$|V_{ub}|$ from Exclusive Semileptonic $B \to \rho$ Decays

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Abstract

We use Omnès representations of the form factors V, A_1 and A_2 for exclusive semileptonic $B \to \rho$ decays, and apply them to combine experimental partial branching fraction information with theoretical calculations of the three form factors to extract $|V_{ub}|$. We find a slightly lower result, $|V_{ub}| = (2.8 \pm 0.2) \times 10^{-3}$, than the values extracted from exclusive semileptonic $B \to \pi$ decays, $(3.47 \pm 0.29 \pm 0.03) \times 10^{-3}$ [1], $(3.36 \pm 0.23) \times 10^{-3}$ [2], $(3.38 \pm 0.35) \times 10^{-3}$ [3], and using all other inputs in CKM fits, $(3.55 \pm 0.15) \times 10^{-3}$ [4,5]. The disagreement is greater when we compare to the result extracted from inclusive $B \to X_u l \nu$ decays, $|V_{ub}| = (4.10 \pm 0.30_{\text{exp}} \pm 0.29_{\text{th}}) \times 10^{-3}$ [6].

1 Introduction

The magnitude of the element V_{ub} of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix plays a critical role in testing the consistency of the Standard Model of particle physics and, in particular, the description of CP violation. Any inconsistency could be a sign of new physics beyond the standard model. V_{ub} is currently the least well-known element of the CKM matrix and improvement in the precision of its determination is highly desirable and topical.

$|V_{ub}|$ can be determined using inclusive or exclusive charmless semileptonic B decays. The inclusive method has historically provided a more precise result, but recent experimental [7–12] and theoretical developments [2, 13–22] are allowing the exclusive semileptonic $B \to \pi$ method to approach the same level of precision.

Recently [1] we extracted $|V_{ub}|$ from combined experimental partial branching fraction information and theoretical [lattice QCD (LQCD) and Light cone sum rules (LCSR)] information on exclusive semileptonic $B \to \pi$ decays. The Omnès representation was employed to provide parametrisations of the form factors. The extracted value turned out to be in striking agreement with that extracted using all other inputs in CKM fits and in some disagreement with $|V_{ub}|$ extracted from inclusive semileptonic decays.

The aim of this letter is to extend the above formalism to study the exclusive semileptonic $B \to \rho$ decay and independently extract $|V_{ub}|$ from the recent measurements of the partially integrated branching fraction by BABAR [8], Belle [9] and CLEO [11, 12]. We will make use of quenched LQCD form factor results [23, 24] for the high q^2 region, and LCSR values [25] at $q^2 = 0$. Thanks to the Omnès representation of the form-factors, we are able to combine all these inputs, as we previously showed for $B \to \pi$ decays.

2 Fit Procedure

2.1 Form-factors and differential decay width

The semileptonic decay $B^0 \to \rho^- \ell^+ \nu_l$ is determined by the matrix element of the $V-A$ weak current between a B meson and a ρ meson. The matrix element is

$$\langle \rho(k,\eta)|\bar{b}\gamma^\mu(1-\gamma_5)u|B(p)\rangle = \eta^*_\beta T^{\mu\beta}, \tag{1}$$

with form factor decomposition

$$\begin{aligned} T_{\mu\beta} &= \frac{2V(q^2)}{m_B+m_\rho}\varepsilon_{\mu\gamma\delta\beta}p^\gamma k^\delta - \mathrm{i}(m_B+m_\rho)A_1(q^2)g_{\mu\beta} \\ &\quad + \mathrm{i}\frac{A_2(q^2)}{m_B+m_\rho}(p+k)_\mu q_\beta - \mathrm{i}\frac{2A(q^2)}{q^2}m_\rho q_\mu (p+k)_\beta, \end{aligned} \tag{2}$$

where $q = p-k$ is the four-momentum transfer and η is the ρ polarisation vector. The meson masses are $m_B = 5279.5$ MeV and $m_\rho = 775.5$ MeV for B^0 and ρ^-, respectively. In the helicity basis each of the form factors corresponds to a transition amplitude with definite spin-parity quantum numbers in the center of mass frame of the lepton pair. This relates the form factors V, A_1 and A_2 to the total angular momentum and parity quantum numbers of the $B\rho$ meson pair, $J^P = 1^-, 1^+$ and 1^+, respectively [26]. The physical region for the squared four-momentum transfer is $0 \le q^2 \le q^2_{\max} \equiv (m_B - m_\rho)^2$. If the lepton mass can be ignored ($l = e$ or μ), the total decay rate is given by

$$\Gamma\left(B^0 \to \rho^-\ell^+\nu_l\right) = \frac{G_F^2|V_{ub}|^2}{192\pi^3 m_B^3}\int_0^{q^2_{\max}} dq^2 q^2 \left[\lambda(q^2)\right]^{\frac{1}{2}} \left(|H^+(q^2)|^2 + |H^-(q^2)|^2 + |H^0(q^2)|^2\right) \tag{3}$$

where $G_F = 1.16637 \times 10^{-5}$ GeV^{-2} is the Fermi constant and $\lambda(q^2) = (m_B^2 + m_\rho^2 - q^2)^2 - 4m_B^2 m_\rho^2$. H^0 comes from the contribution of the longitudinally polarised ρ and is given by

$$H^0(q^2) = -\frac{1}{2m_\rho\sqrt{q^2}}\left\{\left(m_B^2 - m_\rho^2 - q^2\right)\left(m_B + m_\rho\right)A_1(q^2) - \frac{4m_B^2|\vec{k}|^2}{m_B+m_\rho}A_2(q^2)\right\} \tag{4}$$

where $\vec{k}$ is the momentum of the ρ in the B-meson rest frame. $H^\pm$ correspond to the contribution of the transverse polarisations of the vector meson and are given by [1]

$$H^\pm = -\left\{(m_B+m_\rho)A_1(q^2) \mp \frac{2m_B|\vec{k}|}{m_B+m_\rho}V(q^2)\right\} \tag{5}$$

The CLEO Collaboration has also measured partial branching fractions of the differential distribution [11,12]

$$\begin{aligned} \frac{d\Gamma(B^0 \to \rho^-\ell^+\nu)}{dq^2\, d\cos\theta_{W\ell}} &= \frac{G_F^2|V_{ub}|^2}{512\pi^3 m_B^3}q^2\left[\lambda(q^2)\right]^{\frac{1}{2}}\left\{2\sin^2\theta_{W\ell}|H^0(q^2)|^2\right. \\ &\quad \left. + (1-\cos\theta_{W\ell})^2|H^+(q^2)|^2 + (1+\cos\theta_{W\ell})^2|H^-(q^2)|^2\right\} \end{aligned} \tag{6}$$

with $\theta_{W\ell}$ the angle between the charged lepton direction in the virtual $W-$gauge boson rest frame and the virtual W in the B-meson rest frame.

[1] Note a typo in Eq. (1.7) of Ref. [23], the $\pm$ sign should be $\mp$, as used in previous papers of the UKQCD Collaboration [27,28].

2.2 Omnès parametrisations

We have previously [1,16,17,19,20] used a multiply subtracted Omnès dispersion relation [29,30], based on unitarity and analyticity properties, to describe $B \to \pi$ semileptonic decays. Here, we apply these ideas to $B \to \rho$ decays and use for $(n+1)$ subtractions [20]

$$F(q^2) = \frac{1}{s_0 - q^2} \prod_{i=0}^{n} [F(s_i)(s_0 - s_i)]^{\alpha_i(q^2)}, \quad \alpha_i(s) \equiv \prod_{j=0, j\neq i} \frac{s - s_j}{s_i - s_j}, \qquad F = V, A_1, A_2 \qquad (7)$$

where s_0 corresponds to a pole of the form factor F. We fix $s_0 = m_{B^*}^2$ and $s_0 = s_{\rm th} = (m_B + m_\rho)^2$ for V and A_1 and A_2 form factors, respectively. In principle, for the axial form factors one should use the square of the 1^+ B-meson mass. The mass of this latter hadron is not well established yet, but it appears to be heavier than the 1^- B^* resonance. Thus and for the purposes of this exploratory work, since it would be reasonably far from $\sqrt{q^2_{\rm max}}$, it is sufficient to employ $s_{\rm th}$. The parametrisation of Eq. (7) amounts to finding an interpolating polynomial for $\ln[(s_0 - q^2)F(q^2)]$ passing through the points $(s_0^2 - s_i)F(s_i)$. While one could always propose a parametrisation using an interpolating polynomial for $\ln[g(q^2)F(q^2)]$ for a suitable function $g(q^2)$, the derivation using the Omnès representation shows that taking $g(q^2) = s_0^2 - q^2$ is physically motivated [20].

2.3 Theoretical and experimental inputs

We have used experimental partial branching fraction data from CLEO [11, 12], Belle [9] and BABAR [8]. CLEO and BABAR combine results for neutral and charged B-meson decays using isospin symmetry, while Belle give separate values for $B^0 \to \rho^- \ell^+ \nu_l$ and $B^+ \to \rho^0 \ell^+ \nu_l$ decays. Belle use three q^2 intervals, and we have added in quadrature the two different systematic errors quoted for each q^2 bin, and combined charged and neutral B-meson results. We take the resulting systematic errors to be fully correlated. BABAR's untagged analysis also uses three q^2 bins and we have assumed that the quoted percentage systematic errors for the partial branching fractions divided by total branching fraction are representative for the partial branching fractions alone and, following BABAR, took them to be fully correlated. CLEO determines partial branching fractions as a function of both q^2 and of $\cos\theta_{W\ell}$ (see Eq. (6)) and complete correlation matrices are given in [11] for both statistical uncertainties and systematic errors that we have used in our fits.

When computing partial branching fractions, we have used $\tau_{B^0} = 1/\Gamma_{\rm Tot} = (1.527 \pm 0.008) \times 10^{-12}\,$s [31] for the B^0 lifetime. All the branching fraction inputs are listed in Table 1.

For theoretical form-factor inputs (listed in Table 2), we use the lightcone sumrule (LCSR) results at $q^2 = 0$ of Ref. [25] and lattice QCD results from the UKQCD [23] and SPQcdR [24] Collaborations, near $q^2_{\rm max}$. LQCD inputs have been obtained in the quenched approximation. There is therefore an uncontrolled systematic error, which is not fully included in the errors given in Table 2.

2.4 Definition of χ^2

We implement the following fitting procedure. Choose a set of subtraction points spanning the physical range to use in the Omnès formula of equation (7). Now find the best-fit value of $|V_{ub}|$ and the form factors at the subtraction points to match both theoretical input form factor values and the

	q^2 range $[\mathrm{GeV}^2]$	$\cos\theta_{W\ell}$ range	$10^4 B_k^{\mathrm{in}}$	$10^4 B_k^{\mathrm{Omnès}}$
BELLE [9]	$0-8$	$[-1,1]$	$0.62 \pm 0.14 \pm 0.06$	0.69 ± 0.12
	$8-16$	$[-1,1]$	$1.20 \pm 0.23 \pm 0.11$	1.12 ± 0.15
	>16	$[-1,1]$	$0.53 \pm 0.20 \pm 0.12$	0.53 ± 0.08
BABAR [8]	$0-10$	$[-1,1]$	$0.73 \pm 0.17 \pm 0.21$	0.96 ± 0.15
	$10-15$	$[-1,1]$	$0.82 \pm 0.10 \pm 0.13$	0.71 ± 0.10
	>15	$[-1,1]$	$0.59 \pm 0.07 \pm 0.16$	0.68 ± 0.10
CLEO [11]	$0-2$	$[-1,1]$	$0.45 \pm 0.20 \pm 0.15$	0.08 ± 0.03
	$2-8$	$[-1,1]$	$0.96 \pm 0.20 \pm 0.29$	0.61 ± 0.10
	$8-16$	$[0,1]$	$0.75 \pm 0.16 \pm 0.14$	0.74 ± 0.10
	>16	$[0,1]$	$0.35 \pm 0.07 \pm 0.05$	0.39 ± 0.06
	>8	$[-1,0]$	$0.42 \pm 0.18 \pm 0.31$	0.51 ± 0.07

Table 1 Experimental branching fraction inputs for the χ^2 function defined in Eq. (8). Statistical and systematic errors are shown. We also give branching fractions calculated using our fitted form factors and $|V_{ub}|$.

experimental partial branching fraction inputs. The χ^2 function for the fit is :

$$\chi^2 = \sum_{i,j=1}^{60} \left[F_i^{\mathrm{in}} - F_i^{\mathrm{Omnès}}(q_i^2, F_0^i, F_1^i, F_2^i)\right] C_{ij}^{-1} \left[F_j^{\mathrm{in}} - F_j^{\mathrm{Omnès}}(q_j^2, F_0^j, F_1^j, F_2^j)\right] + \sum_{k,l=1}^{11} \left[B_k^{\mathrm{in}} - B_k^{\mathrm{Omnès}}(|V_{ub}|, F_0, F_1, F_2)\right] C_{Bkl}^{-1} \left[B_l^{\mathrm{in}} - B_l^{\mathrm{Omnès}}(|V_{ub}|, F_0, F_1, F_2)\right], \qquad (8)$$

where F_i^{in} are input LCSR or lattice QCD values for $V(q_i^2)$, $A_1(q_i^2)$ and $A_2(q_i^2)$, and B_k^{in} are input experimental partial branching fractions. Moreover, $F_i^{\mathrm{Omnès}}(q_i^2, F_0^i, F_1^i, F_2^i)$ stands for each of the form factors $F = V, A_1, A_2$ at $q^2 = q_i^2$, and it is given by equation (7) with three subtractions $(s_l, F(s_l))$ at $(0, F_0)$, $(2q^2_{\mathrm{max}}/3, F_1)$ and $(q^2_{\mathrm{max}}, F_2)$. The branching fractions $B^{\mathrm{Omnès}}$ are calculated using $F^{\mathrm{Omnès}}$, for V, A_1 and A_2 form factors . There are in total 10 fit parameters: $V(0)$, $V(2q^2_{\mathrm{max}}/3)$, $V(q^2_{\mathrm{max}})$, $A_1(0)$, $A_1(2q^2_{\mathrm{max}}/3)$, $A_1(q^2_{\mathrm{max}})$, $A_2(0)$, $A_2(2q^2_{\mathrm{max}}/3)$, $A_2(q^2_{\mathrm{max}})$ and $|V_{ub}|$. The latter parameter is used when computing $B^{\mathrm{Omnès}}$.

We have assumed that the LCSR and LQCD form factor values have independent statistical uncertainties and treated the errors listed in Table 2 for the SPQcdR inputs as purely statistical. For the UKQCD data we have put the form factor values in the centre of their systematic range and use half that range as the systematic error. We have built a covariance matrix where the statistical uncertainties (σ_i) are uncorrelated and the systematic errors (ε_i) are fully correlated, leading to a 60×60 covariance matrix with three diagonal blocks. The first 3×3 and second 21×21 blocks are for the LCSR and SPQcdR results and have the form $C_{ij} = \sigma_i^2 \delta_{ij}$. The third block is for the UKQCD data and has the form $C_{ij} = \sigma_i^2 \delta_{ij} + \varepsilon_i \varepsilon_j$. We will further discuss the effect of the UKQCD systematic errors on $|V_{ub}|$ below.

The covariance matrix, C_B, for the partial branching fraction inputs is constructed as follows. For Belle and BABAR input data, we have assumed independent statistical uncertainties and fully-correlated systematic errors leading to an 6×6 covariance matrix with two diagonal blocks of the form $C_{Bij} = \sigma_i^2 \delta_{ij} + \varepsilon_i \varepsilon_j$. For the CLEO input, we use an 5×5 covariance matrix $C_{Bij}^{\mathrm{CLEO}} = \sigma_i \sigma_j \mathscr{C}_{Bij}^{\mathrm{CLEO-stat}} + \varepsilon_i \varepsilon_j \mathscr{C}_{Bij}^{\mathrm{CLEO-sys}}$, where we have read off the statistical and systematic correlation

	q^2 [GeV^2]	V	A_1	A_2
LCSR [25]	0	0.323 ± 0.029	0.242 ± 0.024	0.221 ± 0.023
UKQCD [23]	12.67	$0.684 \pm 0.162^{+0.00}_{-0.56}$	$0.439 \pm 0.067^{+0.000}_{-0.080}$	$0.70 \pm 0.49^{+0.08}_{-0.03}$
	13.01	$0.714 \pm 0.162^{+0.00}_{-0.50}$	$0.448 \pm 0.065^{+0.000}_{-0.079}$	$0.71 \pm 0.46^{+0.08}_{-0.03}$
	13.51	$0.763 \pm 0.155^{+0.00}_{-0.40}$	$0.460 \pm 0.063^{+0.000}_{-0.075}$	$0.72 \pm 0.43^{+0.10}_{-0.02}$
	14.02	$0.818 \pm 0.147^{+0.00}_{-0.31}$	$0.472 \pm 0.059^{+0.000}_{-0.073}$	$0.73 \pm 0.42^{+0.12}_{-0.01}$
	14.52	$0.883 \pm 0.141^{+0.00}_{-0.24}$	$0.485 \pm 0.055^{+0.000}_{-0.070}$	$0.76 \pm 0.42^{+0.14}_{-0.03}$
	15.03	$0.967 \pm 0.137^{+0.00}_{-0.20}$	$0.498 \pm 0.051^{+0.000}_{-0.068}$	$0.78 \pm 0.46^{+0.16}_{-0.05}$
	15.53	$1.057 \pm 0.134^{+0.00}_{-0.19}$	$0.513 \pm 0.049^{+0.000}_{-0.067}$	$0.81 \pm 0.54^{+0.18}_{-0.06}$
	16.04	$1.164 \pm 0.150^{+0.10}_{-0.21}$	$0.529 \pm 0.047^{+0.000}_{-0.066}$	$0.84 \pm 0.71^{+0.20}_{-0.07}$
	16.54	$1.296 \pm 0.184^{+0.21}_{-0.25}$	$0.544 \pm 0.043^{+0.000}_{-0.062}$	$0.87 \pm 0.97^{+0.23}_{-0.08}$
	17.05	$1.46 \pm 0.26^{+0.34}_{-0.30}$	$0.560 \pm 0.043^{+0.000}_{-0.059}$	$0.90 \pm 1.35^{+0.27}_{-0.07}$
	17.55	$1.67 \pm 0.40^{+0.49}_{-0.36}$	$0.577 \pm 0.043^{+0.000}_{-0.058}$	$0.90 \pm 1.89^{+0.33}_{-0.03}$
	18.17	$2.02 \pm 0.68^{+0.73}_{-0.48}$	$0.599 \pm 0.052^{+0.000}_{-0.058}$	$0.9 \pm 2.9^{+0.4}_{-0.1}$
SPQcdR [24]	10.69	0.51 ± 0.26	0.354 ± 0.085	0.38 ± 0.26
	12.02	0.61 ± 0.28	0.384 ± 0.087	0.49 ± 0.30
	13.35	0.74 ± 0.30	0.421 ± 0.089	0.65 ± 0.35
	14.68	0.93 ± 0.31	0.465 ± 0.092	0.93 ± 0.41
	16.01	1.20 ± 0.32	0.519 ± 0.097	1.41 ± 0.56
	17.34	1.61 ± 0.33	0.588 ± 0.108	2.39 ± 1.23
	18.67	2.26 ± 0.55	0.678 ± 0.134	4.7 ± 4.1

Table 2 Form factor inputs for the χ^2 function defined in Eq. (8). For UKQCD we show both statistical (symmetrized) and systematical errors, while SPQcdR errors include both systematic and statistical uncertainties (we are indebted with C.M. Maynard and F. Mescia for providing us with these form factors).

matrices ($\mathscr{C}_{Bij}^{\text{CLEO}-\text{stat/sys}}$) from tables X and XI, respectively, of Ref. [11].

We do not consider any correlation between measurements from different experiments, or between different sources of theoretical inputs. Nor do we consider correlations between experimental and theoretical inputs.

3 Results and discussion

The best fit parameters are

$$\begin{aligned}
|V_{ub}| &= (2.76 \pm 0.21) \times 10^{-3} \\
V(0) &= 0.322 \pm 0.030 \\
V(2q^2_{\max}/3) &= 0.681 \pm 0.073 \\
V(q^2_{\max}) &= 4.21 \pm 0.76 \\
A_1(0) &= 0.223 \pm 0.021 \\
A_1(2q^2_{\max}/3) &= 0.449 \pm 0.020 \\
A_1(q^2_{\max}) &= 0.657 \pm 0.055 \\
A_2(0) &= 0.231 \pm 0.022 \\
A_2(2q^2_{\max}/3) &= 0.679 \pm 0.098 \\
A_2(q^2_{\max}) &= 2.76 \pm 1.38
\end{aligned} \tag{9}$$

The fit has $\chi^2/\text{d.o.f.} = 0.21$ for 61 degrees of freedom, while the Gaussian correlation matrix can be found in the appendix A. In figure 1 we show the fitted form factors and the differential decay rate calculated from our fit. Partial branching fractions calculated for the same bins as used experimentally are given in the last column of Table 1. Our calculated total branching ratio turns out to be $(2.30^{+0.24}_{-0.26}) \times 10^{-4}$, in reasonable agreement with $(2.80 \pm 0.18 \pm 0.16) \times 10^{-4}$ quoted by the Heavy Flavours Averaging Group (HFAG) [31].

We have further investigated the effect of the highly asymmetric UKQCD systematic errors on $|V_{ub}|$. First, we have completely dropped them and used only the statistical uncertainties on the UKQCD points. We find $|V_{ub}| = (2.68 \pm 0.19) \times 10^{-3}$. Second, we have performed a Monte Carlo where we randomly choose each UKQCD form factor value within its systematic error range, with complete correlation between all systematic shifts. For each trial we perform a fit like our original one, but setting to zero the systematic errors on the UKQCD inputs. In this case, we find $|V_{ub}| = (2.85 \pm 0.10) \times 10^{-3}$. Note that this last result is the mean and the standard deviation of the fit result for $|V_{ub}|$ over all the trials, whereas the result above and that quoted in Eq. (9) are the fit result and error from a single fit. Thus, the result from this second procedure should be understood as a shift of $+0.09 \pm 0.10$ in the value of $|V_{ub}|$ in Eq. (9). Finally, we have repeated the latter procedure, but taking the A_2 systematic error to be anticorrelated with those of the V and A_1. This results in $|V_{ub}| = (2.86 \pm 0.15) \times 10^{-3}$.

From the above discussion, we estimate

$$|V_{ub}| = 2.8 \pm 0.2 \tag{10}$$

which constitutes our main result. Quenched approximation systematic effects from LQCD are not accounted for by the 0.2 error quoted above. These are difficult to quantify and are a limitation here. However, unquenched lattice simulations are now standard and future lattice QCD results will address this limitation (although they will also face the problem of an unstable ρ meson for light enough simulated up and down quark masses). Nevertherless, we see that the Omnès framework used here provides a fair description of all available experimental and theoretical results for semileptonic $B \to \rho$ decays, leading to a further independent determination of $|V_{ub}|$. The result is lower than the values obtained in the most recent studies of the exclusive semileptonic $B \to \pi$ decay, $(3.47 \pm 0.29 \pm 0.03) \times 10^{-3}$ [1], $(3.36 \pm 0.23) \times 10^{-3}$ [2], $(3.38 \pm 0.35) \times 10^{-3}$ [3], $(3.5 \pm 0.4_{\text{th}} \pm 0.2_{\text{shape}} \pm 0.1_{\text{BR}}) \times 10^{-3}$ [21], and using all other inputs in CKM fits, $(3.55 \pm 0.15) \times 10^{-3}$ [4, 5]. The disagreement is greater when we compare to the most precise result extracted

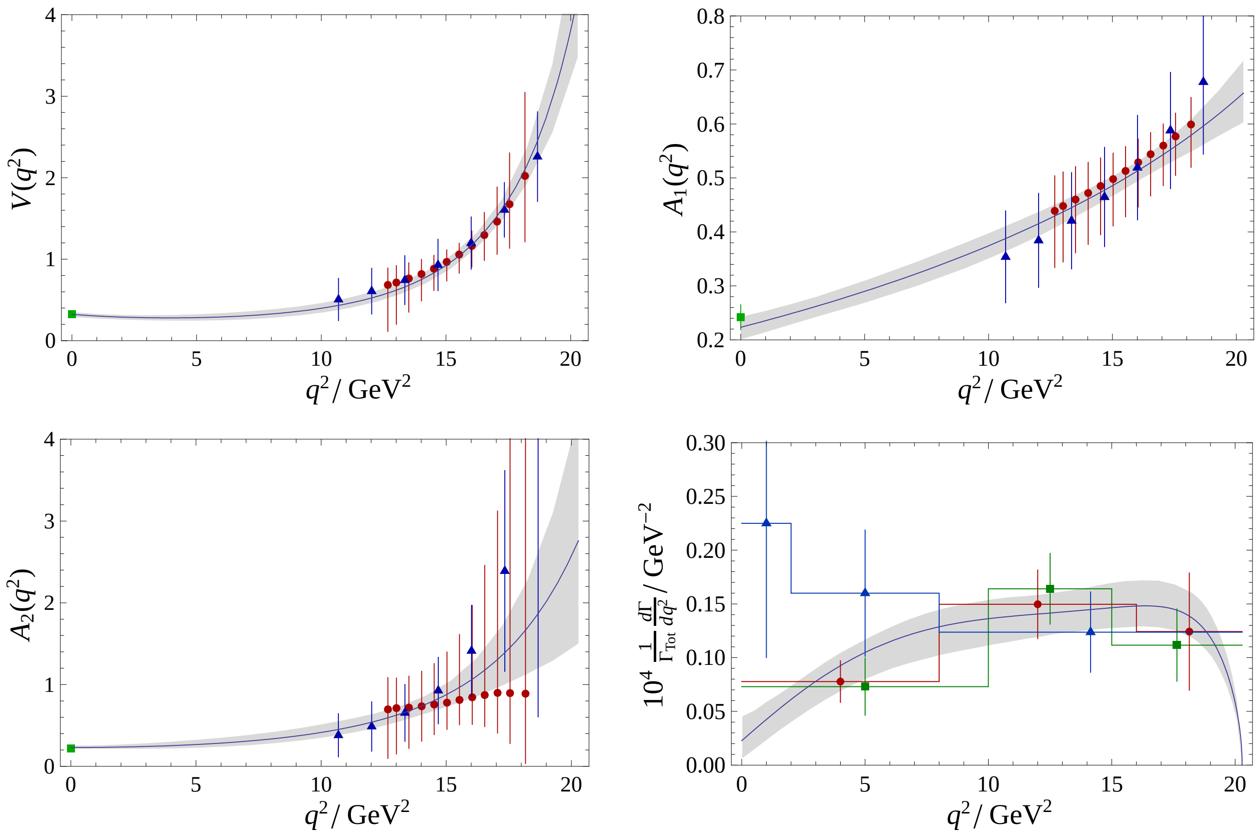

Figure 1 Results obtained from the fit to experimental partial branching fraction data and theoretical form factor calculations. The top and the left bottom plots show the three form factors with their 68% CL bands (shaded) together with the lattice and LCSR input points (green square LCSR, red dots UKQCD, blue triangles SPQcdR). The bottom right plot shows the differential decay rate with 68% CL band (shaded) together with the experimental partial branching fractions divided by the appropriate bin-width (histograms and points). Green squares, red dots and blue triangles denote BABAR, Belle and CLEO results, respectively.

from inclusive $B \to X_u l \nu$ decays, $|V_{ub}| = (4.10 \pm 0.30_{\text{exp}} \pm 0.29_{\text{th}}) \times 10^{-3}$ [6]. Thus, the hints of disagreement between inclusive and exclusive/global-CKM-fit determinations are strengthened.

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A Gaussian correlation matrix

Here, we give the correlation matrix of fitted parameters corresponding to the best-fit parameters in Eq. (9)

$$\begin{pmatrix} 1.00 & -0.01 & -0.21 & -0.16 & -0.10 & -0.31 & -0.23 & 0.05 & 0.34 & 0.01 \\ & 1.00 & 0.01 & 0.04 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -0.01 \\ & & 1.00 & -0.44 & 0.05 & 0.28 & 0.21 & -0.03 & 0.16 & 0.07 \\ & & & 1.00 & -0.05 & 0.09 & -0.08 & 0.02 & -0.08 & 0.05 \\ & & & & 1.00 & 0.10 & 0.03 & 0.17 & 0.12 & -0.46 \\ & & & & & 1.00 & -0.32 & -0.04 & 0.34 & -0.30 \\ & & & & & & 1.00 & 0.04 & -0.27 & 0.47 \\ & & & & & & & 1.00 & -0.04 & 0.21 \\ & & & & & & & & 1.00 & 0.18 \\ & & & & & & & & & 1.00 \end{pmatrix} \qquad (11)$$

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