

Deriving the existence of $B\bar{B}^*$ bound states from the $X(3872)$ and Heavy Quark Symmetry

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We discuss the possibility and the description of bound states between B and \bar{B}^* mesons. We argue that the existence of such a bound state can be deduced from (i) the weakly bound $X(3872)$ state, (ii) certain assumptions about the short range dynamics of the $D\bar{D}^*$ system and (iii) heavy quark symmetry. From these assumptions the binding energy of the possible $B\bar{B}^*$ bound states is determined, first in a theory containing only contact interactions which serves as a straightforward illustration of the method, and then the effects of including the one pion exchange potential are discussed. In this latter case three isoscalar states are predicted: a positive and negative C-parity $^3S_1 - ^3D_1$ state with a binding energy of 20 MeV and 6 MeV below threshold respectively, and a positive C-parity 3P_0 shallow state located almost at the $B\bar{B}^*$ threshold. However, large uncertainties are generated as a consequence of the $1/m_Q$ corrections from heavy quark symmetry. Finally, the newly discovered isovector $Z_b(10610)$ state can be easily accommodated within the present framework by a minor modification of the short range dynamics.

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I. INTRODUCTION

The existence of heavy meson bound states, a possibility first theorized by Voloshin and Okun [1], is grounded on the observation that meson-exchange forces can arise between two heavy mesons as a consequence of their light quark content. In analogy with the nuclear forces in the two-nucleon system, these exchange forces may eventually be strong enough as to generate bound states and resonances in two-meson systems. In particular, we expect the one pion exchange (OPE) potential to drive the long range behaviour of two-meson systems and therefore to play a very special role in the formation and the description of their corresponding bound states. The seminal works of Törnqvist [2, 3], Manohar and Wise [4] and Ericson and Karl [5], which already stressed the importance of pion exchanges in the formation of deuteron-like meson bound states, indicated that these states are more probable (i.e. more bound) in the bottom sector than in the charm one.

The discovery of the $X(3872)$ state by the Belle collaboration [6], later confirmed by CDF [7], D0 [8] and BABAR [9], probably represents the most obvious candidate for a heavy meson bound state. With a mass of $m_X = 3871.56 \pm 0.22$ MeV [10] the $X(3872)$ lies very close to the $D^0\bar{D}^{*0}$ threshold, $m_{D^0} + m_{D^{*0}} = 3871.79 \pm 0.21$ MeV, suggesting the natural interpretation of a shallow bound or virtual state. The average separation between the D^0 and D^{*0} mesons, $\sqrt{\langle r^2 \rangle} \sim 10$ fm, will enhance strongly the role of the molecular component at low energies in comparison with the more compact multi-quark components. It should be noted however that the molecular interpretation is only possible if the quantum

numbers of the $X(3872)$ are $J^{PC} = 1^{++}$, a value compatible with available experimental information [11, 12]. In this regard, the new experimental analysis of Ref. [13], which prefers the quantum numbers 2^{-+} , raises the likelihood of the tetraquark and charmonium interpretations over the molecular hypothesis. On a related note, the recent finding of the $Z_b(10610)$ and $Z_b(10650)$ states by the Belle collaboration [14], very close to the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds respectively, might also prove to be heavy meson bound states.

The molecular descriptions of the $X(3872)$ can be classified in two broad types: (i) the short-range interaction picture [15–21], in which the $X(3872)$ is the result of contact interactions between the D and \bar{D}^* mesons, and (ii) the potential picture [22–28], in which the $X(3872)$ is bound as the result of meson-exchanges. A middle path is provided by X-EFT [29], in which the $X(3872)$ is bound due to the effect of the short range dynamics, but pions are included as perturbations.

The physical motivation behind the short-range description of the $X(3872)$ is that the two particle conforming a low-lying bound state are too far apart from each other as to distinguish the specific details of the interaction binding them, a phenomenon called *universality* [17]. However, although universality demystifies the success of contact theories, there are observables which cannot be explained without the explicit consideration of shorter range components of the $X(3872)$. A clarifying example is the large branching ratio [11],

$$\frac{Br(X \rightarrow J/\psi\pi^+\pi^-\pi^0)}{Br(X \rightarrow J/\psi\pi^+\pi^-)} = 1.0 \pm 0.4 \pm 0.3, \quad (1)$$

which finds a most natural explanation by considering the charged D^+D^{*-} component in the $X(3872)$ [20, 21].

The potential description of the $X(3872)$, which was pioneered by Törnqvist [3], is constructed in analogy with the traditional meson theory of the nuclear forces [30, 31].

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In this picture, the D and \bar{D}^* mesons interact via a series of exchanges of increasingly heavy mesons, conforming a long range potential. The pion is always included, while more sophisticated descriptions also consider the effect of two pion [28] and heavier meson exchanges [25, 26]. However, pseudoscalar and vector meson exchanges give rise to a singular tensor interaction, which diverges as $1/r^3$ at short enough distances. This singularity requires the regularization of the potential, usually by the inclusion of a form factor. Usually, these approaches do not only consider the $D\bar{D}^*$ system, but also other heavy meson systems, in particular $B\bar{B}^*$.

The conclusions which are derived from the potential picture are easy to summarize: in first place, meson exchange forces, specially the pion, are too weak to be able to bind the D and \bar{D}^* mesons together (unless the cut-off in the form factor takes very large values); second, if the argument is extended to the B and \bar{B}^* meson system, the conclusion is that they are probably bound. However, it is very difficult to determine the binding energy of this two-body system. This is just a restatement of the argument of Ericson and Karl [5]: a system with a higher reduced mass is more likely to bind, as a result of the lower kinetic energy, meaning that the existence of a bound state between B mesons is highly likely.

In this work we employ two different effective field theory (EFT) descriptions of heavy meson systems at lowest order: (i) a pionless theory, in which the mesons interact through local contact interactions, and (ii) a pionfull theory, in which we include the pion exchanges explicitly (for reviews, see [32–35]). The EFT approach we follow is cut-off EFT [36, 37], in which the potential is expanded according to naive dimensional analysis

$$V_{\text{EFT}}(\vec{q}) = V^{(0)}(\vec{q}) + V^{(2)}(\vec{q}) + \mathcal{O}(P^3), \quad (2)$$

where P represents the low energy scales of the system, in this case the pion mass and the relative momenta of the heavy mesons. Finally, a cut-off is included in the computation to regularize the ultraviolet divergences. The resulting potential is iterated in the Schrödinger or Lippmann-Schwinger equation in order to compute the observables. If pions are included, the approach basically coincides with the Weinberg counting [38, 39] as applied in the two nucleon system, which is known to be very successful phenomenologically [40, 41]. The optimal cut-off choice is to be taken of the order of the high energy scale of the system [42], that is, the scale at which the short range effects not explicitly taken into account in the EFT start to manifest. This corresponds, for example, with the energy scale at which the composite nature of the heavy mesons can be probed or for which the exchange of heavier mesons (like the ρ) needs to be taken into account, suggesting a value for the hard scale of $\Lambda_0 \sim 0.5 - 1 \text{ GeV}$. In this work, the natural value of cut-off will be roughly interpreted as the inverse size of the heavy mesons.

As a consequence of the scarce phenomenological input available about the $D\bar{D}^*$ and $B\bar{B}^*$ interactions, we limit

ourselves to a leading order (LO or order P^0) EFT, in which there is only one low energy contact operator (C_0). The long range interaction at LO only contains the one pion exchange potential, resulting in a very compact and simple description of the two meson system. In orthodox implementations of the EFT concept, the counterterm is a running coupling constant $C_0 = C_0(\Lambda)$, which can be determined for a given cut-off Λ by reproducing, for example, the binding energy of the system (or any other observable). However the previous idea, although well suited for the $D\bar{D}^*$ case, cannot be employed in the $B\bar{B}^*$ system as there is no experimental information available, posing a problem for the present EFT formulation.

The alternative we follow is the approach of Ref. [21]: we assume the counterterm C_0 to be saturated by short-range dynamics, while the cut-off is determined by reproducing the binding energy in the $D\bar{D}^*$ system. The extension of the previous scheme to the bottom sector is straightforward once we consider heavy quark symmetry (HQS) [43–46], which allows us (i) to extrapolate the saturation condition for the counterterm from the $D\bar{D}^*$ to the $B\bar{B}^*$ system and (ii) to expect the value of the cut-off to be similar in both cases. With these two pieces of information we will be able to make predictions in the bottom sector. This approach, which can be labelled EFT *mapping* or *saturation* method, does not correspond to the standard formulation of the EFT concept. However, if the determination of the cut-off results in a natural value, the saturation condition will not contradict either any of the usual EFT tenets. The fact is that the saturation of the low energy constants by short-range physics is a sensible prospect within the EFT context, as exemplified in Ref. [47] for the two-nucleon sector. The important point is that saturation and HQS allow us to correlate the charm and bottom sectors.

The manuscript is organized as follows: in Sect.II we present the formalism to describe bound states between heavy mesons, in Sect.III we show the prediction for a $B\bar{B}^*$ bound state based on a contact (i.e. pionless) theory, which serves as a straightforward exposition of our approach. In Sect.IV we discuss the role played by the inclusion of the one pion exchange potential, which implies the appearance of a new $B\bar{B}^*$ P-wave bound state. Finally, in Sect.V we present our conclusions.

II. FORMALISM

In this section we will describe the two heavy meson system as a non-relativistic two-body problem in quantum mechanics, for which the potential between the heavy mesons is a well-defined object. We formulate the bound state problem in momentum space, where the wave functions are obtained by solving the Lippmann-Schwinger equation. We also consider in detail the explicit treatment of tensor forces, which will appear as a consequence of pseudoscalar meson exchange, and coupled channels.

A. Bound State Equation

The Lippmann-Schwinger equation for a bound state reads

$$|\Psi_B\rangle = G_0(E_B)V|\Psi_B\rangle, \quad (3)$$

where $|\Psi_B\rangle$ represents the wave function of the bound state system, V the two-body potential and $G_0(E)$ is the resolvent operator, which is given by

$$G_0(E) = \frac{1}{E - H_0}, \quad (4)$$

where $E < 0$ (> 0) for bound (scattering) states. In the expression above, H_0 is the free Hamiltonian of the two particle system, which is defined as

$$H_0|\vec{k}\rangle = \frac{\vec{k}^2}{2\mu}|\vec{k}\rangle, \quad (5)$$

where the center-of-mass motion has been removed, μ is the reduced mass of the system and $|\vec{k}\rangle$ represents a two-particle state with relative momentum \vec{k} in the center-of-mass system. Projecting the previous equation onto plane waves, we get an explicit representation of the bound state equation

$$\begin{aligned} \Psi_B(\vec{k}) = \\ -\frac{2\mu}{k^2 + \gamma^2} \int \frac{d^3\vec{k}'}{(2\pi)^3} \langle \vec{k}|V|\vec{k}'\rangle \Psi_B(\vec{k}'), \end{aligned} \quad (6)$$

with $\Psi_B(\vec{k}) = \langle \vec{k}|\Psi_B\rangle$, $\gamma^2 = -2\mu E_B$ and where $E_B (< 0)$ is the bound state energy. Finally, the solution is subjected to the normalization condition

$$\langle \Psi_B|\Psi_B\rangle = \int \frac{d^3\vec{k}}{(2\pi)^3} |\Psi_B(\vec{k})|^2 = 1. \quad (7)$$

B. Central Potential

If we assume a central potential, the previous equation can be projected onto the partial wave basis, defined as

$$|\vec{k}\rangle = \sqrt{4\pi} \sum_{lm} |k, lm\rangle Y_{lm}^{(*)}(\hat{k}), \quad (8)$$

from which we can expand the bound state wave function as follows

$$\Psi_B(\vec{k}) = \sqrt{4\pi} \sum_{lm} \langle k, lm|\Psi_B\rangle Y_{lm}(\hat{k}). \quad (9)$$

For a central potential the angular momentum of the two-body system is conserved and the matrix elements of the potential in the $|k, lm\rangle$ basis fulfill the relation

$$\langle k, lm|V|k', l'm'\rangle = \langle k|V_l|k'\rangle \delta_{ll'} \delta_{mm'}, \quad (10)$$

in which $\delta_{ll'}$ and $\delta_{mm'}$ take into account the fact that the potential is spherically symmetric. The relationship

between the projected potential and the potential in the plane wave basis is given by

$$\langle k|V_l|k'\rangle = \frac{1}{4\pi} \int d\hat{k} d\hat{k}' Y_{lm}^{(*)}(\hat{k}) \langle \vec{k}|V|\vec{k}'\rangle Y_{lm}(\hat{k}'), \quad (11)$$

where the result does not depend on the third component of the angular momentum m .

For a central potential the bound state has well-defined quantum numbers l and m and the partial wave expansion of $\Psi_B(\vec{k})$, Eq. (9), simplifies to

$$\Psi_{B(lm)}(\vec{k}) = \sqrt{4\pi} \Psi_{B,l}(k) Y_{lm}(\hat{k}), \quad (12)$$

where the partial wave projected wave function $\Psi_{B,l}(k)$ does not depend on m as a consequence of Eq. (10). In such a case, the bound state equation simplifies to

$$\begin{aligned} \Psi_{B,l}(k) = \\ -\frac{2\mu}{k^2 + \gamma^2} \int \frac{k'^2 dk'}{2\pi^2} \langle k|V_l|k'\rangle \Psi_{B,l}(k'), \end{aligned} \quad (13)$$

where the partial wave projected wave function obeys the normalization condition

$$\int \frac{k^2 dk}{2\pi^2} |\Psi_{B,l}(k)|^2 = 1. \quad (14)$$

C. Tensor Forces

The longest range term of the interaction between a heavy pseudoscalar and vector meson (both of which contain a light quark) is due to one pion exchange, which in turn implies the existence of a tensor component in the potential. This tensor force will mix channels with different angular momenta. However, the total angular momentum and the parity of the two meson system will be conserved. Therefore, instead of projecting into the $|k, lm\rangle$ partial waves, we will rather consider the following states

$$|k, ljm\rangle = \sum_{m_l, m_s} |k, lm_l\rangle |1\lambda\rangle \langle 1\lambda lm_l|jm\rangle, \quad (15)$$

where $|1\lambda\rangle$ is the intrinsic spin state of the the pseudoscalar-vector meson system (which can be identified with the polarization vector of the vector meson), and $\langle 1\lambda lm_l|jm\rangle$ is a Clebsh-Gordan coefficient. In the previous basis, the partial wave expansion of the plane wave reads

$$|\vec{k}, 1\lambda\rangle = \sqrt{4\pi} \sum_{ljm} |k, ljm\rangle \mathcal{Z}_{jm}^{l\lambda}(\hat{k}), \quad (16)$$

where the sum over l runs from $|j-1|$ to $|j+1|$, and with

$$\mathcal{Z}_{jm}^{l\lambda}(\hat{k}) = \langle 1\lambda lm_l|jm\rangle Y_{lm_l}(\hat{k}). \quad (17)$$

Conservation of the total angular momentum implies that the matrix elements of the potential are diagonal in j and m , that is

$$\langle k, ljm|V|k', l'j'm'\rangle = \langle k|V_{lj}|k'\rangle \delta_{jj'} \delta_{mm'}. \quad (18)$$

In addition, as a consequence of parity conservation, the matrix elements between odd l and even l' , and viceversa, are zero. The relationship between the projected and unprojected potential reads

$$\begin{aligned} \langle k|V_{l'l}j|k'\rangle &= \frac{1}{4\pi} \sum_{\lambda,\lambda'} \int d\hat{k} d\hat{k}' \mathcal{Z}_{jm}^{l\lambda}(\hat{k}) \\ &\times \langle \vec{k}, 1\lambda|V|\vec{k}', 1\lambda'\rangle \mathcal{Z}_{jm}^{l'\lambda'}(\hat{k}'), \end{aligned} \quad (19)$$

where we have taken into account that the unprojected potential between the pseudoscalar and vector mesons depends on the polarizations λ and λ' of the initial and final states.

Owing to the properties of the tensor force, the bound state between a pseudoscalar and a vector meson has well-defined total angular momentum and parity. In such a case, the partial wave projection of the bound state wave function can be written as

$$\Psi_{B(pjm)}(\vec{k}) = \sqrt{4\pi} \sum_{\{l\}_p} \Psi_{B,lj}(k) \mathcal{Y}_{lm}(\hat{k}) \quad (20)$$

with $\Psi_{B,lj}$ the projected wave function, and where the notation $\{l\}_p$ is used to denote the set of angular momenta with parity p , and with

$$\mathcal{Y}_{lm}(\hat{k}) = \sum_{\lambda} \mathcal{Z}_{jm}^{l\lambda}(\hat{k}) |1\lambda\rangle. \quad (21)$$

It should be noted that the projected wave function $\Psi_{B,lj}$ does not depend on the third component of the total angular momentum (m) as a consequence of Eq. (18). The partial wave projection of the bound state equation reads in this case as

$$\begin{aligned} \Psi_{B,lj}(k) &= \\ &- \frac{2\mu}{k^2 + \gamma^2} \sum_{l'} \int \frac{k'^2 dk'}{2\pi^2} \langle k|V_{l'l}j|k'\rangle \Psi_{B,l'j}(k'), \end{aligned} \quad (22)$$

As can be seen, parity conservation implies that channels with $l = j$ are uncoupled while channels with $l = j \pm 1$ are coupled. Finally, the normalization condition for the wave function reads

$$\sum_{\{l\}_p} \int \frac{k^2 dk}{2\pi^2} |\Psi_{B,lj}(k)|^2 = 1. \quad (23)$$

In general we will prefer the spectroscopic notation $^{2S+1}L_J$ to the (pjm) notation in this work. In the spectroscopic notation, S refers to the total intrinsic spin of the two body system, which for the particular case of a D and \bar{D}^* mesons is always $S = 1$, L is the orbital angular momentum and J is the total angular momentum. For coupled angular momentum channels, we will employ a dash to indicate the different angular momentum components. For example, the $p = +1, j = 1$ state, which couples the S- and D-waves, will be denoted by $^3S_1 - ^3D_1$; on the contrary, the $p = -1, j = 1$ state, which only contains a P-wave, will be simply the 3P_1 channel in this notation.

D. Coupled Channels

In certain cases we will need to consider the existence of different channels in the description of heavy meson bound states, mainly due to isospin breaking effects. For example, the $X(3872)$ contains a neutral and a charged component

$$\begin{aligned} |X(3872)\rangle &= \frac{1}{\sqrt{2}} [|D^0 \bar{D}^{*0}\rangle - |D^{*0} \bar{D}^0\rangle] |X_0\rangle \\ &+ \frac{1}{\sqrt{2}} [|D^+ D^{*-}\rangle - |D^{*+} D^-\rangle] |X_C\rangle, \end{aligned} \quad (24)$$

where, if isospin were conserved, we would have $|X_0\rangle = |X_C\rangle$ for the isoscalar case ($I = 0$). The previous representation assumes that the $X(3872)$ is a positive C-parity state¹. Owing to isospin breaking effects, the interaction can vary slightly depending on the channel and in addition the channels can have different kinematic thresholds which need to be taken into account.

The extension to the coupled-channel case is trivial, and only requires to consider the existence of different components in the wave function

$$|\Psi_B\rangle = \sum_{\alpha} |\Psi_B^{\alpha}\rangle, \quad (25)$$

where we use upper indices to denote the different channels (lower indices are reserved for angular momentum). For the previous wave function, the bound state equation reads

$$|\Psi_B^{\alpha}\rangle = G_0^{\alpha}(E_B) \sum_{\beta} V^{\alpha\beta} |\Psi_B^{\beta}\rangle, \quad (26)$$

where the potential $V^{\alpha\beta}$ is now a matrix, and $G_0^{\alpha}(E)$ takes the form

$$G_0^{\alpha}(E) = \frac{1}{E - H_0^{\alpha}}, \quad (27)$$

with H_0^{α} the eigenvalue of the free Hamiltonian operator in the plane wave basis for the α channel

$$H_0 |\vec{k}, \alpha\rangle = H_0^{\alpha} |\vec{k}, \alpha\rangle = \left[\frac{k^2}{2\mu_{\alpha}} - \Delta_{\alpha} \right] |\vec{k}, \alpha\rangle. \quad (28)$$

We employ Δ_{α} to indicate the existence of different kinematical thresholds. For example, in the $X(3872)$ the charged channel ($D^+ D^{*-}$) lies 8.06 MeV above the neutral channel ($D^0 \bar{D}^{*0}$). However, the binding energy is referred relative to the neutral channel. Therefore in this case we take $\Delta_0 = 0$ and $\Delta_C = 8.06$ MeV.

¹ We remind that C-parity is a good quantum number for a meson-antimeson system. We are also following the negative C-parity convention for the vector meson, in which $\hat{C}|D^*\rangle = -|\bar{D}^*\rangle$, which is the most natural one.

The previous equation can be easily projected into the plane wave basis $|\vec{k}, \alpha\rangle$ yielding

$$\Psi_B^\alpha(\vec{k}) = -\frac{2\mu_\alpha}{k^2 + \gamma_\alpha^2} \sum_\beta \int \frac{d^3\vec{k}'}{(2\pi)^3} \langle \vec{k} | V^{\alpha\beta} | \vec{k}' \rangle \Psi_B^\beta(\vec{k}'), \quad (29)$$

with $\Psi_B^\alpha(\vec{k}) = \langle \vec{k}, \alpha | \Psi_B \rangle$ and $\gamma_\alpha^2 = -2\mu_\alpha(E_B + \Delta_\alpha)$, where we have assumed all the channels to be below threshold ($E_\alpha + \Delta_\alpha \leq 0$) for simplicity. The normalization condition for the wave function is given by

$$\sum_\alpha \int \frac{d^3\vec{k}}{(2\pi)^3} |\Psi_B^\alpha(\vec{k})|^2 = 1. \quad (30)$$

The partial wave projection of the bound state equation can be done as in the previous cases. We will directly consider the general case of an interaction containing a tensor operator, that is

$$\Psi_{B,l_j}^\alpha(k) = -\frac{2\mu_\alpha}{k^2 + \gamma_\alpha^2} \sum_{\beta,l'} \int \frac{k'^2 dk'}{2\pi^2} \langle k | V_{l'l}^{\alpha\beta} | k' \rangle \Psi_{B,l'_j}^\beta(k'), \quad (31)$$

where the wave function is subjected to the normalization condition

$$\sum_{\alpha,\{l\}_p} \int \frac{k^2 dk}{2\pi^2} |\Psi_{B,l}^\alpha(k)|^2 = 1. \quad (32)$$

III. CONTACT THEORY

In this section we propose a pionless effective field theory (EFT) description of the interaction between the D and \bar{D}^* mesons conforming the $X(3872)$. The formulation of a suitable EFT requires the existence of a separation of scales in the physical system under consideration. In the particular case of a two heavy meson system and for momenta which are not able to resolve the finite size of the heavy mesons ($p < 0.5 - 1$ GeV), the system can be described in terms of the non-relativistic D and \bar{D}^* meson fields, the pion field and the local interactions between these fields, as far as they are compatible with the known symmetries of the system like parity, time reversal, rotational invariance and chiral symmetry. This description can also be applied for the B and \bar{B}^* system.

The purpose of this section is to illustrate in a simple and amenable manner the general EFT approach followed in the present work. For this reason, we will intentionally ignore the pion as an explicit degree of freedom and employ instead a contact EFT theory at leading order (LO). If pion exchanges are weak, as happens in the $X(3872)$ [29], the omission of this degree of freedom is not crucial and the effect will be similar to neglecting the Coulomb interaction between charged heavy

mesons. But if pion exchanges are relevant for the description of the system, as probably happens in the bottom sector, the decision of ignoring them will reduce the range of validity of the present EFT formulation to momenta below the pion mass ($p < 140$ MeV). The previous means that, if the energy of a $B\bar{B}^*$ bound state is above $B_{\max} \sim m_\pi^2/2\mu_{B\bar{B}^*} \sim 4$ MeV (with m_π the pion mass and $\mu_{B\bar{B}^*}$ the reduced mass), the predictions of the contact EFT will no longer be reliable. In such a case, the contact theory should be regarded as a model.

We describe the $X(3872)$ as a bound state of a D and \bar{D}^* mesons in a positive C-parity configuration which interact through a momentum- and energy-independent contact interaction of the type

$$\langle \vec{k} | V_C | \vec{k}' \rangle = C_0, \quad (33)$$

which is regularized with a suitable regulator function. However, the different masses of the neutral and charged meson pairs, $m_{D^0} + m_{\bar{D}^{*0}} = 3871.79$ MeV and $m_{D^+} + m_{\bar{D}^{*-}} = 3879.85$ MeV, plus the low lying nature of the $X(3872)$, require the treatment of the neutral ($D^0\bar{D}^{*0}$) and charged ($D^+\bar{D}^{*-}$) components of the X bound state as independent and separate channels. In the neutral-charged basis, the contact interaction can be expressed as the following 2×2 matrix

$$\langle \vec{k} | V_C | \vec{k}' \rangle = C_0^{D\bar{D}^*} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad (34)$$

where we have assumed that the strength of the contact interaction is independent on whether we have a pair of neutral or charged mesons.

For determining the value of the $C_0^{D\bar{D}^*}$ counterterm, we follow the formulation of Ref. [21], where a good description of the branching ratio $Br(X \rightarrow J/\psi\pi^+\pi^-\pi^0)/Br(X \rightarrow J/\psi\pi^+\pi^-)$ was achieved. In agreement with the phenomenological model of Refs. [48, 49], the previous work assumes that the contact interaction between the D and \bar{D}^* mesons are saturated by the D meson weak decay constant f_D , that is ²

$$C_0^{D\bar{D}^*} \simeq -\frac{1}{2f_D^2}. \quad (35)$$

In the language of Refs. [48, 49] the previous condition is equivalent to assuming that the value of the contact operator is saturated by the exchange of a heavy vector meson in the t-channel. In Ref. [21], the counterterm $C_0^{D\bar{D}^*}$ is regularized with a cut-off Λ in momentum space and iterated in the Lippmann-Schwinger equation. The value of the cut-off is determined by fixing the binding energy of the $D\bar{D}^*$ bound state. This scheme generates a small isospin violation at short distances which explains the previous branching ratio.

² Notice that Refs. [48, 49] employ a different normalization for f_D , which is related to ours by a factor of $\sqrt{2}$.

It should be noticed that the phenomenological model of Refs. [48, 49] requires the coupling of the neutral and charged channels to the strange one, conformed by the $D_s^+ D_s^{*-}$ mesons. In such a case, the contact interaction can be written as the following 3×3 matrix

$$\langle \vec{k} | V_C | \vec{k}' \rangle = C_0^{D\bar{D}^*} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad (36)$$

However, the energy of the $X(3872)$ state is about 210 MeV below $D_s^+ D_s^{*-}$ the threshold. This means that the wave number of the strange component in the $X(3872)$ is approximately $\gamma_s = 653$ MeV, representing a very short range contribution to the wave function, and approximately of the order of the natural hard scale for hadronic processes. Therefore we can safely ignore this contribution and the related strange channel in the EFT formulation proposed in this work³.

The formulation of the pionless EFT for the negative C-parity states can be constructed by considering the low energy limit of the phenomenological model of Refs. [48, 49], which predicts a contact interaction identical to the positive C-parity case

$$\langle \vec{k} | V_C | \vec{k}' \rangle_{C=-1} = C_0^{D\bar{D}^*} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad (37)$$

with $C_0^{D\bar{D}^*}$ given by Eq. (35). In principle, the previous potential implies the existence of a negative C-parity partner of the $X(3872)$. However, this conclusion depends on whether we can apply the pionless EFT of Ref. [21] to the $C = -1$ case without substantial modifications. In this regard, the model of Refs. [48, 49] requires the negative C-parity $D\bar{D}^*$ state to couple with (among others) the $J/\Psi \eta$ and $J/\Psi \eta'$ channels. The first lies 227 MeV below the $D_0 \bar{D}_0^*$ threshold and the second 189 MeV above. This means that the $C = -1$ state can decay into $J/\Psi \eta$ with a center-of-mass momentum of $k_{J/\Psi \eta} = 460$ MeV, and has a $J/\Psi \eta'$ component in the wave function with wave number $\gamma_{J/\Psi \eta'} = 517$ MeV. In addition, the $J/\Psi \eta'$ contribution to the wave function has a remarkable tendency to decay into $J/\Psi \eta$ due to $\eta - \eta'$ mixing. The aforementioned momentum scales are slightly below or of the order of the typical hard scale of hadronic processes ($\Lambda_0 \simeq 0.5 - 1.0$ GeV), and may require the inclusion of the J/Ψ , the η and the η' as explicit degrees of freedom in a pionless EFT applicable for the negative C-parity state. If the J/Ψ , η and η' fields are not considered, the EFT treatment of this channel may not be reliable unless a cut-off below $k_{J/\Psi \eta} \sim \gamma_{J/\Psi \eta'} \sim 0.5$ GeV is employed. Yet there is

no reason to believe that the structure of the LO counterterm is similar in the negative and positive C-parity sectors. This observation gets support from the lack of clear experimental evidence for the existence of a negative C-parity $D\bar{D}^*$ state in the region where the $X(3872)$ is located.

This suspicion is confirmed by the results of Ref. [20], in which the negative C-parity $D\bar{D}^*$ state either (i) disappears, if the same regulator is used as in the $X(3872)$ state, or (ii) moves about $\Delta E_{\text{cm}} \simeq -i 26$ MeV in the complex plane, if the regulator is modified as to keep the real part of the energy of the negative C-parity state slightly below the $D_0 \bar{D}_0^*$ threshold. The previous figures serve as a demonstration of the limits of an EFT formulation containing only D and \bar{D}^* mesons for the $C = -1$ case. An additional limitation is provided by the fact that the negative C-parity partner of the $X(3872)$ has not been observed experimentally, as commented in the previous paragraph. That is, contrary to the $C = +1$ case, in which the explanation of the $J/\Psi \omega$ over $J/\Psi \rho$ branching ratio provides a test of the model employed for saturation, we have no analogous external check of the model of Refs. [48, 49] when $C = -1$. In this regard, there is no compelling reason to trust the saturation condition given by Eq. (37). We will therefore ignore for the moment the negative C-parity case.

The extension of the previous EFT formulation to the bottom sector is straightforward: we expect the form of the contact interaction to be identical in the charm and bottom sectors, that is

$$\langle \vec{k} | V_C | \vec{k}' \rangle = C_0^{B\bar{B}^*} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \quad (38)$$

In addition, as a consequence of the tiny mass splitting between the neutral ($B_0 \bar{B}_0^*$) and charged ($B^+ B^{*-}$) channels, the coupled channel structure of the contact potential simplifies to

$$\langle \vec{k} | V_C | \vec{k}' \rangle \simeq 2 C_0^{B\bar{B}^*}. \quad (39)$$

The strange channel ($B_s^0 \bar{B}_s^{*0}$) lies about 180 MeV above the $B_0 \bar{B}_0^*$ and $B^+ B^{*-}$ threshold. However, as a consequence of the heavier reduced mass of the $B\bar{B}^*$ system, the related wave number of a strange component is more short-ranged than the corresponding one in the charm sector: assuming a low lying bound state, we obtain a wave number of $\gamma_s \simeq 980$ MeV. Therefore we can safely ignore this degree of freedom.

The problem in predicting the existence of bound states in the bottom sector is how to determine the value of the counterterm and the cut-off. The key observation in this regard is that we can make sensible estimations of $C_0^{B\bar{B}^*}$ and Λ_B from $C_0^{D\bar{D}^*}$ and Λ_D by invoking HQS. In particular, we expect that (i) the saturation condition for the counterterm between the B and \bar{B}^* mesons is given by

$$C_0^{B\bar{B}^*} \simeq -\frac{1}{2f_B^2}, \quad (40)$$

³ In particular, the strange channel probability within the $X(3872)$ is completely negligible. The contribution of the strange channel to the binding energy of the $B\bar{B}^*$ state is also quite small, as we will see.

in analogy with Eq. (35), and (ii) the value of the counterterms, which roughly represents the inverse size of the heavy mesons, or equivalently, the binding energy between the light and heavy quark, is similar in both cases, modulo $1/m_Q$ corrections, which leads to

$$\Lambda_B = \Lambda_X + \mathcal{O}\left(\frac{1}{m_Q}\right), \quad (41)$$

where m_Q is the mass of the heavy quark and Λ_X the value of the cut-off in the $D\bar{D}^*$ system. Provided the previous assumptions hold, it is trivial to determine whether there exists a bottom counterpart of the $X(3872)$.

A. Description of the $X(3872)$ State

We describe the $X(3872)$ state as a $D\bar{D}^*$ system with C-parity $C = +1$. Following Ref. [21], we include both the neutral and charged components, which means that the wave function reads

$$\begin{aligned} \Psi^X(k) = & \frac{1}{\sqrt{2}} [|D^0\bar{D}^{*0}\rangle - |D^{*0}\bar{D}^0\rangle] \Psi^{X_0}(k) \\ & + \frac{1}{\sqrt{2}} [|D^+\bar{D}^{*-}\rangle - |D^{*+}\bar{D}^-\rangle] \Psi^{X_C}(k), \end{aligned} \quad (42)$$

where we are assuming Ψ^{X_0} and Ψ^{X_C} to be S-wave. We will determine the wave functions of the system by solving a two-channel bound state equation with the following interaction

$$\langle k | V_C^{\alpha,\beta} | k' \rangle = f\left(\frac{k}{\Lambda}\right) C_0^{D\bar{D}^*}(\Lambda) f\left(\frac{k'}{\Lambda}\right), \quad (43)$$

where $\alpha, \beta = 0, C$, depending on whether we are considering the neutral or charged channel. This contact interaction is equivalent to Eq. (34). In addition, we have regularized the contact potential of Eq. (43) with a regulator $f(x)$ fulfilling the conditions: (i) $f(x) \rightarrow 1$ for $x \ll 1$ and (ii) $f(x) \rightarrow 0$ for $x \gg 1$. For simplicity we will use a sharp cut-off regulator, $f(x) = \theta(1 - x)$, along this work.

For the previous contact potential, the bound state equation reads

$$\Psi^{X_0}(k) = \frac{2\mu_{X,0}}{k^2 + \gamma_{X_0}^2} f\left(\frac{k}{\Lambda}\right) F(\Lambda), \quad (44)$$

$$\Psi^{X_C}(k) = \frac{2\mu_{X,C}}{k^2 + \gamma_{X_C}^2} f\left(\frac{k}{\Lambda}\right) F(\Lambda), \quad (45)$$

with $F(\Lambda)$ a function independent of the momentum k , which is given by

$$\begin{aligned} F(\Lambda) = & -C_0^{D\bar{D}^*}(\Lambda) \int \frac{q^2 dq}{2\pi^2} f\left(\frac{q}{\Lambda}\right) \\ & \times [\Psi^{X_0}(q) + \Psi^{X_C}(q)]. \end{aligned} \quad (46)$$

We have taken $\mu_{X_0} = 966.6$ MeV, $\mu_{X_C} = 968.7$ MeV for the reduced mass of the neutral and charged subsystems. The wave numbers are defined as $\gamma_{X_0}^2 =$

$-2\mu_{X_0}(E_X + \Delta_{X_0})$ and $\gamma_{X_C}^2 = -2\mu_{X_C}(E_X + \Delta_{X_C})$. Traditionally the $X(3872)$ bound state energy is referred with respect to the threshold of the neutral component. In accordance with this prescription, we take $\Delta_{X_0} = 0$ and $\Delta_{X_C} = (m_{D^+} + m_{D^{*-}}) - (m_{D^0} + m_{D^{*0}}) = 8.06$ MeV for the kinematical thresholds. We take $B_X = -E_X = 0.1 - 0.6$ MeV.

The bound state equation is trivial to solve, yielding the solutions

$$\Psi^{X_0}(k) = \mathcal{N} \frac{2\mu_{X_0}}{k^2 + \gamma_{X_0}^2} f\left(\frac{k}{\Lambda}\right), \quad (47)$$

$$\Psi^{X_C}(k) = \mathcal{N} \frac{2\mu_{X_C}}{k^2 + \gamma_{X_C}^2} f\left(\frac{k}{\Lambda}\right), \quad (48)$$

where \mathcal{N} is a normalization constant which can be determined with the normalization condition

$$\int \frac{k^2 dk}{2\pi^2} [|\Psi^{X_0}(k)|^2 + |\Psi^{X_C}(k)|^2] = 1. \quad (49)$$

The eigenvalue equation, which describes the running of the counterterm with the cut-off Λ , is obtained by inserting the explicit solution to the wave functions inside the bound state equations, Eqs. (44), (45) and (46), yielding

$$\begin{aligned} -\frac{1}{C_0^{D\bar{D}^*}(\Lambda)} = \\ \int \frac{q^2 dq}{2\pi^2} f^2\left(\frac{q}{\Lambda}\right) \left[\frac{2\mu_{X_0}}{q^2 + \gamma_{X_0}^2} + \frac{2\mu_{X_C}}{q^2 + \gamma_{X_C}^2} \right]. \end{aligned} \quad (50)$$

As previously stated, we follow the approach of Ref. [21] in which the counterterm is saturated by f_D . If the binding energy has been fixed, the saturation condition determines the value of the cut-off

$$C_0^{D\bar{D}^*}(\Lambda = \Lambda_X) = -\frac{1}{2f_D^2}, \quad (51)$$

where we take the value $f_D = 210 \pm 10$ MeV, a value compatible with recent lattice simulations. The saturation condition can only be fulfilled for a specific value of the cut-off $\Lambda = \Lambda_X$. For the range of bound state energies considered in this work, and a sharp cut-off function, we obtain

$$\Lambda_X = 555_{-44}^{+47} \text{ MeV}, \quad (52)$$

where the error accounts for the binding energy window and the uncertainty in f_D ⁴. The cut-off can be physically interpreted as the scale at which the system starts to resolve the composite nature of the D and D^* mesons. That is, the cut-off is related to the size of the heavy mesons.

⁴ In contrast, Ref. [21] takes $f_D = \sqrt{2} \times 165$ MeV (i.e. 233 MeV) and $B_X = 0.1$ MeV, for which the cut-off $\Lambda_X = 653$ MeV is obtained.

B. The $B\bar{B}^*$ Bound State

The description of the theoretical $B\bar{B}^*$ bound state involves a neutral and a charged component with positive C-parity, $C = +1$, that is

$$\begin{aligned} \Psi^{B\bar{B}^*}(k) &= \frac{1}{\sqrt{2}} [|B^0\bar{B}^{*0}\rangle - |B^{*0}\bar{B}^0\rangle] \Psi^{B\bar{B}^*(0)}(k) \\ &+ \frac{1}{\sqrt{2}} [|B^+\bar{B}^{*-}\rangle - |B^{*+}\bar{B}^-\rangle] \Psi^{B\bar{B}^*(C)}(k). \end{aligned} \quad (53)$$

As we will see later, the isospin breaking effects are negligible in this case, and the neutral and charged wave functions are approximately equal. In analogy with the $D\bar{D}^*$ case, the $B\bar{B}^*$ interaction is taken to be

$$\langle k | V_C^{\alpha,\beta} | k' \rangle = f\left(\frac{k}{\Lambda}\right) C_0^{B\bar{B}^*}(\Lambda) f\left(\frac{k'}{\Lambda}\right), \quad (54)$$

where $\alpha, \beta = 0, C$. The bound state equation reads

$$\Psi^{B\bar{B}^*(0)}(k) = \frac{2\mu_{B\bar{B}^*(0)}}{k^2 + \gamma_{B\bar{B}^*(0)}^2} f\left(\frac{k}{\Lambda}\right) F(\Lambda), \quad (55)$$

$$\Psi^{B\bar{B}^*(C)}(k) = \frac{2\mu_{B\bar{B}^*(C)}}{k^2 + \gamma_{B\bar{B}^*(C)}^2} f\left(\frac{k}{\Lambda}\right) F(\Lambda), \quad (56)$$

with F given by

$$\begin{aligned} F(\Lambda) &= -C_0^{B\bar{B}^*}(\Lambda) \int \frac{q^2 dq}{2\pi^2} f\left(\frac{q}{\Lambda}\right) \\ &\times \left[\Psi^{B\bar{B}^*(0)}(q) + \Psi^{B\bar{B}^*(C)}(q) \right]. \end{aligned} \quad (57)$$

We take $\mu_{B\bar{B}^*,0} = 2651.1$ MeV and $\mu_{B\bar{B}^*,C} = 2651.01$ MeV for the reduced masses. The wave numbers are $\gamma_{B\bar{B}^*(0)}^2 = -2\mu_{B\bar{B}^*(0)}(E_{B\bar{B}^*} + \Delta_{B\bar{B}^*,0})$ and $\gamma_{B\bar{B}^*(C)}^2 = -2\mu_{B\bar{B}^*(C)}(E_{B\bar{B}^*} + \Delta_{B\bar{B}^*(C)})$. In this case, the kinematical threshold gaps are given by $\Delta_{B\bar{B}^*(0)} = 0$ and $\Delta_{B\bar{B}^*(C)} = (m_{B^+} + m_{B^{*-}}) - (m_{B^0} + m_{B^{*0}}) \simeq -0.35$ MeV (with an error of 0.27 MeV).

The solution of the bound state equation is analogous to that found in the $D\bar{D}^*$ system. Taking into account the size of the threshold gap, $\Delta_{B\bar{B}^*(C)} \simeq 0.35$ MeV, we can make the approximation

$$\begin{aligned} \Psi^{B\bar{B}^*(0)}(k) &= \Psi^{B\bar{B}^*(C)}(k) = \\ &\frac{1}{\sqrt{2}} \Psi^{B\bar{B}^*}(k) \left[1 + \mathcal{O}\left(\frac{\Delta_{B\bar{B}^*}}{E_{B\bar{B}^*}}\right) \right], \end{aligned} \quad (58)$$

where the relative error will be fairly small for bound state energies above 3 – 4 MeV. The isospin symmetric wave function is given by

$$\Psi^{B\bar{B}^*}(k) = \mathcal{N} \frac{2\mu_{B\bar{B}^*}}{k^2 + \gamma_{B\bar{B}^*}^2} f\left(\frac{k}{\Lambda}\right), \quad (59)$$

where \mathcal{N} is a normalization constant which may be obtained by

$$\int \frac{k^2 dk}{2\pi^2} |\Psi^{B\bar{B}^*}(k)|^2 = 1. \quad (60)$$

In the isospin symmetric limit, the the eigenvalue equation reads

$$-\frac{1}{C_0^{B\bar{B}^*}(\Lambda)} = 2 \int \frac{q^2 dq}{2\pi^2} f^2\left(\frac{q}{\Lambda}\right) \frac{2\mu_{B\bar{B}^*}}{q^2 + \gamma_{B\bar{B}^*}^2}, \quad (61)$$

and finally, the saturation condition is

$$C_0^{B\bar{B}^*}(\Lambda = \Lambda_B) = -\frac{1}{2f_B^2}, \quad (62)$$

with $f_B = 195 \pm 10$ MeV. From HQS we expect the value of the cut-off in the $B\bar{B}^*$ system to be similar to the $D\bar{D}^*$ system, that is $\Lambda_B \simeq \Lambda_X$, as already mentioned. This assumption is equivalent to presuming the size of the D and B mesons not to depend strongly on the mass of the heavy quark.

In this case the known and unknown parameters are different than in the $D\bar{D}^*$ system. Contrary to the $X(3872)$ case, the bound state energy of the theoretical $B\bar{B}^*$ state is unknown and needs to be determined. In this regard, we will use the eigenvalue equation Eq. (61), the saturation condition Eq. (62) and the similar cut-off assumption to predict the expected bound state energy of the $B\bar{B}^*$ system.

The calculation has three error sources: (i) the bound state energy of the $X(3872)$, which ranges from $B_X = 0.1 - 0.6$ MeV, (ii) the uncertainties in the value of f_B , and (iii) the error in the relation $\Lambda_B \simeq \Lambda_X$. The third source of error is unknown, but can be estimated from heavy quark symmetry: we expect the cut-offs to be equal modulo $1/m_Q$ corrections

$$\Lambda_B = \Lambda_X + \mathcal{O}\left(\frac{1}{m_Q}\right), \quad (63)$$

being m_Q the heavy quark mass. The problem is how to evaluate the size of the $1/m_Q$ corrections. A possible estimation comes from the observation that $\sqrt{m_D}f_D$ and $\sqrt{m_B}f_B$ also differ by $1/m_Q$ corrections

$$\sqrt{m_B}f_B = \sqrt{m_D}f_D + \mathcal{O}\left(\frac{1}{m_Q}\right). \quad (64)$$

Assuming that the relative size of these corrections is the same for the constants and the cut-off, we obtain

$$\left| \frac{\Lambda_B - \Lambda_X}{\Lambda_B} \right| \sim \left| \frac{\sqrt{m_D}f_D}{\sqrt{m_B}f_B} - 1 \right|, \quad (65)$$

which yields a $\sim 10 - 15\%$ relative error to the naive relation $\Lambda_B = \Lambda_X$. Therefore, we take $\Lambda_B = \Lambda_X (1 \pm 0.15)$ as a conservative estimate. For the $X(3872)$, we obtained from the saturation condition $\Lambda_X = 555_{-44}^{+47}$ MeV, in which the uncertainties coming from B_X and f_D are already taken into account. Considering the $1/m_Q$ corrections and quadratic error propagation, the cut-off window for the $B\bar{B}^*$ raises to $\Lambda_B = 555_{-94}^{+96}$ MeV, where the errors are dominated by the $1/m_Q$ term.

For the previous estimations of f_B and Λ_B , we obtain a bound state energy of $B = 45_{-35}^{+24}$ MeV. The

binding energy is clearly beyond the limits of a contact range theory, which means that the inclusion of pion exchanges is mandatory from the EFT viewpoint. The explicit inclusion of isospin breaking shifts the bound state energy about $\Delta B = -0.2$ MeV for the central value $B = 45$ MeV, much below the error in the determination of the bound state energy, and is therefore a negligible effect. The effect of including the strange component in the charm and bottom sectors is small, generating a shift of $\Delta B = -4$ MeV of the $B\bar{B}^*$, again much smaller than other uncertainty sources; the probability of the strange state in such a case is $P_s \simeq 2 - 3\%$.

IV. THE ROLE OF ONE PION EXCHANGE

In this section we consider the role of one pion exchange in the $D\bar{D}^*$ and $B\bar{B}^*$ systems. As in the two nucleon system, we expect the long range interaction between two heavy mesons to be driven by one pion exchange. From the EFT perspective, the inclusion of multiple pion exchanges, which are in turn constrained by the requirement of broken chiral symmetry, extends the range of validity of the effective description up to momenta of the order of the ρ mass or the binding energy of the heavy and light quarks conforming the heavy meson. Both scales are of the same order of magnitude, yielding a breakdown scale of $\Lambda_0 \sim 0.5 - 1$ GeV. In this respect, we expect the EFT formulation to be able to describe bound states up to a maximum binding energy of $B_{\max} \sim \Lambda_0^2/2\mu_{B\bar{B}^*} \sim 50 - 200$ MeV for the particular case of the $B\bar{B}^*$ system.

As already noticed in Ref. [3], an interesting difference with respect to the two nucleon case lies in the role of the short range interaction, which does not involve a strongly repulsive core in the heavy meson case. In fact, the short range force is attractive in S-waves, as can be deduced from the existence of the $X(3872)$ state. This factor will help the formation of S-wave bound states in the $B\bar{B}^*$ system.

A. The One Pion Exchange Potential

1. Derivation of the One Pion Exchange Potential

The one pion exchange potential between a pseudoscalar (P) and a vector (P^*) meson (both containing a light quark) can be derived from the $PP^*\pi$ vertex of Fig. (1). In Heavy Hadron Chiral Effective Field Theory [50, 51], the non-relativistic amplitude corresponding to this vertex in the isospin basis is given by

$$\mathcal{A}(P^{*i} \rightarrow P\pi^a) = \frac{g}{f_\pi} \frac{\tau^a}{\sqrt{2}} \vec{\epsilon}_i \cdot \vec{q} \quad (66)$$

with τ_a the isospin operator (i.e. the Pauli matrices) for the pion π^a in the cartesian basis and $\vec{\epsilon}_i$ the polariza-

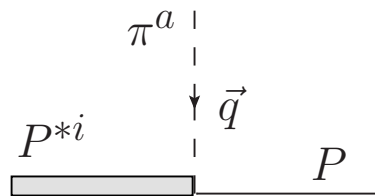


FIG. 1. Diagram representing the pion-vector-pseudoscalar-meson vertex. The thin line represents a pseudoscalar meson, the thick line a vector meson with polarization i , and the dashed line an incoming pion with isospin a and momentum \vec{q} .

tion vector of the P^{*i} meson. For the pion weak decay constant, we use the normalization $f_\pi \simeq 130$ MeV.

The value of g represents the coupling of the pion to the light quark in the pseudoscalar/vector meson. In non-relativistic quark models, we expect $g = 1$, while in chiral quark models, $g = 0.75$ (see, for example, Ref. [52]). However, the best way to determine the value of g is from the D^* decay width. For the particular cases of the decays $D^{*+} \rightarrow D^0\pi^+$ and $D^{*+} \rightarrow D^+\pi^0$, the previous amplitude reduces to

$$\mathcal{A}(D^{*+} \rightarrow D^0\pi^+) = \frac{g}{f_\pi} \vec{\epsilon}_i \cdot \vec{q}, \quad (67)$$

$$\mathcal{A}(D^{*+} \rightarrow D^+\pi^0) = \frac{g}{\sqrt{2}f_\pi} \vec{\epsilon}_i \cdot \vec{q}, \quad (68)$$

as can be trivially checked, yielding the well-known decay rates

$$\Gamma(D^{*+} \rightarrow D^0\pi^+) = g^2 \frac{|\vec{q}_{\pi^+}|^3}{6\pi f_\pi^2}, \quad (69)$$

$$\Gamma(D^{*+} \rightarrow D^+\pi^0) = g^2 \frac{|\vec{q}_{\pi^0}|^3}{12\pi f_\pi^2}. \quad (70)$$

From these decays (and taking into account the additional electromagnetic decay $D^{*+} \rightarrow D^+\gamma$), Refs. [53, 54] obtain the value $g = 0.59 \pm 0.01 \pm 0.07$. For simplicity, we will take $g = 0.6 \pm 0.1$ in the $D\bar{D}^*$ sector.

For the case of the $B\bar{B}^*\pi$ vertex, the value of g cannot be determined from pion decay. However, from heavy quark symmetry we should expect a value similar to that in the $D\bar{D}^*\pi$ vertex. This expectation seems to be confirmed by lattice simulations [55–58], which suggest a value in the range 0.5–0.6. We will take $g = 0.55 \pm 0.10$ in the $B\bar{B}^*$ sector. However there are also other determinations suggesting a smaller value of g . For example, Ref. [59] obtains $g = 0.37_{-0.02}^{+0.04}$ based on a study of the Dyson-Schwinger equations. Therefore, we will also discuss the consequences of having smaller values of g in the $B\bar{B}^*$ system at the end of this section.

In the static limit, the one pion exchange potential in momentum space between a pseudoscalar and vector meson (see Fig. (2)) takes the form

$$\tilde{V}_{\text{OPE}}(\vec{q}) = -\frac{g^2}{2f_\pi^2} \vec{\tau}_2 \cdot \vec{\tau}_1 \frac{\vec{\epsilon}_2^* \cdot \vec{q} \vec{\epsilon}_1 \cdot \vec{q}}{q^2 + \mu^2 + i\epsilon}, \quad (71)$$

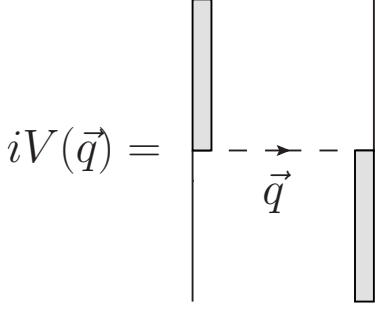


FIG. 2. Diagram corresponding to the one pion exchange potential between a pseudoscalar and a vector meson.

where $\tau_{1(2)}$ is the isospin operator in vertex 1(2), $\vec{\epsilon}_{1(2)}$ is the P^* meson polarization vector and $\mu^2 = m_\pi^2 - (m_{P^*} - m_P)^2$, with m_{P^*} and m_P the masses of the vector and pseudoscalar mesons. For the pion propagator we have used $D_{ab}(q) = \delta_{ab}/(q^2 - m_\pi^2 - i\epsilon)$, where a, b are isospin indices. In the isospin symmetric limit, we will take for the pion mass the value $m_\pi = 138.03$ MeV (which corresponds to $(m_{\pi^0} + 2m_{\pi^\pm})/3$). The small imaginary piece ($i\epsilon$) is added for dealing with the $\mu^2 < 0$ case.

The form of the OPE potential in configuration space can be obtained by Fourier transforming the momentum space representation of Eq. (71), in which case we obtain

$$\begin{aligned} V_{\text{OPE}}(\vec{r}) &= \int \frac{d^3 q}{(2\pi)^3} \tilde{V}_{\text{OPE}}(\vec{q}) e^{-i\vec{q}\cdot\vec{r}} \\ &= \frac{g^2}{2f_\pi^2} \vec{\tau}_2 \cdot \vec{\tau}_1 (\vec{\epsilon}_2^* \cdot \vec{\nabla}) (\vec{\epsilon}_1 \cdot \vec{\nabla}) \frac{e^{-\mu r}}{4\pi r}. \end{aligned} \quad (72)$$

The previous expression can be rewritten as

$$\begin{aligned} V_{\text{OPE}}(\vec{r}) &= -\vec{\tau}_2 \cdot \vec{\tau}_1 \vec{\epsilon}_2^* \cdot \vec{\epsilon}_1 \frac{g^2}{6f_\pi^2} \delta(\vec{r}) \\ &+ \vec{\tau}_2 \cdot \vec{\tau}_1 \left[\vec{\epsilon}_2^* \cdot \vec{\epsilon}_1 W_C(r) \right. \\ &\left. + (3\vec{\epsilon}_2^* \cdot \hat{r} \vec{\epsilon}_1 \cdot \hat{r} - \vec{\epsilon}_2^* \cdot \vec{\epsilon}_1) W_T(r) \right], \end{aligned} \quad (73)$$

where the central and tensor components of the potential, W_C and W_T , are defined as follows

$$W_C(r) = \frac{g^2 \mu^3}{24\pi f_\pi^2} \frac{e^{-\mu r}}{\mu r}, \quad (74)$$

$$W_T(r) = \frac{g^2 \mu^3}{24\pi f_\pi^2} \frac{e^{-\mu r}}{\mu r} \left(1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2} \right). \quad (75)$$

It should be noted that by making the replacements $\vec{\epsilon}_i \rightarrow \vec{\sigma}_i$, $g \rightarrow g_A (= 1.26)$ and $\mu \rightarrow m_\pi$ we recover the one pion exchange potential for the two-nucleon system. This representation will be useful for determining the channels in which one pion exchange is more attractive.

A particular problem which arises with the treatment of the one pion exchange potential in the DD^* and DD^* systems is that μ^2 is negative, generating a singularity

in the potential. This problem is overcome by taking a principal value prescription

$$\frac{1}{\bar{q}^2 - \tilde{\mu}^2 + i\epsilon} = \mathcal{P} \left(\frac{1}{\bar{q}^2 - \tilde{\mu}^2} \right) - \frac{i\pi}{2q} \delta(q - \tilde{\mu}), \quad (76)$$

where $\tilde{\mu}^2 = -\mu^2$ and \mathcal{P} denotes the principal value. This is equivalent to the change $\mu \rightarrow i\tilde{\mu}$ in the coordinate space potentials. We will ignore the imaginary piece of the potential, which is related to the decay of the $X(3872)$ to $D\bar{D}\pi$. A more complete treatment of the $\mu^2 < 0$ case, in which the $D\bar{D}\pi$ channel is included explicitly, can be found in Ref. [60].

2. C-parity

We are interested in meson-antimeson systems where C-parity plays an important role. In this regard, it should first be noted that the previous OPE potential has been obtained for a meson-meson system. The corresponding potential for the meson-antimeson system in the isospin basis can be obtained by means of a G-parity transformation, in which case the potential remains unchanged⁵

$$V_{\text{OPE}}^{P\bar{P}^*}(\vec{r}) = V_{\text{OPE}}(\vec{r}), \quad (77)$$

where $V_{\text{OPE}}(\vec{r})$ is the OPE potential as given in Eq. (73).

Now we take into account that the previous meson-antimeson potential is written in the following basis

$$\begin{aligned} V_{\text{OPE}}^{P\bar{P}^*}(\vec{r}) &= \langle P\bar{P}^* | V | P^*\bar{P} \rangle \\ &= \langle \bar{P}P^* | V | \bar{P}^*P \rangle, \end{aligned} \quad (78)$$

where C-parity plays no role. Therefore, we project the potential into a basis with well-defined C-parity, that is

$$|P\bar{P}^*(\eta)\rangle = \frac{1}{\sqrt{2}} [|P\bar{P}^*\rangle - \eta |P^*\bar{P}\rangle], \quad (79)$$

where η represents the intrinsic C-parity of the system⁶. In this basis, the potential reads

$$\langle P\bar{P}^*(\eta_{\text{out}}) | V | P\bar{P}^*(\eta_{\text{in}}) \rangle = -\frac{\eta_{\text{in}} + \eta_{\text{out}}}{2} V_{\text{OPE}}^{P\bar{P}^*}(\vec{r}), \quad (80)$$

that is, the OPE potential conserves C-parity and the sign of the potential depends on the intrinsic C-parity of the system.

⁵ Of course this depends on the C-parity convention for the heavy vector meson. In the present work we have taken $\hat{C}|P^*\rangle = -|\bar{P}^*\rangle$, which also implies an extra minus sign when performing the G-parity transformation on the vector meson. If we take into account the negative G-parity of the pion, the potential in the isospin basis does not change sign for antiparticles. If we had chosen the plus sign convention in the C- and G-parity transformations of the heavy vector mesons, the sign of the potential would have changed. However, in such a case we would also have obtained an additional minus sign after projecting into well-defined C-parity states in Eq. (80), thus leaving the final form of the potential in the C-parity basis unchanged.

⁶ The total C-parity of the heavy meson-antimeson system will be given by $C = (-1)^l \eta$, where l is the orbital angular momentum.

3. Isospin Breaking Effects

In the previous sections we have derived the potential in the isospin symmetric basis. However, isospin may be broken due to two effects: (i) the different masses of the charged and neutral pions and (ii) the different masses of the charged and neutral heavy mesons. For the $X(3872)$ (the $D\bar{D}^*$ system), the most important effect is (ii), due to the weakly bound nature of the $X(3872)$. On the contrary, for the $B\bar{B}^*$ system we expect both types of isospin breaking effects to play a relatively minor role.

To take into account isospin breaking, we consider the charged and neutral components of the system separately, i.e.

$$|P\bar{P}^*(\eta)\rangle = a_0 |(P\bar{P}^*(\eta))_0\rangle + a_{\pm} |(P\bar{P}^*(\eta))_{\pm}\rangle. \quad (81)$$

In this neutral-charged basis, we make the substitution ⁷

$$\vec{\tau}_1 \cdot \vec{\tau}_2 \rightarrow \begin{pmatrix} -1 & -2 \\ -2 & -1 \end{pmatrix}, \quad (82)$$

and the OPE potential reads

$$V_{\text{OPE}}^{P\bar{P}^*}(\vec{q}) \rightarrow -\eta \begin{pmatrix} -V_{\pi^0}(\vec{q}, \mu_0) & -2\bar{V}_{\pi^{\pm}}(\vec{q}, \mu_{\pm}) \\ -2\bar{V}_{\pi^{\pm}}(\vec{q}, \mu_{\pm}) & -V_{\pi^0}(\vec{q}, \mu'_0) \end{pmatrix}, \quad (83)$$

with $V_{\pi}(\vec{q}, \mu)$ given by

$$V_{\pi}(\vec{q}, \mu) = -\frac{g^2}{2f_{\pi}^2} \frac{\vec{\epsilon}_2^* \cdot \vec{q} \vec{\epsilon}_1 \cdot \vec{q}}{q^2 + \mu^2}, \quad (84)$$

where μ must be evaluated for the particular channel under consideration. For the neutral-neutral channel we have $\mu_0^2 = m_{\pi^0}^2 - (m_{P^*0} - m_{P0})^2$, while in the charged-charged channel $\mu'_0{}^2 = m_{\pi^0}^2 - (m_{P^*\pm} - m_{P\pm})^2$. For the term of the potential connecting the charged-neutral channels there are also two possible definitions for the value of μ_{\pm} , which are $\mu_{\pm}^2 = m_{\pi^{\pm}}^2 - (m_{P^*\pm} - m_{P0})^2$ and $\mu'_{\pm}{}^2 = m_{\pi^{\pm}}^2 - (m_{P^*0} - m_{P\pm})^2$. They are different as a consequence of $m_{P^*\pm} - m_{P0} \neq m_{P^*0} - m_{P\pm}$. To include this correction, we simply average over the two possible definitions of μ_{\pm}

$$\bar{V}_{\pi^{\pm}}(\vec{q}, \mu_{\pm}) = \frac{1}{2} (V_{\pi^{\pm}}(\vec{q}, \mu_{\pm}) + V_{\pi^{\pm}}(\vec{q}, \mu'_{\pm})) . \quad (85)$$

4. The Relative Strength of One Pion Exchange

The strength of the OPE potential in the $P\bar{P}^*$ system is different in each partial wave (i.e. ${}^{2S+1}L_J$ channel). If

⁷ We need to take into account that the \bar{D}^0 and \bar{D}^{*0} mesons have an extra minus sign when expressed as isospinors, that is \bar{D}^0 and $\bar{D}^{*0} = -|\frac{1}{2} \frac{1}{2}\rangle$.

we ignore the effect of zero-range contributions, bound states between two heavy mesons will be more probable in channels for which the long range potential (in this case, OPE) is most attractive. For this purpose we employ the coordinate space formulation, which simplifies the identification of the attractive channels. In addition, to avoid the problems derived from the existence of a neutral and charged component of the wave function, which will obscure the analysis, we take the isospin symmetric limit in the following discussion. The methods employed in this subsection were originally developed in Refs. [61–63] within the context of the renormalizability of nuclear forces in chiral EFT.

The configuration space wave function of the heavy meson pair (with total angular momentum j) can be decomposed into a radial and angular piece

$$\Psi_{B(pjm)}(\vec{r}) = \sum_{\{l\}_p} \frac{u_{lj}(r)}{r} \mathcal{Y}_{ljm}(\hat{r}), \quad (86)$$

where u_{lj} are the reduced wave functions of the $P\bar{P}^*$ system and $\{l\}_p$, j , m and \mathcal{Y}_{ljm} were already defined in Sect. II C. Ignoring zero range contributions, the reduced Schrödinger equation for the wave functions $u_{lj}(r)$ reads

$$-u_{lj}''(r) + 2\mu_{P\bar{P}^*} \sum_{l'} V_{ll'}(r) u_{l'j}(r) + \frac{l(l+1)}{r^2} u_{lj}(r) = -\gamma_{P\bar{P}^*}^2 u_{lj}(r), \quad (87)$$

where $\mu_{P\bar{P}^*}$ is the reduced mass of the $P\bar{P}^*$ system, $\gamma_{P\bar{P}^*} = \sqrt{-2\mu_{P\bar{P}^*} E_{P\bar{P}^*}}$ the wave number of the bound state, and $E_{P\bar{P}^*}$ the center-of-mass energy of the two meson system. The partial wave projection of the potential can be obtained from the following expression

$$V_{ll'}(r) = \sum_{\lambda, \lambda'} \int d\hat{r} \mathcal{Z}_{jm}^{l\lambda}(\hat{r}) \times \langle 1\lambda | V(\vec{r}) | 1\lambda' \rangle \mathcal{Z}_{jm}^{l'\lambda'}(\hat{r}), \quad (88)$$

as can be checked by adapting the methods of Sect. II C to coordinate space.

For the particular case of the OPE potential, $V_{ll'}$ reads

$$V_{ll'}(r) = -\eta \tau [\delta_{ll'} W_C(r) + S_{ll'} W_T(r)], \quad (89)$$

where η is the intrinsic C-parity and $\tau = \vec{\tau}_2 \cdot \vec{\tau}_1 = 2I(I+1) - 3$, with I the total isospin of the two meson system. W_C and W_T are the central and tensor pieces of the potential, see Eqs. (74) and (75). The $\delta_{ll'}$ and $S_{ll'}$ represent the partial wave projection of the $\vec{\epsilon}_2^* \cdot \vec{\epsilon}_1$ and $S_{12}(\hat{r}) (= 3\vec{\epsilon}_2^* \cdot \hat{r} \vec{\epsilon}_1 \cdot \hat{r} - \vec{\epsilon}_2^* \cdot \vec{\epsilon}_1)$ operators respectively. The $\delta(\vec{r})$ contribution to the OPE potential, see Eq. (73), has been ignored as it is a zero range contribution.

The matrix elements of the tensor operator in the partial wave basis are the following: (i) for uncoupled channels ($l = j$) we have $S_{jj} = -1$, (ii) for coupled channels ($l = j \pm 1$) we obtain

$$\mathbf{S}_j = \frac{1}{2j+1} \begin{pmatrix} j-1 & -3\sqrt{j(j+1)} \\ -3\sqrt{j(j+1)} & j+2 \end{pmatrix}, \quad (90)$$

where the matrix notation stands for $(\mathbf{S}_j)_{ll'} = S_{ll'j}$ ⁸; finally, (iii) for the 3P_0 channel, which is uncoupled (but for which $l = j + 1$), we have $S_{110} = 2$.

Naively we expect the strength of the OPE potential to depend on the interplay between the central and tensor components. However, at distances below the pion wavelength ($\mu r \leq 1$), the central piece contribution is small in comparison with the tensor component⁹. In particular, we expect the $1/r^3$ behaviour of the tensor force to determine the relative strength of OPE effects. At short enough distances, $\mu r \ll 1$, we have

$$2\mu_{P\bar{P}^*} W_T(r) \rightarrow \frac{R_T}{r^3}, \quad (91)$$

where R_T is a length scale related to the strength of tensor OPE. For the OPE potential, we expect in turn

$$2\mu_{P\bar{P}^*} V_{ll'j}(r) \rightarrow \lambda_{ll'j} \frac{R_T}{r^3}, \quad (92)$$

where $\lambda_{ll'j} = -\eta\tau S_{ll'j}$. That is, the $S_{ll'j}$ matrix determines which channels are attractive.

For the uncoupled channels the identification of the most attractive channels is trivial as it only requires the multiplication of the matrix element of the tensor force by C and τ . For the coupled channels, however, the relevant observation is that at short enough distances, the tensor force ($\sim 1/r^3$) overcomes the centrifugal barrier ($\sim 1/r^2$) and the central force ($\sim 1/r$), in which case we can approximate the full Schrödinger equation by

$$\begin{aligned} -u_{lj}''(r) - \eta\tau \frac{R_T}{r^3} \sum_{l'} S_{ll'j} u_{l'j}(r) \\ = -\gamma_{P\bar{P}^*}^2 u_{lj}(r) + \mathcal{O}\left(\frac{u_{lj}}{r^2}\right), \end{aligned} \quad (93)$$

where we have ignored the centrifugal barrier and potential contributions which diverge less strongly than $1/r^3$. If we define the following vector, which contains the two components of the wave function

$$\mathbf{u}_j = \begin{pmatrix} u_{j-1,j} \\ u_{j+1,j} \end{pmatrix}, \quad (94)$$

we can rewrite the Schrödinger equation above as

$$-\mathbf{u}_j''(r) - \eta\tau \frac{R_T}{r^3} \mathbf{S}_j \mathbf{u}_j = -\gamma_{P\bar{P}^*}^2 \mathbf{u}_j + \mathcal{O}\left(\frac{\mathbf{u}_j}{r^2}\right). \quad (95)$$

In this notation, it is clear that any transformation which diagonalizes \mathbf{S}_j also diagonalizes the Schrödinger equation at short distances. This means that the attractive or

$I(\eta)$	${}^3S_1 - {}^3D_1$	3P_0	3P_1	${}^3P_2 - {}^3F_2$	3D_2
0(+1)	{+6, -3}	+6	-3	{+6, -3}	-3
0(-1)	{-6, +3}	-6	+3	{-6, +3}	+3
1(+1)	{-2, +1}	-2	+1	{-2, +1}	+1
1(-1)	{+2, -1}	+2	-1	{+2, -1}	-1

TABLE I. Relative strength of the tensor component of OPE for the different partial waves in the isospin symmetric basis. The $I(\eta)$ notation indicates the isospin and the intrinsic C-parity subchannel under consideration. Negative values in the table denote channels in which the tensor force is attractive. In the coupled channel case, we show the two eigenvalues of the tensor force. The strength of tensor OPE in the peripheral uncoupled (coupled) waves coincides with that of the 3P_1 (${}^3S_1 - {}^3D_1$) channel.

repulsive character of the tensor force is determined by the eigenvalues of \mathbf{S}_j , that is

$$\mathbf{R}_j \mathbf{S}_j \mathbf{R}_j^T = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}, \quad (96)$$

where \mathbf{R}_j is the rotation matrix which brings \mathbf{S}_j into the diagonal basis. Independently of the values of η and τ , there is always an attractive and a repulsive eigenchannel. In this regard, at short enough distances the coupled wave function contains an attractive component which may generate a bound state.

The relative strength of the tensor force for the different partial waves, that is, the eigenvalues of $\lambda_{ll'j}$, can be found in Table I. The most attractive combination, $\lambda = -6$, is achieved in isoscalar ($I = 0$) channels with negative C-parity ($\eta = -1$), and in particular the uncoupled 3P_0 channel and all the coupled channels (${}^3S_1 - {}^3D_1$, ${}^3P_2 - {}^3F_2$, etc.). Next in attractiveness, the isoscalar ($I = 0$) channels with positive C-parity ($\eta = +1$) are to be found, such as the uncoupled 3P_1 and the coupled ${}^3S_1 - {}^3D_1$. However, owing to the repulsive role of the centrifugal barrier, we only expect the lower partial waves, such as the ${}^3S_1 - {}^3D_1$, the 3P_0 and the 3P_1 , to be the most promising candidates for a bound state.

In Table II we list the minimum value of the momentum space cut-off for having a bound state in the lowest partial waves. We consider the full OPE potential, as defined in Eqs. (71) or (73). That is, we include the zero-range δ contribution. As we can see, the previous expectations are approximately fulfilled except for the S-waves: the deviations in the ${}^3S_1 - {}^3D_1$ channel are caused by the zero range piece of the OPE potential. In the ${}^3P_2 - {}^3F_2$ channel we do not find bound states for values of the cut-off below 2 GeV, probably due to the strong centrifugal barrier of the F-wave subchannel, ruling out the possibility of a molecular interpretation in case the $J^{PC} = 2^{-+}$ assignment [13] turns out to be confirmed by future works.

⁸ As a curious fact, the matrix elements of the tensor operator for the $P\bar{P}^*$ system have opposite sign and half the strength of the corresponding matrix elements in the two-nucleon system.

⁹ Indeed this is the case in the two nucleon system, where only the tensor component of the OPE potential needs to be treated non-perturbatively, while the central piece can always be regarded as a perturbation.

$D\bar{D}^*$	Partial Wave		
$I(\eta)$	${}^3S_1 - {}^3D_1$	3P_0	3P_1
0(+1)	840^{+330}_{-200}	–	$> 2 \text{ GeV}$
0(-1)	1710^{+710}_{-430}	1050^{+440}_{-260}	–
1(+1)	$> 2 \text{ GeV}$	$> 2 \text{ GeV}$	–
1(-1)	$> 2 \text{ GeV}$	–	$> 2 \text{ GeV}$
$B\bar{B}^*$	Partial Wave		
$I(\eta)$	${}^3S_1 - {}^3D_1$	3P_0	3P_1
0(+1)	420^{+150}_{-100}	–	940^{+420}_{-240}
0(-1)	720^{+330}_{-180}	470^{+210}_{-120}	–
1(+1)	$> 2 \text{ GeV}$	1330^{+650}_{-370}	–
1(-1)	1040^{+470}_{-270}	–	$> 2 \text{ GeV}$

TABLE II. Cut-off (in MeV) at which the first bound state appears for the OPE potential in different partial waves. The $I(\eta)$ column specifies the isospin and the intrinsic C-parity subchannel (for the $D\bar{D}^*$, where isospin breaking has been taken into account, I is to be interpreted as "mostly a $I = 0/1$ state with some small admixture of $I = 1/0$ "). Partial waves without bound states below $\Lambda = 2 \text{ GeV}$ are not displayed. Errors take into account the uncertainty in the $P\bar{P}^*\pi$ coupling g .

5. The Negative C-parity States and the $P^*\bar{P}^*$ System

An interesting feature of the OPE potential, which we have ignored until now, is that it can mix the $P\bar{P}^*$ and the $P^*\bar{P}^*$ heavy meson systems. The reason is the following $P^*\bar{P}^*\pi$ vertex

$$\mathcal{A}(P^{*i} \rightarrow P^{*j}\pi^a) = \frac{g}{f_\pi} \frac{\tau_a}{\sqrt{2}} i(\vec{\epsilon}_i \times \vec{\epsilon}_j^*) \cdot \vec{q}, \quad (97)$$

which allows the $PP^* \rightarrow P^*P^*$ transition to happen via the one pion exchange mechanism depicted in Fig. (3). However, owing to the energy separation between the $P\bar{P}^*$ and the $P^*\bar{P}^*$ thresholds, the coupling between these two systems will not become evident unless the energy of the $P\bar{P}^*$ bound state is comparable to the mass difference between the P and P^* heavy mesons. In the bottom sector, this requires a binding energy of about 50 MeV: for a $B\bar{B}^*$ bound state at threshold, the wave number of the $B^*\bar{B}^*$ component is $\gamma_{B^*\bar{B}^*} \simeq 0.5 \text{ GeV}$, which is of the order of the hard scale of the system ($\Lambda_0 \sim 0.5 - 1.0 \text{ GeV}$). Consequently we can ignore the eventual $B^*\bar{B}^*$ short range component of the wave function of a $B\bar{B}^*$ bound state at the price of setting the hard scale at the lower bound $\Lambda_0 \sim 0.5 \text{ GeV}$ ¹⁰. From the power counting perspective this prescription can be taken into account by noticing that the mixing between

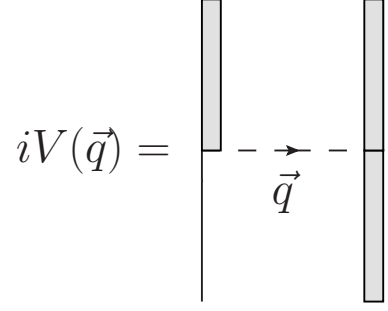


FIG. 3. One pion exchange diagram mixing the pseudoscalar-vector (PP^*) and the vector-vector (P^*P^*) heavy meson systems.

the $P\bar{P}^*$ and $P^*\bar{P}^*$ channels is suppressed as the ratio between the $P^*\bar{P}^*$ and $P\bar{P}^*$ propagators near the $P\bar{P}^*$ threshold, that is, by two orders in the EFT expansion

$$\frac{G_0^{P^*\bar{P}^*}(E)}{G_0^{P\bar{P}^*}(E)} = \frac{q^2 - 2\mu_{P\bar{P}^*} E}{q^2 + 2\mu_{P\bar{P}^*} \Delta - 2\mu_{P\bar{P}^*} E} \sim \left(\frac{P}{\Lambda_0}\right)^2, \quad (98)$$

where $q \sim \sqrt{2\mu_{P\bar{P}^*} E} \sim P$ are considered to be light scales, while $\sqrt{2\mu_{P\bar{P}^*} \Delta} \sim \Lambda_0$ is a heavy scale. In this regard, we expect the mixing of the channels to be an effect similar in size to chiral two pion exchange (TPE), which also enters at $\mathcal{O}(P^2)$.

If we are interested in a more detailed account of the effect of the $P^*\bar{P}^*$ channel in the description of the $P\bar{P}^*$ bound states, it should be noted that the conservation of parity, C-parity and total angular momentum requires that the coupling between the $P\bar{P}^*$ and $P^*\bar{P}^*$ channels only occurs between states with the same J^{PC} quantum numbers. In particular the intrinsic negative C-parity $P\bar{P}^*$ states will tend to have a stronger mixing with the $P^*\bar{P}^*$ system than the $\eta = +1$ states. The reason is that the ${}^{2S+1}L_J(P\bar{P}^*)$ partial wave with $\eta = -1$ has the same quantum numbers as the ${}^{2S+1}L_J(P^*\bar{P}^*)$ wave¹¹. In this regard, the $J^{PC} = 1^{+-} P\bar{P}^*$ state is expected to have the strongest mixing with the $P^*\bar{P}^*$ system, as the coupling can happen via S-wave (${}^3S_1(P\bar{P}^*) - {}^3S_1(P^*\bar{P}^*)$). In the 0^{-+} states, the ${}^3P_0(P\bar{P}^*)$ and ${}^3P_0(P^*\bar{P}^*)$ channels can also mix, although being a P-wave the ${}^3P_0(P^*\bar{P}^*)$ component is expected to be suppressed. The overall effect of this mixing will be attractive: the short range $P^*\bar{P}^*$ component of the wave function can always take a configuration which minimizes the energy of the system, thus increasing the binding.

¹⁰ The situation is in fact very similar to ignoring the Δ isobar resonance in the low energy description of the two nucleon system, where a similar wave number is obtained for the eventual $N\Delta$ short range component of the deuteron.

¹¹ We remind that the C-parity of a PP^* and $P^*\bar{P}^*$ systems is $C = (-1)^l \eta$ and $C = (-1)^{l+s}$ respectively. This implies that intrinsic negative C-parity ${}^{2S+1}L_J(P\bar{P}^*)$ states can always mix with the ${}^{2S+1}L_J(P^*\bar{P}^*)$ states.

On the other hand, the intrinsic positive C-parity $P\bar{P}^*$ states will tend to couple much more weakly with the $P^*\bar{P}^*$ channel. The extreme case is the ${}^3P_0(P\bar{P}^*)$ partial wave ($J^{PC} = 0^{--}$), which does not contain any $P^*\bar{P}^*$ component as there is no $P^*\bar{P}^*$ state with such quantum numbers. In the 1^{++} state, the mixing is expected to be small: the ${}^3S_1 - {}^3D_1(P\bar{P}^*)$ channel can only couple with the ${}^5D_1(P^*\bar{P}^*)$ partial wave. In these cases we can probably ignore the $P\bar{P}^*$ mixing without significantly reducing the convergence of the theory.

B. The $X(3872)$ with One Pion Exchange

We describe the $X(3872)$ as a ${}^3S_1 - {}^3D_1$, $C = +1$ state, where the charged and neutral components are treated independently. The wave function reads

$$\begin{aligned} \Psi_{(pjm)}^X(\vec{k}) &= \frac{1}{\sqrt{2}} [|D^0\bar{D}^{*0}\rangle - |D^{*0}\bar{D}^0\rangle] \Psi_{(pjm)}^{X_0}(\vec{k}) \\ &+ \frac{1}{\sqrt{2}} [|D^+D^{*-}\rangle - |D^{*+}D^-\rangle] \Psi_{(pjm)}^{X_C}(\vec{k}), \end{aligned} \quad (99)$$

where $p = +1$, $j = 1$ and $\{l\}_p = 0, 2$. The $\Psi_{(pjm)}^{X_0}(\vec{k})$ and $\Psi_{(pjm)}^{X_C}(\vec{k})$ wave functions can be decomposed into an S-wave and D-wave component

$$\begin{aligned} \Psi_{(+1jm)}^{X_\alpha}(\vec{k}) &= \sqrt{4\pi} \left[\Psi_S^{X_\alpha}(k) \mathcal{Y}_{01m}(\hat{k}) \right. \\ &\left. + \Psi_D^{X_\alpha}(k) \mathcal{Y}_{21m}(\hat{k}) \right], \end{aligned} \quad (100)$$

where the subscript $\alpha = 0, C$ denotes the neutral and charged components, and where we have used S and D instead of $l = 0, 2$ for concreteness. The normalization condition for the wave function reads

$$\sum_{X_\alpha} \int \frac{k^2 dk}{2\pi^2} \left[|\Psi_S^{X_\alpha}(k)|^2 + |\Psi_D^{X_\alpha}(k)|^2 \right] = 1. \quad (101)$$

The wave functions are obtained by solving numerically a four channel Lippmann-Schwinger equation, Eq. (31), with the OPE potential and a contact term. The OPE potential in the neutral-charged basis is taken from Eqs. (83) and (84). For $\mu^2 < 0$, we employ the principal value prescription and ignore the imaginary piece of the OPE potential. The partial wave projection of the OPE potential can be obtained from Eq. (19). For the contact piece, the partial wave projection is trivial, yielding

$$\langle p, lj | V_C^{\alpha\beta} | p', l' j \rangle = C_0^{D\bar{D}^*} \delta_{l,0} \delta_{l',0}. \quad (102)$$

The full potential is regularized with a suitable regulator function $f(x)$

$$\langle p, lj | V^{\alpha\beta} | p', l' j \rangle \rightarrow f\left(\frac{k}{\Lambda}\right) \langle p, lj | V^{\alpha\beta} | p', l' j \rangle f\left(\frac{k'}{\Lambda}\right), \quad (103)$$

where $\alpha, \beta = 0, C$ and $V^{\alpha\beta} = V_{\text{OPE}}^{\alpha\beta} + V_C^{\alpha\beta}$. We will employ a sharp cut-off regulator, $f(x) = \theta(1-x)$.

As in the contact theory case, the value of the cut-off is determined from the saturation condition, Eq. (51). For the central values $g = 0.6$, $B_X = 0.35 \text{ MeV}$ and $f_D = 210 \text{ MeV}$, and a sharp cut-off regulator, the saturation cut-off

$$\Lambda_X = 396_{-43}^{+43} \text{ MeV} \quad (104)$$

is obtained, where the spread in the cut-off value comes from the uncertainty in g , B_X and f_D ¹².

1. The Negative C-Parity State

An interesting advantage of including pion exchanges explicitly is that the EFT framework can be now applied to the negative C-parity sector. With OPE the saturation cut-off takes a smaller size than the typical momenta associate to the $J/\Psi\eta$ and $J/\Psi\eta'$ channels, which means that we are within the range of applicability of the theory. In this regard, while the contact theory seems to predict a negative C-parity partner of the $X(3872)$, the inclusion of OPE in the $I(J^{PC}) = 0(1^{+-})$ channel prevents the formation of a bound state unless the cut-off Λ is greater than 760 MeV. However Λ_X turns out to be too small, meaning that OPE explains in a natural manner why the negative C-parity state does not exist.

C. $B\bar{B}^*$ Bound States with One Pion Exchange

The number of bound states between the $B\bar{B}^*$ mesons depends on the natural value of the cut-off for this system. Assuming the relation $\Lambda_B \simeq \Lambda_X$, and taking into account the results from Table II, we expect to find two states in the $B\bar{B}^*$ system: a $I(J^{PC}) = 0(1^{++})$ and a $I(J^{PC}) = 0(0^{-+})$ state. The $0(1^{++})$, ${}^3S_1 - {}^3D_1$ state is almost universally predicted in works investigating $B\bar{B}^*$ molecular states, see for example Refs. [19, 25–27]. The prediction of a 3P_0 resonant state is less usual, as most works concentrate in the S-wave states. In addition, owing to the fundamental role of contact interactions in the formation of heavy meson bound states, we will find that the negative C-parity $0(1^{+-})$ state is bound too. In Table III there is a summary of the binding energies of the bound states and resonances predicted in the present work.

¹² In particular, we have $\Lambda_X = 396_{-37}^{+38} {}_{-10}^{+7} {}_{-19}^{+19} \text{ MeV}$, where the errors come from g , B_X and f_D respectively. For the final value, we have assumed that the errors add quadratically.

Theory	$^{2s+1}L_J$	$I(J^{PC})$	$B (E_{\text{cm}})$	Λ
Contact	$^3S_1 - ^3D_1$	$0(1^{++})$	45_{-35}^{+24}	555_{-94}^{+96}
OPE	$^3S_1 - ^3D_1$	$0(1^{++})$	20_{-12}^{+18}	396_{-73}^{+73}
OPE	$^3S_1 - ^3D_1$	$0(1^{+-})$	$6.4_{-4.6}^{+7.3}$	396_{-73}^{+73}
OPE	3P_0	$0(0^{-+})$	$0.7_{-1.9}^{+\infty} - 1.1_{-1.6}^{+\infty} \frac{i}{2}$	396_{-73}^{+73}

TABLE III. Summary of bound states and resonances in the bottom sector. For the bound states we show the binding energy ($B = -E_{\text{cm}}$), while for the 3P_0 resonance we indicate the center of mass energy of the state.

1. The Positive C-parity $^3S_1 - ^3D_1$ Bound State

In first place, we consider a theoretical $B\bar{B}^*$ bound state in the $^3S_1 - ^3D_1$ channel with positive C-parity, total isospin $I = 0$ and $J^{PC} = 1^{++}$. For this system, the wave function reads

$$\Psi_{(pjm)}^{B\bar{B}^*}(\vec{k}) = \frac{1}{2} \left[|B^0\bar{B}^{*0}\rangle - |B^{*0}\bar{B}^0\rangle + |B^+B^{*-}\rangle - |B^{*+}B^-\rangle \right] \Psi_{(pjm)}(\vec{k}), \quad (105)$$

where $p = +1$, $j = 1$, $\{l\}_p = 0, 2$ and we have assumed isospin symmetry to hold. As in the $X(3872)$ case, the wave function can be decomposed into an S- and D-wave piece

$$\Psi_{(+11m)}(\vec{k}) = \sqrt{4\pi} \left[\Psi_S(k) \mathcal{Y}_{01m}(\hat{k}) + \Psi_D(k) \mathcal{Y}_{21m}(\hat{k}) \right], \quad (106)$$

subjected to the normalization condition

$$\int \frac{k^2 dk}{2\pi^2} [|\Psi_S(k)|^2 + |\Psi_D(k)|^2] = 1. \quad (107)$$

For the $B\bar{B}^*$ system, as a consequence of isospin symmetry, we solve a two channel Lippmann-Schwinger equation, Eq. (31). The full potential results from adding the OPE contribution, as described in Eq. (71), and the contact theory contribution, which in the partial wave basis reads

$$\langle p, lj | V_C | p', l' j \rangle = 2 C_0^{B\bar{B}^*} \delta_{l,0} \delta_{l',0}, \quad (108)$$

where a factor of 2 needs to be included as we are working in the isospin symmetric basis. We assume the contact interaction to be determined by the saturation condition, Eq. (62), where we take $f_B = 195 \pm 10$ MeV. The full, partial wave projected potential is then regulated with a regulator function $f(x)$,

$$\langle p, lj | V | p', l' j \rangle \rightarrow f\left(\frac{k}{\Lambda}\right) \langle p, lj | V | p', l' j \rangle f\left(\frac{k'}{\Lambda}\right), \quad (109)$$

with $V = V_{\text{OPE}} + V_C$. As in previous cases, we will employ a sharp cut-off, $f(x) = \theta(1 - x)$. The cut-off is supposed to be $\Lambda_B = \Lambda_X$, modulo $1/m_Q$ corrections, which we estimate to be of the order of 15%. This translates into the value $\Lambda_B = 396_{-73}^{+73}$ MeV, where the combined uncertainty is the result of quadratic error propagation. The cut-off error is dominated by the estimation of the size of the $1/m_Q$ corrections.

For the values of g , f_B and Λ_B we are using, we obtain an estimation of the bound state energy of $B = 20_{-12}^{+18}$ MeV. The error estimation is completely dominated by the cut-off uncertainty, which alone would generate $B = 20_{-11}^{+18}$ MeV. The other sources of error estimation yield $B = 20.0_{-2.7}^{+3.4} + 2.9_{-2.4}$ MeV, corresponding to the g and f_B uncertainty respectively.

It is also interesting to notice that Ref. [27] predicts a total of two 1^{++} states in the bottom sector: a shallow bound state just below the $B\bar{B}^*$ threshold and a deeper one with a binding energy about 140 MeV. The reason for the appearance of the two bound states is a unusually big value of the $B\bar{B}^*\pi$ coupling. Within the present framework, and taking $g = 0.55$, a second bound state does not appear up to $\Lambda_B \simeq 1.2$ GeV: for this cut-off, there is a low-lying bound state at $B \simeq 0$ and a second bound state at $B \simeq 400$ MeV. If we take $g = 0.7$, which is a relatively large value for the $B\bar{B}\pi$ coupling, the appearance of the second bound state happens at $\Lambda_B \simeq 1$ GeV with a binding energy $B \simeq 175$ MeV, not far away from the results of Ref. [27]. From the aforementioned values the possibility of a second bound state in the $^3S_1 - ^3D_1$ channel seems quite unlikely, in view of the values of the cut-off obtained in the present work. However, the required cut-off and the position of a second bound state may depend on higher order contributions to the potential.

2. The Negative C-parity $^3S_1 - ^3D_1$ Bound State

In second place we consider the negative C-parity partner of the previous state, that is, a $B\bar{B}^*$ bound state in the $^3S_1 - ^3D_1$ channel with total isospin $I = 0$ and $J^{PC} = 1^{+-}$. The complete wave function reads in this case

$$\Psi_{(pjm)}^{B\bar{B}^*}(\vec{k}) = \frac{1}{2} \left[|B^0\bar{B}^{*0}\rangle + |B^{*0}\bar{B}^0\rangle + |B^+B^{*-}\rangle + |B^{*+}B^-\rangle \right] \Psi_{(pjm)}(\vec{k}), \quad (110)$$

where $p = +1$, $j = 1$ and $\{l\}_p = 0, 2$. The spatial wave function, $\Psi_{(pjm)}(\vec{k})$, is normalized and decomposed into an S- and D-wave component in exactly the same way as its positive C-parity partner. The short range potential takes the form given by Eq. (108) and is subjected to the saturation condition, see Eq. (62), with $f_B = 195 \pm 10$ MeV. The OPE potential has a overall minus sign with respect to the $C = +1$ case.

If we employ a sharp cut-off regulator with $\Lambda_B = 396_{-73}^{+73}$ MeV, we obtain a binding energy of $B = 6.4_{-4.6}^{+7.3}$ MeV, where the error is again dominated by the $1/m_Q$ corrections (i.e. the cut-off uncertainty). However, contrary to the positive C-parity state, we do not have an external check of the validity of the saturation condition employed in this channel. This represent an additional source of error which cannot be easily estimated and we should therefore expect a much larger uncertainty in the position of this state.

3. The 3P_0 Resonant State

The last state we will consider in detail is a isoscalar ($I = 0$), positive C-parity, 3P_0 resonant state with $J^{PC} = 0^{-+}$. For this state the wave function can be written as

$$\Psi_{(pjm)}^{B\bar{B}^*}(\vec{k}) = \frac{1}{2} \left[|B^0\bar{B}^{*0}\rangle + |B^{*0}\bar{B}^0\rangle + |B^+B^{*-}\rangle + |B^{*+}B^-\rangle \right] \Psi_{(pjm)}(\vec{k}), \quad (111)$$

where $p = -1$, $\{l\}_p = 1$, $j = 0$ and $m = 0$. Even though the intrinsic C-parity of this state is negative, the total C-parity is positive, as interchange of the meson and the antimeson will generate an additional minus sign in the wave function. The wave function only contains a P-wave component, which means that the separation of the radial and angular piece is trivial

$$\Psi_{(-100)}(\vec{k}) = \sqrt{4\pi} \Psi_P(k) \mathcal{Y}_{100}(\hat{k}). \quad (112)$$

Finally, the normalization condition is

$$\int \frac{k^2 dk}{2\pi^2} |\Psi_P(k)|^2 = 1, \quad (113)$$

which only applies when the state is bound.

For obtaining the resonant (bound) state energy we solve a one channel Lippmann-Schwinger equation with the OPE potential in the second (first) Riemann sheet (see Appendix A for details). The potential is regulated with a sharp cut-off function, where the cut-off window is the one determined in the previous section (i.e. $\Lambda_B = 396_{-73}^{+73}$ MeV). With this cut-off and $g = 0.55 \pm 0.1$, we obtain a resonant state energy of

$$E_{\text{cm}} = 0.7_{-1.9}^{+\infty} - 1.1_{-1.6}^{+\infty} \frac{i}{2} \text{ MeV}, \quad (114)$$

where the upper error is to be interpreted as the disappearance of the resonant state, which eventually happens for the smaller values of the cut-off. On the other hand, the lower bound is compatible with a shallow bound state. The interesting feature of the theoretical 3P_0 state is that, at leading order, it does not depend on the value of a contact operator, only on the estimation for the natural value of the cut-off. Although the resonance vanishes for small values of g or of the cut-off, the existence of

this states looks more probable than not. In addition, owing to the intrinsic negative C-parity of this state, the wave function can contain an appreciable $B^*\bar{B}^*$ component: this effect will reduce the energy of the system by a small amount, helping the formation of a bound state in this channel.

4. The Isovector States and the $Z_b(10610)$ Resonance

The recent discovery by the Belle collaboration of two new resonances, the $Z_b(10610)$ and $Z_b(10650)$ [14], which lie a few MeV above the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds respectively, provides two interesting candidates for heavy meson molecular states. The quantum numbers of the two Z_b states are $I^G(J^P) = 1^+(1^+)$. In the particular case of the $Z_b(10610)$, it is natural to interpret this resonance as a low-lying S-wave $B\bar{B}^*$ state. Several theoretical works [64–73] have appeared recently trying to explain the nature and properties of these two states.

The S-wave molecular interpretation is however not completely trivial owing to the location of the $Z_b(10610)$ at 4 ± 2 MeV above the $B\bar{B}^*$ threshold. Within a potential description this requires an explanation in terms of a $B\bar{B}^*$ resonance. In this regard, the existence of a resonant state in non-relativistic scattering depends on two ingredients, namely (i) a repulsive potential barrier at long distances and (ii) some sort of short range attraction. If we consider the form of the OPE potential in the $I = 1$, $C = -1$, ${}^3S_1 - {}^3D_1$ channel, we can appreciate that the OPE potential between two S-wave mesons is weakly repulsive and given by

$$V_{\text{ss}(j=1)}(r) = W_C(r), \quad (115)$$

see Eq. (89). The shorter range attraction is provided by the S- and D-wave mixing, which in fact is able to bind the system for large enough values of the cut-off, see Table II. In addition, being an intrinsic negative C-parity state, the $B\bar{B}^*$ and $B^*\bar{B}^*$ channels can mix. If we take into account that the potential barrier provided by OPE is only about 1 MeV high at $r = 0.5$ fm, which is insufficient for reaching the current position of the $Z_b(10610)$ resonance, it is apparent that a molecular interpretation of this resonant state requires a coupled channel approach in which the $B^*\bar{B}^*$ is explicitly included. In principle the argument above certainly renders difficult the direct extrapolation of the (purely) contact theories employed to describe the $X(3872)$ to the newly discovered $Z_b(10610)$, as pion exchanges and coupled channels are expected to be important. However, the theoretical analysis of $\Upsilon(5S) \rightarrow h_b\pi^+\pi^-$ and $h_b(2P)\pi^+\pi^-$ decays of Ref. [71] suggests that the position of the eventual $B\bar{B}^*$ state corresponding to the Z_b does not need to coincide with the position obtained from the Breit-Wigner parametrization employed in Ref. [14]. In particular the $Z_b(10610)$ can be located below threshold, around 5 MeV according to Ref. [71], reopening the possibility of a bound state interpretation.

$I(J^{PC})$	$B_{gc}^{(n=1)}$	$B_{gc}^{(n=2)}$	$B_{gc}^{(n=3)}$	$B_{gc}^{(n=4)}$	B_{sc}
$0(1^{++})$	29^{+30}_{-18}	23^{+21}_{-14}	21^{+20}_{-13}	21^{+18}_{-13}	20^{+18}_{-12}
$0(1^{+-})$	8^{+10}_{-6}	$6.7^{+8.1}_{-5.0}$	$6.5^{+7.7}_{-4.7}$	$6.5^{+7.4}_{-4.7}$	$6.4^{+7.3}_{-4.6}$
Cut-off (Λ_B)	620^{+120}_{-120}	517^{+96}_{-97}	478^{+89}_{-88}	458^{+84}_{-84}	396^{+73}_{-73}

TABLE IV. Dependence of the binding energy of the two $B\bar{B}^*$ isoscalar bound states with respect to different regulator choices. In particular we consider gaussian regulators with $n = 1, 2, 3, 4$, labeled $B_{gc}^{(n=1,2,3,4)}$ and the sharp cut-off regulator B_{sc} . As can be appreciated the central value of the binding energy can change moderately from one regulator to another. However, these variations are smaller than the uncertainty coming from other sources (in particular the 15% error associated with $1/m_Q$).

In general, isovector heavy meson states are not usually predicted (or even considered) in potential models, the reason being that the expected strength of the potential is three times weaker in isovector than in isoscalar states. From Table II we can see that the $I(J^{PC}) = 1(1^{+-})$ state binds at a cut-off $\Lambda \sim 1$ GeV, which is definitively bigger than the obtained EFT cut-off, $\Lambda_B \sim 0.4$ GeV, or the values of the cut-off which bind the two isoscalar states, $\Lambda \sim 0.4$ and 0.7 GeV respectively¹³. This does not mean however that the current estimation of the size of the cut-off is incorrect. A more natural explanation lies in the modification of the short-range dynamics. In particular, the saturation condition employed in the present work requires the contact interaction to be $C_0 = -1/f_B^2$ in isoscalar channels and $C_0 = 0$ in the isovectors, which penalizes the formation of $I = 1$ states. On the contrary, if we assume a zero energy bound state in the $Z_b(10610)$ channel, the isovector counterterm needs to take the value $C_0 = (-0.3 \pm 0.1)/f_B^2$, which although non-zero, is still relatively small in comparison to the isoscalar counterterm. Moreover the size of the counterterm is expected to decrease further if we take into account that (i) the $Z_b(10610)$ is above threshold (see however the previous discussion) and (ii) the mixing with the $B^*\bar{B}^*$ channel will provide additional attraction. In this regard, the only ingredient to accommodate the $Z_b(10610)$ state within the present framework is a small correction to the short range dynamics of the system in the line of

$$\langle \vec{k} | V_C | \vec{k}' \rangle = C_0^{B\bar{B}^*} \begin{pmatrix} 1 + \delta & 1 - \delta \\ 1 - \delta & 1 + \delta \end{pmatrix}, \quad (116)$$

instead of the form given in Eq. (38), with δ some small

¹³ In a related note, the recent exploration of meson exchange in the $B\bar{B}^*$ and $B^*\bar{B}^*$ systems of Ref. [70] also indicates that the value of the monopolar cut-off which binds the $Z_b(10610)$ channel is twice the size of the required cut-off for binding the $I(J^{PC}) = 0(1^{++})$ state, in qualitative agreement with the results of Table II.

number¹⁴. This will generate a contribution to the contact interaction in the isovector channel, $C_0 = -\delta/f_B^2$, thus providing the missing attraction needed to generate a resonant (or bound) state in the $Z_b(10610)$ channel.

5. Higher Order Corrections

In the previous calculations of the energies of the $B\bar{B}^*$ bound states we have taken into account three sources of error: (i) the $B\bar{B}^*\pi$ coupling constant g , (ii) the weak decay constant f_B and (iii) the uncertainty in the size of the $1/m_Q$ corrections for determining a suitable cut-off window. An additional error source in the EFT formulation is the size of the contributions from higher order terms. From the EFT viewpoint, if an observable is computed at order P^ν , the relative error for this observable is expected to be $\mathcal{O}(P^{\nu+1}/\Lambda_0^{\nu+1})$, where P and Λ_0 are generic notation for the soft and high scales of the system. For a LO calculation (i.e. P^0), the previous estimation will yield an expected error of $\mathcal{O}(P/\Lambda_0)$. However, the EFT with pions and heavy mesons fields does not contain any correction at order P^1 , the reason being parity conservation. Therefore we expect the calculations of the binding energy to be accurate up to

$$B^{\text{EFT}} = B^{(0)} + \mathcal{O}\left(\frac{P^2}{\Lambda_0^2}\right), \quad (117)$$

where B^{EFT} is the full value of the binding energy including all the EFT corrections and $B^{(0)}$ is the LO approximation. In this context, P can be interpreted as the wave number of the bound state, that is $P \sim \gamma^{(0)} = \sqrt{2\mu_{P\bar{P}^*} B^{(0)}}$, with $\mu_{P\bar{P}^*}$ the reduced mass of the heavy meson-antimeson system. This means in particular that the relative error is proportional to the binding energy; therefore, we can write

$$B^{\text{EFT}} = B^{(0)} + \mathcal{O}\left(\frac{B^{(0)}}{B_0^{\text{max}}}\right), \quad (118)$$

where B_0^{max} is the maximum bound state energy which can be described by the EFT, which corresponds to

$$B_0^{\text{max}} = \frac{\Lambda_0^2}{2\mu_{P\bar{P}^*}}. \quad (119)$$

Assuming a breakdown scale in the range $\Lambda_0 = 0.5 - 1.0$ GeV, we can estimate $B_0^{\text{max}} = 120 - 500$ MeV for the $D\bar{D}^*$ system and $B_0^{\text{max}} = 45 - 190$ MeV for $B\bar{B}^*$. As we are ignoring the mixing with the $D^*\bar{D}^*$ and $B^*\bar{B}^*$ channels, we expect the true breakdown scale to lie closer to $\Lambda_0 = 0.5$ GeV than 1 GeV.

¹⁴ The model of Gamermann and Oset [20] already contains this kind of corrections. However, their size may be far too small in the case of the $B\bar{B}^*$ system, giving $\delta \simeq (m_\rho/m_\Upsilon)^2 \sim 0.006$.

As we can see, the lower estimation for the breakdown of the EFT description of the BB^* bound states suggests moderate corrections to the estimation of the binding energy of the $I(J^{PC}) = 0(1^{++})$ state. This situation requires further analysis in order to check the reliability of the results. In a standard cut-off EFT formulation, a practical way to estimate the size of the higher order contributions is to vary the cut-off within a sensible range¹⁵. The underlying idea is that cut-off uncertainties are a higher order effect. In this sense, varying the cut-off mimics the effect of including (or excluding) the higher order contributions. However, in the non-standard formulation employed in this work, cut-off variations are used for estimating the size of $1/m_Q$ corrections. In principle, this may be interpreted as the necessity of going to subleading orders to explicitly check the size of the higher order corrections. Nevertheless there is a second way of doing things which is to consider how the results vary with different regulators. In particular we can check the effect of using gaussian cut-offs of the type

$$f\left(\frac{k}{\Lambda}\right) = e^{-\frac{k^{2n}}{\Lambda^{2n}}}, \quad (120)$$

for different values of n . As in the sharp cut-off case, we determine Λ by fixing the location of the $X(3872)$ state.

The results for changing the regulator are shown in Table IV, where we have considered the cases $n = 1, 2, 3, 4$ ¹⁶. We have only considered the two S-wave isoscalar bound states, as the calculation of the P-wave resonant state energy is much more involved. As can be seen, the biggest change happens when the $n = 1$ gaussian regulator is used, for which the central value of the cut-off is raised to $\Lambda = 620$ MeV and which generates a change of 10 MeV in the binding energy of the $0(1^{++})$ ${}^3S_1 - {}^3D_1$ state (and 2.5 MeV in its negative C-parity partner). The other regulators generate however a much smaller change in the results. If we take the $n = 1$ gaussian regulator as an upper bound of the LO uncertainties, we can appreciate that the EFT error is a bit smaller than the other error sources (in particular, the $1/m_Q$ corrections). In addition, the range in which we expect the binding energy to lie does not change so much. The uncertainties follow the expectations of Eq. (118), that is, the relative error grows with the binding energy. On the other hand, the 40 – 50% relative error in the $0(1^{++})$ state is consistent with the lower estimations of the breakdown scale, indicating that we are well within the range of validity of the EFT with pions.

¹⁵ See for example Ref. [41] for the application of this idea in the context of nucleon-nucleon scattering

¹⁶ It should be noticed that $2n$ must be higher than the order P^ν at which the computations are done to avoid contamination of the auxiliary cut-off scale at orders below that of the EFT calculation. For the LO calculation of the present work, this condition does not translate into a constraint for the value of n . However, if we go to order P^2 , we should employ at least $n \geq 2$.

6. Further Uncertainties in the $BB^*\pi$ Coupling

As previously mentioned in this section, a particular problem we encounter when considering the OPE potential in the bottom sector is the determination of a suitable value of the $BB^*\pi$ coupling. The value we have employed, $g = 0.55 \pm 0.10$, approximately encompasses most results from lattice QCD, which usually range from $g = 0.44$ [58] to $g = 0.63$ [57]¹⁷. However, this value of the $BB^*\pi$ coupling does not reflect all the theoretical uncertainties involved in the determination of this quantity (see, for example, Refs. [59, 74] for a compilation of values). In particular, smaller values of this coupling may be possible, even in the range $g = 0.3 - 0.4$, a case which we will consider here. A small $BB^*\pi$ coupling translates into a weaker OPE potential, as its strength scales as g^2 , and a much weaker chiral TPE ($\propto g^4$), meaning that the higher order corrections owing to pion exchanges will be strongly suppressed.

If we take $g = 0.37_{-0.03}^{+0.04}$ as a reference value, which was obtained in Ref. [59] from the Dyson-Schwinger equations of QCD, we predict the binding energies $B(1^{++}) = 16_{-9}^{+14}$ MeV and $B(1^{+-}) = 9.2_{-6.0}^{+9.5}$ MeV for the positive and negative C-parity ${}^3S_1 - {}^3D_1$ isoscalar states respectively. These new values mostly overlap with the predictions corresponding to $g = 0.55 \pm 0.10$, which is consistent with the observation that contact operators are the dominating mechanism in the formation of S-waves BB^* bound states. On the contrary, the 3P_0 (0^{-+}) resonant state disappears, as it depends crucially on the strength of the OPE potential. For the isovector $Z_b(10610)$ state, the results do not significantly change: the contact term still needs to be of the order of $C_0 \simeq -0.3/f_B^2$ to bind the state.

V. CONCLUSIONS

We have considered the $D\bar{D}^*$ and $B\bar{B}^*$ two meson systems within the framework of a pionless and a pionfull (or chiral) cut-off EFT at LO. In the charmed sector, the existence of the $X(3872)$ state, together with the saturation hypothesis for the low energy constant $C_0^{D\bar{D}^*}$, sets the conditions for the applicability of the EFT formulation employed in this work and results in a natural value of the cut-off Λ_X . The determination of the low energy constant and the value of the cut-off in the $B\bar{B}^*$ case requires invoking HQS to overcome the absence of experimental information for this two body system. In this regard HQS is able to correlate the charm and bottom sectors, and, in addition, provides error estimates for the

¹⁷ It should be noted that in most of lattice QCD calculations the value of the $BB^*\pi$ coupling is computed in the $m_Q \rightarrow \infty$ limit, probably with the exception of Ref. [55], in which $g = 0.58$ is obtained. In general, we expect $g(m_Q = m_b) > g(m_Q \rightarrow \infty)$.

contact term and the cut-off in the $B\bar{B}^*$ system. This in turn allows to assign errors to the resulting binding energies of the possible $B\bar{B}^*$ bound states.

The present framework predicts the existence of three isoscalar $B\bar{B}^*$ states with positive C-parity: two $^3S_1 - ^3D_1$ states with positive and negative C-parity with a binding energy of 20 MeV and 6 MeV respectively, and a 3P_0 (0^{-+}) resonant state which lies almost at the $B\bar{B}^*$ threshold. The different error sources result in relatively large uncertainties in the previous estimations. In addition, we expect moderate corrections from subleading order contributions to the chiral potential between the two heavy mesons. However, the higher order corrections are probably smaller than the current uncertainties stemming from the use of the approximate HQS. Nevertheless, the existence of the bound states is a conclusion that will likely remain unchanged: if the binding energy is lowered, the subleading order corrections are expected to decrease, resulting in a stabilization of the results.

The applicability of the present approach to negative C-parity states is not entirely free of problems, one of them being that there is no direct experimental evidence supporting the short-range dynamics employed in the present work. This feature translates into an additional error source in the isoscalar $C = -1$ $^3S_1 - ^3D_1$ bound state which we are not able to estimate. In addition, the existence of the $Z_b(10610)$ state points out to small but significant deviations from the form of the short range interaction employed in the present work. A second issue with the intrinsic negative C-parity states is that they tend to couple more strongly with the $B^*\bar{B}^*$ system than their positive C-parity counterparts, an effect that is expected to increase the overall attraction of the system and help the formation of bound states, specifically the aforementioned $Z_b(10610)$.

The extension of the present framework to higher orders in the EFT formulation, or to other heavy meson systems, such as $P\bar{P}$ and $P^*\bar{P}^*$, is left for future research.

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Appendix A: Resonant State Equation

In this section we extend the Lippmann-Schwinger equation to the second Riemann sheet for finding resonant and virtual states. For simplicity we will work in the partial wave decomposition corresponding to the central potential case; the extension to coupled channels and

tensor forces is straightforward. The starting point is to use the vertex function, instead of the more usual wave function. The vertex function is related with the residue of the T-matrix at the pole energy, that is

$$\lim_{E \rightarrow E_B} (E - E_B) \langle k | T_l(E) | k' \rangle = \phi_{B,l}(k) \phi_{B,l}(k'), \quad (\text{A1})$$

consequently the relation between the vertex and the wave function is given by

$$\Psi_B(k) = G_0 \phi_B(k). \quad (\text{A2})$$

Inserting this relationship into the bound state equation, Eq. (13), and extending the equation to arbitrary energies, we obtain

$$\phi_{B,l}(k) = -\frac{\mu}{\pi^2} \int \frac{k'^2 dk'}{k'^2 - q^2} \langle k | V_l(q) | k' \rangle \phi_{B,l}(k'), \quad (\text{A3})$$

where $q^2 = -\gamma^2 = 2\mu E$. The advantage of the vertex equation is that it contains the resolvent operator within the integral, which makes it possible to select the Riemann sheet by deforming the integration contour around the $k^2 = q^2$ singularity, see Ref. [75]. The vertex equation only has solutions for the energies at which the T-matrix has a pole.

In principle, to find the position of resonant and virtual states it is enough to analytically extend the solutions of the vertex equation to the second Riemann sheet. However, in numerical calculations the previous is not trivial. The numerical evaluation of the resolvent operator, $G_0(E)$, always chooses the first Riemann sheet, preventing us from finding either resonant or virtual states. The solution is to force the selection of the second Riemann sheet, for example, by changing the integration contour as previously commented. Here we will use instead a more informal derivation for extending Eq. (A3) to the second Riemann sheet. We will assume that the physical scattering region corresponds to $E + i\epsilon$, with E real and positive. If we consider $E - i\epsilon$ instead, the resolvent operator takes the form

$$\frac{1}{E - \frac{k^2}{2\mu}} = \mathcal{P}\left(\frac{1}{E - \frac{k^2}{2\mu}}\right) + i\pi \frac{\mu}{q} \delta(k - q), \quad (\text{A4})$$

where \mathcal{P} denotes that the principal value should be taken and with $E = \frac{q^2}{2\mu}$. However, if we were in the second Riemann sheet, we would need the imaginary piece of the resolvent operator to be negative instead of positive: we are moving from $E + i\epsilon$ to $E - i\epsilon$ in a continuous manner, which means that the imaginary piece should be the same in $E + i\epsilon$ as in $E - i\epsilon$. A practical solution is to add the imaginary piece directly into the resolvent operator, that is, in the second Riemann sheet we substitute the original resolvent operator $G_0^{(I)}$ by a new resolvent operator $G_0^{(II)}$

$$G_0^{(I)}(E) \rightarrow G_0^{(II)} = G_0^{(I)}(E) - i2\pi \frac{\mu}{q} \delta(k - q). \quad (\text{A5})$$

From the point of view of the bound state equation, the previous changes amounts to the substitution

$$\frac{1}{k'^2 - q^2} \rightarrow \frac{1}{k'^2 - q^2} + i\frac{\pi}{q} \delta(k' - q), \quad (\text{A6})$$

within the integral in Eq. (A3). This change leads to the following set of equations

$$\begin{aligned} \phi_{B,l}^{(II)}(k) = & -i\frac{\mu q}{\pi} \langle k|V_l|q \rangle \phi_{B,l}^{(II)}(q) \\ & - \frac{\mu}{\pi^2} \int \frac{k'^2 dk'}{k'^2 - q^2} \langle k|V_l|k' \rangle \phi_{B,l}^{(II)}(k'), \end{aligned} \quad (\text{A7})$$

$$\begin{aligned} \phi_{B,l}^{(II)}(q) = & -i\frac{\mu q}{\pi} \langle q|V_l|q \rangle \phi_{B,l}^{(II)}(q) \\ & - \frac{\mu}{\pi^2} \int \frac{k'^2 dk'}{k'^2 - q^2} \langle q|V_l|k' \rangle \phi_{B,l}^{(II)}(k'), \end{aligned} \quad (\text{A8})$$

which are equivalent to the analogous set of equations obtained in Ref. [75]. Rearranging the different terms in the previous equation to eliminate the $\phi_{B,l}^{(II)}(q)$ term, we arrive at

$$\begin{aligned} \phi_{B,l}^{(II)}(k) = \\ - \frac{\mu}{\pi^2} \int \frac{k'^2 dk'}{k'^2 - q^2} \langle k|W_l(q)|k' \rangle \phi_{B,l}^{(II)}(k'), \end{aligned} \quad (\text{A9})$$

where $W_l(q)$ is defined as

$$\langle k|W_l(q)|k' \rangle = \langle k|V_l|k' \rangle - i\frac{\mu q}{\pi} \frac{\langle k|V_l|q \rangle \langle q|V_l|k' \rangle}{1 + i\frac{\mu q}{\pi} \langle q|V_l|q \rangle}. \quad (\text{A10})$$

Depending on the value of q^2 , that is, of E , the previous equation will look for resonances ($\text{Re}(E) > 0$ and $\text{Im}(E) < 0$) or virtual states ($\text{Re}(E) < 0$ and $\text{Im}(E) = 0$).

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