

# Study of the strong $\Sigma_b \rightarrow \Lambda_b \pi$ and $\Sigma_b^* \rightarrow \Lambda_b \pi$ in a non-relativistic quark model

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We present results for the strong widths corresponding to the  $\Sigma_b \rightarrow \Lambda_b \pi$  and  $\Sigma_b^* \rightarrow \Lambda_b \pi$  decays. We apply our model in Ref. Phys. Rev. D 72, 094022 (2005) where we previously studied the corresponding transitions in the charmed sector. Our non-relativistic constituent quark model uses wave functions that take advantage of the constraints imposed by heavy quark symmetry. Partial conservation of axial current hypothesis allows us to determine the strong vertices from an analysis of the axial current matrix elements.

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In this work we extend our model in Ref. [1] to the bottom sector by evaluating strong one-pion decays of the  $\Sigma_b$  and  $\Sigma_b^*$  baryons into  $\Lambda_b^0 \pi$ . The  $\Lambda_b^0$  was first observed in 1991 by the UA1 Collaboration at the CERN proton-antiproton collider in the  $\Lambda_b^0 \rightarrow J/\Psi \Lambda$  decay channel [2]. A precise determination of its mass was performed by the CDF Collaboration in 2006 analyzing exclusive  $J/\Psi$  decays [3]. In 2007 the CDF Collaboration observed for the first time the  $\Sigma_b^\pm$  and  $\Sigma_b^{*\pm}$  baryons and their masses were also measured [4]. New results on the  $\Sigma_b^{(*)\pm}$  masses by the CDF Collaboration are compiled in their Public Note 10286 (See the CDF Collaboration web page). The  $\Xi_b^-$  baryon was first observed by the D0 Collaboration [5] and its mass was later measured with precision by the CDF Collaboration [6]. The CDF Collaboration has just reported the observation of its isospin partner the  $\Xi_b^0$  [7], but there is no evidence yet for the  $\Xi_b^{*-}$ ,  $\Xi_b^{*0}$  states. New precise results for bottom baryons are expected from the different LHC Collaborations at CERN.

Similarly to the charm quark sector,  $\Sigma_b^{(*)}$  decays are dominated by the strong decay channel  $\Lambda_b^0 \pi$ . These strong decays have been analyzed before in Ref. [8], using an approach that combined chiral dynamics with the MIT bag model, in Ref. [9], where the authors used the Bethe-Salpeter formalism under the covariant instantaneous approximation, in Refs. [10–12] within the non-relativistic  ${}^3P_0$  quark model, and in Refs. [13–15] using light cone QCD sum rules (LCSR). There is also an estimate by the CDF Collaboration [4] obtained using the expected initial and final baryon mass differences and the heavy quark effective theory (HQET) approach of Ref. [16].

In our model we use the heavy quark symmetry (HQS) constrained wave functions that we evaluated in Ref. [17] and the pion emission amplitude is obtained with the use of partial conservation of axial current hypothesis (PCAC). HQS [18–21] tell us that in the infinite heavy quark mass limit the dynamics of the light quark degrees of freedom becomes independent of the heavy quark flavor and spin. This allows to take the light degrees of freedom with well defined quantum numbers which simplifies the solution of the three-body problem for ground state ( $L=0$ ) baryons with a heavy quark. In Ref. [17] we solved the three-body problem using a variational approach and our results for the spectrum agreed with a previous Faddeev calculation [22] using the same interquark potentials.

As in the charm sector the phase space available in these reactions is very limited, and for that reason, we need to use precise mass values. When available, we shall use physical masses taken from Ref. [23]. For the  $\Sigma_b^{*0}$  strong decay we shall give an estimate of the decay width taking, following Ref. [8],  $M_{\Sigma_b^{*0}} = \frac{1}{2}(M_{\Sigma_b^*+} + M_{\Sigma_b^{*-}})$ . For the  $\Sigma_b^0$  case, corrections to the analogous relation, due to the electromagnetic interaction between the two light quarks in the heavy baryon, have been evaluated in Ref. [24] using HQET and in Ref. [25] in chiral perturbation theory to leading one-loop order. Based on the known experimental data they get  $M_{\Sigma_b^0} = 5810.5 \pm 2.2 \text{ MeV}$  [24] and  $M_{\Sigma_b^0} = 5810.3 \pm 1.9 \text{ MeV}$  [25], their central values being 1 MeV lower than the value one would obtain from the less accurate relation  $M_{\Sigma_b^0} = \frac{1}{2}(M_{\Sigma_b^+} + M_{\Sigma_b^-})$ . To give an estimate of the  $\Sigma_b^0$  decay width we shall use the value  $M_{\Sigma_b^0} = 5810.5 \text{ MeV}$  given in Ref. [24]. As shown below the decay width is proportional to the cube of the final baryon momentum and a one MeV increase in the  $\Sigma_b^0$  mass induces an approximate 4% increase in the decay width. This correction can be considered as an estimate of the uncertainties on the widths evaluation due to the uncertainties in the experimental baryon masses.

As mentioned, we determine the pion emission amplitude  $\mathcal{A}_{BB'\pi}^{(s,s')}(P_B, P_{B'})$  through the use of PCAC that allows to relate that amplitude to the matrix element of the divergence of the axial current. For a strong  $B \rightarrow B'\pi^+$  decay we have that <sup>1</sup>

$$\left\langle B', s' \vec{P}' \mid q^\mu J_{A\mu}^{du}(0) \mid B, s \vec{P} \right\rangle_{\text{non-pole}} = i f_\pi \mathcal{A}_{BB'\pi^+}^{(s,s')}(P', P') \quad (1)$$

where  $s, s'$  are the third component of the spin of the  $B, B'$  baryons in their respective center of mass systems,  $P = (E, \vec{P})$ ,  $P' = (E', \vec{P}')$  are their respective four-momenta and  $q = P - P'$ .  $J_{A\mu}^{du}(0)$  is the axial current for the  $u \rightarrow d$  transition, and  $f_\pi = 130.41 \text{ MeV}$  [23] is the pion decay constant. The baryon states are normalized as  $\left\langle B, s' \vec{P}' \mid B, s \vec{P} \right\rangle = \delta_{s,s'} (2\pi)^3 2E \delta(\vec{P} - \vec{P}')$ .

For the particular case of the  $\Sigma_b^{(*)+} \rightarrow \Lambda_b^0$  decays we have that

$$\mathcal{A}_{\Sigma_b^+ \Lambda_b^0 \pi^+}^{(s,s')}(P, P') = \frac{-i}{f_\pi} \left\langle \Lambda_b^0, s' \vec{P}' \mid q^\mu J_{A\mu}^{du}(0) \mid \Sigma_b^+, s \vec{P} \right\rangle_{\text{non-pole}} = -i \frac{g_{\Sigma_b^+ \Lambda_b^0 \pi^+}}{M_{\Sigma_b^+} + M_{\Lambda_b^0}} q_\nu \bar{u}_{\Lambda_b^0 s'}(\vec{P}') \gamma^\nu \gamma_5 u_{\Sigma_b^+ s}(\vec{P}) \quad (2)$$

$$\mathcal{A}_{\Sigma_b^{*+} \Lambda_b^0 \pi^+}^{(s,s')}(P, P') = \frac{-i}{f_\pi} \left\langle \Lambda_b^0, s' \vec{P}' \mid q^\mu J_{A\mu}^{du}(0) \mid \Sigma_b^{*+}, s \vec{P} \right\rangle_{\text{non-pole}} = i \frac{g_{\Sigma_b^{*+} \Lambda_b^0 \pi^+}}{2M_{\Lambda_b^0}} q_\nu \bar{u}_{\Lambda_b^0 s'}(\vec{P}') u_{\Sigma_b^{*+} s}(\vec{P}) \quad (3)$$

<sup>1</sup> Note that we give the expression corresponding to the non-pole part of the matrix element. If the pion pole contribution is included then the relation is given by  $\left\langle B', s' \vec{P}' \mid q^\mu J_{A\mu}^{du}(0) \mid B, s \vec{P} \right\rangle = -i f_\pi \frac{m_\pi^2}{q^2 - m_\pi^2} \mathcal{A}_{BB'\pi^+}^{(s,s')}(P, P')$ .

where the  $g_{\Sigma_b^{(*)} + \Lambda_b^0 \pi^+}$  are dimensionless strong coupling constants.  $u_{\Sigma_b^+ s}(\vec{P})$  and  $u_{\Lambda_b^0 s'}(\vec{P}')$  are Dirac spinors and  $u'_{\Sigma_b^{*+} s}(\vec{P})$  is a Rarita-Schwinger spinor all normalized to twice the energy.

Taking  $\vec{P} = \vec{0}$  and  $\vec{P}' = -|\vec{q}|\vec{k}$  in the  $z$  direction, the width is given in each case by

$$\Gamma(\Sigma_b^+ \rightarrow \Lambda_b^0 \pi^+) = \frac{|\vec{q}| g_{\Sigma_b^+ \Lambda_b^0 \pi^+}^2 (E_{\Lambda_b^0} - M_{\Lambda_b^0})}{4\pi M_{\Sigma_b^+}} = \frac{|\vec{q}| g_{\Sigma_b^+ \Lambda_b^0 \pi^+}^2}{8\pi M_{\Sigma_b^+}^2} \left[ (M_{\Sigma_b^+} - M_{\Lambda_b^0})^2 - m_{\pi^+}^2 \right] \approx \frac{|\vec{q}|^3 g_{\Sigma_b^+ \Lambda_b^0 \pi^+}^2}{8\pi M_{\Lambda_b^0} M_{\Sigma_b^+}} \quad (4)$$

$$\Gamma(\Sigma_b^{*+} \rightarrow \Lambda_b^0 \pi^+) = \frac{|\vec{q}|^3 g_{\Sigma_b^{*+} \Lambda_b^0 \pi^+}^2 E_{\Lambda_b^0} + M_{\Lambda_b^0}}{12\pi M_{\Sigma_b^{*+}} 4M_{\Lambda_b^0}^2} = \frac{|\vec{q}|^3 g_{\Sigma_b^{*+} \Lambda_b^0 \pi^+}^2 (M_{\Sigma_b^{*+}} + M_{\Lambda_b^0})^2 - m_{\pi^+}^2}{24\pi M_{\Sigma_b^{*+}}^2 4M_{\Lambda_b^0}^2} \quad (5)$$

The final momentum is given by  $|\vec{q}| = \frac{1}{2M_{\Sigma_b^+}} \lambda^{1/2}(M_{\Sigma_b^+}^2, M_{\Lambda_b^0}^2, m_{\pi^+}^2)$  with  $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$  the Källén function. The  $|\vec{q}|^3$  behavior makes the widths very sensitive to the actual baryon masses used. That is the reason to use experimental masses. The test of the model comes through the evaluation of the corresponding coupling constants.

From the PCAC relation in Eqs. (2-3), taking  $\vec{P} = \vec{0}$  and  $\vec{P}' = -|\vec{q}|\vec{k}$  in the  $z$  direction, and  $s = s' = 1/2$ , we have

$$g_{\Sigma_b^+ \Lambda_b^0 \pi^+} = \frac{-1}{f_\pi} \frac{\sqrt{E_{\Lambda_b^0} + M_{\Lambda_b^0}}}{|\vec{q}| \sqrt{2M_{\Sigma_b^+}}} \left[ (M_{\Sigma_b^+} - E_{\Lambda_b^0}) A_{\Sigma_b^+ \Lambda_b^0, 0}^{1/2, 1/2} + |\vec{q}| A_{\Sigma_b^+ \Lambda_b^0, 3}^{1/2, 1/2} \right]$$

$$g_{\Sigma_b^{*+} \Lambda_b^0 \pi^+} = \frac{\sqrt{3}}{f_\pi \sqrt{2}} \frac{2M_{\Lambda_b^0}}{|\vec{q}| \sqrt{2M_{\Sigma_b^{*+}} (E_{\Lambda_b^0} + M_{\Lambda_b^0})}} \left[ (M_{\Sigma_b^{*+}} - E_{\Lambda_b^0}) A_{\Sigma_b^{*+} \Lambda_b^0, 0}^{1/2, 1/2} + |\vec{q}| A_{\Sigma_b^{*+} \Lambda_b^0, 3}^{1/2, 1/2} \right] \quad (6)$$

with

$$A_{\Sigma_b^{(*)+} \Lambda_b^0, \mu}^{1/2, 1/2} = \left\langle \Lambda_b^0, 1/2 - |\vec{q}|\vec{k} \mid J_{A\mu}^{du}(0) \mid \Sigma_b^{(*)+}, 1/2 \vec{0} \right\rangle_{non-pole} \quad (7)$$

The  $A_{\Sigma_b^{(*)+} \Lambda_b^0, \mu}^{1/2, 1/2}$  weak matrix elements are easily evaluated in the model using one-body current operators and their expressions can be found in the appendix.

In Table I we present the results for  $g_{\Sigma_b^{(*)+} \Lambda_b^0 \pi^+}$  and the widths  $\Gamma(\Sigma_b^{(*)+} \rightarrow \Lambda_b^0 \pi^+)$ ,  $\Gamma(\Sigma_b^{(*)0} \rightarrow \Lambda_b^0 \pi^0)$  and  $\Gamma(\Sigma_b^{(*)-} \rightarrow \Lambda_b^0 \pi^-)$ . To get the values for  $\Gamma(\Sigma_b^{(*)0} \rightarrow \Lambda_b^0 \pi^0)$  and  $\Gamma(\Sigma_b^{(*)-} \rightarrow \Lambda_b^0 \pi^-)$ , we use  $g_{\Sigma_b^{(*)+} \Lambda_b^0 \pi^+}$  and make the appropriate mass changes in the rest of the factors in Eqs. (4-5). Our results for the decay widths are in good agreement with the estimation by the CDF Collaboration. We systematically get larger results than in Ref. [8] by some 17 – 38% depending on the transition. In Ref. [9] no attempt is made to get the widths for individual isospin states. Their theoretical uncertainties result from the unknown parameters in the model, the scalar and vector diquark masses and the parameter that describes the confinement interaction between the heavy quark and the light diquark. Our results lie in the lower part of the interval determined in Ref. [9]. Our results are also some 10 – 20% larger than the ones obtained in Refs. [11, 12] using the  $^3P_0$  production model. In this latter case the results depend on the parameters that control the strength of the  $^3P_0$  creation vertex and the size of the baryons, which in Refs. [11, 12] are fitted to reproduce the strange sector  $\Sigma(1385) \rightarrow \Lambda\pi$  decay. Two different fits are quoted in Ref. [12]. Much smaller results are obtained in the  $^3P_0$  calculation of Ref. [10] where a different size parameter and a different strength are used. In all three  $^3P_0$  calculations the same size parameter is used to describe the relative motion of the two light quarks and the relative motion of the heavy quark with respect to the center of mass of the two light quark system. As shown in Figure 4 of Ref. [26], those sizes can be significantly different for  $b$ -baryons. A mild dependence on the size parameters is claimed in Ref. [10] though. When compared to the LCSR calculations in Refs. [13] and [14] we find our  $g_{\Sigma_b \Lambda_b \pi}$  decay constant is a factor 2.2 and 3.4 larger respectively. The decay constants in Refs. [13, 14] would give rise to rather small decay widths when compared to other determinations. The value of  $\Gamma(\Sigma_b \rightarrow \Lambda_b \pi) = 3.93 \pm 1.5 \text{ MeV}$  quoted in Ref. [13] is a factor 3.3 larger than one would expect from their value for the decay constant due to the approximation  $(M_{\Sigma_b} - M_{\Lambda_b})^2 - m_\pi^2 \rightarrow (M_{\Sigma_b} - M_{\Lambda_b})^2$ , used in their Eq. (24), and the approximation  $|\vec{q}| = \frac{1}{2M_{\Sigma_b}} \lambda^{1/2}(M_{\Sigma_b}^2, M_{\Lambda_b}^2, m_\pi^2) \rightarrow \frac{M_{\Sigma_b}^2 - M_{\Lambda_b}^2}{2M_{\Sigma_b}}$  used in their evaluation of the final momentum. Those approximations are not good because of the small mass difference between the initial and final baryons. The agreement with the LCSR calculation is better for the  $g_{\Sigma_b^* \Lambda_b \pi}$  coupling constant, being our value larger than the central value obtained in Ref. [15] by a factor 1.3. This implies however that our predictions for the decay widths are some 70% larger.

	$g_{\Sigma_b^+ \Lambda_b^0 \pi^+}$	$\Gamma(\Sigma_b^+ \rightarrow \Lambda_b^0 \pi^+)$ [MeV]	$\Gamma(\Sigma_b^0 \rightarrow \Lambda_b^0 \pi^0)$ [MeV]	$\Gamma(\Sigma_b^- \rightarrow \Lambda_b^0 \pi^-)$ [MeV]
This work	51.4	6.0	6.6 <sup>¶</sup>	7.7
[4, 16]		$\approx 7^\dagger$		$\approx 7^\dagger$
[8]		4.35	5.65 <sup>‡</sup>	5.77
[9]		6.73 – 13.45	6.73 – 13.45	6.73 – 13.45
[10]		3.5		4.7
[11]				6.7
[12]		4.82, 4.94		6.31, 6.49
[13]	$23.5 \pm 4.9$			
[14]	$15.0 \pm 4.9$			

  

	$g_{\Sigma_b^{*+} \Lambda_b^0 \pi^+}$	$\Gamma(\Sigma_b^{*+} \rightarrow \Lambda_b^0 \pi^+)$ [MeV]	$\Gamma(\Sigma_b^{*0} \rightarrow \Lambda_b^0 \pi^0)$ [MeV]	$\Gamma(\Sigma_b^{*-} \rightarrow \Lambda_b^0 \pi^-)$ [MeV]
This work	87.6	11.0	12.1 <sup>‡</sup>	13.2
[4, 16]		$\approx 13^\dagger$		$\approx 13^\dagger$
[8]		8.50	10.20 <sup>‡</sup>	10.44
[9]		10.00 – 17.74	10.00 – 17.74	10.00 – 17.74
[10]		7.5		9.2
[11]				12.3
[12]		9.68, 10.06		11.81, 12.34
[15]	$67 \pm 12$ <sup>§</sup>			

TABLE I. Coupling constants  $g_{\Sigma_b^{(*)+} \Lambda_b^0 \pi^+}$  and total widths  $\Gamma(\Sigma_b^{(*)+} \rightarrow \Lambda_b^0 \pi^+)$ ,  $\Gamma(\Sigma_b^{(*)0} \rightarrow \Lambda_b^0 \pi^0)$  and  $\Gamma(\Sigma_b^{(*)-} \rightarrow \Lambda_b^0 \pi^-)$  (See text for details). The result with a <sup>¶</sup> is an estimation using the  $\Sigma_b^0$  mass prediction obtained in the HQET evaluation of Ref [24]. Results with a <sup>‡</sup> are estimations obtained assuming a mass given by  $M_{\Sigma_b^{(*)0}} = (M_{\Sigma_b^{(*)+}} + M_{\Sigma_b^{(*)-}})/2$ . Results with a <sup>†</sup> are estimations by the CDF Collaboration [4] based on the expected masses for the  $\Sigma_b^{(*)}$  baryons and the HQET predictions in Ref. [16]. Result with a <sup>§</sup> our estimation from the value given in Ref. [15] where the coupling constant is defined differently.

To end this discussion we compare the present results for the dimensionless coupling constants  $\frac{f_\pi g_{\Sigma_b^+ \Lambda_b^0 \pi^+}}{M_{\Sigma_b^+} + M_{\Lambda_b^0}}$  and  $\frac{f_\pi g_{\Sigma_b^{*+} \Lambda_b^0 \pi^+}}{2M_{\Lambda_b^0}}$  with the corresponding ones in the charm sector that we obtained in Ref. [1]. According to HQS one would expect them to be equal to the extent that one can consider the  $b$  and  $c$  quark masses heavy enough. What we get is

$$\begin{aligned}
\frac{f_\pi g_{\Sigma_b^+ \Lambda_b^0 \pi^+}}{M_{\Sigma_b^+} + M_{\Lambda_b^0}} &= 0.586 \approx 0.598 = \frac{f_\pi g_{\Sigma_c^+ \Lambda_c^+ \pi^+}}{M_{\Sigma_c^+} + M_{\Lambda_c^+}} \\
\frac{f_\pi g_{\Sigma_b^{*+} \Lambda_b^0 \pi^+}}{2M_{\Lambda_b^0}} &= 1.02 \approx 1.03 = \frac{f_\pi g_{\Sigma_c^{*+} \Lambda_c^+ \pi^+}}{2M_{\Lambda_c^+}}
\end{aligned} \tag{8}$$

that coincide at the level of 1-2%.

We have obtained accurate predictions of the widths for the bottom-baryon decays  $\Sigma_b \rightarrow \Lambda_b \pi$  and  $\Sigma_b^* \rightarrow \Lambda_b \pi$  within a constituent quark model framework. We have used wave functions constrained by HQS and that were obtained after solving the non-relativistic three-body problem with the help of a simple variational ansatz. The quality of our wave functions has been tested in the study of the  $\Lambda_b$  and  $\Xi_b$  semileptonic decays performed in Ref. [27], with our results for partially integrated decay widths being in agreement with lattice QCD data by the UKQCD Collaboration [28]. To evaluate the pion emission amplitude, we have made use of PCAC from the analysis of weak current matrix elements. A similar procedure was carried out in [29] to evaluate the strong coupling constants  $g_{B^* B \pi}$  and  $g_{D^* D \pi}$  that turned out to be in agreement with the experimental determination<sup>2</sup> of the latter constant and with lattice results [30] for the former one. Finally, we have explicitly shown that the corresponding strong couplings  $g_{\Sigma_Q \Lambda_Q \pi}$  and  $g_{\Sigma_Q^* \Lambda_Q \pi}$  scale

<sup>2</sup> From the  $D^* \rightarrow D \pi$  width.

like  $m_Q$ , with  $Q = b$  or  $c$ , being the corrections to the infinite mass limit predictions unexpectedly small, and certainly much smaller than those found in the meson sector [30].

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### Appendix A: Description of baryon states and expressions for the $A_{BB',\mu}^{1/2,1/2}$ weak matrix elements

The state of a heavy baryon  $B$  with three-momentum  $\vec{P}$  and spin projection  $s$  in its center of mass is given as

$$\begin{aligned} |B, s \vec{P}\rangle_{NR} = \sqrt{2E} \int d^3Q_1 \int d^3Q_2 \frac{1}{\sqrt{2}} \sum_{\alpha_1, \alpha_2, \alpha_3} \hat{\psi}_{\alpha_1, \alpha_2, \alpha_3}^{(B, s)}(\vec{Q}_1, \vec{Q}_2) \frac{1}{(2\pi)^3 \sqrt{2E_{f_1} 2E_{f_2} 2E_{f_3}}} \\ \times \left| \alpha_1 \vec{p}_1 = \frac{m_{f_1}}{M} \vec{P} + \vec{Q}_1 \right\rangle \left| \alpha_2 \vec{p}_2 = \frac{m_{f_2}}{M} \vec{P} + \vec{Q}_2 \right\rangle \left| \alpha_3 \vec{p}_3 = \frac{m_{f_3}}{M} \vec{P} - \vec{Q}_1 - \vec{Q}_2 \right\rangle \end{aligned} \quad (A1)$$

where the normalization factor  $\sqrt{2E}$  has been introduced for later convenience.  $\alpha_j$  represents the quantum numbers of spin  $s$ , flavor  $f$  and color  $c$  ( $\alpha_j \equiv (s_j, f_j, c_j)$ ) of the  $j$ -th quark, while  $(E_{f_j}, \vec{p}_j)$  and  $m_{f_j}$  represent its four-momentum and mass.  $M$  stands for  $M = m_{f_1} + m_{f_2} + m_{f_3}$ . In our case we choose the third quark to be the  $b$  quark while the first two will be the light ones. The normalization of the quark states is  $\langle \alpha' \vec{p}' | \alpha \vec{p} \rangle = \delta_{\alpha', \alpha} (2\pi)^3 2E_f \delta(\vec{p}' - \vec{p})$ . Besides,  $\hat{\psi}_{\alpha_1, \alpha_2, \alpha_3}^{(B, s)}(\vec{Q}_1, \vec{Q}_2)$  is the non-relativistic momentum space wave function for the internal motion, being  $\vec{Q}_1$  and  $\vec{Q}_2$  the momenta conjugate to the relative positions  $\vec{r}_1$  and  $\vec{r}_2$  of the two light quarks with respect to the heavy one. This wave function is antisymmetric under the simultaneous exchange  $\alpha_1 \longleftrightarrow \alpha_2, \vec{Q}_1 \longleftrightarrow \vec{Q}_2$ , being also antisymmetric under an overall exchange of the color degrees of freedom. It is normalized such that

$$\int d^3Q_1 \int d^3Q_2 \sum_{\alpha_1, \alpha_2, \alpha_3} \left( \hat{\psi}_{\alpha_1, \alpha_2, \alpha_3}^{(B, s')}(\vec{Q}_1, \vec{Q}_2) \right)^* \hat{\psi}_{\alpha_1, \alpha_2, \alpha_3}^{(B, s)}(\vec{Q}_1, \vec{Q}_2) = \delta_{s', s} \quad (A2)$$

and, thus, the normalization of our non-relativistic baryon states is

$${}_{NR} \langle B, s' \vec{P}' | B, s \vec{P} \rangle_{NR} = \delta_{s', s} (2\pi)^3 2E \delta(\vec{P}' - \vec{P}) \quad (A3)$$

For the particular case of ground state  $\Lambda_b, \Sigma_b$ , and  $\Sigma_b^*$ , we assume the orbital angular momentum to be zero. We will also take advantage of HQS and assume the light-degrees of freedom quantum numbers are well defined (For quantum numbers see, for instance, Table 1 in Ref [17]). In that case we have<sup>3</sup>

$$\begin{aligned} \hat{\psi}_{\alpha_1, \alpha_2, \alpha_3}^{(\Lambda_b^0, s)}(\vec{Q}_1, \vec{Q}_2) &= \frac{\varepsilon_{c_1 c_2 c_3}}{\sqrt{3!}} (1/2, 1/2, 0; s_1, s_2, 0) \\ &\quad \times \frac{\delta_{f_3, b} \delta_{s_3, s}}{\sqrt{2}} \left( \delta_{f_1, u} \delta_{f_2, d} \tilde{\phi}_{u, d, b}^{\Lambda_b^0}(\vec{Q}_1, \vec{Q}_2) - \delta_{f_1, d} \delta_{f_2, u} \tilde{\phi}_{d, u, b}^{\Lambda_b^0}(\vec{Q}_1, \vec{Q}_2) \right) \\ \hat{\psi}_{\alpha_1, \alpha_2, \alpha_3}^{(\Sigma_b^+, s)}(\vec{Q}_1, \vec{Q}_2) &= \frac{\varepsilon_{c_1 c_2 c_3}}{\sqrt{3!}} \tilde{\phi}_{u, u, b}^{\Sigma_b^+}(\vec{Q}_1, \vec{Q}_2) \delta_{f_1, u} \delta_{f_2, u} \delta_{f_3, b} \sum_m (1/2, 1/2, 1; s_1, s_2, m) (1, 1/2, 1/2; m, s_3, s) \\ \hat{\psi}_{\alpha_1, \alpha_2, \alpha_3}^{(\Sigma_b^{*+}, s)}(\vec{Q}_1, \vec{Q}_2) &= \frac{\varepsilon_{c_1 c_2 c_3}}{\sqrt{3!}} \tilde{\phi}_{u, u, b}^{\Sigma_b^{*+}}(\vec{Q}_1, \vec{Q}_2) \delta_{f_1, u} \delta_{f_2, u} \delta_{f_3, b} \sum_m (1/2, 1/2, 1; s_1, s_2, m) (1, 1/2, 3/2; m, s_3, s) \end{aligned} \quad (A4)$$

Here  $\varepsilon_{c_1 c_2 c_3}$  is the fully antisymmetric tensor on color indices being  $\frac{\varepsilon_{c_1 c_2 c_3}}{\sqrt{3!}}$  the antisymmetric color wave function, the  $(j_1, j_2, j; m_1, m_2, m_3)$  are Clebsch-Gordan coefficients and the  $\tilde{\phi}_{f_1, f_2, f_3}^B(\vec{Q}_1, \vec{Q}_2)$  are the Fourier transform of the

<sup>3</sup> We only give the wave function for the baryons involved in  $\pi^+$  emission. Wave functions for other isospin states of the same baryons are easily constructed.

corresponding normalized coordinate space wave functions obtained in Ref. [17] using the AL1 potential or Ref. [22, 31]. Their dependence on momenta is through  $|\vec{Q}_1|$ ,  $|\vec{Q}_2|$  and  $\vec{Q}_1 \cdot \vec{Q}_2$  alone, and they are symmetric under the simultaneous exchange  $f_1 \longleftrightarrow f_2$ ,  $\vec{Q}_1 \longleftrightarrow \vec{Q}_2$ .

Within the model we evaluate the  $A_{BB',\mu}^{1/2,1/2}$  as

$$A_{\Sigma_b^{(*)+}\Lambda_b^0,\mu}^{1/2,1/2} =_{NR} \left\langle \Lambda_b^0, 1/2 - |\vec{q}| \vec{k} \mid J_{A\mu}^{du}(0) \mid \Sigma_b^{(*)+}, 1/2 \vec{0} \right\rangle_{NR \text{ non-pole}} \quad (\text{A5})$$

Using one-body current operators their expressions are given by

$$A_{\Sigma_b^+\Lambda_b^0,\mu}^{1/2,1/2} = \frac{\sqrt{2}}{\sqrt{3}} \sqrt{2M_{\Sigma_b^+} 2E_{\Lambda_b^0}} \int d^3Q_1 d^3Q_2 \phi_{u,u,b}^{\Sigma_b^+}(\vec{Q}_1, \vec{Q}_2) \left( \phi_{d,u,b}^{\Lambda_b^0}(\vec{Q}_1 - \frac{m_u+m_b}{M_{\Lambda_b^0}} |\vec{q}| \vec{k}, \vec{Q}_2 + \frac{m_u}{M_{\Lambda_b^0}} |\vec{q}| \vec{k}) \right)^* \\ \times \sum_{s_1} (1/2, 1/2, 1; s_1, -s_1, 0) (1/2, 1/2, 0; s_1, -s_1, 0) \frac{\bar{u}_{d s_1}(\vec{Q}_1 - |\vec{q}| \vec{k}) \gamma_\mu \gamma_5 u_{u s_1}(\vec{Q}_1)}{\sqrt{2E_d(|\vec{Q}_1 - |\vec{q}| \vec{k}|) 2E_u(|\vec{Q}_1|)}} \quad (\text{A6})$$

where the quark Dirac spinors are normalized to twice the energy. For  $\mu = 0, 3$  we get the final expressions

$$A_{\Sigma_b^+\Lambda_b^0,0}^{1/2,1/2} = \frac{\sqrt{2}}{\sqrt{3}} \sqrt{2M_{\Sigma_b^+} 2E_{\Lambda_b^0}} \int d^3Q_1 d^3Q_2 \phi_{u,u,b}^{\Sigma_b^+}(\vec{Q}_1, \vec{Q}_2) \left( \phi_{d,u,b}^{\Lambda_b^0}(\vec{Q}_1 - \frac{m_u+m_b}{M_{\Lambda_b^0}} |\vec{q}| \vec{k}, \vec{Q}_2 + \frac{m_u}{M_{\Lambda_b^0}} |\vec{q}| \vec{k}) \right)^* \\ \times \sqrt{\frac{(E_d(|\vec{Q}_1 - |\vec{q}| \vec{k}|) + m_d) (E_u(|\vec{Q}_1|) + m_u)}{2E_d(|\vec{Q}_1 - |\vec{q}| \vec{k}|) 2E_u(|\vec{Q}_1|)}} \left( \frac{Q_1^z}{E_u(|\vec{Q}_1|) + m_u} + \frac{Q_1^z - |\vec{q}|}{E_d(|\vec{Q}_1 - |\vec{q}| \vec{k}|) + m_d} \right) \quad (\text{A7})$$

$$A_{\Sigma_b^+\Lambda_b^0,3}^{1/2,1/2} = -\frac{\sqrt{2}}{\sqrt{3}} \sqrt{2M_{\Sigma_b^+} 2E_{\Lambda_b^0}} \int d^3Q_1 d^3Q_2 \phi_{u,u,b}^{\Sigma_b^+}(\vec{Q}_1, \vec{Q}_2) \left( \phi_{d,u,b}^{\Lambda_b^0}(\vec{Q}_1 - \frac{m_u+m_b}{M_{\Lambda_c^+}} |\vec{q}| \vec{k}, \vec{Q}_2 + \frac{m_u}{M_{\Lambda_c^+}} |\vec{q}| \vec{k}) \right)^* \\ \times \sqrt{\frac{(E_d(|\vec{Q}_1 - |\vec{q}| \vec{k}|) + m_d) (E_u(|\vec{Q}_1|) + m_u)}{2E_d(|\vec{Q}_1 - |\vec{q}| \vec{k}|) 2E_u(|\vec{Q}_1|)}} \left( 1 + \frac{2(Q_1^z)^2 - |\vec{Q}_1|^2 - Q_1^z |\vec{q}|}{(E_d(|\vec{Q}_1 - |\vec{q}| \vec{k}|) + m_d) (E_u(|\vec{Q}_1|) + m_u)} \right) \quad (\text{A8})$$

For  $A_{\Sigma_b^{*+}\Lambda_b^0,\mu}^{1/2,1/2}$  we get similar relations with an extra  $-\sqrt{2}$  factor.

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