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Chiral perturbation theory study of the axial $N \to \Delta(1232)$ transition

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We have performed a theoretical study of the axial Nucleon to Delta(1232) ($N \to \Delta$) transition form factors up to one-loop order in covariant baryon chiral perturbation theory within a formalism in which the unphysical spin-1/2 components of the Δ fields are decoupled.

Keywords: $N \to \Delta$ transition form factors; neutrino-nucleon(nucleus) interaction; Chiral perturbation theory.

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1. Introduction

Nowadays, it is generally accepted that Quantum Chromodynamics (QCD) is the theory of the strong interaction. It has been very successful and tested to great precision at high energies; however, its application in the low energy region of ~ 1 GeV is quite problematic due to the large running coupling constant. The advent of chiral perturbation theory (χPT) and lattice QCD approach has made possible a model independent study of the low-energy strong phenomena for the first time.

Neutrino physics has made remarkable progress in recent years, as evidenced by the 2002 Nobel prize in physics (awarded partly to Raymond Davis Jr and Masatoshi Koshiba "for pioneering contributions to astrophysics, in particular for the detection of cosmic neutrinos"). After many years of experimental (and theoretical) efforts, two facts have been firmly established: (i) neutrino have masses and (ii) different flavors of neutrino can oscillate into each other. Presently, one of the main goals in the field is to measure accurately the masses and oscillation parameters. A good understanding of pion production is relevant to reduce systematic uncertainties in oscillation experiments. The axial nucleon to $\Delta(1232)$ transition, characterized by four form factors, plays an important role in this reaction at low Q^2 transfer.[1]

Most of our current (experimental) knowledge of the $N \to \Delta$ axial transition form factors comes from neutrino bubble chamber data.[2] The possibility to extract them using parity-violating electron scattering at Jefferson Lab has been extensively studied,[3] and could shed new light on the nature of these form factors. Present and

future neutrino experiments (MiniBoone, K2K, Fermilab) could also provide further information.

In the past, the theoretical descriptions have been done using different approaches, mostly quark models (for a review, see Ref. 4). In recent years, there has been an increasing interest on these form factors. They have been calculated, for instance, using the chiral constituent quark model and light cone QCD sum rules. State of the art calculations within lattice QCD have also become available.[5]

While the axial $N \rightarrow \Delta$ form factors have been addressed in (tree level) HBχPT,[6] *no calculation has been performed up to now within the relativistic framework*. With lattice QCD results becoming available and in view of the many ongoing experimental efforts to extract these form factors from electron- and neutrino-induced reactions, it is timely to study the axial $N \rightarrow \Delta$ transition form factors within covariant baryon χPT.[7]

2. Theoretical framework: covariant baryon χPT with explicit Δ's

The study of the $N \rightarrow \Delta$ transition form factors using covariant baryon χPT is much more complicated than it seems to be. To begin with, one has to address the following three questions: power counting, chiral Lagrangians, and the appropriate form of the Δ propagator.

(1) A proper power counting scheme is at the center of effective field theories. To include the $\Delta(1232)$ explicitly, one has to count the N-Δ mass difference $\Delta \equiv M_\Delta - M_N \sim 0.3\,\text{GeV}$ properly. In the present work, we adopt the δ expansion scheme, which counts $m_\pi/\Lambda_{\chi\text{SB}}$ as δ^2 to maintain the scale hierarchy $m_\pi \ll \Delta \ll \Lambda_{\chi\text{SB}}$.[8]

(2) The pion-nucleon and pion-pion Lagrangians are rather standard. The $N\Delta$ and $\Delta\Delta$ Lagrangians, on the other hand, require more attention. The $\Delta(1232)$ is a spin-3/2 resonance and, therefore, its spin content can be described in terms of the Rarita-Schwinger (RS) field, Δ_μ, where μ is the Lorentz index. This field, however, contains unphysical spin-1/2 components. In order to tackle this problem, we follow Ref. 9 and adopt the "consistent" couplings, which are gauge-invariant under the transformation

$$\Delta_\mu(x) \rightarrow \Delta_\mu(x) + \partial_\mu \epsilon(x). \tag{1}$$

(3) Different forms of the spin-3/2 propagator have been used in the literature, some of which may lead to serious theoretical problems.[10] Due to the spin-3/2 gauge symmetric nature of the consistent couplings, we can use the most general spin-3/2 free field propagator.[11]

A more detailed discussion of these issues and the relevant $N\Delta$ and $\Delta\Delta$ Lagrangians can be found in Ref. 7.
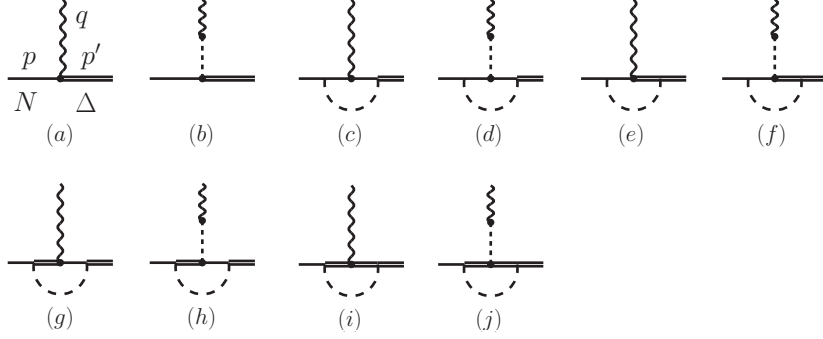
Fig. 1.   Feynman diagrams contributing to the $N \to \Delta$ axial transition form factors up to order $\delta^{(3)}$. The double, solid, and dashed lines correspond to the delta, nucleon, and pion, respectively; while the wiggly line denotes the external pseudovector source.

3. Results and Discussions

The $N \to \Delta$ axial transition form factors can be parametrized in terms of the usually called Adler form factors:[12,13]

$$\langle \Delta_\alpha^+(p')| - A^{\alpha\mu,3}|P(p)\rangle = \bar{\Delta}_\alpha^+(p')\bigg\{ \frac{C_3^A(q^2)}{M_N}\big(g^{\alpha\mu}\gamma\cdot q - q^\alpha\gamma^\mu\big)$$
$$+\frac{C_4^A(q^2)}{M_N^2}\big(q\cdot p' g^{\alpha\mu} - q^\alpha p'^\mu\big) + C_5^A(q^2)g^{\alpha\mu} + \frac{C_6^A(q^2)}{M_N^2}q^\alpha q^\mu\bigg\}N,$$

where $A^{\alpha\mu,3}$ is the third isospin component of the axial current.

In the δ expansion scheme, up to order $\delta^{(3)}$, all diagrams contributing to the axial $N \to \Delta$ transition form factors are displayed in Fig. 1. The corresponding results in terms of low-energy constants (LEC) and loop functions are summarized in Table 1.

We can easily see that at $\delta^{(1)}$, $C_5^A(0) = \sqrt{\frac{2}{3}}\frac{h_A}{2} \approx 1.16$, where h_A is the $\pi N\Delta$ coupling determined from the Δ width, which is close to the Kitagaki-Adler value[2] of 1.2. The Kitagaki-Adlder assumption $C_6^A = C_5^A\frac{M_N^2}{m_\pi^2-q^2}$, on the other hand, is satisfied only up to $\delta^{(2)}$, i.e., non pion-pole contributions appear at order $\delta^{(3)}$.[7]

Table 1.   The $N \to \Delta$ axial transition form factors in covariant baryon χPT; d_1, d_2, d_3, d_4 are order 2 LEC (in GeV^{-1}) while f_1, f_2, f_3, f_4, f_5, f_6, f_7 are order 3 LEC (in GeV^{-2}); $g_3(q^2)$, $g_4(q^2)$, $g_5(q^2)$, and $g_6(q^2)$ are the one-loop contributions as defined in Eq. (31) of Ref. 7.

FF	$\delta^{(1)}$	$\delta^{(2)}$	$\delta^{(3)}$
$-\sqrt{\frac{3}{2}}\frac{C_3^A(q^2)}{M_N}$	0	$-d_2$	$f_3\Delta + g_3(q^2)$
$-\sqrt{\frac{3}{2}}\frac{C_4^A(q^2)}{M_N^2}$	0	$-d_1/M_\Delta$	$(f_4+f_6)\Delta/M_\Delta + g_4(q^2)$
$-\sqrt{\frac{3}{2}}C_5^A(q^2)$	$-\frac{h_A}{2}$	$-(d_3+d_4)\Delta$	$(f_5+f_7)\Delta^2 + (f_1+f_2)q^2 + g_5(q^2)$
$-\sqrt{\frac{3}{2}}\frac{C_6^A(q^2)}{M_N}$	$\frac{h_A/2}{q^2-m_\pi^2}$	$\frac{(d_3+d_4)\Delta}{q^2-m_\pi^2}$	$-f_1 + g_6(q^2) + \frac{-(f_5+f_7)\Delta^2-f_2q^2-(g_5(q^2)+g_6(q^2)q^2)}{q^2-m_\pi^2}$

The form factors C_3^A and C_4^A both start at chiral order 2 and get their q^2 dependence at order 3 from the loops. For C_3^A, we find a small q^2 dependence, which is quite sensitive to the $\pi\Delta\Delta$ coupling constant. On the other hand, its imaginary part, coming mainly from the N-N internal diagram, is finite (~ 0.03 at $q^2 = 0$) and has a mild q^2 dependence. This suggest that C_3^A is small (compared to $C_{4,5,6}^A$) but not necessarily zero. The C_4^A dependence on q^2 is also found to be rather mild at order $\delta^{(3)}$.

In covariant χPT up to $\delta^{(3)}$, four $\delta^{(2)}$ and seven $\delta^{(3)}$ LEC appear in the results. However, some of them appear in particular combinations. Therefore, effectively we have only five unknown constants. They can be fixed by fitting either to the phenomenological form factors obtained from neutrino bubble chamber data (with several assumptions), to the results of other approaches, such as those of various quark models, or to the lattice QCD results[5]. For a more detailed discussion, see Ref. 7.

Acknowledgments

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