Production of Θ^+ Hypernuclei with the (K^+, π^+) reaction

H. Nagahiro[a], S. Hirenzaki[a], E. Oset[b] and M.J. Vicente Vacas[b]

[a]Department of Physics, Nara Women's University, Nara 630-8506, Japan

[b]Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC, Institutos de Investigación de Paterna, Aptd. 22085, 46071 Valencia, Spain

February 8, 2008

Abstract

We present results on the production of bound states of Θ^+ in nuclei using the (K^+, π^+) reaction. By taking into account the states obtained within a wide range of strength of the Θ^+ nucleus optical potential, plus the possibility to replace different nucleons of the nucleus, we obtain an excitation spectra with clearly differentiated peaks. The magnitude of the calculated cross sections is well within reachable range.

The discovery of the Θ^+ at SPring-8/Osaka [1], followed by its confirmation in different other experiments, has made a substantial impact in hadronic physics (see [2] for a compilation of experimental and theoretical works on the issue). The possibility that there would be Θ^+ bound states in nuclei has not passed unnoticed and in [3] the Θ^+ selfenergy in the nucleus was evaluated, however with only the part tied to the KN decay, which is known experimentally to be very small. As a consequence, the Θ^+ potential obtained was too weak to bind Θ^+ in nuclei. Suggestions of possible bound states within a schematic model for quark pair interaction with nucleons were made in Ref. [4]. In that work a Θ^+ selfenergy in the nucleus was obtained of the same order as the contribution of the same mechanisms to the Θ^+ mass. The (γ, Σ) and (K, π) reactions were also suggested in that work as a means to produce Θ^+ bound states in nuclei.

In a recent paper [5] the possibility of having Θ^+ bound states in nuclei, tied to the $K\pi N$ content of the Θ^+, was investigated and it was concluded that there is an attractive Θ^+ potential, which, within uncertainties, is strong enough to bind the Θ^+ in nuclei. Restrictions from Pauli blocking and binding reduce the Θ^+ width in nuclei to about one third or less of the free width, and with attractive Θ^+ nucleus potentials ranging from 60 to 120 MeV at normal nuclear matter density, the separation between the deeper Θ^+ levels in light and medium nuclei is larger than the width, even in the case that the free Θ^+ width were as big as 15 MeV. This is a desirable experimental situation in which clear peaks could be observed provided an appropriate reaction is used.

In [5] the Θ^+ selfenergy tied to the KN decay was also studied and found to be very small like in [3]. The large attraction found in [5] is tied to the coupling of the Θ^+ to two mesons and a baryon which was related to the strong decay of the $N^*(1710)$ resonance to a nucleon and two pions.

In the present paper we investigate the reaction (K^+, π^+) in nuclei, which leads to clear peaks and a fair strength in the spectra of Θ^+ nuclear states. The reaction is of the substitutional type, in which one of the nucleons in the nucleus will be substituted by the Θ^+. Hence, on top of the different Θ^+ bound states, we shall also have to take into account the binding energies of the nucleon levels in the nucleus when we look for the spectra of this reaction.

The analogous (K^-, π^-) reaction has been a standard tool to produce Λ hypernuclei [6] and it was also used to see the only Σ hypernucleus known so far [7, 8]. There is an added difficulty here since one would wish to have a kinematics as close as possible to a recoilless condition, which makes the production cross sections larger. This is the case of the $(d,^3He)$ reaction, which was used with success to find the deeply bound pionic atoms [9, 10]. In the present case, given the fact that one has to create the excess mass of the Θ^+ over the nucleon from the kinetic energy of the kaon, this induces inevitably a momentum transfer, which reach a minimal value of the order of 450 MeV/c for an initial K^+ momentum of around 620 MeV/c. Although this quantity is larger than the Fermi momentum, it can still be accommodated in the Θ^+ wave function without much penalty, such that the cross sections obtained are still sizable.

We show the momentum transfer of the forward (K^+,π^+) reactions for the Θ^+ nuclei formation as a function of the incident K energy in Fig.1. We can also expect to produce Θ^+ nuclear states by the (γ,K^-) and (π^-, K^-) reactions. However, the typical momentum transfer of these reactions are much larger than that of the (K^+,π^+) reaction, and the formation rate of these reactions will be suppressed significantly. Also the distortion of the K^+ is substantially weaker than that of the K^-.

The cross section for the (K^+,π^+) reaction for the formation of the Θ^+ bound states is given by,

$$\frac{d\sigma}{d\Omega_\pi d\omega_\pi} = \frac{1}{16\pi^2}\frac{p}{k}|t|^2 \sum_{\substack{\text{p}-\text{state,}\\ \Theta-\text{state}}} \frac{\Gamma}{2\pi}\frac{1}{(\Delta E)^2+\Gamma^2/4}\left|\int d^3\vec{r}\psi^\dagger_\Theta(\vec{r})e^{i(\vec{k}-\vec{p})\cdot\vec{r}}D(\vec{b},z)\psi_p(\vec{r})\right|^2, \tag{1}$$

where $\psi_\Theta(\vec{r})$ and $\psi_p(\vec{r})$ are the Θ and proton wavefunctions in bound states, $\vec{k}$ and $\vec{p}$ are the momenta of the incident kaon and the emitted pion, respectively, and Γ is the decay width of the final Θ^+ nuclear states. In Eq. (1) ΔE is defined as,

$$\Delta E = \omega(K^+) + (M_p - S_p) - (M_\Theta - B_\Theta) - \omega(\pi^+), \tag{2}$$

where $\omega(K^+)$ and $\omega(\pi^+)$ are the relativistic energies of K^+ and π^+, S_p is the proton separation energy from the target nucleus, B_Θ is the Θ binding energy in the final states and M_p, M_Θ are the proton and Θ masses, respectively. We sum up all contributions of proton single particle states in the target nucleus and Θ bound states in the final states in Eq. (1).

In order to take into account the distortion affecting the pion and kaon, we use the eikonal approximation, in which the distorted waves are approximated by plane waves with a distortion factor. The distortion factor $D(\vec{b},z)$ appearing in Eq. (1) is defined as,

$$D(\vec{b},z) = \exp\left[-\frac{1}{2}i\int_{-\infty}^{z}\frac{\Pi_K}{k}dz' - \frac{1}{2}i\int_{z}^{\infty}\frac{\Pi_\pi}{p}dz'\right], \tag{3}$$

where Π_K and Π_π are the kaon and pion selfenergies in the nuclear medium. The real part of Π_K is taken from the $t\rho$ approximation [11, 12]

$$Re\,\Pi_K = 0.13\, m_K^2\rho/\rho_0, \tag{4}$$

which accounts for the largely dominant $s-$wave interaction and the imaginary part, also from the $t\rho$ approximation, is obtained using the optical theorem

$$\frac{Im\,\Pi_K}{k} = -(\sigma_{Kp}\rho_p + \sigma_{Kn}\rho_n)\,. \tag{5}$$

The pion selfenergy Π_π is taken from Refs. [13, 14, 15].

The matrix element t in Eq. (1) is the $K^+p \to \pi^+\Theta^+$ transition $t-matrix$. We get this magnitude from the same Lagrangian used in [5] to obtain the Θ^+ nuclear potential. The Θ^+ selfenergy in [5] was obtained studying the excitation of intermediate states KN and $K\pi N$, and incorporating the medium effects in the mesons and the nucleon. The part of the selfenergy tied to the KN channel was found very small, but the one tied to the $K\pi N$ channel led to a sizable attraction.

In Ref. [5] the following two Lagrangians were used coupling the Θ^+ to $K\pi N$

$$\mathcal{L} = ig_{\bar{1}\bar{0}}\epsilon^{ilm}\bar{T}_{ijk}\gamma^\mu B^j_l (V_\mu)^k_m, \tag{6}$$

$$\mathcal{L} = \frac{1}{2f}\tilde{g}_{\bar{1}\bar{0}}\epsilon^{ilm}\bar{T}_{ijk}(\phi\cdot\phi)^j_l B^k_m, \tag{7}$$

with V_μ the two mesons vector current, $V_\mu = \frac{1}{4f^2}(\phi\partial_\mu\phi - \partial_\mu\phi\phi)$, f the pion decay constant and T_{ijk}, B^j_l, ϕ^j_l SU(3) tensors which account for the antidecuplet states, the octet of $\frac{1}{2}^+$ baryons and the octet of 0^- mesons, respectively.

In Ref.[16] a study was done of the possible Lagrangians coupling the antidecuplet states to two meson and one baryon under the assumptions of $SU(3)$ symmetry and minimal number of derivatives for each possible $SU(3)$ structure. In addition, a possible chiral symmetric term was also studied as well as a mass term breaking explicitly chiral symmetry. It was found there that the two relevant structures were those in Eqs. (6) and (7). The chiral Lagrangian gave results remarkably similar to those of Eq. (6) and other terms as the chiral symmetry breaking term and one coming from the 27 $SU(3)$ representation had to have small strength from physical grounds. The amplitude provided by the first Lagrangian, in the nonrelativistic limit, is proportional to the difference of the meson energies in the $\Theta^+ \to N\pi K$ process. Here, since we have an incoming and an outgoing meson, the difference of energies is substituted by their sum. Hence, the $K^+p \to \Theta^+\pi^+$ amplitude is written as,

$$t = -\frac{1}{4f^2}(-\sqrt{6})g_{\bar{1}\bar{0}}(\omega(K^+) + \omega(\pi^+)), \tag{8}$$

where $f = 93$ MeV and $g_{\bar{1}\bar{0}} = \alpha\left[\frac{m^2_{K^*}}{m^2_{K^*}-(k-p)^2}\right]$ with $\alpha = 0.315$ [16], which incorporates a form factor to explicitly account for the exchange of a $K^*(892)$ in the $t-$channel.

The amplitude provided by the second Lagrangian, Eq. (7), is

$$t = \frac{1}{2f}\sqrt{6}\tilde{g}_{\bar{10}}, \tag{9}$$

with $\tilde{g}_{\bar{10}} = 1.88$.

The strength of these two terms is similar and they interfere, but their relative phase is unknown. That phase is not relevant for the evaluation of the selfenergies in Ref. [16] since there one studies the πK production where the meson pair comes in $p-$wave and $s-$wave from the Lagrangians of Eqs. (6) and (7) respectively.

Hence, in the present reaction we must admit a quite large uncertainty. There is a hint on which relative sign to take, based on the small upper bound for Θ^+ production in the preliminary results of K. Imai et al.[17] for the reaction (π^-, K^-) on the proton. This small cross section could be qualitatively understood in base to a negative interference of the two terms that we have. For the case of (K^+, π^+) the sign of the amplitude for the vector Lagrangian of Eq. (6) is opposite to that of the (π^-, K^-) reaction, while the amplitudes from Eq. (7) do not change. Based upon this, we take the positive relative sign between the amplitudes.

The wave functions of the Θ^+ are obtained by solving the Schrödinger equation with two potentials, one with a strength

$$V(r) = -60\frac{\rho(r)}{\rho_0}[\mathrm{MeV}], \tag{10}$$

and the other one

$$V(r) = -120\frac{\rho(r)}{\rho_0}[\mathrm{MeV}]. \tag{11}$$

The calculated binding energies with these potentials are reported in Ref. [5]. In this exploratory level, we only take into account a volume type potential and we have no LS splitting in the Θ energy spectrum. As to the width, it was found in [5] that assuming the free width to be 15 MeV the width in the medium was smaller than 6-7 MeV, due to Pauli blocking effects mostly. We evaluate the level widths using the imaginary part of the Θ selfenergies calculated in Ref. [5] at the appropriate Θ energies.

With this range of values we intend to account for different uncertainties discussed in Ref.[5], like the experimental uncertainties in the input used to fix the $\tilde{g}_{\bar{10}}$ and $g_{\bar{10}}$ couplings, uncertainties in the nucleon selfenergies, etc.

We show the calculated spectra for the formation of the Θ^+ bound states in Fig. 2 and Fig. 3. We selected carbon as target since the level spacing of each subcomponent is expected to be comfortably large to observe the isolated peaks structure. The incident kinetic energy of the kaon beam is fixed at 300 MeV to minimize the momentum transfer of the reaction.

The results with the shallow Θ potential $V(r) = -60\rho/\rho_0$ (MeV) are shown in Fig. 2. We find three isolated peaks in the spectrum. We also find that the magnitude of the formation cross section is around a few [μb/sr MeV] which is expected to be reachable in experiments. The results with the deep Θ potential $V(r) = -120\rho(r)/\rho_0$ (MeV) are shown in Fig. 3, where we find the separated peaks in the cross section again. In this case, we have six clear peaks in the spectrum according to the existence of more Θ bound states due to the deeper potential. The magnitude of the cross section is around 10 times larger than with the shallow potential case because the width of the Θ state is smaller for the deeper bound states which makes the peaks higher. The number of subcomponents of the spectrum is increased for the case of the deeper potential and we find again reasonably large cross section for the reaction. We should mention here that the real parts of the distortion potentials in Eq. (3) are relevant and make the cross sections larger by about a factor two for both Θ potential cases.

In these calculated results, we have not included the quasi-free Θ production, which will have certain contribution to the spectrum above the Θ production threshold. The threshold is shown by the vertical lines in Fig. 2 and 3. The spectrum for ω_π below the threshold would be modified by the inclusion of the quasi-free Θ processes. However, the spectrum in the bound Θ region will not be affected by them.

In summary, the results of [5] indicate that there should be bound states of Θ^+ in nuclei, with separation energies reasonably larger than the width of the states. In view of that, we investigated the (K^+, π^+) reaction to produce these states and obtained excitation spectra of Θ^+ states for a ^{12}C target with two different potentials which cover the likely range of the Θ^+ nucleus optical potential according to the calculations of [5]. With the caveat about the uncertainties in the interference of the two terms discussed above, we obtain reasonable production rates in spite of the fact that the momentum transfer is not too small.

Measurements of binding energies and partial decay widths in nuclei would provide precise information on the coupling of the Θ^+ to two meson channels and about the $K\pi N$ component in the Θ^+ wave function. The results obtained here should strongly encourage to do this experiment which could open the doors to the new field of Θ^+ hypernuclei.

Acknowledgments

This work is partly supported by DGICYT contract number BFM2003-00856, and the E.U. EURIDICE network contract no. HPRN-CT-2002-00311. This research is part of the EU Integrated Infrastructure Initiative Hadron Physics Project under contract number RII3-CT-2004-506078.

References

[1] T. Nakano *et al.* [LEPS Collaboration], neutron," Phys. Rev. Lett. **91** (2003) 012002 [arXiv:hep-ex/0301020].

[2] T. Hyodo, http://www.rcnp.osaka-u.ac.jp∼hyodo/research/Thetapub.html

[3] H. C. Kim, C. H. Lee and H. J. Lee, arXiv:hep-ph/0402141.

[4] G. A. Miller, Phys. Rev. C **70** (2004) 022202 [arXiv:nucl-th/0402099].

[5] D. Cabrera, Q.B. Li, W. Magas, E. Oset and M.J. Vicente Vacas, Phys. Lett. B **608** (2005) 231 [arXiv:nucl-th/0407007].

[6] A. Sakaguchi *et al.*, Nucl. Phys. A **721** (2003) 979.

[7] T. Nagae *et al.*, Reaction At Phys. Rev. Lett. **80** (1998) 1605.

[8] S. Bart *et al.*, Phys. Rev. Lett. **83** (1999) 5238.

[9] H. Toki, S. Hirenzaki and T. Yamazaki, Pionic Nucl. Phys. A **530** (1991) 679.

[10] S. Hirenzaki, H. Toki, T. Yamazaki, Phys. Rev. C **44** (1991) 2472.

[11] N. Kaiser, T. Waas and W. Weise, Nucl. Phys. A **612** (1997) 297 [arXiv:hep-ph/9607459].

[12] E. Oset and A. Ramos, Nucl. Phys. A **679**, 616 (2001) [arXiv:nucl-th/0005046].

[13] E. Oset and M.J. Vicente-Vacas, Nucl. Phys. A**454** (1986) 637-652.

[14] J. Nieves, E. Oset and C. Garcia-Recio, Nucl. Phys. A**554** (1993) 554-579.

[15] D. Cabrera, E. Oset, and M.J. Vicente-Vacas, Nucl. Phys. A**705** (2002) 90.

[16] A. Hosaka, T. Hyodo, F. J. Llanes-Estrada, E. Oset, J. R. Pelaez and M. J. Vicente Vacas, Phys. Rev. C, in print, arXiv:hep-ph/0411311.

[17] K. Imai at the SNP04 Conference, Osaka, July 2004.

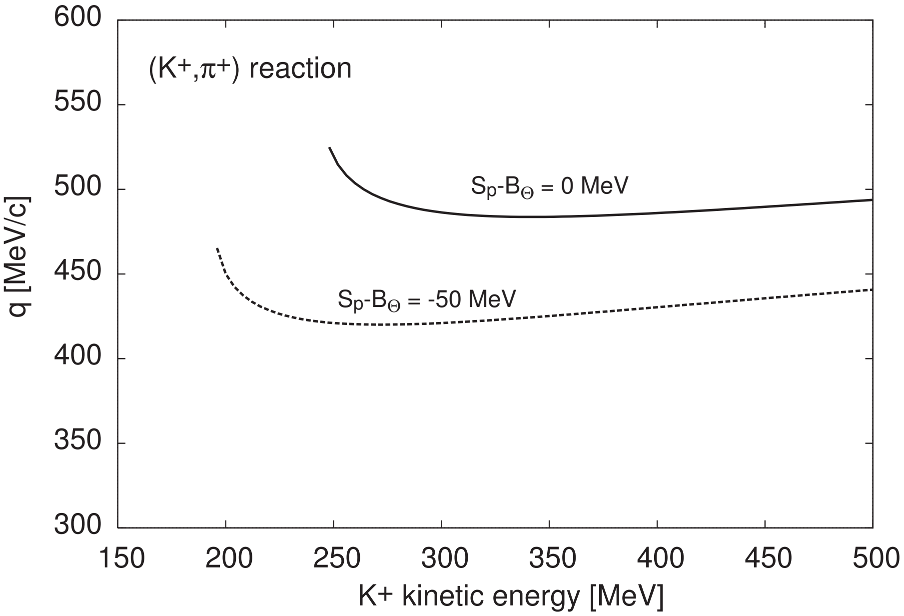

Figure 1: Momentum transfer of the (K^+,π^+) reaction for the formation of the Θ^+ nuclear states plotted as a function of the incident kaon kinetic energy. The solid line shows the result with $S_p - B_\Theta = 0$ and the dashed line with $S_p - B_\Theta = -50$ MeV, where S_p and B_Θ are the proton separation energy and Θ binding energy, respectively.

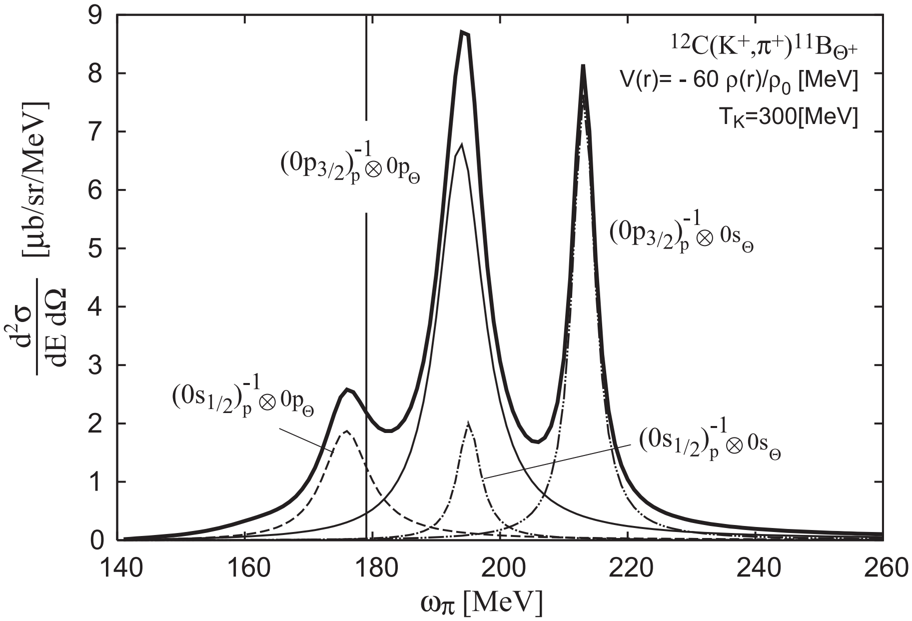

Figure 2: Calculated Θ bound states formation cross section shown as a function of the emitted pion energy ω_π at forward angles for a ^{12}C target. The incident kaon kinetic energy, T_K, is 300 MeV, and the shallow Θ nuclear potential $V(r) = -60\rho(r)/\rho_0$ MeV is used. The total spectrum is shown by the thick-solid line and the dominant subcomponents are also shown by the thin lines as indicated in the figures. The Θ production threshold leaving the residual nucleus in its ground state is shown by the vertical line.

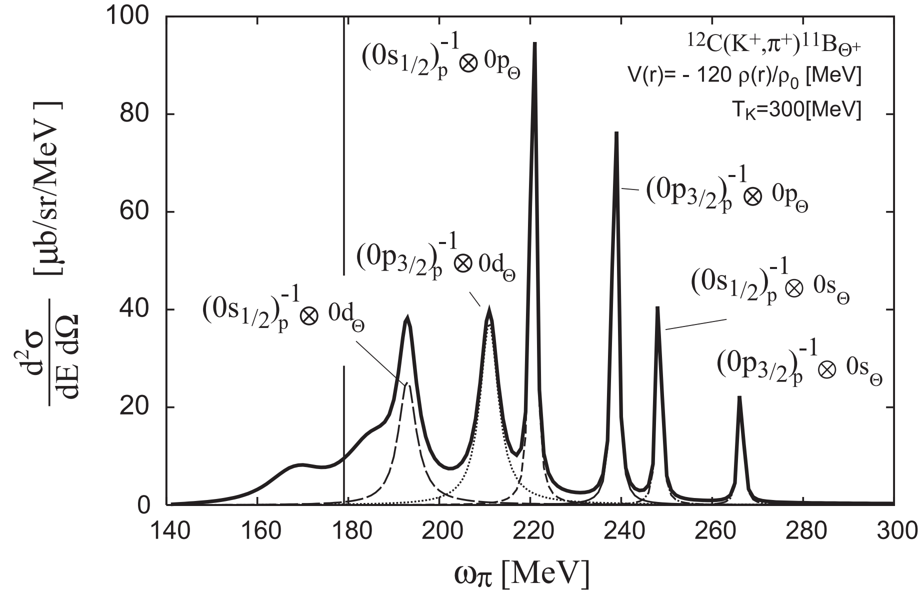

Figure 3: Calculated Θ bound states formation cross section shown as a function of the emitted pion energy ω_π at forward angles for a ^{12}C target. The incident kaon kinetic energy, T_K, is 300 MeV, and the deep Θ nuclear potential $V(r) = -120\rho(r)/\rho_0$ MeV is used. The total spectrum is shown by the thick-solid line and the dominant subcomponents are also shown by the thin lines as indicated in the figure. The Θ production threshold leaving the residual nucleus in its ground state is shown by the vertical line.