24, 25]. Its extension to higher energies is a highly non-trivial task. Here, we are not so much concerned with the size of the cross sections but rather with the outgoing pion angular and energy dependence. For this latter purpose, it might be sufficient to use σ_{inel} to compute the distortion profile. It is true that in this way, we do neither account for pion absorption nor for pion quasielastic distortion (induced by pion–nucleon elastic scattering). However, this is partially compensated since the use of σ_{inel} leads to a larger distortion than it would be expected, if the inelastic processes were considered in the nuclear medium and Fermi statistics was taken into account. The same discussion applies to the profile function, $\Gamma(b, z)$, relevant for the neutrino coherent pion production reaction, and defined in Eq. (27).

VI. NEUTRINO-INDUCED COHERENT π^0 PRODUCTION RESULTS

Once our framework has been satisfactorily tested against elastic pion-nucleus differential cross section data, we give here results for neutrino-induced coherent π^0 production. We work in the neutrino-nucleus LAB frame. For the x, y, t differential coherent π^0 production cross section we use

$$\frac{d\sigma}{dx dy dt} = \frac{G^2 ME}{\pi^2} f_\pi^2 (1 - y) \frac{H[1 - |\cos\theta|]}{(1 - q^2/1\text{GeV}^2)^2} |F_A^{distort}(|\vec{q}|, |\vec{k}_\pi|, \vec{q} \cdot \vec{k}_\pi)|^2 \frac{d\sigma_{nsf}(\pi^0 N \rightarrow \pi^0 N)}{dt} \Big|_{E_\pi=q^0}, \quad (35)$$

with $H[\dots]$ the step function and θ the center of mass pion angle in the free pion–nucleon elastic reaction (see Eq. (21)).

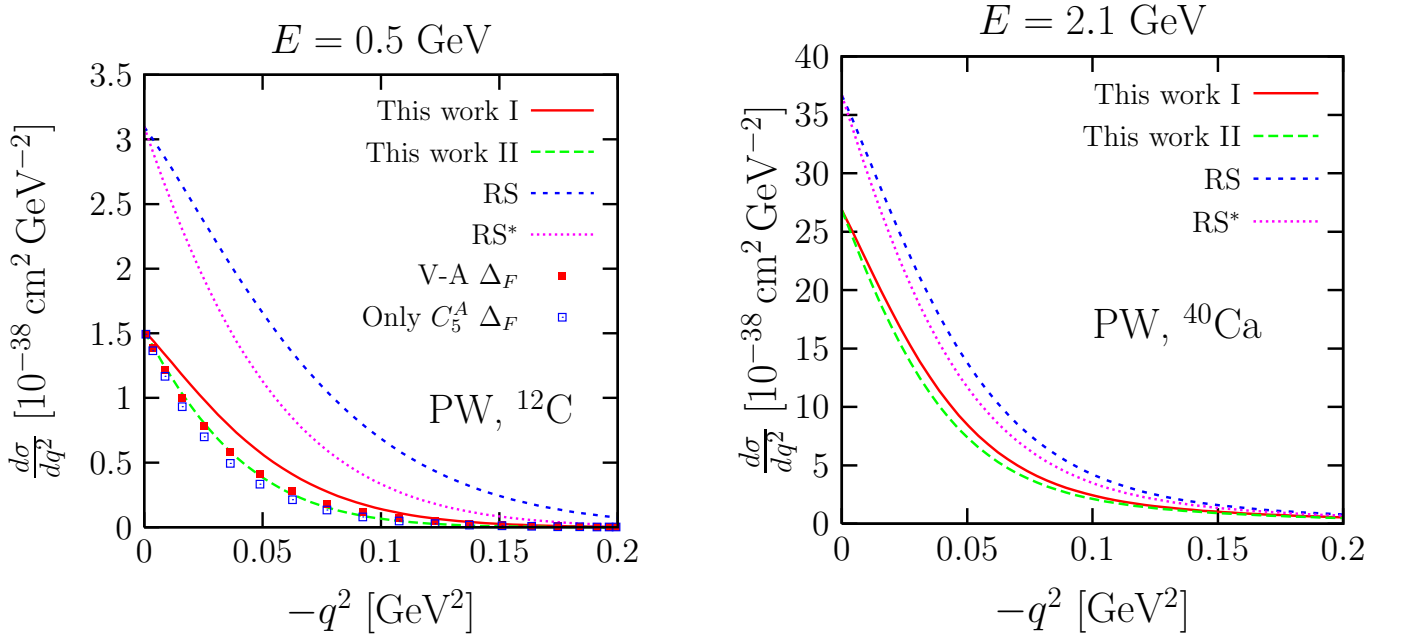


FIG. 3: Plane Wave (no pion distortion) q^2 distributions obtained from models I and II, together with those deduced from the RS and RS* models. In addition for $E=0.5$ GeV in carbon, we also display results from the microscopic approach described in the text. Filled (open) squares stand for the calculation with the full V-A (only with the C_5^A contribution) $N\Delta$ current.

That defines our model I. We will also consider a second model (II), where some kinematical corrections, recently proposed by Berger and Sehgal [18], are incorporated. Those corrections, and their degree of arbitrariness, were already discussed in Section II and they amount to replace in Eq. (35)

$$(1-y) \leftrightarrow \frac{q^0}{|\vec{q}|} u v = \frac{q^0}{|\vec{q}|} \left(1 - y + \frac{q^2}{4E^2} \right). \quad (36)$$

We also incorporate these corrections in the original RS model performing the above replacement in Eq. (19). In what follows, we will refer to this latter model as RS*. Although we will look at the effect of the kinematical corrections proposed in Ref. [18], we will not present results obtained from the full approach followed in that reference. The reason being that in Ref. [18] the use of the experimental pion-nucleus elastic cross section is advocated. This is not fully correct because of the strong distortion of the incoming pion, implicit in the experimental cross section data, that should not be taken into account in neutrino-induced coherent pion production.

A. q^2 -distributions

We readily obtain $d\sigma/dq^2$ from Eq. (35) by integrating over y and t and performing the change of variables $x \leftrightarrow q^2$. In Fig. 3, we show results obtained without distortion of the final pion. Eliminating distortion allows us to check, by comparison with a microscopical calculation, the validity of the $t=0$ approximation used in the RS and RS* models. With that aim, in carbon and for $E=0.5$ GeV, we also display results from the microscopic model of Ref. [25]. At this energy, neutrino-induced coherent pion production is dominated by the excitation of the $\Delta(1232)$ resonance and its subsequent decay into a $\pi^0 N$ pair. Thus, for simplicity, we have neglected the tiny effects of the non-resonant background and the crossed- Δ term included in Ref. [25]. Besides, as all discussed models are based upon the free pion nucleon cross section, nuclear medium effects that modify the Δ propagator have not been included. We consider two cases, one that takes into account the full $N\Delta$ weak current (V-A Δ_F), and another that only includes the dominant axial term proportional to the C_5^A form factor⁴ (Only $C_5^A \Delta_F$). PCAC based models rely on Eq. (14)

⁴ See for instance Eq. (1) of Ref. [44] for a form factor decomposition of the $N\Delta$ weak current.

which relates, at $q^2 = 0$, a weak matrix element with a purely hadronic one. In the case of the $N\Delta$ transition, the only term of the divergence of the axial current that survives at $q^2 = 0$ is that proportional to the form factor C_5^A . Thus, to make meaningful the comparison between the PCAC based models examined here and the microscopic approach including explicitly a Δ , requires fixing $C_5^A(0)$ to the value predicted by the non-diagonal Goldberger–Treiman relation ($C_5^A(0) \equiv \sqrt{\frac{2}{3}} f_\pi \frac{f^*}{m_\pi} = 1.2$, with the $\pi N\Delta$ coupling $f^* = 2.2$ fixed to the Δ width). We see that I and II calculations are in reasonable agreement with the microscopical model, whereas the RS and RS* ones predict much larger and wider differential cross sections. As mentioned above, this is mainly due to the $t = 0$ approximation assumed in those models⁵. The effects due to the $t = 0$ approximation decrease with the neutrino energy and atomic number, but they are still important for $E=2.1$ GeV in a medium sized nucleus like calcium. At $q^2 = 0$, and in the absence of distortion, our model should essentially match the microscopical calculation (as it was discussed in Sect. IVB-1 of Ref. [25]), and indeed the two models nicely agree at this kinematical point.

Far from $q^2 = 0$, PCAC based models just consider the axial part of the current, that given the tiny effect of the vector part in the microscopical model (as can be seen by comparing “V–A Δ_F ” and “Only $C_5^A \Delta_F$ ” microscopic results), it turns out to be an excellent approximation here. Our model II describes the “Only $C_5^A \Delta_F$ ” results better than model I, which make us conclude that the kinematical corrections recently proposed by Berger and Sehgal [18] are sound.

The small differences at finite q^2 can be due to some discrepancies between the experimental pion-nucleon cross section and that deduced from the Δ mechanism alone (with f^* fixed from the Δ width), to offshellness ($q^2 \neq m_\pi^2$) effects, or to different treatments of the nucleon dynamics inside the nuclear medium. Besides, there are other effects that vanish at $q^2 = 0$ and that are present in this case. For instance, in the microscopical model the σ_S structure function (Eq. (11)) is not proportional to $q^\mu q^\nu H_{\mu\nu}$ and hence σ_S cannot be exactly related to the pion-nucleus cross section for $q^2 \neq 0$ values. Similarly, corrections due to the σ_R , σ_L structure functions in Eq. (10) are not taken into account within the PCAC approximation. Other sources of discrepancies come from the adopted q^2 behaviour of $C_5^A(q^2)$ [25], which gives rise to a faster q^2 -decrease than that provided by the propagator term $1/(1 - q^2/m_\Delta^2)^2$ included in the PCAC based models. Nonetheless, at least for this observable, the effect of all these differences is minor and model II gives a good approximation to the microscopical calculation⁶.

Now we look at the results with distortion of the final pion that we show in Fig. 4. In this case, for comparison, we use the model based on the microscopic approach developed in Refs. [24, 25] including both distortion and the nuclear medium effects affecting the $\Delta(1232)$. Again, and for simplicity, we have neglected the tiny effect of the non-resonant background and the crossed- Δ term included in Ref. [25].

⁵ The used nuclear form-factors are also different (see text after Eqs. (19) and (23)), but their effect can not account for the large differences observed. As a matter of example in ^{12}C and for $E=0.5$ GeV, the plane wave RS model gives an integrated cross section (in units of 10^{-38} cm²) of 0.206, while our model I predicts a value of 0.070 in the same units. If we use model I with the nuclear form factor proposed in the original RS model, we find an integrated cross section of $0.089 \cdot 10^{-38}$ cm².

⁶ A word of caution must be said here. The basic PCAC formula in Eq. (14) might suffer from corrections. For instance, in the Δ region, a value for $C_5^A(0)$ significantly smaller (0.87) than 1.2 was used in Ref. [25]. This reduced coupling would lead to cross sections around a factor of two $[(1.2/0.87)^2]$ smaller than those presented in Fig. 3. This violation of the off-diagonal Goldberger-Treiman relation by about 30% was proposed in Ref. [9] after fitting the Argonne bubble chamber $\nu_\mu + p \rightarrow \mu^- \pi^+ p$ cross section data [10], including a non-resonant background. Non-resonant background terms, though important at the nucleon level, turn out to be negligible in the neutrino coherent pion process [25]. The lattice QCD results shown in Figure 4 of Ref. [45] might also support a value for $C_5^A(0)$ smaller than $\sqrt{\frac{2}{3}} f_\pi \frac{f^*}{m_\pi}$. This is indeed a serious problem that deserves a joined theoretical and experimental effort to sort it out.

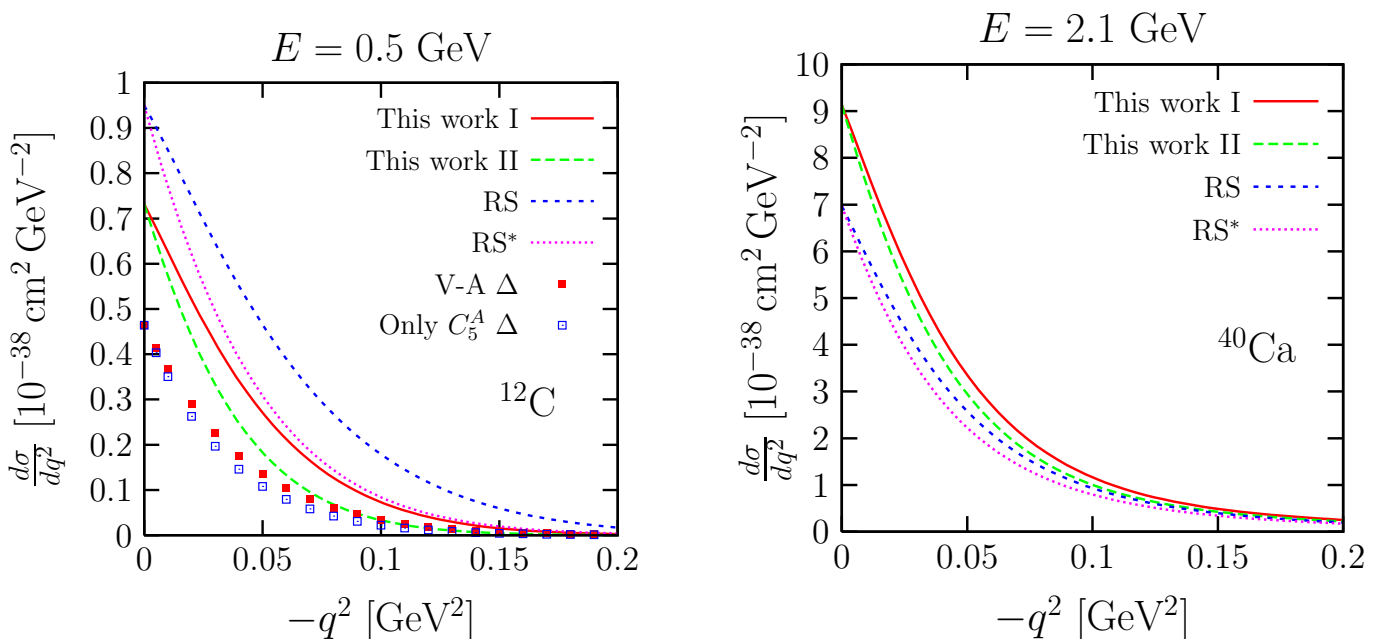


FIG. 4: Full q^2 distributions (including distortion of the outgoing pion). For $E=0.5$ GeV in carbon we also display results from the microscopic approach of Ref. [25] with $C_5^A(0) = 1.2$. Filled (open) squares stand for the calculation with the full V-A (only with the C_5^A contribution) $N\Delta$ current.

Distortion reduction is significantly larger in the RS and RS* models than in our scheme. That diminishes the differences in the full calculation in carbon, and explains the reverse pattern observed in calcium. Note, however, that the pion distortion in the RS and RS* schemes is too strong. Indeed, it leads to forward pion-nucleus elastic cross sections smaller than both data and those deduced within our framework, as can be appreciated in Fig. 2. In spite of that, both the RS and RS* models still overestimate the microscopical calculation in carbon by a large factor.

On the other hand, the situation is somehow puzzling when we compare microscopical results with those of model I or II. As discussed above, within the PW approximation both types of approach nicely agree, but distorted results notably disagree. This is despite the fact that the Klein-Gordon Δ -hole model, in which the microscopical results depicted in Fig. 4 are based, leads to elastic pion nucleus cross sections of similar quality as those deduced from the eikonal framework developed here, and that our model I and II use (see for instance the comparison performed in the right top panel of Fig. 2 for a typical pion energy). This contradiction deserves some discussion, and we believe it is related to the inadequacy of the use of eikonal distortion for low energy pions and to the modifications of the Δ resonance properties in the nuclear medium.

Describing correctly the elastic pion-nucleus cross section is a necessary condition that should satisfy any model aiming at a proper description of the coherent pion production process induced by neutrinos. However, that constraint is not sufficient, and the situation in Fig. 4 is an example of that.

One of the major sources of differences between our model of Eq. (32) and the microscopical calculation is due to in-medium modifications of the free pion-nucleon cross section in the latter case (see for instance Ref. [25]). Within the eikonal model assumed here to construct the elastic pion-nucleus cross section, there are two competing factors: the pion-nucleon elastic non spin flip σ_{nsf} cross section and the pion distortion, controlled by σ_{inel} . As a consequence, the pion-nucleus scattering amplitude does not linearly depend on the corresponding pion-nucleon one. That explains why different approaches for the in medium pion-nucleon amplitude could lead to similar pion-nucleus cross sections. For instance, let us suppose that in medium modifications reduce (enhance) the pion-nucleon cross sections. We have that pion distortion effects become less (more) important with decreasing (increasing) σ_{inel} and this reduced (enhanced) suppression can compensate a decrease (an enhancement) in σ_{nsf} . The outcome is that you can predict similar pion-nucleus cross sections starting from quite different pion-nucleon cross sections. It seems that induced differences in the pion distortion compensate the changes due to the medium in the elastic pion-nucleon cross section in pion-nucleus scattering, but not in neutrino induced coherent pion production where distortion becomes smaller and affects only the outgoing pion.

For low energy pions, of interest in MiniBooNE and T2K, we believe that the distorted results from the microscopical models of Refs. [24, 25] are more realistic than those based on the eikonal approximation presented here, where the pion-nucleon interaction is not modified in the medium and a simple model based on σ_{inel} is used to distort the

outgoing pion waves. The framework of Ref. [22], despite the use of the eikonal approximation, greatly overcomes these shortcomings, since there, the modification of the Δ properties in the medium are taken into account, and the imaginary part of a realistic pion-nucleus optical potential is used to distort the outgoing pion. The model becomes as complicated as that of Refs. [24, 25], as difficult as this latter one to extrapolate at higher pion energies, and it is still less reliable, since multiple scattering is not taken into account within the eikonal approximation as accurately as by solving the Klein Gordon equation [24, 25].

B. Pion distributions

For NC driven processes, the q^2 distribution can not be easily measured because of the obvious difficulty in detecting the outgoing neutrino or the nucleus recoil. Thus, it is of great interest to examine pion distributions.

1. Pion energy distributions

In Fig. 5, we show the outgoing pion spectrum from oxygen, with and without distortion effects, predicted by the different models considered in the previous subsection. The incoming neutrino energy is 0.65 GeV. The integrated cross sections are compiled in Table I.

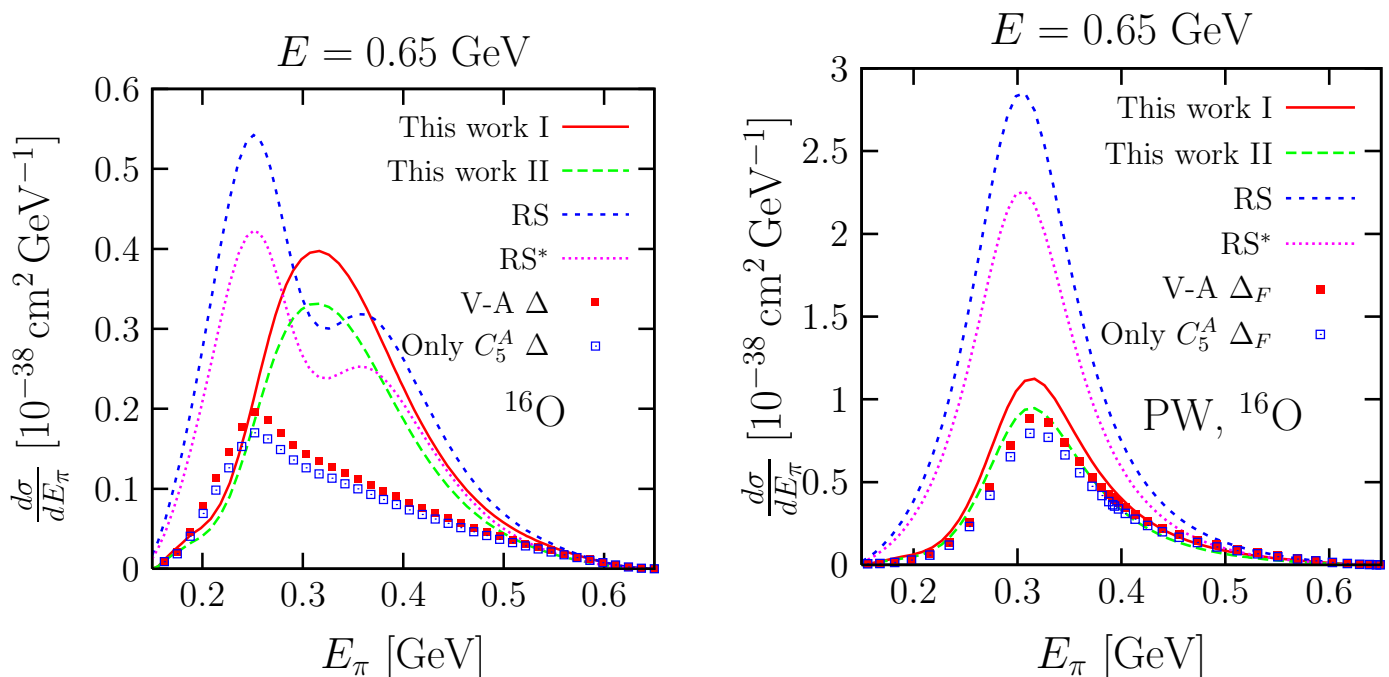


FIG. 5: Distorted (left panel) and plane wave (right panel) $d\sigma/dE_\pi$ results in oxygen obtained from our models I and II, together with those deduced from the original RS and the RS* models. The incoming neutrino energy is 0.65 GeV. In addition, we also display results from the microscopic approach of Ref. [25] with $C_5^A(0) = 1.2$. Filled (open) squares stand for the calculation with the full V-A (only with the C_5^A contribution) $N\Delta$ current.

We see once more that, for the case without distortion, our model II has a reasonable agreement with the “V-A Δ_F ” and “Only $C_5^A \Delta_F$ ” microscopic calculations. Not only the shapes, which peak at around $E_\pi = 320$ MeV, but also the integrated cross sections are similar (see the table). Again the RS and RS* models give much larger results and the position of the peak is slightly displaced towards lower energies.

In the distortion case we find totally different behaviours. Results obtained with our models I and II show a shape similar to the case without distortion, while the RS and RS* calculations show a double peak structure. In the microscopical calculation a single peak appears, but now at a lower pion energy $E_\pi \approx 260$ MeV. This change in the position of the peak in the microscopical results can be traced to the modification of the properties of the Δ in the medium and to nonlocal effects in the amplitude (see for instance Fig. 2a in Ref. [25]). These non-localities reflect the

	Microscopical					
	This work I	This work II	V-A Δ	Only C_5^A Δ	RS	RS*
PW	0.152	0.127	0.117	0.106	0.389	0.305
DW	0.072	0.060	0.037	0.033	0.100	0.078

TABLE I: Integrated cross sections (in units of 10^{-38} cm²) corresponding to the different curves displayed in Fig.5. PW and DW stand for plane wave and distorted results, respectively.

fact that the pion three-momentum is only well defined asymptotically when the pion-nucleus potential vanishes [25]. These effects are neither present in any PCAC based model where one uses the free-space pion nucleon cross sections, nor in any microscopic calculation with plane waves.

At low and intermediate neutrino energies, once the distortion is considered, we must again conclude that the microscopic model predictions are most reliable than those deduced from the PCAC based model examined here. The original RS predictions have little reliability and the hope is that at higher energies, where we expect nuclear medium modifications of the elementary pion–nucleon interaction to be less relevant, the model I and II could become better suited.

2. Pion angular distributions

The starting point of the PCAC based models is the x, y, t differential cross section. Firstly, we would like to stress that for $q^2 \neq 0$, the knowledge of $\frac{d\sigma}{dx dy dt}$ is not sufficient to compute the angular distribution of the outgoing pion with respect to the direction of the incoming neutrino. Let us take here as z-axis the direction of the incoming neutrino. The pion and transferred momenta are given by

$$\vec{k}_\pi = |\vec{k}_\pi| (\sin \theta_\pi \cos \phi_\pi, \sin \theta_\pi \sin \phi_\pi, \cos \theta_\pi), \quad (37)$$

$$\vec{q} = (-|\vec{k}'| \sin \theta', 0, |\vec{k}| - |\vec{k}'| \cos \theta'), \quad (38)$$

with θ' the outgoing neutrino scattering angle, θ_π the angle between the incoming neutrino and the outgoing pion and ϕ_π the azimuthal pion angle in this frame. By means of a rotation that takes \vec{q} along the positive z -axis one can easily obtain

$$\cos \theta_\pi = \hat{k} \cdot \hat{k}_\pi = \frac{|\vec{k}'|}{|\vec{q}|} \sin \theta' \sin \theta_{k_\pi q} \cos \phi_{k_\pi q} + \frac{|\vec{k}| - |\vec{k}'| \cos \theta'}{|\vec{q}|} \cos \theta_{k_\pi q}, \quad (39)$$

where $\theta_{k_\pi q}$ and $\phi_{k_\pi q}$ are the pion polar and azimuthal angles in that frame where the positive z -axis is taken in the direction of \vec{q} .

The incoming neutrino energy and the variables x and y determine $|\vec{k}'|$, $|\vec{q}|$ and θ' , while, within the $E_\pi = q^0$ approximation, t fixes $\theta_{k_\pi q}$ [$t = -\vec{q}^2 - \vec{k}_\pi^2 + 2|\vec{q}||\vec{k}_\pi| \cos \theta_{k_\pi q}$], remaining $\phi_{k_\pi q}$ undetermined. Thus, as stated before, the knowledge of $d\sigma/dx dy dt$ is not sufficient to compute the angular distribution of the outgoing pion with respect to the direction of the incoming neutrino, and it would be necessary to know $\frac{d\sigma}{dx dy dt d\phi_{k_\pi q}}$. Only for $q^2 = 0$, $\theta_{k_\pi q} = \theta_\pi$, since $\theta' = 0$ (both the outgoing neutrino and the momentum transfer go along the incoming neutrino direction) and $\frac{d\sigma}{dx dy dt} \Big|_{q^2=0}$ determines $\frac{d\sigma}{dx dy d \cos \theta_\pi} \Big|_{q^2=0}$.

In PCAC based models it is assumed,

$$\frac{d\sigma}{dx dy dt d\phi_{k_\pi q}} = \frac{1}{2\pi} \frac{d\sigma}{dx dy dt} = \frac{1}{2\pi} \int d\phi_{k_\pi q} \frac{d\sigma}{dx dy dt d\phi_{k_\pi q}}, \quad (40)$$

which leads to [12]

$$\frac{d\sigma}{dE_\pi d\eta d \cos \theta' d\phi_\pi} = \frac{E - E_\pi}{ME_\pi^2} \frac{|\vec{k}_\pi| |\vec{q}|}{\pi} H [1 - |\cos \theta_\pi|] \frac{d\sigma}{dx dy dt}, \quad (41)$$

where $\eta = E_\pi(1 - \cos \theta_\pi)$ is the variable proposed by the MiniBooNE Collaboration in its recent analysis of coherent π^0 production of Ref. [28].

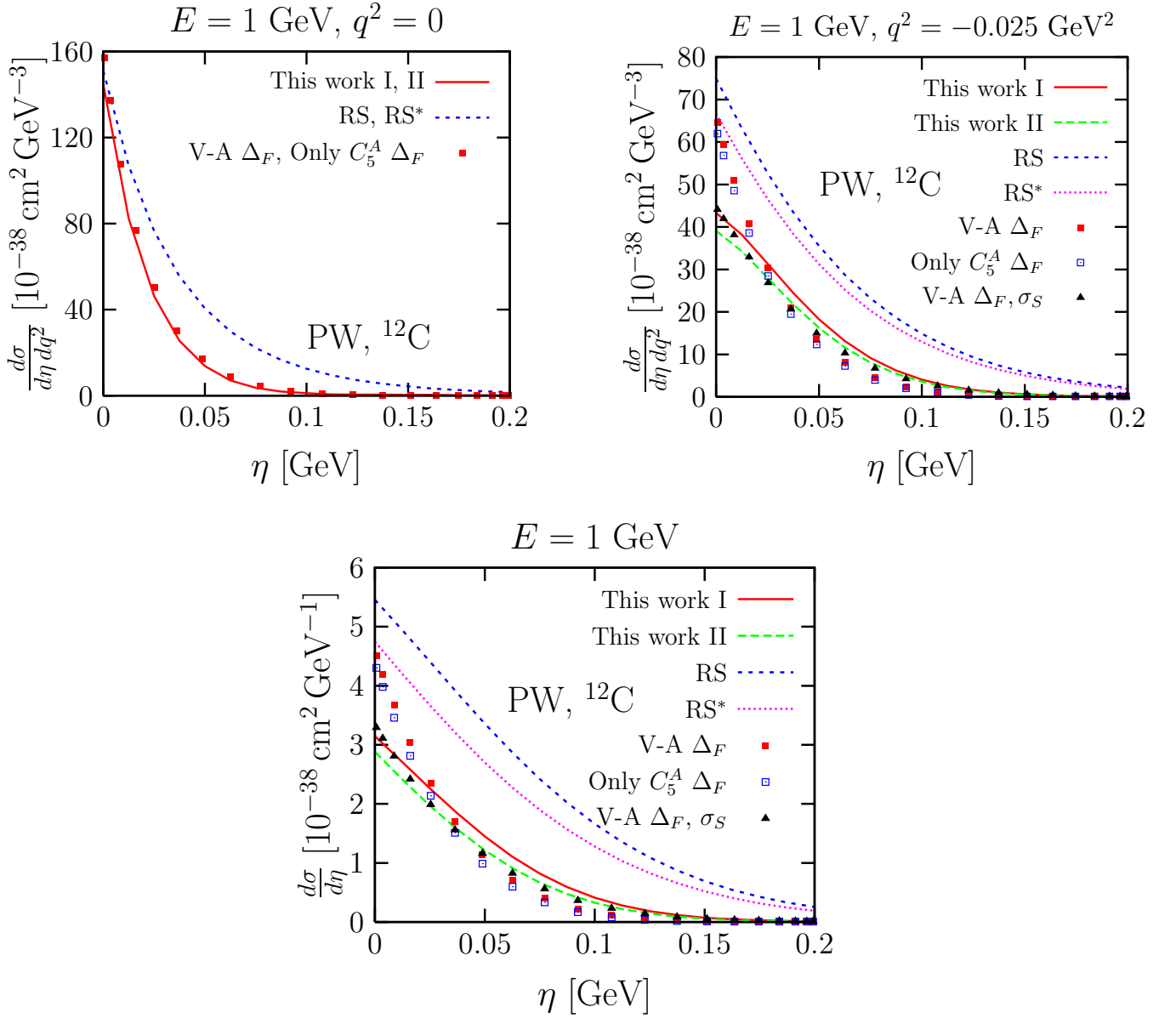


FIG. 6: Undistorted $d\sigma/d\eta dq^2$ (top panels) and $d\sigma/d\eta$ (bottom panel) differential cross sections obtained from models I, II, RS and RS* and the microscopic approach of Ref. [25] with $C_5^A(0) = 1.2$. Filled (open) squares stand for the calculation with the full V-A (only with the C_5^A contribution) $N\Delta$ current. In the top-right and bottom panels we also show results (triangles) for the “V-A Δ_F ” microscopic calculation including only the σ_S contribution.

For non-vanishing q^2 values, Eq. (40) is incorrect, and therefore Eq. (41) is wrong as well. The problem arises because $\frac{d\sigma}{dx dy dt d\phi_{k\pi q}}$ depends in general on $\phi_{k\pi q}$, through the $\frac{dA}{dt d\phi_{k\pi q}}$ term in Eq. (4). When q^2 is zero that term does not contribute and thus Eqs. (40) and (41) are correct only in this limit.

Let us start by looking at undistorted results in Fig. 6. In the two upper panels we show $\frac{d\sigma}{d\eta dq^2}$ differential cross sections for two different q^2 values, while in the lower panels we show q^2 integrated distributions. The nucleus is carbon and the incident neutrino energy is $E = 1$ GeV. For $q^2 = 0$ (top-left panel) we find a very nice agreement between our models I and II calculation and the microscopic model results for $\frac{d\sigma}{d\eta dq^2}$. On the other hand, RS and RS* models give rise to significantly flatter distributions. In the top-right panel, we show results for $q^2 = -0.25$ GeV. We find now a disagreement of the models I and II with the microscopic calculation. This disagreement goes over to the q^2 integrated $\frac{d\sigma}{d\eta}$ differential cross section shown in the bottom panel. The microscopic calculation is much more peaked at $\eta = 0$ than either models I and II or the RS and RS* models, the latter two showing the flattest

distributions. The reason for the discrepancy between the microscopic model and the PCAC based ones is due to two approximations made in the latter models. The first approximation made in PCAC based models is the one encoded in Eq. (40) which leads to Eq. (41) instead of the correct expression

$$\frac{d\sigma}{dE_\pi d\eta d\cos\theta' d\phi_\pi} = 2 \frac{E - E_\pi}{ME_\pi^2} |\vec{k}_\pi| |\vec{q}| H [1 - |\cos\theta_\pi|] \frac{d\sigma}{dx dy dt d\phi_{k_\pi q}} \quad (42)$$

As mentioned above, the approximation in Eq. (40) amounts to neglect the $\frac{d\mathcal{A}}{dt d\phi_{k_\pi q}}$ term in Eq. (4), term that the microscopic calculation properly takes into account.

Besides, PCAC based models only consider the σ_S contribution in Eq. (4) while the microscopic one also takes into account the contributions from σ_R and σ_L . To see the effects associated to these neglected terms we also show in the top-right and bottom panels the “V–A Δ_F ” microscopic calculation considering only the σ_S contribution. What we see is that the differential cross sections become much flatter than before, being now in a nice agreement with our model II calculation. The fact that the RS and RS* models give the flattest distributions is explained by the further $t = 0$ approximation used there.

The conclusion is that in PCAC models σ_R , σ_L and \mathcal{A} structure functions in Eq. (4) cannot be taken into account. The omission of these contributions leads to significantly flatter $d\sigma/d\eta$ differential cross sections than those predicted by a microscopic calculation. This affects both our models I and II and the RS and RS* models. In the latter two cases the $t = 0$ approximation enhances this behaviour.

This is also true for distorted results that are shown in Fig. 7. Again the microscopic calculation is more peaked close to $\eta = 0$, and the RS and RS* results show the flattest distributions due to the $t = 0$ approximation. In any case, model RS* and even more model II represent a major improvement as compared to the original RS model for this observable.

These findings have immediate consequences for the results published by the MiniBooNE Collaboration in its recent analysis of coherent π^0 production of Ref. [28], which rely on the RS model. The distribution for $E_\pi(1 - \cos\theta_\pi)$ given there should be definitely much narrower and much more peaked around zero, and thus it might improve the description of the first bin value in Fig. 3b of this reference. This was already suggested in Ref. [25].

VII. SUMMARY AND CONCLUSIONS

We have critically reviewed the commonly used Rein–Sehgal model for NC neutrino coherent pion production [12]. We have unambiguously pointed out the main deficiencies of this model, which induce important uncertainties for pions of relatively low energy, as those of relevance in the MiniBooNE and T2K experiments. Among others, the more relevant ones are:

1. The t dependence of the coherent production is fully ascribed to the nuclear form factor $F_A(t)$, while further and significant t –dependences induced by the pion–nucleon interaction are ignored (see Fig. 1).

The recent works of Refs. [18, 19] try to overcome this problem by using experimental information on the t –dependence of the elastic pion–nucleus cross section. Apart from the obvious limitation coming from the lack of experimental data for many pion energies and nuclei, this might not be appropriate either for energies where the process is dominated by the weak excitation of the $\Delta(1232)$ resonance. As discussed in Sect. II there might be a non-trivial off-shell behaviour for $\sigma_{\pi^0\mathcal{N}}$ [34]. In a microscopic approach one would argue that because of the strong distortion of the incoming pion in the on-shell elastic pion nucleus process at energies in the $\Delta(1232)$ resonance region, one cannot directly relate its amplitude to the pion production induced by a weak current. That takes us naturally to the second caveat.

2. The treatment of the outgoing pion distortion within the original RS model is quite poor, and it turns out to be certainly insufficient for resonant pions. An unquestionable evidence for that can be seen in Fig. 2, where it is shown that both the size and the angular dependence predicted by a model derived from the RS approach strongly disagree with the elastic pion–nucleus differential cross section data.
3. Far from the $q^2 = 0$ kinematical point, any PCAC based model, and in particular the RS one, cannot be used to determine the angular distribution of the outgoing pion with respect to the direction of the incoming neutrino (see right top and bottom panels of Fig. 6). Terms that vanish at $q^2 = 0$, and that are not considered in PCAC based models, provide much more forward peaked outgoing pion–incoming neutrino angular distributions.

PCAC models can only determine the distribution on the angle formed by the pion and the lepton transferred momentum, \vec{q} . Experimentally, one can have access to this latter differential cross section in the case of CC driven processes, but not when the reaction takes place through the weak neutrino NC.

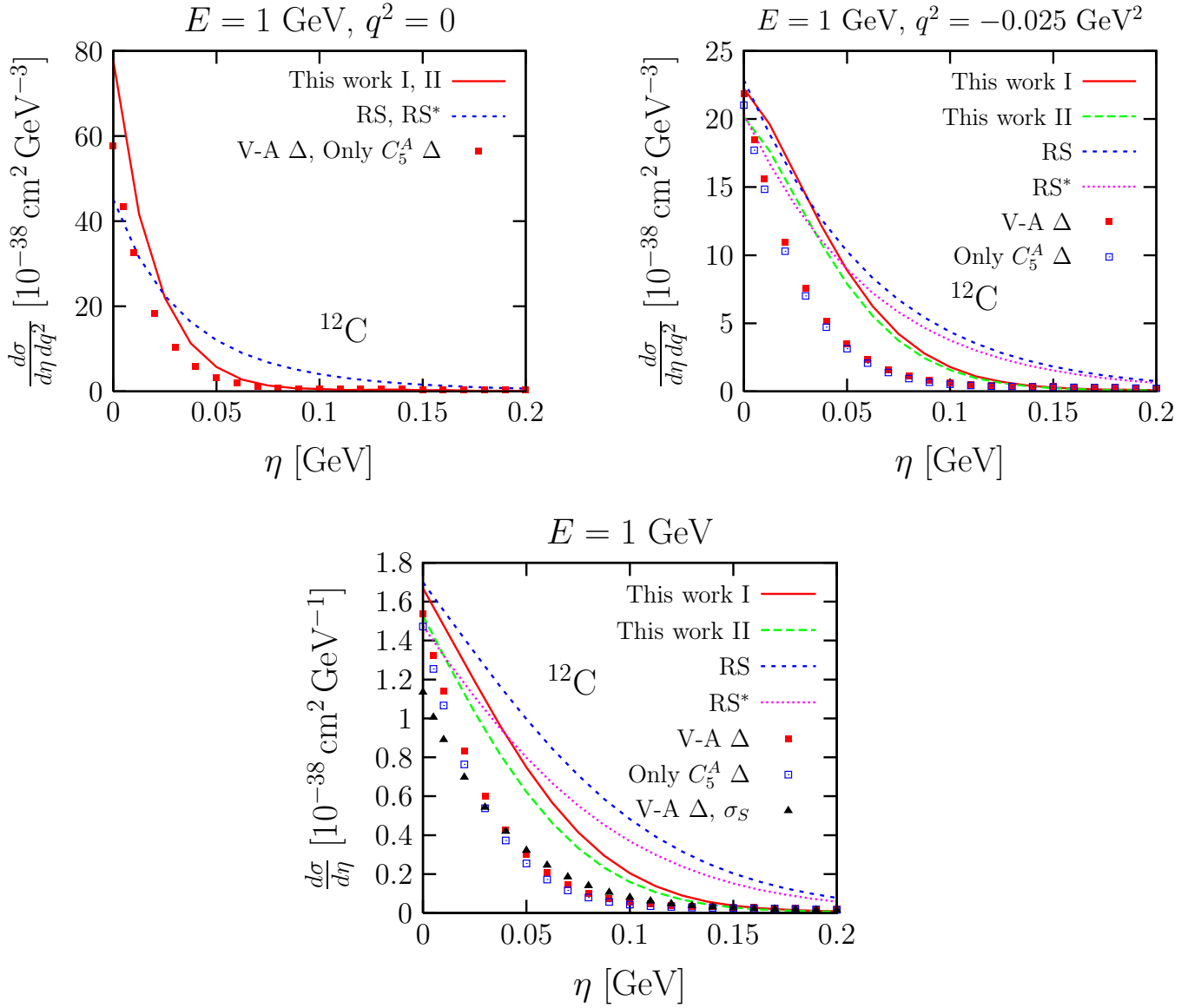


FIG. 7: Same as Fig. 6 including distortion.

We address the first of these deficiencies, and we succeed to improve the original RS model by incorporating the t -dependence of the pion-nucleon cross section.

However, there is not an easy solution for the other two caveats at low and intermediate pion energies.

We have tried to improve the treatment of the distortion of the outgoing pion, while keeping the model still reasonably simple. Although we have managed to describe the pion-nucleus elastic cross section, we still find significant discrepancies in the case of neutrino induced processes when we compare with the accurate microscopical model of Refs. [24, 25] (see Figs. 4 and 5). This illustrates a further interesting point: describing correctly the elastic pion-nucleus cross section is a necessary condition that should satisfy any model aiming at a proper description of the coherent pion production process induced by neutrinos, but however it is not a sufficient one. Discrepancies are due to the modification of the elementary processes when they take place inside the nuclear medium, and to the highly non-linear character of the strong driven processes.

Altogether, we have only been able to quantify the unavoidable systematic error associated to these two last caveats for low and intermediate energy pions by comparing with the model of Refs. [24, 25].

All its limitations notwithstanding, the improved models I and II give a much better description of low and intermediate energies coherent pion production than the widely used RS model. Of course, as the neutrino energy

increases, the $q^2 = 0$ kinematics becomes much more dominant, and on the other hand we expect nuclear medium modifications of the elementary pion–nucleon interaction to be less relevant. Under these circumstances, our improved models become even more appropriate. Nevertheless, we should point out the existence of a real problem: for neutrino energies in the region 1 to 2 GeV, there does not exist a reliable model to describe the coherent pion production process. This is because, these neutrino energies are not large enough for PCAC models to properly work, and they are certainly in the limit of applicability of microscopical models which only include the dynamics of the $\Delta(1232)$ –resonance.

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