

Neutrino Energy Reconstruction and the Shape of the CCQE-like Total Cross Section

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We show that because of the multinucleon mechanism effects, the algorithm used to reconstruct the neutrino energy is not adequate when dealing with quasielastic-like events, and a distortion of the total flux unfolded cross section shape is produced. This amounts to a redistribution of strength from high to low energies, which gives rise to a sizable excess (deficit) of low (high) energy neutrinos. This distortion of the shape leads to a good description of the MiniBooNE unfolded CCQE-like cross sections published in Ref. [1]. However, these changes in the shape are artifacts of the unfolding process that ignores multinucleon mechanisms.

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I. INTRODUCTION

In most theoretical works the name Quasielastic (QE) scattering is used for processes where the gauge boson W is absorbed by just one nucleon, which together with a lepton is emitted (see Fig. 1(a)). However, in the MiniBooNE measurement of Ref. [1], QE is related to processes in which only a muon is detected in the final state. Though this definition could make sense because ejected nucleons are not detected in that experiment, it includes multinucleon processes (see Fig. 1(b))¹ and others like pion production followed by absorption. However, it discards pions coming off the nucleus, since they will give rise to additional leptons after their decay (see Fig. 1(c)). The MiniBooNE analysis of the data corrects (through a Monte Carlo estimate) for some of these events, where in the neutrino interaction a real pion is produced, but it escapes detection because it is reabsorbed in the nucleus, leading to multinucleon emission.

As firstly pointed out by M. Martini et al. [3, 4], and corroborated by our group [2, 5], the data of Ref. [1] correspond to the sum of the QE (absorption by just one nucleon), and the multinucleon contributions. For this reason, we will use the name QE-like to quote the MiniBooNE data of Ref. [1]. Also for simplicity, we will often refer to the multinucleon mechanism contributions, though they include effects beyond gauge boson absorption by a nucleon pair, as 2p2h (two particle-hole) effects. The 2p2h contributions allows to describe [5, 6] the CCQE-like flux averaged double differential cross section $d\sigma/dE_\mu d\cos\theta_\mu$ measured by MiniBooNE with values of M_A (nucleon axial mass) around 1.03 ± 0.02 GeV that is usually quoted as the world average [9, 10]. This is re-assuring from the theoretical point of view and more satisfactory than the situation envisaged by some other works that described these CCQE-like data in terms of a larger value of M_A of around 1.3–1.4 GeV [1, 11–13].

For the QE cross-sections, the predictions of the model that we employed in [5] agree quite well with those obtained/used in Refs. [3, 4, 6], and both groups also agree on the relevant role played by the 2p2h mechanisms to describe the MiniBooNE data. We, however, differ considerably in the size (about a factor of two) of the multinucleon effects [7]. Thus, although Martini et al., predictions look consistent with MiniBooNE data, however our predictions, when the 2p2h contribution is included, would favor a global normalization scale of about 0.9 (see [5]). This would be consistent with the MiniBooNE estimate of a total normalization error of 10.7%. In view of the disagreement, we should emphasized here that our evaluation in [2, 5], of these pionless multinucleon emission contributions to the cross section is fully microscopical and it contains terms, which were either not considered or only approximately taken into account in [3, 4, 6]. Indeed, the results of these latter works rely on some computation of the 2p2h mechanisms for the (e, e') inclusive reaction ([8]), whose results are used for neutrino induced processes. Thus, it is clear that these latter calculations do not contain any information on axial or axial-vector contributions.

¹ Note that the intermediate pion in this figure is virtual and it is part of the $\Delta N \rightarrow NN$ interaction inside of the nucleus. Indeed, one should consider a full interaction model for the in medium baryon–baryon interaction. Thus, for instance, the model of Ref. [2] contains, besides pion exchange, ρ –exchange, and short and long range (RPA) correlations.

We would also like to point out that the simple phenomenological approach adopted in [14] to account for the 2p2h effects also reinforces the picture that emerges from the works of Refs. [5, 6]. Yet, a partial microscopical calculation of the 2p2h contributions to the CCQE cross section has been also presented in Refs. [15] and [16], for neutrino and antineutrino induced reactions, respectively. In these works, the contribution of the vector meson exchange currents in the 2p2h sector is added to the QE neutrino or antineutrino cross section predictions deduced from a phenomenological model (SuSA) [17] based on the super-scaling behavior of electron scattering data. In [18], and for the neutrino case, the SuSA+2p2h results were also compared with those obtained from a relativistic mean field approach. Although, all these schemes do not account for the axial part of the 2p2h effects yet, the preliminary results also corroborate that 2p2h meson exchange currents play an important role in both CCQE neutrino and antineutrino scattering, and that they may help to resolve the controversy on the nucleon axial mass raised by the recent MiniBooNE data. This is not surprising, since these two-body currents, that arise from microscopic relativistic modeling performed for inclusive electron scattering reactions, are known to result in a significant increase in the vector-vector transverse response function in QE electron scattering data [19–21].

The study and comparison in detail of the different models used to describe 2p2h effects, though of great interest, is left for future research. We aim here at determining the possible influence of the 2p2h excitations on the process needed to extract neutrino energy unfolded cross sections from the measured flux-average data.

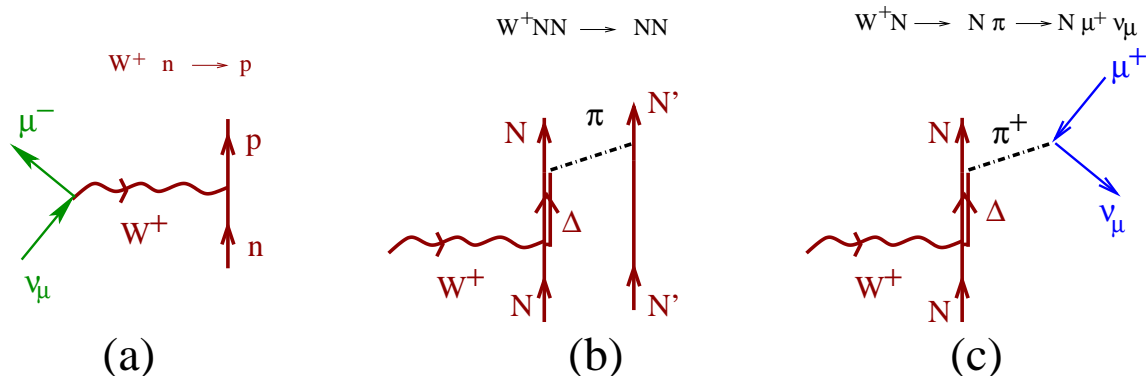


FIG. 1: Mechanisms for W absorption inside of a nucleus.

Indeed, there still exists another feature of neutrino physics which deserves attention and that has motivated this work. Neutrino beams are not monochromatic. For QE-like events, only the charged lepton is observed and the only measurable quantities are then its direction² (scattering angle θ_μ with respect to the neutrino beam direction) and its energy E_μ . The energy of the neutrino that has originated the event is unknown. Then, it is common to define a reconstructed neutrino energy E_{rec} as,

$$E_{\text{rec}} = \frac{ME_\mu - m_\mu^2/2}{M - E_\mu + |\vec{p}_\mu| \cos \theta_\mu} \quad (1)$$

which will correspond to the energy of a neutrino that emits a muon, of energy E_μ and three-momentum \vec{p}_μ , and a gauge boson W that is being absorbed by a nucleon of mass M at rest. Namely, the usual reconstruction procedure assumes that we are dealing with a genuine quasielastic event on a nucleon at rest, ie. E_{rec} is determined by the QE-peak condition $q^0 = -q^2/2M$, where q^μ is the W four momentum. Note that each event contributing to the flux averaged double differential cross section $d\sigma/dE_\mu d\cos\theta_\mu$ defines unambiguously a value of E_{rec} . The actual (“true”) energy, E , of the neutrino that has produced the event will not be exactly E_{rec} . Actually, for each E_{rec} , there exists a distribution of true neutrino energies that could give rise to events whose muon kinematics would lead to the given value of E_{rec} . Several effects can influence this distribution. Firstly, the Fermi motion which broadens the QE peak and the Pauli blocking which cuts the low momentum response. These effects are well known³, usually are under control and lead to very minor changes in the process of expressing observables as a function of the true neutrino energy. This is because genuine QE events produce true energy distributions quite narrow and strongly

² From now on, we will always identify the charged lepton with a muon.

³ It is also common to consider some corrections in the definition of E_{rec} in Eq. (1) to account for the binding energy of the target nucleon in the nucleus, but these corrections turn out to be irrelevant for our discussion.

peaked around the expected E_{rec} values [22]. However multinucleon mechanisms, relevant for QE-like processes, can indeed distort the expected (QE-based) (E_{rec}, E) distributions, since they produce distributions quite flat and that do not peak around E_{rec} [22]. The effects of the inclusion of multinucleon processes on the energy reconstruction have been investigated in Ref. [22], within their 2p2h model and also estimated in Ref. [39], using some simplified model for the multinucleon mechanisms.

We will show in this work that 2p2h effects sizably distort the shape of the total CCQE-like flux unfolded cross section, as a function of the neutrino energy. Indeed, we will see that these multinucleon mechanisms produce a redistribution of strength from high energy to low energies, which gives rise to a sizable enhancement of the number of events attributed to low energy neutrinos leading to a good description of the unfolded cross section given in [1]. However, we will show that these changes in the shape are artifacts of the unfolding process that ignores multinucleon mechanisms.

II. EXCESS OF LOW ENERGY NEUTRINOS IN THE MINIBOONE CCQE-LIKE FLUX UNFOLDED CROSS SECTION DATA

The QE+multinucleon mechanism model of Ref. [5], with $M_A = 1.049$ GeV, provides an excellent description of the MiniBooNE neutrino flux folded CCQE-like $d\sigma/dT_\mu d\cos\theta_\mu$ differential cross sections given in [1], even when no parameters have been fitted to data, beyond a global scale, $\lambda \sim 0.9$. This scale λ is consistent with the global normalization uncertainty of around 10% acknowledged in [1].

The QE contribution used in [5] was derived in Ref. [23], and it incorporates several nuclear effects. The main one is the medium polarization (RPA), including Δ -hole degrees of freedom and explicit π and ρ meson exchanges in the vector-isovector channel of the effective nucleon-nucleon interaction. The model for multinucleon mechanisms has been fully discussed in Ref. [2] and it is based on a model for neutrino pion production derived in Refs. [24, 25]. The whole model constitutes a natural extension of previous studies of photon, electron, and pion interactions with nuclei [26–29].

The prediction of the model, used in Ref. [5], for the total flux unfolded neutrino CCQE-like cross section is depicted in Fig. 2. Several remarks are in order here:

- The 2p2h contributions clearly improve the description of the data in Fig. 2, which are totally missed by the QE prediction. Though the model provides a reasonable description, we observe a sizable excess of low energy neutrinos in the data, that is not even covered by the theoretical error band.
- As discussed above, the flux folded double differential cross-section data is well described in Ref. [5], except for the global scale, $\lambda \sim 0.9$, which needs to be introduced there. It is to say, predicted cross sections in Ref. [5] are globally around 10% smaller than the measured ones. This disagreement could be due to a theoretical underestimation of the absolute number of neutrinos in the MiniBooNE flux. Thus, we should expect our predictions for the total unfolded cross section to under-estimate the data points by about 10%, as well. However, we do not see this in Fig. 2. There is a problem in the neutrino-energy shape, as pointed out above, which would not be improved by increasing the size our predictions by a global factor.

In any case, given that the actually measured quantity it is the double differential cross-section and that observable is well described by our model, any difference on the unfolded cross section must come from the unfolding procedure.

- Finally, we should mention that the QE theoretical results for the cross section shown here (model from [5]) slightly differ from those in Fig. 18 (left) of our previous work of Ref. [2]. The main difference is the inclusion of relativistic corrections. We discuss this point in some detail in Appendix A, since we do not want to deviate here the attention from the main point of this work: because of the 2p2h effects, the algorithm used to reconstruct the neutrino energy is not adequate when dealing with QE-like events, and that it produces a distortion of the total CCQE-like flux unfolded cross section shape.

Nevertheless, we should mention here that in Ref. [2] and to account for Final State Interactions (FSI), we used the non-relativistic QE model of Ref. [23], while in Fig. 2, the results depicted are those obtained from the relativistic model of [23] for the QE process, without the inclusion of FSI effects⁴. This improvement in the model to account for the relativistic effects is the reason for the differences mentioned above. As a final remark,

⁴ In Ref. [2], FSI effects are being treated within the non-relativistic scheme derived in Ref. [30]. A non-relativistic treatment is unsuitable for the large momenta transferred that are reached in the MiniBooNE neutrino flux folded $d\sigma/dT_\mu d\cos\theta_\mu$ differential cross sections, but

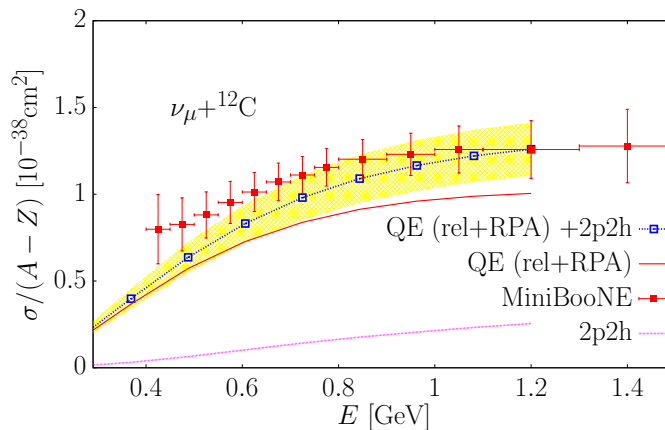


FIG. 2: Flux-unfolded MiniBooNE ν_μ CCQE-like cross section per neutron as a function of neutrino energy (data points) from Ref. [1], together with the predictions derived from the model used in Ref. [5]. The yellow band accounts for theoretical uncertainties, as discussed in Ref. [2], while the QE contribution includes relativistic effects, and some nuclear corrections, among other those due to long range RPA correlations.

we stress that the results of Ref. [5] for the double differential cross section and those displayed in Fig. 2 have been calculated with the same model.

We see in Fig. 2 that the proportion of multinucleon events contributing to the QE-like signal is quite large in the whole energy range relevant in the MiniBooNE experiment. This questions the validity of the algorithm used to reconstruct the neutrino energy in Eq. (1). In the next section, we will explore in detail this problem and we will find out the shape distortion effects induced by the use of Eq. (1) in the MiniBooNE data.

III. THE DISTRIBUTION OF RECONSTRUCTED ENERGIES IN CCQE-LIKE EXPERIMENTS

Let $P(E_{\text{rec}}^0|E)$ be the conditional probability density of measuring an event with reconstructed energy (combination of muon energy and scattering angle given in Eq. (1)) comprised in the interval $[E_{\text{rec}}^0, E_{\text{rec}}^0 + dE_{\text{rec}}]$ and induced by the interaction with the nuclear target of a neutrino of energy E (it is to say, conditional probability density of obtaining E_{rec} “given” E). This probability density can be computed theoretically as:

$$P(E_{\text{rec}}^0|E) = \frac{1}{\sigma(E)} \frac{d\sigma}{dE_{\text{rec}}}(E; E_{\text{rec}} = E_{\text{rec}}^0) \quad (2)$$

where $\sigma(E)$ is the integrated CCQE-like cross section for neutrinos of energy E , and the distribution $d\sigma/dE_{\text{rec}}$ is obtained from the double differential cross section, with respect the energy and scattering angle of the outgoing muon, as

$$\frac{d\sigma}{dE_{\text{rec}}}(E; E_{\text{rec}}^0) = \int_{m_\mu}^E dE_\mu \frac{d^2\sigma}{dE_{\text{rec}}dE_\mu}(E; E_{\text{rec}}^0) = \int_{m_\mu}^E dE_\mu \left| \frac{\partial(\cos\theta_\mu)}{\partial E_{\text{rec}}} \right| \frac{d^2\sigma}{d(\cos\theta_\mu)dE_\mu}(E; E_{\text{rec}}^0) \quad (3)$$

for a fix value of the reconstructed energy $E_{\text{rec}} = E_{\text{rec}}^0$ and “true” neutrino energy E . Eq. (1) can be used to express $\cos\theta_\mu$ in terms of E_μ and E_{rec} . Besides, the Jacobian can be trivially computed also from Eq. (1) and it reads

$$\frac{\partial(\cos\theta_\mu)}{\partial E_{\text{rec}}} = -\frac{ME_\mu - m_\mu^2/2}{E_{\text{rec}}^2|\vec{p}_\mu|} \quad (4)$$

it is more appropriated for the total unfolded cross section as long as the neutrino energy is sufficiently small. A final consideration, in Ref. [11], it was found that the main effect of FSI is a shift of ~ 10 MeV of the QE peak for neutrino energies closer to the MiniBooNE neutrino flux mean energy, $\langle E \rangle \sim 800$ MeV and that it has little impact on the integrated cross section (we will illustrate, within the model of Ref. [23], this latter affirmation also in the Appendix A). However, we cannot discard the possibility that these effects could be more important in the angle and energy distributions for low energy neutrinos. Moreover, it has been also pointed out that some relativistic approaches to account for FSI lead to larger variations of the total cross section [31].

We would like to stress that $d\sigma/dE_{\text{rec}}(E; E_{\text{rec}}^0)$ is an observable, but this distribution is not accessible to experiments where only the kinematics of the outgoing muon is measured.

On the other hand, let $P_{\text{rec}}(E_{\text{rec}})$ be the probability density of measuring an event with reconstructed energy E_{rec} ,

$$P_{\text{rec}}(E_{\text{rec}}) = \int P(E_{\text{rec}}|E)P_{\text{true}}(E)dE \quad (5)$$

where

$$P_{\text{true}}(E) = \frac{1}{\langle\sigma\rangle}\Phi(E)\sigma(E), \quad \langle\sigma\rangle = \int \Phi(E')\sigma(E')dE' \quad (6)$$

is the density probability of having an event due to the interaction of a neutrino, with energy between E and $E + dE$, with the nuclear target. Φ is the neutrino flux normalized to one, and $\langle\sigma\rangle$ is the total flux averaged cross section. It trivially follows,

$$P_{\text{rec}}(E_{\text{rec}}) = \frac{1}{\langle\sigma\rangle} \int \frac{d\sigma}{dE_{\text{rec}}}(E; E_{\text{rec}})\Phi(E)dE \quad (7)$$

a magnitude that can be measured in a CCQE-like experiment.

IV. THE DISTRIBUTION OF TRUE NEUTRINO ENERGIES $P_{\text{true}}(E)$ FROM CCQE-LIKE EXPERIMENTS

We use Bayes's theorem to estimate $P_{\text{true}}(E)$ from the measured density probability $P_{\text{rec}}(E_{\text{rec}})$. To that end, let us introduce $P(E|E_{\text{rec}})$ that is, given an event of reconstructed energy E_{rec} , the conditional density probability of being produced by a neutrino of energy E . It follows,

$$P_{\text{true}}(E) = \int dE_{\text{rec}}P_{\text{rec}}(E_{\text{rec}})P(E|E_{\text{rec}}) \quad (8)$$

Now, since Bayes's theorem reads

$$P(E|E_{\text{rec}}) = \frac{P(E_{\text{rec}}|E)P_{\text{true}}(E)}{P_{\text{rec}}(E_{\text{rec}})} \quad (9)$$

we deduce

$$P(E|E_{\text{rec}}) = \frac{\Phi(E)d\sigma/dE_{\text{rec}}(E; E_{\text{rec}})}{\int dE''\Phi(E'')d\sigma/dE_{\text{rec}}(E''; E_{\text{rec}})} \quad (10)$$

from Eqs. (2), (6) and (7). The recent work of M. Martini et al. [22] pays an special attention to this distribution that, as we observe, depends on the neutrino flux. The above equation implies

$$P_{\text{true}}(E) = \int dE_{\text{rec}}P_{\text{rec}}(E_{\text{rec}})\frac{\Phi(E)d\sigma/dE_{\text{rec}}(E; E_{\text{rec}})}{\int dE''\Phi(E'')d\sigma/dE_{\text{rec}}(E''; E_{\text{rec}})} \quad (11)$$

Finally and attending to the existing relation between $P_{\text{true}}(E)$ and $\sigma(E)$ in Eq. (6), we could write

$$\sigma(E) = \int dE_{\text{rec}} \left[\langle\sigma\rangle P_{\text{rec}}(E_{\text{rec}}) \right] \times \left[\frac{d\sigma/dE_{\text{rec}}(E; E_{\text{rec}})}{\int dE''\Phi(E'')d\sigma/dE_{\text{rec}}(E''; E_{\text{rec}})} \right] \quad (12)$$

A consistency check is obtained if we substitute Eq. (7) in Eq. (12) which leads to

$$\sigma(E) = \int dE_{\text{rec}} \frac{d\sigma}{dE_{\text{rec}}}(E; E_{\text{rec}}) \quad (13)$$

that it is trivially satisfied thanks to the definition of $d\sigma/dE_{\text{rec}}(E; E_{\text{rec}})$ in Eq. (3).

Eq. (12) might be used to estimate the integrated flux unfolded cross section from data. CCQE-like experiments measure the quantities that appear in the first bracket of this equation, namely, $\langle\sigma\rangle \times P_{\text{rec}}(E_{\text{rec}})$. We have already discussed about $P_{\text{rec}}(E_{\text{rec}})$, while the total flux averaged cross section $\langle\sigma\rangle$ is determined by the ratio of the total

number of events (N_{event}) over the number of incident neutrinos per unit of area (N_{inc}). N_{event} is directly measured while for N_{inc} there exist, in principle, accurate theoretical predictions. Note that the flux $\Phi(E)$, normalized to one, gives only the shape of the neutrino flux, but it is independent of the total number of incident neutrinos.

Thus, if one had a theoretical model for the second bracket $[d\sigma/dE_{rec}(E; E_{rec})/\int dE''\Phi(E'')d\sigma/dE_{rec}(E''; E_{rec})]$ in Eq. (12) or equivalently for $P(E|E_{rec})$, one could extract the flux unfolded cross section $\sigma(E)$ after folding it with the measured data ($\langle\sigma\rangle \times P_{rec}(E_{rec})$). This method reproduces [32] the data unfolding used by the MiniBooNE collaboration in Ref. [1] and described in Ref. [36]. The iterative unfolding method in MiniBooNE is needed due to the statistical fluctuations in the data and the lack of "a priori" knowledge of the $P_{true}(E)$ probability in Eq. (9), both are not relevant for theoretical calculations. However, as a proof of the validity of this approach we have checked that the iterative method in [36] yields to identical results. As discussed in the introduction, the works of Refs. [2-6] show that the QE-like (and/or differential) cross section is given by the sum

$$\sigma(E) = \sigma^{QE}(E) + \sigma^{2p2h}(E) \quad (14)$$

of the genuine QE and the multinucleon contributions. Up to now, experimental analysis have completely neglected the latter (2p2h) cross section, while well established nuclear corrections, like RPA correlations, have also been ignored in the computation of the former one (QE). As a consequence, a high value of $M_A > 1.3$ GeV is required in the MiniBooNE analysis to describe the flux folded $d\sigma/dq^2$ or $d\sigma/dT_\mu d\cos\theta_\mu$ distributions [1]. But, if this approximate model is used in Eq. (12) to extract the unfolded cross section,

$$\sigma_{appx}(E) = \int dE_{rec} \left[\langle\sigma\rangle P_{rec}(E_{rec}) \right]_{Exp} \times \left[\frac{d\sigma/dE_{rec}(E; E_{rec})}{\int dE''\Phi(E'')d\sigma/dE_{rec}(E''; E_{rec})} \right]_{QE \text{ no RPA}, M_A > 1.3 \text{ GeV}} \quad (15)$$

the resulting estimate $\sigma_{appx}(E)$ might significantly differ from the real QE-like neutrino-nucleus cross section $\sigma(E)$. Actually, we will show that a redistribution of strength from high energy to low neutrino energies is being produced. To illustrate this, we will focus on the total cross section data on carbon published by the MiniBooNE in [1]. We take in Eq. (15)

$$\left[\langle\sigma\rangle P_{rec}(E_{rec}) \right]_{Exp} \sim \int \left(\frac{d\sigma}{dE_{rec}}(E'; E_{rec}) \Big|_{QE+RPA}^{M_A=1.049 \text{ GeV}} + \frac{d\sigma^{2p2h}}{dE_{rec}}(E'; E_{rec}) \right) \Phi(E') dE' \quad (16)$$

where we have used Eq. (7) with our best theoretical model, since we should mimic the experiment. Indeed, this is the same model we employed in Ref. [5] to successfully describe the MiniBooNE CCQE-like flux averaged double differential cross section $d\sigma/dE_\mu d\cos\theta_\mu$, up to a global normalization scale $\lambda (= 0.89 \pm 0.01)$. This latter parameter is the only one which is fitted to data ($\chi^2/dof = 53/137$). There are no free parameters in the description of nuclear effects, since they were fixed in previous studies of photon, electron, and pion interactions with nuclei [26-29, 37, 38]. Besides, form factors are determined in independent analysis of the experimental data on nucleons. In particular, the model uses a value for the nucleon axial mass of $M_A = 1.049$ GeV, that agrees within errors with the world average [9, 10] value of 1.03 ± 0.02 GeV. As already mentioned, the genuine QE piece was computed in Ref. [2], it includes relativistic effects, and some other nuclear corrections in addition to the RPA ones, explicitly included in the label of Eq. (16), though it does not include FSI effects. The multinucleon cross section is taken from [2]. The QE and 2p2h contributions to the integrated cross section $\sigma(E)$ in carbon were displayed in Fig. 2. For simplicity, in what follows, we will label this QE model as "QE (rel+RPA)" as in that figure.

On the other hand, to compute the second factor in the right hand side of Eq. (15) we need to mimic the input used in the experimental analysis. To that end, we use a simple Fermi gas model with $M_A = 1.32$ GeV that only accounts for the genuine QE contribution and that does not include RPA corrections. The value of M_A in this model was fitted to the flux-folded double differential cross section $d\sigma/dE_\mu d\cos\theta_\mu$ in Ref. [5], leading to an excellent value of $\chi^2/dof = 35/137$. This model should be quite similar to the one originally used in the MiniBooNE analysis. The main difference being that we use a local rather than global Fermi gas in the calculation. Gathering the different terms, we have

$$\sigma_{appx}(E) \sim \sigma_{appx}^{QE \text{ (rel+RPA)}}(E) + \sigma_{appx}^{2p2h}(E) \quad (17)$$

$$\sigma_{appx}^{QE \text{ (rel+RPA)}}(E) \sim \int dE_{rec} \int \frac{\overbrace{d\sigma^{QE \text{ (rel+RPA)}}}^{M_A=1.049 \text{ GeV}}}{dE_{rec}}(E'; E_{rec}) \Phi(E') dE' \times \overbrace{\left[\frac{d\sigma/dE_{rec}(E; E_{rec})}{\int dE''\Phi(E'')d\sigma/dE_{rec}(E''; E_{rec})} \right]}^{QE \text{ no RPA}, M_A=1.32 \text{ GeV}} \quad (18)$$

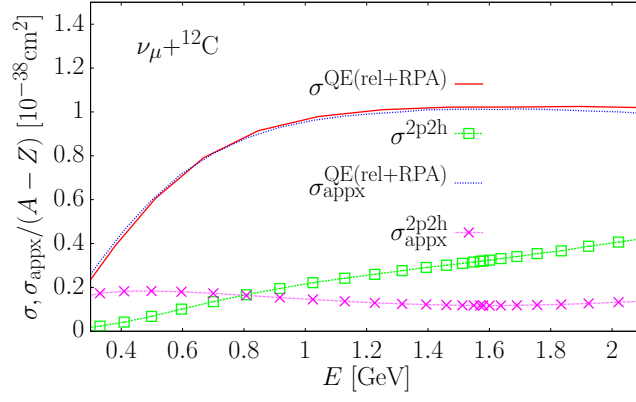


FIG. 3: Theoretical (σ) and approximate (σ_{appx} , defined in Eqs. (17)–(19)) CCQE-like integrated cross sections in carbon as a function of the neutrino energy.

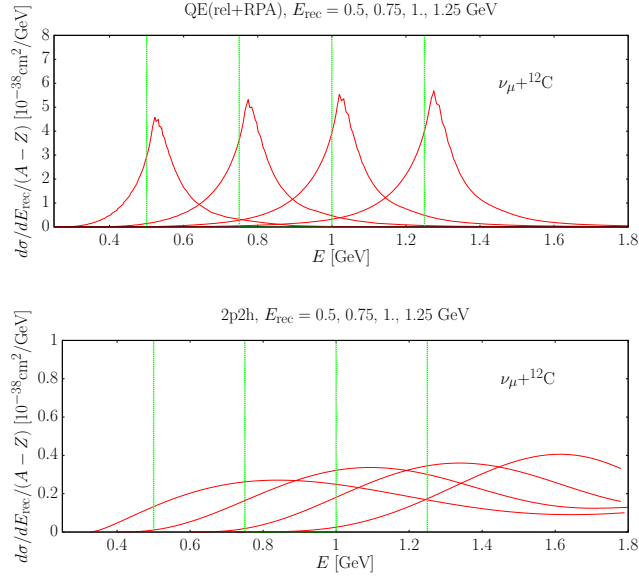


FIG. 4: Differential cross section $d\sigma/dE_{\text{rec}}(E; E_{\text{rec}})$ as a function of the “true” neutrino energy (E) and four different values of $E_{\text{rec}} = 0.5, 0.75, 1$ and 1.25 GeV, which are indicated by the vertical lines. QE contributions, from the (rel+RPA) model, are displayed in top panel, while 2p2h ones are shown in the bottom plot.

$$\sigma_{\text{appx}}^{2\text{p}2\text{h}}(E) \sim \int dE_{\text{rec}} \int \frac{d\sigma^{2\text{p}2\text{h}}(E'; E_{\text{rec}})\Phi(E')dE'}{\underbrace{\int dE''\Phi(E'')d\sigma/dE_{\text{rec}}(E''; E_{\text{rec}})}_{\text{QE no RPA, } M_A=1.32 \text{ GeV}}} \quad (19)$$

Results both for the QE and the 2p2h contributions to σ_{appx} are shown in Fig. 3. As can be appreciated there, $\sigma_{\text{appx}}(E)$ is an excellent approximation to the real $\sigma(E)$ cross section in the case of the QE contribution. The reason is that for genuine QE processes, the distribution $d\sigma^{\text{QE}}/dE_{\text{rec}}(E; E_{\text{rec}})$ is strongly peaked around $E \approx E_{\text{rec}}$, as seen

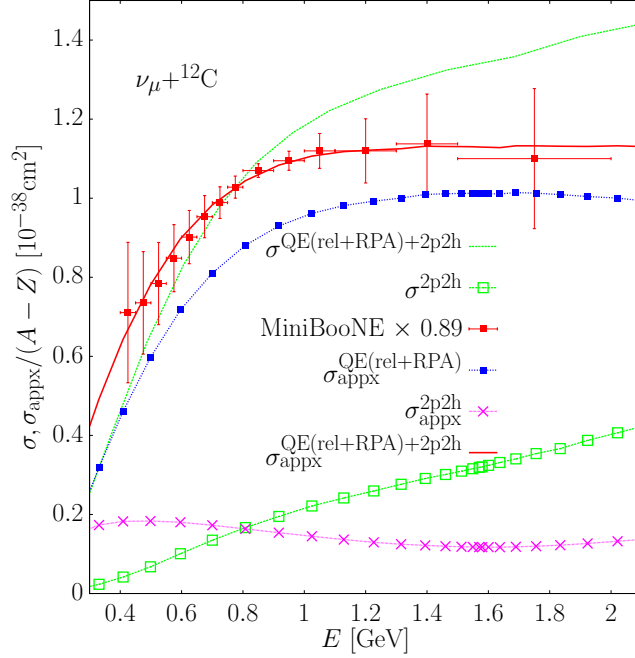


FIG. 5: Theoretical σ and approximate σ_{appx} , (defined in Eqs. (17)–(19)) CCQE-like integrated cross sections in carbon as a function of the neutrino energy. For consistency with our previous results for the flux-folded double differential cross section $d\sigma/dE_\mu d\cos\theta_\mu$ in Ref. [5], the MiniBooNE data [1] and errors have been re-scaled by a factor 0.89. The shape errors are shown for the MiniBooNE data.

in the top panel⁵ of Fig. 4.⁶

Though the actual $d\sigma^{\text{QE}}/dE_{\text{rec}}$ distributions have some widths, these differential cross sections are sufficiently peaked to render the width effects on the ratio $\left(\sigma_{\text{appx}}^{\text{QE (rel+RPA)}}/\sigma^{\text{QE (rel+RPA)}}\right)$ quantitatively irrelevant, as the results of the Fig. 3 indicate. Thus, we could conclude that when dealing only with genuine QE events the procedure outlined in Eq. (15) to obtain the flux unfolded cross section is quite accurate. This is despite of the fact that RPA correlations and other nuclear effects were not considered in the ansatz for $P(E|E_{\text{rec}})$ (second bracket in Eq. (12)). Note however, that all nuclear effects are included in the first factor $\left[\langle\sigma\rangle P_{\text{rec}}(E_{\text{rec}})\right]_{\text{Exp}}$ in Eq. (15).

However, the situation is drastically different for the 2p2h contribution case, as one can also observe in Fig. 3. Indeed, it turns out that $\sigma_{\text{appx}}^{2\text{p}2\text{h}}(E)$ is a poor estimate of the actual multinucleon mechanism contribution $\sigma^{2\text{p}2\text{h}}(E)$. As before, if we approximate

$$\sigma_{\text{appx}}^{2\text{p}2\text{h}}(E) \approx \int dE_{\text{rec}} \frac{\delta(E - E_{\text{rec}})}{\Phi(E_{\text{rec}})} \int \frac{d\sigma^{2\text{p}2\text{h}}(E'; E_{\text{rec}})\Phi(E')dE'}{dE_{\text{rec}}} = \frac{1}{\Phi(E)} \int \frac{d\sigma^{2\text{p}2\text{h}}(E'; E_{\text{rec}} = E)\Phi(E')dE'}{dE_{\text{rec}}} \quad (22)$$

⁵ Actually the peak is shifted about 25 MeV up to higher energies since the bound energy of the target nucleon is not considered in Eq.(1). On the other hand, RPA correlations modify the size but do not affect significantly this peak structure.

⁶ If for illustration purposes, we use a Dirac's delta to approximate $\frac{d\sigma^{\text{QE}}}{dE_{\text{rec}}}(E; E_{\text{rec}}) \approx \sigma^{\text{QE}}(E) \times \delta(E - E_{\text{rec}})$ then, we will have

$$\overbrace{\left[\frac{d\sigma/dE_{\text{rec}}(E; E_{\text{rec}})}{\int dE'' \Phi(E'') d\sigma/dE_{\text{rec}}(E''; E_{\text{rec}})} \right]}^{\text{QE no RPA, } M_A=1.32 \text{ GeV}} \approx \frac{\delta(E - E_{\text{rec}})}{\Phi(E_{\text{rec}})} \quad (20)$$

and therefore, independently of the nuclear model for QE, within this limit

$$\sigma_{\text{appx}}^{\text{QE (rel+RPA)}}(E) \approx \int dE_{\text{rec}} \frac{\delta(E - E_{\text{rec}})}{\Phi(E_{\text{rec}})} \int \overbrace{\sigma^{\text{QE (rel+RPA)}}(E')}^{M_A=1.049 \text{ GeV}} \delta(E' - E_{\text{rec}})\Phi(E')dE' = \sigma^{\text{QE (rel+RPA)}}(E) \quad (21)$$

Taking into account that $d\sigma^{2p2h}/dE_{\text{rec}}(E; E_{\text{rec}})$ is almost negligible for $E \leq E_{\text{rec}}$ and that it shows a quite long tail above this energy (see bottom panel of Fig. 4), is then easy to understand the redistribution of strength from high to low neutrino energies observed in $\sigma_{\text{appx}}^{2p2h}(E)$ when it is compared to the actual $\sigma^{2p2h}(E)$ cross section. Up to some approximation the area is conserved, though.

Finally, in Fig. 5 we compare the MiniBooNE CCQE-like data with both σ and σ_{appx} . We see an excellent agreement between the latter one and the data scaled down by a factor 0.89. As mentioned, our QE(rel+RPA)+2p2h model successfully describes the MiniBooNE CCQE-like flux averaged double differential cross section $d\sigma/dE_{\mu}d\cos\theta_{\mu}$ data, up to a global scale λ ($= 0.89 \pm 0.01$) [5]. This is the best observable to compare with theoretical models because both the muon angle and energy are directly measured quantities, and thus the shape of this distribution is readily obtained from the number of events measured for each muon kinematical bin. To obtain the absolute normalization of the distribution, however, it is necessary to rely on some estimate for the number of incident neutrinos per unit of area (N_{inc}). We believe the obtained value for $\lambda \sim 0.89$ in [5] indicates that the actual number of incident neutrinos per unit of area might be larger than the central value assumed in the MiniBooNE analysis. This would be still consistent with the MiniBooNE estimate of a total normalization error of 10.7% [1]. The value of N_{inc} is needed to estimate $\langle\sigma\rangle_{\text{Exp}}$ in the expression of Eq. (15) for $\sigma_{\text{appx}}(E)$, and thus the predictions of our model in Eq. (16) would have to be multiplied by $1/\lambda$, or equivalently the data have to be scaled down by a factor λ .

Coming back to the results displayed in Fig. 5, we should conclude that the unfolded cross section published in [1] appreciably differs from the real one $\sigma(E)$. Actually, it is not a very clean observable after noticing the importance of multinucleon mechanisms, because the unfolding itself is model dependent and assumes that the events are purely QE. The same limitation occurs for the differential cross section $d\sigma/dq^2$, given that q^2 is also deduced assuming the events are QE. When compared with the “real“ $\sigma(E)$, the MiniBooNE unfolded cross section exhibits an excess (deficit) of low (high) energy neutrinos, which is an artifact of the unfolding process that ignores multinucleon mechanisms.

The semi-phenomenological model of Refs. [3, 4] predicts a theoretical cross section $\sigma(E)$ that provides a good description of the MiniBooNE unfolded cross section of Ref. [1]. However, we have shown here that these data do not correspond to the actual cross section because the unfolding process is biased. To compare with these data, the authors of [3, 4] should carry out a procedure similar to that proposed in Eq. (15). Since the model of these works includes strong 2p2h contributions, we would expect an appreciable change in the shape, as discussed above, that might distort quantitatively and qualitatively the agreement with the MiniBooNE unfolded cross section found in Refs. [3, 4].

In Ref. [22], Martini et al. have paid special attention to the flux dependent $P(E|E_{\text{rec}})$ probability. We however believe that $P(E_{\text{rec}}|E)$ (or equivalently the differential cross section $d\sigma/dE_{\text{rec}}$) could be more illuminating. First, because it does not depend on the neutrino flux, and second because it can be used by experiments to determine $\sigma(E)$ thanks to Bayes’s theorem. Finally, we should also mention a recent and quite comprehensive work on neutrino-nucleus observables [39] has also found, using some simple models for multinucleon mechanisms, that 2p2h interactions lead to a downward shift of the reconstructed energy in agreement with our results.

V. CONCLUSIONS

We have shown that because of the multinucleon mechanism effects, the algorithm used to reconstruct the neutrino energy is not adequate when dealing with quasielastic-like events. This effect is relevant to neutrino oscillation experiments that uses the CCQE samples to compute the neutrino energy. The assumption of a pure CCQE interaction introduces biases in the determination of Δm^2 and mixing angle. Moreover, 2p2h contributions put also limitations on the validity of the flux unfolding procedure used in [1]. The MiniBooNE unfolded cross section exhibits an excess (deficit) of low (high) energy neutrinos, which is an artifact of the unfolding process that ignores multinucleon mechanisms. Actually, σ_{appx} , defined in Eqs. (17)–(19), provides an excellent description of the data of [1]. This, together with our previous results in Ref. [5] for the CCQE-like flux averaged double differential cross section $d\sigma/dE_{\mu}d\cos\theta_{\mu}$, make us quite confident on the reliability of our QE(rel+RPA)+2p2h microscopical model derived in Refs. [2, 23]. Furthermore, because it is just a natural extension of previous successful studies of photon, electron, and pion interactions with nuclei [26–29].

Appendix A: Relativistic vs non-relativistic QE total CC neutrino cross sections within the scheme of Ref. [23]

The left panel of Fig. 18 in Ref. [2] corresponds to the flux-unfolded MiniBooNE ν_{μ} CCQE-like cross section per neutron as a function of the neutrino energy. There, both the QE and the 2p2h contributions to the total cross section were also shown separately. The latter ones were computed fully relativistically, while the QE predictions were taken

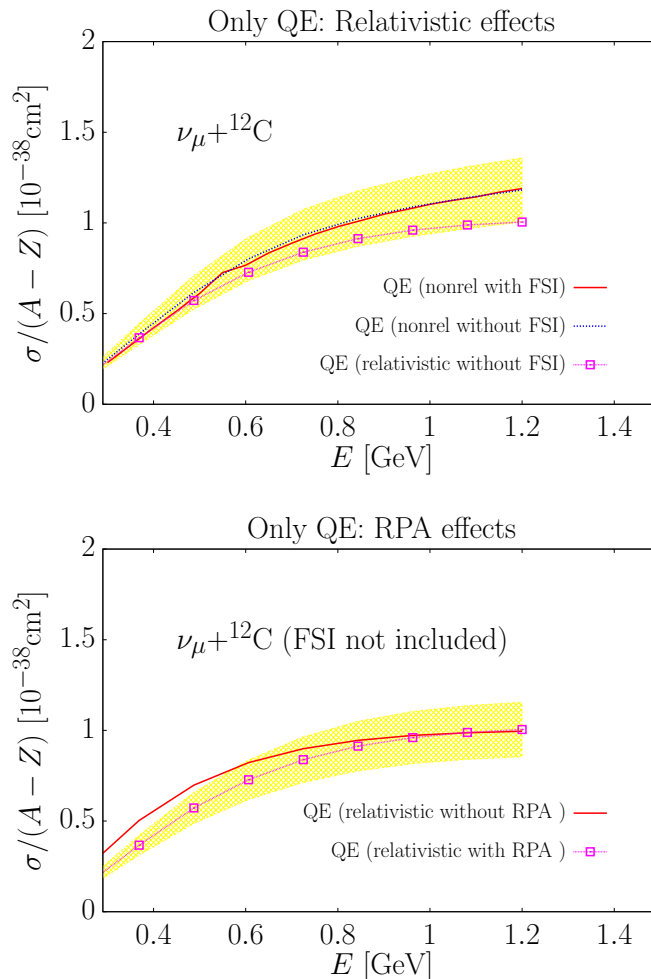


FIG. 6: Different theoretical predictions for neutrino CCQE total cross section off ^{12}C obtained from the model of Ref. [23]. Yellow bands account for a 15% theoretical uncertainties that might affect the nuclear corrections included in the model of Ref. [23], as discussed in [35].

from a previous work [23]. Concretely, results that included RPA and FSI effects, were selected and displayed in the Fig. 18. Since the approach used in [23] to account for FSI effects was not relativistic, the QE curves displayed in (both panels) Fig. 18 of Ref. [2] neglect some relativistic effects for the nucleons. In particular, those results are based on a non-relativistic approximation for the nucleon (particle and hole) propagators⁷. This is one of the sources of systematic errors, among others, that should be accounted for by the bands of theoretical uncertainties displayed in Fig. 18 (see the discussion of the third paragraph of pag. 16 in [2]). The use of non-relativistic nucleon propagators is responsible of the odd behaviour of the QE results in the Fig.18 of Ref. [2] when the neutrino energy increases. Indeed, the QE cross sections are too big for neutrino energies above 0.5-0.6 GeV and these predictions clearly depart [33] from the common pattern exhibited by different models collected in a review talk presented in the NUINT 2009 Workshop [34].

In the top panel of Fig. 6, we show the size of FSI and relativistic effects within the QE model of Ref. [23] (note that there, results obtained using relativistic nucleon propagators, but neglecting FSI, were also shown). In all curves of this top panel, RPA effects are taken into account. We see that FSI have little effect on the integrated cross sections, though FSI might affect differential distributions [23], and we should also pointed out that some relativistic approaches to FSI lead to larger increases of the total cross section [31]. On the other hand, for neutrino energies above 1 GeV, relativistic effects for the nucleons reduce the cross sections by around 15%, but still the relativistic QE

⁷ Actually, all QE results of Ref. [2] were obtained with non-relativistic nucleon propagators.

results lie within the theoretical error band assumed for the QE predictions in Ref. [2].

The flux folded CC double differential neutrino cross section, measured by the MiniBooNE Collaboration, was analyzed in Ref. [5] by using the full relativistic model of Ref. [23] without the inclusion of FSI, but taking into account RPA correlations, which effects in the integrated flux-unfolded cross section can be seen in the bottom panel of Fig. 6. This is the model that is used in this work. Note, as can be appreciated in this latter plot, that though RPA effects are quite relevant for low neutrino energies, they are negligible above 1 GeV, and much smaller than the theoretical uncertainties, discussed in [35], for $E > 0.7$ GeV.

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