Θ^+ Hypernuclei

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Abstract

We present results for the selfenergy of the Θ^+ pentaquark in nuclei associated with two sources: the KN decay of the Θ^+ and the two meson baryon decay channels of the Θ^+ partners in an antidecuplet of baryons. The first source is shown to produce a small potential, unable to bind the Θ^+ in nuclei, while the second source gives rise to a large attractive potential. At the same time we show that the width of the Θ^+ in nuclei is small, such that, in light and medium nuclei, many bound Θ^+ states would appear with a separation between levels appreciably larger than the width of the states, thus creating an ideal scenario for pentaquark spectroscopy in nuclei.

1 Introduction

The physics of hypernuclei, Λ, Σ, Ξ is one of the active branches of nuclear physics with steady progress at the experimental and theoretical levels [1–7]. It has brought information on the ΛN interaction, the $\Lambda N \to NN$ weak transition, interesting examples of drastic Pauli blocking effects in the $\Lambda \to \pi N$ decay in nuclei, one of the cleanest examples of the accuracy of the mean field approximation in the case of Λ hypernuclei, a striking example of medium effects with increases of a factor fifty or more in the mesonic Λ decay width due to the interaction of the pion with the nucleus and other topics. So far only hypernuclei with strangeness -1 or -2 have been formed. The discovery of an exotic baryon with positive strangeness, Θ^+ [8] (see also Ref. [9] for a list of experimental and theoretical related works), opens new possibilities of forming exotic Θ^+ hypernuclei which, like in the case of negative strangeness hypernuclei, can provide information unreachable or complementary to that obtained in elementary reactions.

Suggestions that Θ^+ could be bound in nuclei have already been made. In Ref. [10] a schematic model for quark-pair interaction with nucleons was used to describe the Θ^+, which suggested that Θ^+ hypernuclei, stable against strong decay, may exist. In Ref. [11] the Θ^+ selfenergy in the nuclei is calculated, based on the $\Theta^+ \to KN$ decay mode, Pauli

blocking and a mass modification of the nucleon in the nuclear matter. The resulting selfenergy is too weak to bind the Θ^+ in nuclei.

In the present work we redo the calculations of Ref. [11] modifying the assumption of a strong shift of the nucleon mass and renormalizing the kaon cloud in the nucleus. The results are qualitatively similar to those of Ref. [11] and a small potential is obtained from this source. As a novelty, we also evaluate the imaginary part of the potential and show that the Θ^+ width becomes smaller for possible nuclear bound states and would be narrow enough to allow distinct peaks to be seen experimentally, provided some large attraction is obtained from other source. This is the other issue we deal with in this work. Indeed, we show that the in-medium renormalization of the pion in the two meson cloud of the Θ^+ leads to a sizable attraction, enough to produce a large number of bound and narrow Θ^+ states in nuclei.

The coupling of the Θ^+ to two mesons and a nucleon is studied in Ref. [12] where, with the assumption that the $N^*(1710)$ resonance has a large component in the same antidecuplet as the Θ^+, two terms of a $SU(3)$ symmetric Lagrangian are constructed to account for the $N^* \to N$ ($\pi\pi$, $p-$wave, $I = 1$) and $N^* \to N$ ($\pi\pi$, $s-$wave, $I = 0$) partial decay widths of the $N^*(1710)$. With this Lagrangian an attractive selfenergy is obtained for all the members of the antidecuplet coming from the two meson cloud.

In this work we study the nuclear medium effects on the Θ^+ selfenergy diagrams derived from this $\Theta^+ \to K\pi N$ Lagrangian. This is accomplished by modifying the pion, kaon and nucleon propagators in the nuclear medium. We find quite a large attractive potential of the Θ^+ which leads to bound states even for light nuclei. We also investigate a new source of Θ^+ decay width, namely $\Theta^+ \to NKph$, where the ph (particle-hole) comes from the absorption of a virtual pion, and we find it to be rather small. Altogether, the total in-medium Θ^+ width is much smaller than the separation of the deeper Θ^+ energy levels that we obtain for most nuclei, which could open the grounds for Θ^+ spectroscopy in nuclei.

2 The Θ^+ selfenergy from KN decay channel

We begin by evaluating the selfenergy of the Θ^+ related to the KN decay channel in the medium. The Θ^+ selfenergy diagram is depicted in Fig. 1.

We assume first $I = 0$ and $J^P = \frac{1}{2}^-$ for the Θ^+. This implies an $L = 0$ coupling to KN. The KN state in $I = 0$ is

$$|KN, I = 0 >= \frac{1}{\sqrt{2}}(|K^+n > -|K^0p >).\tag{1}$$

The $\Theta^+ KN$ couplings in this case are

$$-i\,t_{\Theta^+K^+n} = -ig_{K^+n}; \quad -i\,t_{\Theta^+K^0p} = ig_{K^+n}.\tag{2}$$

For the $I = 0$, $L = 1$, $J^P = \frac{1}{2}^+$ case, the quantum numbers of the antidecuplet suggested in Ref. [13], we would have

$$-i\,t_{\Theta^+K^+n} = -\bar{g}_{K^+n}\vec{\sigma}\vec{q}; \quad -i\,t_{\Theta^+K^0p} = \bar{g}_{K^+n}\vec{\sigma}\vec{q},\tag{3}$$
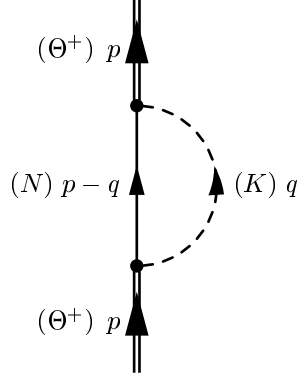
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Figure 1: Θ^+ selfenergy diagram related to the KN decay channel.

with q the outgoing kaon momentum. This amplitude is a nonrelativistic reduction of the relativistic vertices used in Ref. [11]. We have also done the complete relativistic calculations and the differences are negligible.

In the case of $L = 1$ we could also have $J^P = \frac{3}{2}^+$ and the couplings are written in terms of the corresponding spin transition operators, but it is easy to see, following the steps of [14], that the results for the selfenergy would be the same as in the $J^P = \frac{1}{2}^+$ case. Similarly, we could also assume $I = 1$, which would only change the relative sign of the KN components in Eq. (1), which appear squared in the selfenergy. Thus, the Θ^+ selfenergy does not change by assuming $I = 0$ or 1. We have only two independent cases, $L = 0$ and $L = 1$, which we evaluate below.

For the $L = 0$ case the free Θ^+ selfenergy from the diagram in Fig. 1 is given by

$$-i\Sigma_{KN}(p) = 2 \int \frac{d^4q}{(2\pi)^4} (-ig_{K^+n})^2 \frac{M}{E_N(\vec{p}-\vec{q})} \frac{i}{p^0 - q^0 - E_N(\vec{p}-\vec{q}) + i\epsilon} \frac{i}{q^2 - m_K^2 + i\epsilon}, \quad (4)$$

where M is the nucleon mass, $E_N(k) = \sqrt{M^2 + \vec{k}^2}$, and the factor 2 accounts for the K^+n and K^0p channels, which leads to a Θ^+ decay width

$$\Gamma = -2 \operatorname{Im} \Sigma_{KN} = \frac{g_{K^+n}^2}{\pi} \frac{q_{on}}{M_{\Theta^+}}, \quad (5)$$

where q_{on} is the momentum of the kaon in the $\Theta^+ \to KN$ decay. The result for $L = 1$ is obtained by the substitution

$$g_{K^+n}^2 \to \bar{g}_{K^+n}^2 \vec{q}^{\,2},$$

and hence

$$\Gamma = \frac{\bar{g}_{K^+n}^2}{\pi} \frac{q_{on}^3}{M_{\Theta^+}}. \quad (6)$$

We proceed now to evaluate the Θ^+ selfenergy in an infinite nuclear medium with density ρ. First, the nucleon propagator changes in the following way,

$$\frac{1}{p^0 - q^0 - E_N(\vec{p}-\vec{q}) + i\epsilon} \to \frac{1 - n(\vec{p}-\vec{q})}{p^0 - q^0 - E_N(\vec{p}-\vec{q}) + i\epsilon} + \frac{n(\vec{p}-\vec{q})}{p^0 - q^0 - E_N(\vec{p}-\vec{q}) - i\epsilon}, \quad (7)$$
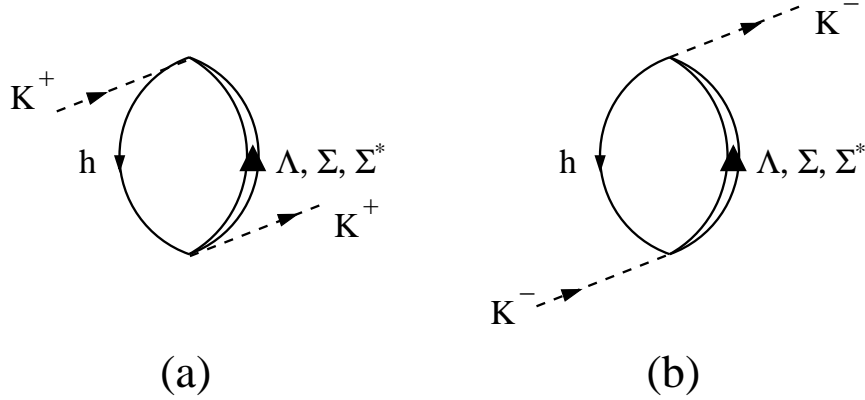
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Figure 2: In-medium kaon $p-$wave selfenergy diagrams: (a) K crossed term; (b) $\bar{K}$ direct term.

where $n(\vec{k})$ is the occupation number of the uncorrelated Fermi sea. On the other hand, the vacuum kaon propagator is replaced by the in-medium one,

$$\frac{1}{q^2 - m_K^2 + i\epsilon} \rightarrow \frac{1}{q^2 - m_K^2 - \Pi_K(q, \rho)}, \tag{8}$$

where $\Pi_K(q^0, |\vec{q}|, \rho)$ is the kaon selfenergy which accounts for $s-$wave and $p-$wave KN interaction. The $s-$wave part of the self energy is well approximated by [15, 16]

$$\Pi_K^{(s)}(\rho) = 0.13\, m_K^2 \rho/\rho_0 \quad [\text{MeV}^2]\,, \tag{9}$$

where ρ_0 is the normal nuclear density. The $p-$wave part is taken such that

$$\Pi_{K^+}^{(p)}(q^0, |\vec{q}|, \rho) = \Pi_{K^-}^{(p)}(-q^0, |\vec{q}|, \rho)\,, \tag{10}$$

and for $\Pi_{K^-}^{(p)}$ we take the model of Refs. [16,17] which accounts for Λh, Σh and $\Sigma^*(1385)h$ excitations, see Fig. 2. Since the $p-$wave selfenergy of the K^+ involves crossed terms of the Yh excitation, this part is small and there is practically no q^0 dependence in the K^+ selfenergy, which makes the quasiparticle approximation accurate. Hence, the pole structure of the free K^+ propagator, with poles in $q^0 = \pm\omega(q) \mp i\epsilon$, is substituted by a similar one with the poles shifted to $\widetilde{\omega}(q)$, such that

$$\widetilde{\omega}(q)^2 - \vec{q}^{\,2} - m_K^2 - \Pi_K(\widetilde{\omega}(q), |\vec{q}|, \rho) = 0\,. \tag{11}$$

This equation is solved selfconsistently and the result is very close to

$$\widetilde{\omega}(q) \simeq \sqrt{m_K^2(1 + 0.13\rho/\rho_0) + \vec{q}^{\,2}}\,. \tag{12}$$

In view of this, the q^0 integration is performed in the modified Eq. (4) leading to

$$\Sigma_{KN}(p^0, \vec{p}; \rho) =$$

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$$
\begin{aligned}
= & \frac{M_\Theta \Gamma}{M q_{on}} \frac{1}{(2\pi)^2} \int d^3 q \frac{M}{E_N(\vec{p}-\vec{q})} \mathcal{F}_L(q) \frac{1}{2\widetilde{\omega}(q)} \frac{1}{p^0 - \widetilde{\omega}(q) - E_N(\vec{p}-\vec{q}) - V_N + i\epsilon} \\
& - \frac{M_\Theta \Gamma}{M q_{on}} \frac{1}{(2\pi)^2} \int d^3 q \frac{M}{E_N(\vec{p}-\vec{q})} \mathcal{F}_L(q) \frac{n(\vec{p}-\vec{q})}{[p^0 - E_N(\vec{p}-\vec{q}) - V_N]^2 - \vec{q}^2 - m_K^2 - \Pi_K(q, \rho)} \tag{13}
\end{aligned}
$$

with $q^0 = p^0 - E_N(\vec{p}-\vec{q}) - V_N$, $V_N = -\frac{k_F^2}{2M}$, and $\mathcal{F}_0 = 1$, $\mathcal{F}_1 = \frac{\vec{q}^2}{q_{on}^2}$. In Eq. (13) we have also taken into account the nucleon binding and, consistently with the posterior use of the results within the local density approximation ($\rho \to \rho(r)$ in the nucleus), we have taken the Thomas-Fermi potential for the nucleons, $V_N = -k_F(r)^2/2M$ with $k_F(r) = (3\pi^2 \rho(r)/2)^{1/3}$. For the calculations we have taken an average value of the momentum of the Θ^+ in eventual bound states of $p = 200$ MeV, similar to that of bound nucleons in nuclei.

The consideration of the Θ^+ momentum is important for the imaginary part of Σ_{KN}, and hence for the width of the Θ^+ in the nucleus. This can be easily understood. Consider a Θ^+ at rest with 1540 MeV of energy decaying into $K^+ n$. The momentum of the neutron is 269.6 MeV/c. Taking $\rho = \rho_0$, $k_F = 269$ MeV/c and hence the decay would be allowed. However, if the Θ^+ has a small initial momentum, it is clear that when boosting the neutron momentum from the Θ^+ rest frame to the Θ^+ moving frame, approximately half of the events would lead to $p_n > k_F$ while the other half would lead to $p_n < k_F$, hence reducing the Θ^+ width to one half its free value.

This result is roughly valid for any momentum of the Θ^+ smaller than k_F. We should expect a reduction by a factor around two of the Θ^+ width only from this source. On the other hand, if the Θ^+ energy is smaller because it is in a bound state, then the width is further reduced. We can see this in Figs. 3, 4 where we assume that $\Gamma = 15$ MeV. This is the upper limit in most experiments coming basically from the experimental resolution. Studies based on $K^+ N$ scattering suggest that the width should be smaller than 5 MeV [18–20] or even of the order of 1 MeV [21–23]. What we see in these figures is that even if $\Gamma = 15$ MeV in free space, inside the nucleus, particularly for the case of $L = 1$ which corresponds to $J^P = \frac{1}{2}^+$ for the Θ^+, the width is small; and for 20 MeV of Θ^+ binding the width would go down from 15 MeV to less than 6 MeV. This width could be reasonably smaller than the separation between different bound levels.

Next, we consider the real part of the Θ^+ selfenergy, shown in Fig. 5, after subtracting the vacuum selfenergy from Eq. (4), for a typical finite momentum of 200 MeV. The subtraction is convergent for the $L = 0$ case, whereas for $L = 1$ we use a cut-off in the momentum of the particles in the loop. As shown in the figure, the cut-off dependence is small. We find, in qualitative agreement with Ref. [11], that the Θ^+ potential in the medium is very small. Note however that the results are not directly comparable since we present the real part of the selfenergy, instead of the in-medium Θ^+ mass change presented in Ref. [11]. According to Eq. (13) the selfenergy scales like Γ, which we have taken as 15 MeV for the results shown for both the real and imaginary parts of the selfenergy and if, as suggested in [21–23], the width is of the order of 1 MeV, the in-medium selfenergy associated to the KN decay channel would be negligible. In any case, up to $\rho = \rho_0$ the real part of the selfenergy is not enough to bind Θ^+ in nuclei.

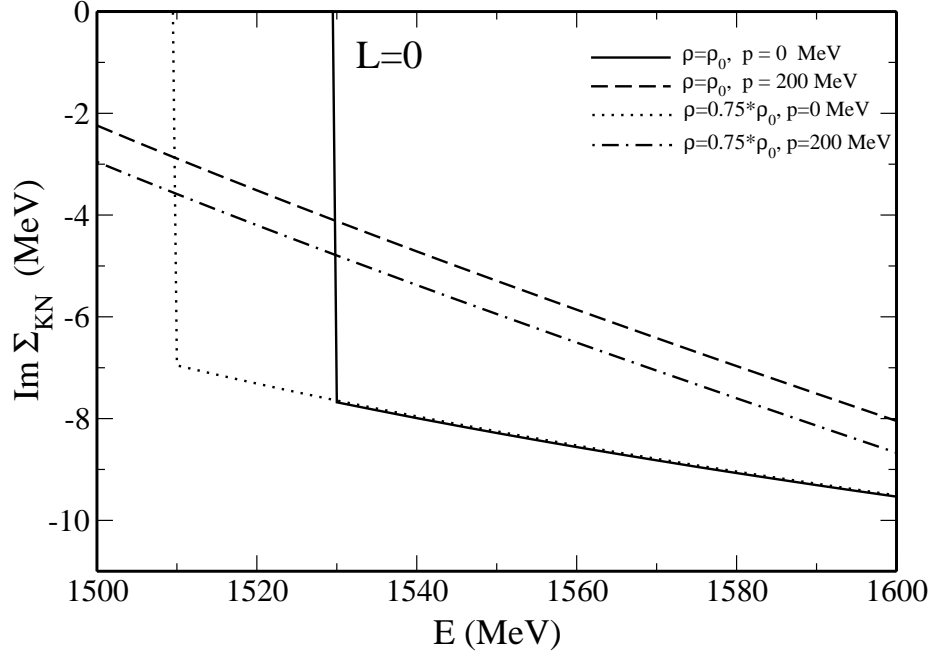In the next section we investigate another source of attraction which leads to larger

Figure 3: Imaginary part of the Θ^+ selfenergy associated to the KN decay channel for $L = 0$.
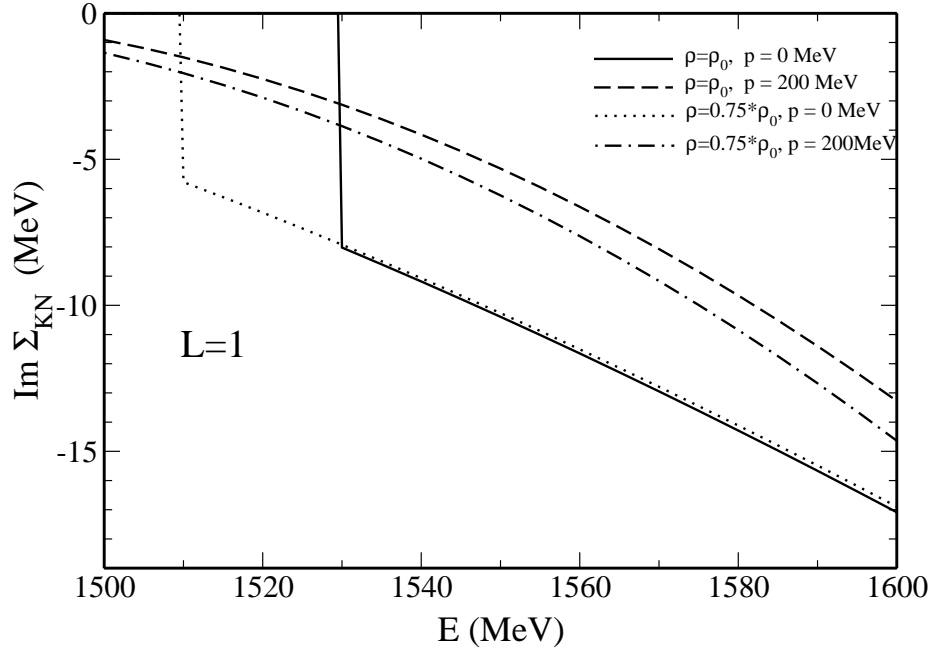

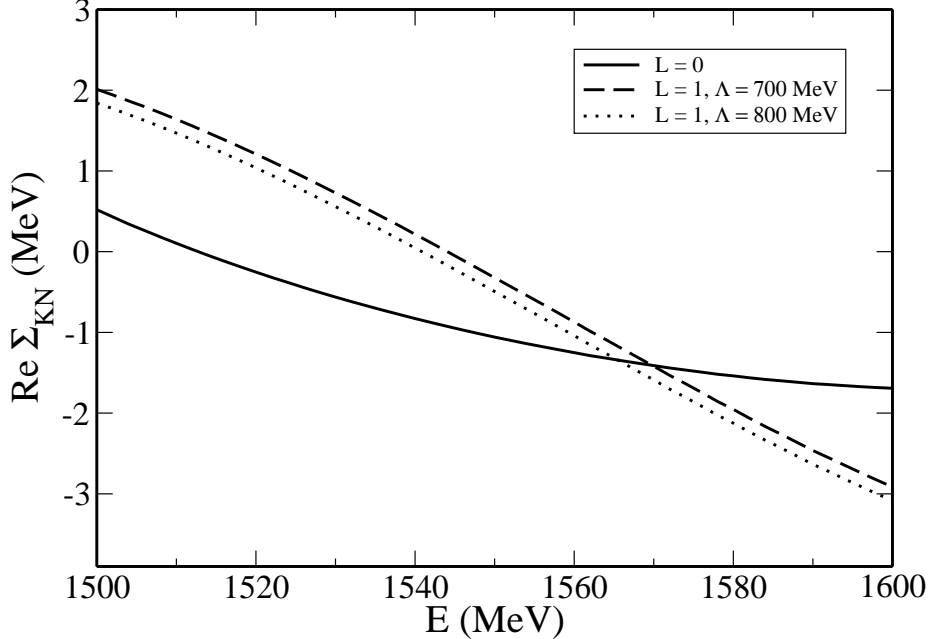Figure 4: Imaginary part of the Θ^+ selfenergy associated to the KN decay channel for $L = 1$.

Figure 5: Real part of the Θ^+ selfenergy associated to the KN decay channel at $\rho = \rho_0$. A momentum of the Θ^+ of 200 MeV is taken.

attractive potentials.

3   The Θ^+ selfenergy tied to the two-meson cloud

In this section we will study contributions to the Θ^+ selfenergy from diagrams in which the Θ^+ couples to a nucleon and two mesons, like the one in Fig. 6. There is no direct information on these couplings since the Θ^+ mass is below the two-meson decay threshold.

From now on we will do several assumptions. The validity of our results depends on them. First, the Θ^+ is assumed to have $J^P = 1/2^+$ associated to an $SU(3)$ antidecuplet, as in Ref. [13]. In addition, the $N^*(1710)$ is supposed to couple largely to this antidecuplet.

From the data on $N^*(1710)$ decays we can determine the couplings to the two-meson channels, and using $SU(3)$ symmetry obtain the corresponding couplings for the Θ^+.

In Ref. [12] two $SU(3)$ symmetric Lagrangian terms, with minimal number of derivatives in the meson fields, are proposed in order to account for the $N^*(1710)$ decay into $N(\pi\pi, p-\text{wave}, I = 1)$ and $N(\pi\pi, s - \text{wave}, I = 0)$. The first term is

$$\mathcal{L}_1 = ig_{\overline{10}}\epsilon^{ilm}\bar{T}_{ijk}\gamma^\mu B_l^j(V_\mu)_m^k, \tag{14}$$

with V_μ the vector current which for two mesons is written as

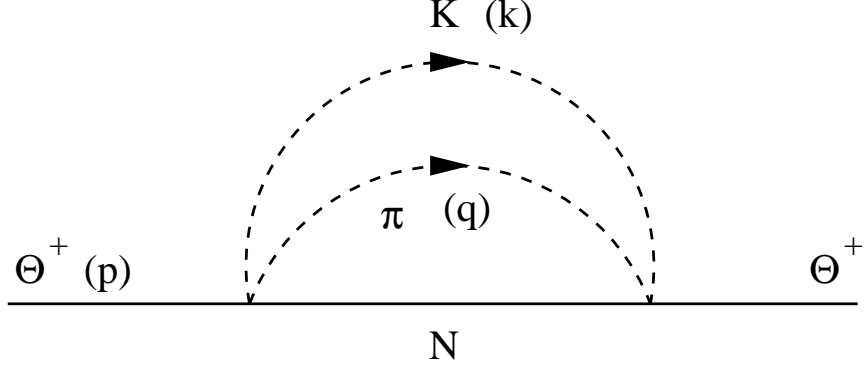$$V_\mu = \frac{1}{4f^2}(\phi\partial_\mu\phi - \partial_\mu\phi\phi), \tag{15}$$

Figure 6: Two-meson Θ^+ selfenergy diagram.

with $f = 93$ MeV the pion decay constant and T_{ijl}, B_l^j, ϕ_m^k $SU(3)$ tensors which account for the antidecuplet states, the octet of $\frac{1}{2}^+$ baryons and the octet of 0^- mesons, respectively [24]. The second term is given by

$$\mathcal{L}_2 = \frac{1}{2f} \tilde{g}_{\overline{10}} \epsilon^{ilm} \bar{T}_{ijk} (\phi \cdot \phi)_l^j B_m^k \,, \tag{16}$$

which couples two mesons in $L = 0$ to the antidecuplet and the baryon and they are in $I = 0$ for the case of two pions. From the Lagrangian terms of Eqs. (14, 16) one can obtain, after some $SU(3)$ algebra, the transition amplitudes from any member of the antidecuplet to the different MMB channels to which it couples, in particular $N^* \to \pi\pi N$. Taking the central values from the PDG [25] for the $N^*(1710) \to N(\pi\pi, p-\text{wave}, I = 1)$ (which we take from the ρN fraction of the $N\pi\pi$ decay) and for the $N^*(1710) \to N(\pi\pi, s - \text{wave}, I = 0)$, the resulting coupling constants are $g_{\overline{10}} = 0.72$ and $\tilde{g}_{\overline{10}} = 1.9$.

The Θ^+ selfenergy associated to the diagram of Fig. 6 is given by

$$\Sigma^{(V)}(p) = 18 \, \Sigma^V(p; K\pi N) + 18 \, \Sigma^V(p; K\eta N) \,,$$
$$\Sigma^{(S)}(p) = 18 \, \Sigma^S(p; K\pi N) + 2 \, \Sigma^S(p; K\eta N) \,, \tag{17}$$

with

$$\Sigma^j(p) = -\int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} |t^j|^2 \frac{1}{k^2 - m_1^2 + i\epsilon} \frac{1}{q^2 - m_2^2 + i\epsilon}$$
$$\frac{M}{E_N(\vec{k} + \vec{q})} \frac{1}{p^0 - k^0 - q^0 - E_N(\vec{k} + \vec{q}) + i\epsilon} \,, \tag{18}$$

where $j \equiv V, S$ and m_1, m_2 are the masses of the mesons in the loop ($K\eta$, $K\pi$). Eq. (18) stands for the Θ^+ selfenergy at rest. In contrast to the KN decay channel, the dependence on the Θ^+ momentum is not relevant. The t^j amplitudes in Eq. (18) are given by

$$|t^S|^2 = \left(\frac{\tilde{g}_{\overline{10}}}{2f} \right)^2 \,,$$

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$$|t^V|^2 = \left(\frac{g_{\bar{1}0}}{4f^2}\right)^2 \frac{1}{2M}\{[E_N(\vec{k}+\vec{q})+M][\omega_1(k)-\omega_2(q)]^2$$
$$+2(\vec{k}^2-\vec{q}^2)[\omega_1(k)-\omega_2(q)]+[E_N(\vec{k}+\vec{q})-M](\vec{k}-\vec{q})^2\} \ . \tag{19}$$

The implementation of the medium effects is done by including the medium selfenergy of the kaon and modifying the nucleon propagator, as done in Section 2. On the other hand, the pion being so light requires a more careful treatment and here we use, as normally done [26–32], the $p-$wave selfenergy from ph and Δh excitation. It is convenient to write the pion propagator in terms of its Lehman representation [27] and we have

$$\frac{1}{q^2-m_\pi^2-\Pi_\pi(q^0,\vec{q},\rho)} = \int_0^\infty d\omega \ 2\omega \ \frac{S_\pi(\omega,\vec{q},\rho)}{q^{02}-\omega^2+i\epsilon} \ , \tag{20}$$

where $S_\pi(\omega,\vec{q},\rho)$ is the pion spectral function

$$S_\pi(\omega,\vec{q},\rho) = -\frac{1}{\pi}\frac{\mathrm{Im}\ \Pi_\pi(\omega,\vec{q},\rho)}{|\omega^2-\vec{q}^2-m_\pi^2-\Pi_\pi(\omega,\vec{q},\rho)|^2} \ . \tag{21}$$

By performing the energy integrals in Eq. (18) after the medium effects are incorporated, we get for the $K\pi N$ intermediate channel the following results

$$\Sigma^j(p) = \int \frac{d^3k}{(2\pi)^3}\int \frac{d^3q}{(2\pi)^3}\int_0^\infty d\omega \ S_\pi(\omega,\vec{q},\rho)\left|t^j\right|^2$$
$$\frac{1}{2\tilde{\omega}(k)}\frac{M}{E_N(\vec{k}+\vec{q})}\frac{1-n(\vec{k}+\vec{q})}{p^0-\tilde{\omega}(k)-\omega-E_N(\vec{k}+\vec{q})-V_N+i\epsilon} \ . \tag{22}$$

The $K\eta$ channel contributes little to the Θ^+ selfenergy and since the changes of η in the medium are very small compared to those of the pion, we disregard this channel to account for the medium contributions.

Once the Θ^+ selfenergy at a density ρ is evaluated, the optical potential felt by the Θ^+ in the medium is obtained by subtracting the free Θ^+ selfenergy, and hence

$$\tilde{\Sigma}(p) = \Sigma(p,\rho)-\Sigma(p,\rho=0) \ . \tag{23}$$

We should also note that while the $\Theta^+ \to K\pi N$ decay is forbidden, in the medium the π can lead to a ph excitation and this opens a new decay channel $\Theta^+N \to NNK$, which is open down to 1432 MeV, quite below the free Θ^+ mass. We will show that the width from this channel is also very small, but should the Θ^+ free width be of the order of 1 MeV as suggested in [22,23] the new decay mode would make the width in the medium larger than the free width.

The integral of Eq. (22) is regularized by means of a cut-off. We use a cut-off of around 700-800 MeV by means of which reasonable results for the vacuum selfenergy of the antidecuplet, of the order of $7-15$ % of the antidecuplet masses, are obtained[1]. More

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[1]In a recent paper where a QCD sum rule is used to obtain the mass of an $S=1$, $I=0$, $J^P=\frac{1}{2}^+$ state made from two diquarks and one antiquark, central masses around 1.64 GeV are obtained [33] which welcome an extra attractive contribution of $100-150$ MeV from the MMB components (heptaquark) as we find.
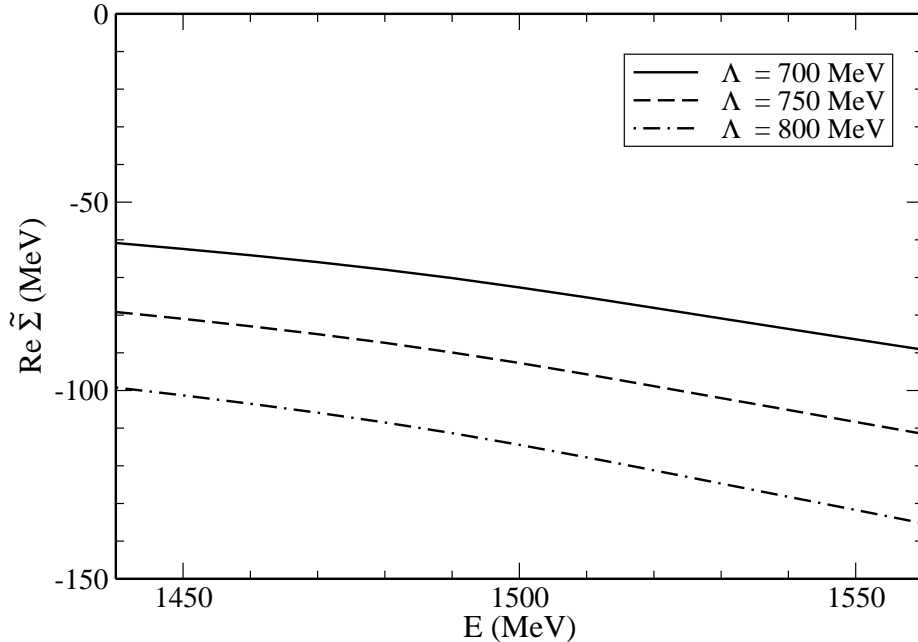
Figure 7: Real part of the two-meson contribution to the Θ^+ selfenergy at $\rho = \rho_0$.

specifically for $p^0 = 1540$ MeV and $\Lambda = 700$ MeV we find a contribution to the free Θ^+ selfenergy of 48 MeV from the scalar Lagrangian and 40 MeV from the vector Lagrangian. We find here that the additional attraction of the Θ^+ in the medium at $\rho = \rho_0$ is of the same order of magnitude as the binding created by the mechanism considered in the Θ^+ mass. Similar conclusions were found in the work of [10] where, although the formalisms used are quite different, the works share some basic features, like the small coupling of the Θ^+ to KN and a sizeable coupling to $KN\pi$, with $K\pi$ with K^* quantum numbers via the vector Lagrangian, which in [10] is realized by a large coupling to K^*N. In Ref. [12] a thorough study is done of the free selfenergy of all states of the antidecuplet due to the two meson cloud, using the same Lagrangians as here as well as other ones which are allowed by $SU(3)$ symmetry considerations. Several constraints from phenomenology lead to the Lagrangians which we use here as the leading ones. The vector Lagrangian is also further modified in [12] to account for the decay of the $N^*(1710) \to N(\pi\pi, p - \text{wave})$ into the actual $N\rho$ channel quoted in the PDG. This reduces the strength of the vector Lagrangian contribution, and hence the numbers obtained here for the binding would be reduced by about 20% from these corrections.

We present the results in Figs. 7 and 8. From Fig. 7 we can see that the potential for $\rho = \rho_0$ is sizable and attractive and goes from -70 MeV using a cut-off of 700 MeV to -120 MeV using 800 MeV.

Even with the large uncertainties we conclude that there is a sizable attraction of the order of magnitude of 50-100 MeV at normal nuclear density, which is more than enough to bind the Θ^+ in any nucleus. In Fig. 8 we show the imaginary part of the Θ^+ selfenergy
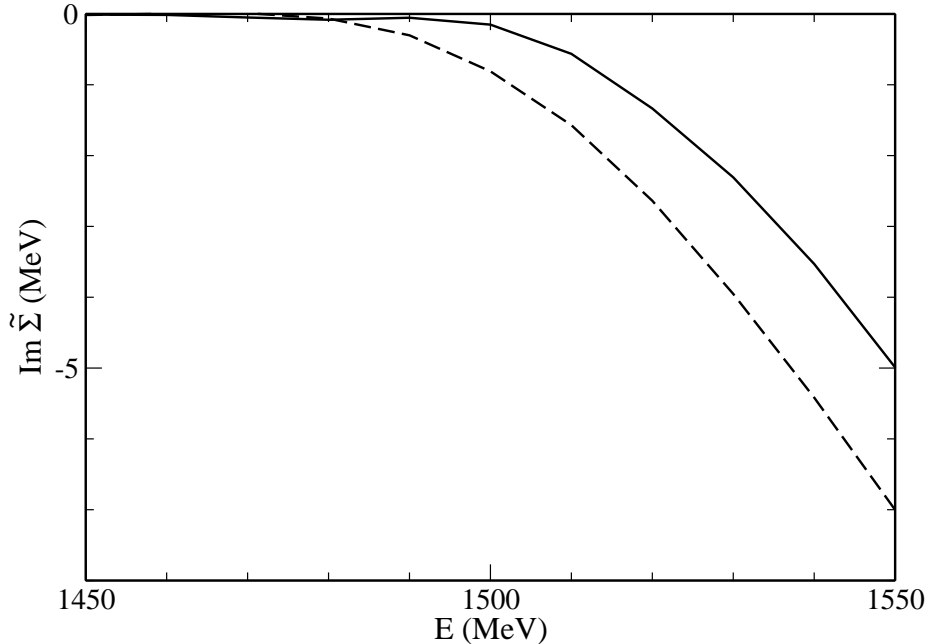
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Figure 8: Imaginary part of the two-meson contribution to the Θ^+ selfenergy at $\rho = \rho_0$, using the Thomas-Fermi potential (solid line) or $V_N(r) = -50\rho(r)/\rho_0$ [MeV] (dashed line) for the nucleon potential.

related to the two-meson decay mechanism for two different nucleon potentials in the nucleus discussed below. We can see that $\Gamma = -2\,\mathrm{Im}\,\Sigma$ would be smaller than 5 MeV for bound states with a binding of $\sim$20 MeV and negligible for binding energies of $\sim$40 MeV or bigger. This, together with the small widths associated to the KN decay channel, would lead to Θ^+ widths below 8 MeV, assuming a free width of 15 MeV, and much lower if the Θ^+ free width is of the order of 1 MeV. In any case, for most nuclei, this width would be smaller than the separation of the deep levels.

The calculations done here are performed for infinite nuclear matter. In order to apply the results to finite nuclei we resort to use the local density approximation, $\rho \rightarrow \rho(r)$, with $\rho(r)$ the realistic density distribution in the nucleus, which is shown to be a good approximation in [34].

In order to illustrate the point about width and separation of levels we solve the Schrödinger equation with two potentials: $V(r) = -120\rho(r)/\rho_0$ (MeV), and $-60\rho(r)/\rho_0$ (MeV). The density $\rho(r)$ is taken from experiment [35] for several nuclei. The results are shown in Table 3. In a light nucleus like ^{12}C we find several bound states separated by around 20 MeV or more with both potentials. For medium and heavy nuclei, as in ^{40}Ca shown in the table, we find more bound states and the energy separation is somewhat smaller.

It is important to remark that the separation of the deep states is reasonably bigger than the upper bounds estimated for the width of these states obtained by considering the

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KN and the $K\pi N$ decay channels in the medium. This would make a clear case for the experimental observation of these states.

We have considered two other sources of uncertainty in the calculations. In the first place we have also included the pion selfenergy due to $2p2h$ excitation which leads to pion absorption. The pion in the loop in Fig. 6 cannot be put on shell simultaneously with the K and the nucleon. The pion can excite a ph and this gives the K^+Nph decay mode of the Θ^+ which we have studied. The pion can also lead to $2p2h$ excitation and this would give a new decay mode, K^+N2p2h. Since this is a $\mathcal{O}(\rho^2)$ correction compared to the $\mathcal{O}(\rho)$ contribution of the ph decay channel, we can think a priori that the $2p2h$ channel will be less relevant than the ph one. The situation is reminiscent of the one nucleon induced and two nucleon induced Λ decay in nuclei [36, 37] where the $\Lambda \to N\pi$ decay is also forbidden by Pauli blocking. There one finds [37] that the two nucleon induced decay represents a fraction smaller that 20% of the ph one. In the present case there is even less energy left for the pion as an average than in Λ decay, and Pauli blocking is also more effective, thus we should expect smaller results. This is indeed the case as we find in actual calculations. For this purpose we include the selfenergy given in [37],

$$\Pi_\pi^{2p2h} = -4\pi \vec{q}^{\,2} \mathcal{C}_0^* \rho^2 \ , \tag{24}$$

with $\mathcal{C}_0^* = (0.105 + i0.096)m_\pi^{-6}$, which is obtained from pionic atoms and is modified in [37] to account for the different phase space offered by the off-shell pions which we find in the present case. The result of the calculation is that the strength of the real part of the Θ^+ selfenergy in the medium decreases by a few MeV and the imaginary part increases less than 5%. Hence, the effect of including this new decay channel is negligible considering the large uncertainty from other sources.

The other element considered has to do with the nucleon binding. In Eq. (13) we took the Thomas-Fermi potential for the nucleon. Now we take a standard potential,

$$V_N = -V_0 \rho/\rho_0 \ ; \ V_0 = 50 \text{ MeV} \ , \tag{25}$$

and recalculate the results. The real part increases by a few MeV and the imaginary part, plotted in Fig. 8, shows some changes with respect to that obtained with the Thomas-Fermi potential, still leading to a width of only 5 MeV for 20 MeV binding.

4   Conclusions

We have evaluated the selfenergy of the Θ^+ in the nuclear medium associated to the KN decay channels and the MMB decay channels of the Θ^+ partners in the antidecuplet. We obtain a small potential associated to the KN decay, even assuming a large free width of around 15 MeV for the Θ^+, but at the same time we also show that Pauli blocking and the decreased phase space from the Θ^+ binding decrease appreciably the Θ^+ width in the nucleus from the KN decay.

On the other hand, we find a large attractive Θ^+ potential in the nucleus associated to the two meson cloud of the antidecuplet. A new decay channel opens for the Θ^+ in the

$V = -60$ MeV ρ/ρ_0		$V = -120$ MeV ρ/ρ_0	
E_i (MeV), ^{12}C	E_i (MeV), ^{40}Ca	E_i (MeV), ^{12}C	E_i (MeV), ^{40}Ca
-34.0 (1s)	-42.6 (1s)	-87.3 (1s)	-98.2 (1s)
-14.6 (1p)	-30.9 (1p)	-59.5 (1p)	-83.3 (1p)
-0.3 (2s)	-18.7 (1d)	-32.0 (2s)	-67.5 (1d)
	-17.9 (2s)	-31.9 (1d)	-65.9 (2s)
	-6.3 (1f)	-8.6 (2p)	-50.8 (1f)
	-5.6 (2p)	-5.6 (1f)	-48.5 (2p)
			-33.5 (1g)
			-31.1 (2d)
			-30.4 (3s)
			-15.9 (1h)
			-14.2 (2f)
			-13.8 (3p)
			-0.5 (4s)

Table 1: Binding energies of Θ^+ in ^{12}C and ^{40}Ca.

medium, $\Theta^+ N \to NNK$, but the width from this new channel, together with the one from KN decay, is still small compared to the separation of the bound levels of the Θ^+ in light and intermediate nuclei (very large nuclei like ^{208}Pb would have the states too packed to prove efficient in the detection of these states).

In reaching the former conclusions there are several assumptions done.

1. The Θ^+ is assumed to be $1/2^+$ associated to an $SU(3)$ antidecuplet;

2. The $N^*(1710)$ is supposed to couple largely to this antidecuplet;

3. The Lagrangians have been chosen to reproduce the $N(\pi\pi, p - \text{wave}, I = 1)$ and $N(\pi\pi, s - \text{wave}, I = 0)$ decay mode of $N^*(1710)$ by imposing $SU(3)$ symmetry with a minimal number of derivatives in the fields.

4. Some values of the cut off have been chosen to obtain reasonable numbers for the free Θ^+ selfenergy;

5. The average value of the $N^*(1710)$ width and the partial decay ratios, which experimentally have large uncertainties, have been taken to fix the couplings of the antidecuplet to the baryon octet and the two meson octets.

It is clear that with all these assumptions one must accept a large uncertainty in the results. So we can not be precise on the binding energies of the Θ^+. However, the order of magnitude obtained for the potential is such that even with a wide margin of uncertainty, the conclusion that there would be bound states is quite safe. In fact, with potentials with a strength of 20 MeV or less one would already get bound states. Furthermore, since the

strength of the real part and the imaginary part from the $NKph$ decay are driven by the same coupling, a reduction on the strength of the potential would also lead to reduced widths such that the principle that the widths are reasonably smaller than the separation between levels would be saved.

The work done here provides thus a sensible case in favor of the existence of bound Θ^+ states in nuclei which should spur experimental work in this area.

Acknowledgments

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