SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$ in covariant baryon chiral perturbation theory

L.S. Geng, J. Martin Camalich, and M.J. Vicente Vacas
Departamento de Física Teórica and IFIC, Universidad de Valencia-CSIC, E-46071 Valencia, Spain
(Dated: December 31, 2013)

We calculate the SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$ up to $\mathcal{O}(p^4)$ in covariant baryon chiral perturbation theory with dynamical octet and decuplet contributions. We find that the decuplet contributions are of similar or even larger size than the octet ones. Combining both, we predict positive SU(3)-breaking corrections to all the four independent $f_1(0)$'s (assuming isospin symmetry), which are consistent, within uncertainties, with the latest results form large N_c fits, chiral quark models, and quenched lattice QCD calculations.

PACS numbers: 13.30.Ce, 12.15.Hh, 12.39.Fe

I. INTRODUCTION

Hyperon semileptonic decays, parameterized by three vector transition form factors (f_1, f_2, and f_3) and three axial form factors (g_1, g_2, and g_3), have received renewed interest in recent years due to various reasons. In particular, they provide an alternative source [1, 2, 3, 4] to allow one to extract the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{us} [5, 6], in addition to kaon semileptonic decays (see, e.g., Ref. [7] for a recent review), hadronic decays of the τ lepton [8] and the ratio $\Gamma(K^+ \to \mu^+\nu_\mu)/\Gamma(\pi^+ \to \mu^+\nu_\mu)$ [9]. The hyperon vector coupling $f_1(0)$ plays an essential role in order to extract V_{us} accurately.

Due to the Conservation of Vector Current (CVC) $f_1(0)$ is known up to SU(3)-breaking effects, which are of subleading-order according to the Ademollo-Gatto theorem [10]. Theoretical estimates of SU(3)-breaking corrections to $f_1(0)$ have been performed in various frameworks, including quark models [11, 12, 13], large-N_c fits [3], and chiral perturbation theory (ChPT) [14, 15, 16, 17, 18]. These SU(3)-breaking corrections have also been studied recently in quenched lattice QCD (LQCD) calculations for two of the four independent channels (assuming isospin symmetry): $\Sigma^- \to n$ [19] and $\Xi^0 \to \Sigma^+$ [20].

In principle, ChPT provides a model independent way to estimate the SU(3)-breaking corrections to $f_1(0)$. However, it is known that ChPT calculations converge slowly in SU(3) flavor space. This problem becomes even more pronounced in the one-baryon sector, where the physics at a certain order can be blurred by the power-counting restoration procedures, as can be clearly seen in the case of the baryon octet magnetic moments [21]. Fortunately, in the case of $f_1(0)$, the Ademollo-Gatto theorem dictates that up to $\mathcal{O}(p^4)$ no unknown LEC's contribute and, therefore, no power-counting-breaking terms appear. Consequently, up to this order there is no need to apply any power-counting restoration procedures and a ChPT calculation is fully predictive.

In a recent $\mathcal{O}(p^4)$ calculation performed in Heavy Baryon (HB) ChPT, it was shown that the chiral series with only the octet contributions converge slowly while the convergence is completely spoiled by the inclusion of the decuplet ones [17]. In a later work [18], the infrared version of baryon chiral perturbation theory (IRChPT) [22] was employed and calculations were performed up to $\mathcal{O}(p^4)$ with only the octet contributions. The slow convergence of the chiral series was confirmed but the importance of relativistic corrections was stressed.

In the present work, we perform the first covariant baryon ChPT calculation of $f_1(0)$ up to $\mathcal{O}(p^4)$, including both octet and decuplet contributions. This article is organized as follows. In Sec. 2, we fix our notation and write down the relevant chiral Lagrangians up to $\mathcal{O}(p^4)$. To study the contributions of the decuplet baryons, we adopt the "consistent" coupling scheme for the Rarita-Schwinger description of the spin-3/2 decuplet fields [23]. In Sec. 3, we present our numerical results order by order, contrast them with the corresponding HBChPT and IRChPT results, and study the convergence of the chiral series. We also compare our full results with those obtained in other approaches, including large N_c fits, quark models, and lattice QCD calculations. Finally, we use our results of $f_1(0)$ to extract V_{us} from the experimental values of the decay rates and $g_1(0)/f_1(0)$. Summary and conclusions follow in Sec. 4.

II. FORMALISM

The baryon vector form factors as probed by the charged ΔS=1 weak current $V^\mu = V_{us}\bar{u}\gamma^\mu s$ are defined by

$$\langle B'|V^\mu|B\rangle = V_{us}\bar{u}(p')\left[\gamma^\mu f_1(q^2) + \frac{2i\sigma^{\mu\nu}q_\nu}{M_{B'}+M_B}f_2(q^2) + \frac{2q^\mu}{M_{B'}+M_B}f_3(q^2)\right]u(p), \tag{1}$$

where $q = p' - p$. In the SU(3)-symmetric limit, $f_1(0)$ is fixed by the conservation of the SU(3)$_V$-charge g_V. Furthermore, the Ademollo-Gatto theorem states that SU(3)-breaking corrections start at second order in the expansion parameter $m_s - m$

$$f_1(0) = g_V + \mathcal{O}((m_s - m)^2), \tag{2}$$

where m_s is the strange quark mass and m is the mass of the light quarks. The values of g_V are $-\sqrt{\frac{3}{2}}$, $-\frac{1}{\sqrt{2}}$, -1, $\sqrt{\frac{3}{2}}$, $\frac{1}{\sqrt{2}}$, 1 for $\Lambda \to p$, $\Sigma^0 \to p$, $\Sigma^- \to n$, $\Xi^- \to \Lambda$, $\Xi^- \to \Sigma^0$, and $\Xi^0 \to \Sigma^+$, respectively. In the isospin-symmetric limit only four of these channels, which we take as $\Lambda \to N$, $\Sigma \to N$, $\Xi \to \Lambda$, and $\Xi \to \Sigma$, provide independent information We will parameterize the SU(3)-breaking corrections order-by-order in the relativistic chiral expansion as follows:

$$f_1(0) = g_V \left(1 + \delta^{(2)} + \delta^{(3)} + \cdots\right), \tag{3}$$

where $\delta^{(2)}$ and $\delta^{(3)}$ are the leading and next-to-leading order SU(3)-breaking corrections induced by loops, corresponding to $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$ chiral calculations.

A. Chiral Lagrangians involving only octet baryons and pseudoscalars

The lowest-order SU(3) chiral Lagrangian describing the pseudo-Goldstone bosons in the presence of an external vector current is:

$$\mathcal{L}_\phi^{(2)} = \frac{F_0^2}{4}\langle \nabla_\mu U (\nabla^\mu U)^\dagger + U\chi^\dagger + \chi U^\dagger \rangle, \tag{4}$$

where the parameter F_0 is the chiral-limit decay constant, U is the SU(3) representation of the meson fields and ∇_μ is its covariant derivative: $\nabla_\mu U = \partial_\mu - i[v_\mu, U]$, with v_μ being the vector source. The explicit breaking of chiral symmetry comes from $\chi = 2B_0\mathcal{M}$ where B_0 measures the strength of the breaking and $\mathcal{M} = \mathrm{diag}(m, m, m_s)$ is the quark mass matrix in the isospin symmetric limit [24]. In the above and forthcoming Lagrangians, the symbol $\langle ... \rangle$ denotes the trace in SU(3) flavor space.

The lowest-order chiral Lagrangian describing octet baryons interacting with pseudoscalars and an external vector source reads:

$$\mathcal{L}_{\phi\mathrm{B}}^{(1)} = \langle \bar{B}\left(i\not{D} - M_0\right)B\rangle + \frac{D/F}{2}\langle \bar{B}\gamma^\mu\gamma_5[u_\mu, B]_\pm\rangle, \tag{5}$$

where B denotes the traceless flavor matrix accounting for the octet-baryon fields, M_0 is the chiral-limit octet-baryon mass, D and F are the axial and vector meson-baryon couplings and $D_\mu B = \partial_\mu B + [\Gamma_\mu, B]$ is the covariant derivative. Furthermore, with $u^2 \equiv U$, u_μ and Γ_μ are the so-called *vielbein* and *connection*:

$$\begin{aligned} u_\mu &= i(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger) + (u^\dagger v_\mu u - u v_\mu u^\dagger), \\ \Gamma_\mu &= \frac{1}{2}(u^\dagger\partial_\mu u + u\partial_\mu u^\dagger) - \frac{i}{2}(u^\dagger v_\mu u + u v_\mu u^\dagger). \end{aligned} \tag{6}$$

The only higher-order chiral Lagrangian that also contributes is through the SU(3)-breaking of the masses of octet baryons[1]

$$\mathcal{L}_{\phi B}^{(2)} = b_{D/F}\langle \bar{B}[\chi_+, B]_\pm\rangle$$

with

$$\chi_+ = 2\chi = 4B_0\mathcal{M} = 2\mathrm{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2), \tag{7}$$

which leads to the following octet baryon masses up to this order:

$$\begin{aligned} M_N &= M_0 + 4m_K^2 b_D - 4(m_K^2 - m_\pi^2)b_F, \qquad M_\Sigma = M_0 + 4m_\pi^2 b_D, \\ M_\Xi &= M_0 + 4m_K^2 b_D + 4(m_K^2 - m_\pi^2)b_F, \qquad M_\Lambda = M_0 + \frac{4}{3}(4m_K^2 - m_\pi^2)b_D. \end{aligned} \tag{8}$$

[1] We have omitted a singlet term indistinguishable from M_0 for our purposes.

Fitting the above masses to their corresponding physical values, with $m_\pi = 0.138$ GeV, $m_K = 0.496$ GeV, and $m_\eta = 0.548$ GeV, yields $M_0 = 1.197$ GeV, $b_D = -0.0661$ GeV^{-1}, and $b_F = 0.2087$ GeV^{-1}, which correspond to $M_N = 0.942(0.939)$ GeV, $M_\Sigma = 1.192(1.193)$ GeV, $M_\Xi = 1.321(1.318)$ GeV, and $M_\Lambda = 1.112(1.116)$ GeV, with the physical values given in parentheses. It is clear that the differences between the second order fits and the physical values are quite small. Using either of them will be numerically equivalent. In the $\mathcal{O}(p^4)$ calculation, we will use the second order fits, Eq. (8), to keep track of the SU(3)-breaking pattern. While at $\mathcal{O}(p^3)$, we use the average mass of the octet baryons, $M_B = 1.151$ MeV, without introducing mass splittings.

B. Chiral Lagrangians involving decuplet baryons

In this work, we adopt the so-called "consistent" couplings [23] to describe the interactions between the decuplet and the octet baryons. Compared to conventional couplings (see, e.g., Refs. [23, 25]), the consistent couplings are more stringent due to the requirement that all interactions have the same type of gauge invariance as the kinetic term of the spin-3/2 fields [23]. To calculate $f_1(0)$ up to $\mathcal{O}(p^3)$, one only needs the following lowest-order chiral Lagrangians [26]:

$$\mathcal{L}^{(1)}_{\rm DD} = \bar{T}_\mu(\gamma^{\mu\nu\alpha} iD_\alpha - M_{D0}\gamma^{\mu\nu})T_\nu, \tag{9}$$

$$\mathcal{L}^{(1)}_{\rm DB} = \frac{i\mathcal{C}}{m_D}(D^\dagger_\mu \bar{T}_\nu \gamma^{\mu\nu\lambda} u_\lambda B + \bar{B} u_\lambda \gamma^{\mu\nu\lambda} D_\mu T_\nu), \tag{10}$$

where M_{D0} is the chiral-limit decuplet-baryon mass, $D^\alpha T^\nu_{abc} = \partial^\alpha T^\nu_{abc} + (\Gamma^\alpha)^d_a T^\nu_{dbc} + (\Gamma^\alpha)^d_b T^\nu_{adc} + (\Gamma^\alpha)^d_c T^\nu_{abd}$, $T^\nu = T_{ade}\psi^\nu$, $\bar{T}^\mu = \bar{T}^{ade}\bar{\psi}^\mu$ with the following associations: $T_{111} = \Delta^{++}$, $T_{112} = \Delta^+/\sqrt{3}$, $T_{122} = \Delta^0/\sqrt{3}$, $T_{222} = \Delta^-$, $T_{113} = \Sigma^{*+}/\sqrt{3}$, $T_{123} = \Sigma^{*0}/\sqrt{6}$, $T_{223} = \Sigma^{*-}/\sqrt{3}$, $T_{133} = \Xi^{*0}/\sqrt{3}$, $T_{233} = \Xi^{*-}/\sqrt{3}$, and $T_{333} = \Omega^-$. The value of the pseudoscalar-baryon-decuplet coupling $\mathcal{C}$ is determined to be $\mathcal{C} \approx 1.0$ from the $\Delta \to \pi N$ decay width.[2] In SU(3) flavor space, the value of $\mathcal{C}$ can be different for different channels. In the present work, as in Ref. [17], we use the same $\mathcal{C}$ for all the channels, assuming that SU(3)-breaking corrections to $f_1(0)$ induced by using channel-specific $\mathcal{C}$'s are of higher order. The spin-3/2 propagator in d dimensions is

$$S^{\mu\nu}(p) = -\frac{\not{p} + M_D}{p^2 - M_D^2 + i\epsilon}\left[g^{\mu\nu} - \frac{1}{d-1}\gamma^\mu\gamma^\nu - \frac{1}{(d-1)M_D}(\gamma^\mu p^\nu - \gamma^\nu p^\mu) - \frac{d-2}{(d-1)M_D^2}p^\mu p^\nu\right] \tag{11}$$

with M_D the decuplet baryon mass.

To calculate $f_1(0)$ at $\mathcal{O}(p^4)$, the following second-order chiral Lagrangian is needed to break the mass degeneracy of the decuplet baryons[3]

$$\mathcal{L}^{(2)}_{\rm DD} = \frac{\gamma_M}{2}\bar{T}^\mu \chi_+ T_\mu, \tag{12}$$

which leads to

$$\begin{aligned} M_\Delta &= M_{D0} + 3m_\pi^2\gamma_M, \qquad & M_{\Sigma^*} &= M_{D0} + (2m_K^2 + m_\pi^2)\gamma_M \\ M_{\Xi^*} &= M_{D0} + (4m_K^2 - m_\pi^2)\gamma_M, \qquad & M_\Omega &= M_{D0} + 3(2m_K^2 - m_\pi^2)\gamma_M. \end{aligned} \tag{13}$$

A fit to the decuplet baryon masses, with the meson masses given above, yields $\gamma_M = 0.3236$ GeV^{-1} and $m_{D0} = 1.216$ GeV, which correspond to $M_\Delta = 1.235(1.232)$ GeV, $M_{\Sigma^*} = 1.382(1.384)$ GeV, $M_{\Xi^*} = 1.529(1.533)$ GeV, and $M_\Omega = 1.676(1.672)$ GeV. As in the octet case, we use the second order fits in our calculation of the $\mathcal{O}(p^4)$ results, while in the $\mathcal{O}(p^3)$ calculation, we use the average of the decuplet baryon masses, $M_D = 1.382$ GeV, for all the decuplet baryons.

---

[2] Note the definition of u_μ in Eq. (6) is a factor of 2 different from that of HBChPT in Refs. [17, 27].
[3] As in the octet case, we have omitted a singlet term indistinguishable from M_{D0} for our purposes.

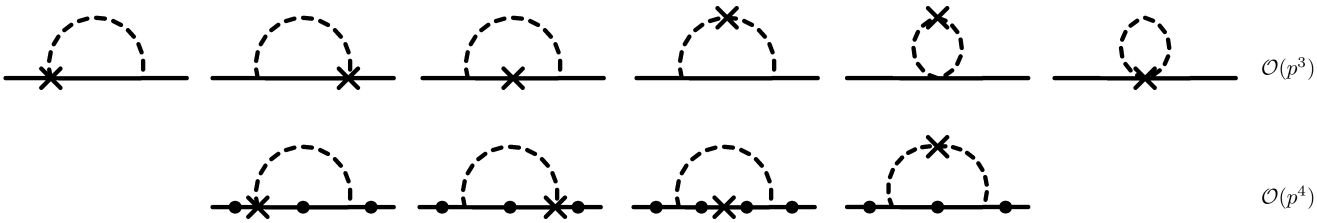

FIG. 1: Feynman diagrams contributing to the SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$ up to $\mathcal{O}(p^4)$. The solid lines correspond to baryons and dashed lines to mesons; crosses indicate the coupling of the external current; black dots denote mass splitting insertions. We have not shown explicitly those diagrams corresponding to wave function renormalization, which have been taken into account in the calculation.

III. RESULTS AND DISCUSSIONS

A. SU(3)-breaking corrections to $f_1(0)$ due to octet contributions up to $\mathcal{O}(p^4)$

All the diagrams contributing to $f_1(0)$ up to $\mathcal{O}(p^4)$ are shown in Fig. 1, where the leading and next-to-leading order SU(3)-breaking corrections are given by the diagrams in the first and second row, respectively.

The $\mathcal{O}(p^3)$ results are quite compact and have the following structure for the transition $i \to j$:

$$\begin{aligned}\delta_B^{(2)}(i\to j) \;=\; & \sum_{M=\pi,\eta,K}\beta_M^{\rm BP}H_{\rm BP}(m_M)+\sum_{M=\pi,\eta}\beta_M^{\rm MP}H_{\rm MP}(m_M,m_K)+\sum_{M=\pi,\eta,K}\beta_M^{\rm KR}H_{\rm KR}(m_M)\\ & -\frac{3}{8}\sum_{M=\pi,\eta}H_{\rm TD1}(m_M,m_K)+\frac{3}{8}\sum_{M=\pi,\eta}H_{\rm TD2}(m_M)+\frac{3}{4}H_{\rm TD2}(m_K)\\ & +\frac{1}{2}\sum_{M=\pi,\eta,K}(\beta_M^{\rm WF}(i)+\beta_M^{\rm WF}(j))H_{\rm WF}(m_M), \end{aligned} \tag{14}$$

where $\beta^{\rm BP}$, $\beta^{\rm MP}$, $\beta^{\rm KR}$, and $\beta^{\rm WF}$ are given in Tables VI, VII, VIII, and IX in the Appendix, and the loop functions $H_{\rm BP}$, $H_{\rm MP}$, $H_{\rm KR}$, $H_{\rm TD1}$, $H_{\rm TD2}$, and $H_{\rm WF}$ are also given there. It is interesting to note that although separately these loop functions are divergent (scale-dependent) and some of them contain power-counting breaking pieces ($H_{\rm KR}$ and $H_{\rm MP}$), the overall contributions are finite and do not break power-counting. This is an explicit manifestation of the Ademollo-Gatto theorem.

In the $\mathcal{O}(p^4)$ calculation, we have implemented mass-splitting corrections in a similar way as Ref. [18] except that we have used the masses obtained from the second-order ChPT fit, as described above, instead of the physical masses. Similar to the IRChPT study of Ref. [18], the $\mathcal{O}(p^4)$ results contain higher-order divergences. We have removed the infinities using the modified minimal-subtraction ($\overline{MS}$) scheme. The analytical results are quite lengthy and will not be shown here. In Fig. 2, we show the scale dependence of the octet contributions, which is rather mild for most cases except for the $\Sigma \to N$ transition. The scale dependence can be used to estimate higher-order contributions by varying μ in a reasonable range. In the following, we present the results by varying μ from 0.7 to 1.3 GeV. It should be mentioned that if we had adopted the same method as Ref. [17] to calculate the $\mathcal{O}(p^4)$ contributions, i.e., by expanding the results and keeping only those linear in baryon mass splittings, our $\mathcal{O}(p^4)$ results would have been convergent.

We have checked that our results up to $\mathcal{O}(p^3)$ are the same as those obtained in Ref. [14], while in the $M_B \sim \Lambda_{\chi SB}$ limit our results recover the HBChPT ones [17] at both $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$ including the $1/M$ recoil corrections. All these are known to explicitly verify the Ademollo-Gatto theorem in the sense of Eq. (2).

Table II shows the SU(3)-breaking corrections in the notation of Eq. (3). For comparison, we also list the numbers obtained in HBChPT [17] and IRChPT [18]. The numerical values are obtained with the parameters given in Table I. As in Ref. [21] we have used an average $F_0 = 1.17 f_\pi$ with $f_\pi = 92.4$ MeV. It should be pointed out that the HBChPT and the IRChPT results are obtained using f_π.

First, we note that in three of the four cases, the $\delta^{(3)}$ numbers are smaller than the $\delta^{(2)}$ ones. The situation is similar in IRChPT but quite different in HBChPT. In the HBChPT calculation [17], the $\delta^{(3)}$ contribution is larger than the $\delta^{(2)}$ one for the four cases.[4] This tells that recoil corrections (in the HBChPT language) or relativistic effects are

[4] What we denote by $\delta^{(3)}$ is the sum of those labeled by $\alpha^{(3)}$ and $\alpha^{(1/M)}$ in Ref. [17].

TABLE I: Values for the masses and couplings appearing in the calculation of the SU(3)-breaking corrections to $f_1(0)$.

D	0.8	M_B	1.151 GeV
F	0.46	M_D	1.382 GeV
f_π	0.0924 GeV	M_0	1.197 GeV
F_0	$1.17 f_\pi$	b_D	-0.0661 GeV^{-1}
m_π	0.138 GeV	b_F	0.2087 GeV^{-1}
m_K	0.496 GeV	M_{D0}	1.216 GeV
m_η	0.548 GeV	γ_M	0.3236 GeV^{-1}
$\mathcal{C}$	1.0		

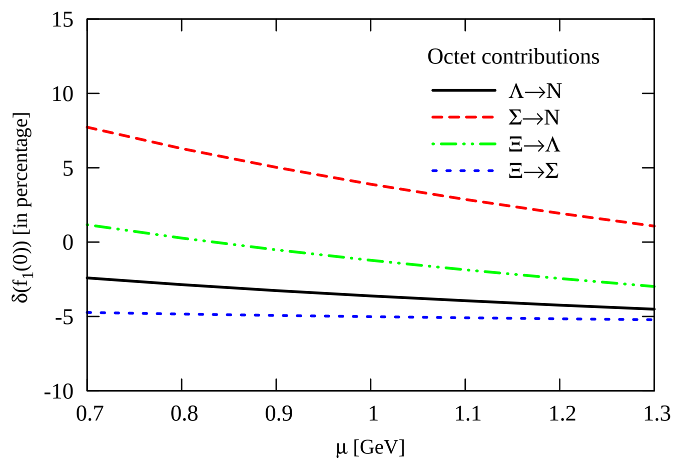

FIG. 2: Scale dependence of the octet contributions to the SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$.

important. On the other hand, the results of the present work and those of IRChPT [18], including the contributions of different chiral orders, are qualitatively similar. They are both very different from the HBChPT predictions, even for the signs in three of the four cases. Obviously, as stressed in Ref. [18], one should trust more the relativistic than the non-relativistic results, which have to be treated with caution whenever $1/M$ recoil corrections become large.

It is clear from Table II that the convergence is slow even in the case of the relativistic calculations, a well known feature of SU(3) baryon ChPT. It is then necessary to have a way to calculate "higher-order" contributions. Going to $\mathcal{O}(p^5)$ one needs to introduce unknown LEC's such that the predictive power of ChPT is lost. An alternative approach is to consider the contributions of dynamical heavier resonances. A basic assumption of ChPT is that these heavier degrees of freedom can be integrated out with their effects incorporated in the LEC's. However, that may not be totally true in the one-baryon sector since the gap between the lowest baryon octet and the lowest baryon decuplet is only ~ 0.3 GeV, not very different from the pion mass and even smaller than the kaon(eta) mass. Therefore, it is necessary to investigate their contributions. In the HBChPT scheme, this task has recently been undertaken in Ref. [17], where it is concluded that the decuplet contributions completely spoil the convergence of the chiral series. We study the contributions of the decuplet baryons in the covariant framework in the following section.

B. SU(3)-breaking corrections to $f_1(0)$ induced by dynamical decuplet baryons up to $\mathcal{O}(p^4)$

Fig. 3 shows the diagrams that contribute to SU(3)-breaking corrections to $f_1(0)$ with dynamical decuplet baryons up to $\mathcal{O}(p^4)$. It should be noted that unlike in the HBChPT case [17], Kroll-Rudermann (KR) kind of diagrams also contribute. In fact, using the consistent coupling scheme of Ref. [23], there are four KR diagrams: Two are from minimal substitution in the derivative of the pseudoscalar fields and the other two are from minimal substitution in the derivative of the decuplet fields (see Eq. (10) and also Ref. [26]).

TABLE II: Octet contributions to the SU(3)-breaking corrections to $f_1(0)$ (in percentage). The central values of the $\mathcal{O}(p^4)$ results are calculated with $\mu = 1$ GeV and the uncertainties are obtained by varying μ from 0.7 to 1.3 GeV.

	present work			HBChPT [17]	IRChPT [18]
	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$
$\Lambda\to N$	-3.8	$0.2^{+1.2}_{-0.9}$	$-3.6^{+1.2}_{-0.9}$	2.7	-5.7 ± 2.1
$\Sigma\to N$	-0.8	$4.7^{+3.8}_{-2.8}$	$3.9^{+3.8}_{-2.8}$	4.1	2.8 ± 0.2
$\Xi\to\Lambda$	-2.9	$1.7^{+2.4}_{-1.8}$	$-1.2^{+2.4}_{-1.8}$	4.3	-1.1 ± 1.7
$\Xi\to\Sigma$	-3.7	$-1.3^{+0.3}_{-0.2}$	$-5.0^{+0.3}_{-0.2}$	0.9	-5.6 ± 1.6

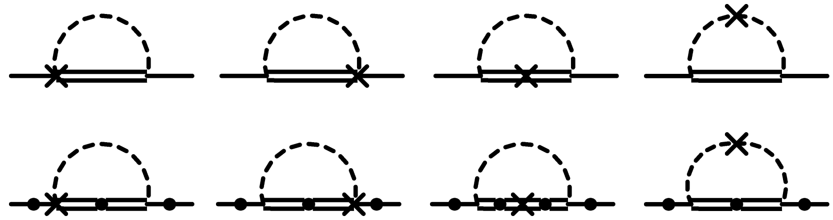

FIG. 3: Feynman diagrams contributing to the leading and next-to-leading order SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$, through dynamical decuplet baryons. The notations are the same as those of Fig. 1 except that double lines indicate decuplet baryons. We have not shown explicitly those diagrams corresponding to wave function renormalization, which have been included in the calculation.

The $\mathcal{O}(p^3)$ results are relatively simple and have the following general structure for the transition $i\to j$:

$$\delta_D^{(2)}(i\to j) = \sum_{M=\pi,\eta,K}\gamma_M^{\mathrm{BP}}D_{\mathrm{BP}}(m_M)+\sum_{M=\pi,\eta}\gamma_M^{\mathrm{MP}}D_{\mathrm{MP}}(m_M,m_K)+\sum_{M=\pi,\eta,K}\gamma_M^{\mathrm{KR}}D_{\mathrm{KR}}(m_M) + \frac{1}{2}\sum_{M=\pi,\eta,K}(\gamma_M^{\mathrm{WF}}(i)+\gamma_M^{\mathrm{WF}}(j))D_{\mathrm{WF}}(m_M), \quad (15)$$

where γ^{BP}, γ^{MP}, γ^{KR}, and γ^{WF} are listed in Tables X, XI, XII, and XIII of the Appendix. The loop functions D^{BP}, D^{MP}, D^{KR}, and D^{WF} can be calculated analytically, but they are quite lengthy. In the Appendix, they are given in terms of Feynman-parameter integrals, which can be easily integrated.

To calculate the $\mathcal{O}(p^4)$ chiral contributions, we implement the decuplet-baryon mass splittings in the same way as in the octet case. The $\mathcal{O}(p^4)$ results contain again higher-order divergences, with the scale dependence shown in

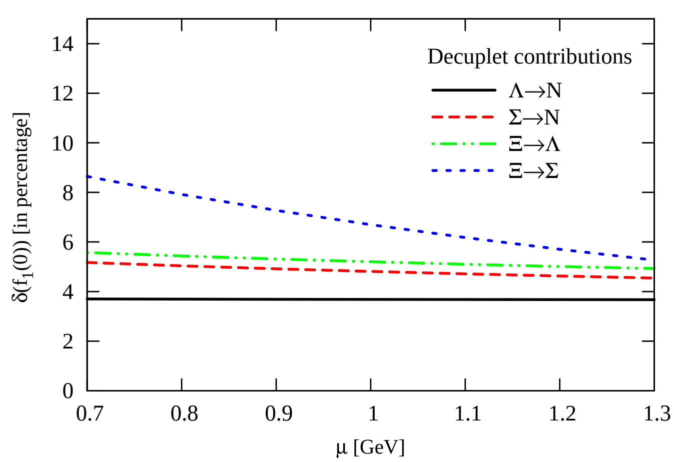

FIG. 4: Scale dependence of the decuplet contributions to the SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$.

TABLE III: Decuplet contributions to the SU(3)-breaking corrections to $f_1(0)$ (in percentage). The central values of the $\mathcal{O}(p^4)$ result are calculated with $\mu = 1$ GeV and the uncertainties are obtained by varying μ from 0.7 GeV to 1.3 GeV.

	Present work			HBChPT		
	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$
$\Lambda\to N$	0.7	$3.0^{+0.1}_{-0.1}$	$3.7^{+0.1}_{-0.1}$	1.8	1.3	3.1
$\Sigma\to N$	−1.4	$6.2^{+0.4}_{-0.3}$	$4.8^{+0.4}_{-0.3}$	−3.6	8.8	5.2
$\Xi\to\Lambda$	−0.02	$5.2^{+0.4}_{-0.3}$	$5.2^{+0.4}_{-0.3}$	−0.05	4.2	4.1
$\Xi\to\Sigma$	0.7	$6.0^{+1.9}_{-1.4}$	$6.7^{+1.9}_{-1.4}$	1.9	−0.2	1.7

TABLE IV: SU(3)-breaking corrections to $f_1(0)$ up to $\mathcal{O}(p^4)$ (in percentage), including both the octet and the decuplet contributions.

	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$
$\Lambda\to N$	−3.1	$3.2^{+1.3}_{-1.0}$	$0.1^{+1.3}_{-1.0}$
$\Sigma\to N$	−2.2	$10.9^{+4.2}_{-3.1}$	$8.7^{+4.2}_{-3.1}$
$\Xi\to\Lambda$	−2.9	$6.9^{+2.8}_{-2.1}$	$4.0^{+2.8}_{-2.1}$
$\Xi\to\Sigma$	−3.0	$4.7^{+2.2}_{-1.6}$	$1.7^{+2.2}_{-1.6}$

Fig. 4. The infinities have been removed by the $\overline{MS}$ procedure. The dependence is found to be rather mild except for the $\Xi\to\Sigma$ transition. In this case, unlike in the octet case, the divergences cannot be removed by expanding and keeping only terms linear in baryon and decuplet mass splittings. The full $\mathcal{O}(p^4)$ analytical results are quite involved and, therefore, they will not be shown here.

The numerical results obtained with the parameter values given in Table I are summarized in Table III. It can be seen that at $\mathcal{O}(p^3)$, the decuplet contributions are relatively small compared to the octet ones at the same order. On the other hand, the $\mathcal{O}(p^4)$ contributions are sizable and all of them have positive signs.

Using the conventional Lagrangians of Ref. [25], one obtains different numbers and different μ dependence. In the heavy-baryon limit, however, the results obtained with both coupling schemes are found to be the same and convergent, confirming the fact that the differences induced by the "consistency" procedure are of higher chiral order [23] (see also Ref. [26]).

In Table III, the numbers denoted by HBChPT differ from those of Ref. [17]. The $\delta^{(2)}$ column would have coincided if we had used the same values for the couplings $\mathcal{C} = 0.8$ and $F_0 = 0.0933$ GeV. On the other hand, our $\delta^{(3)}$ contributions due to the octet baryon mass splittings are much smaller than those of Ref. [17]. It is interesting to note that unlike in the octet case, the HBChPT results are similar to the relativistic ones.[5]

As the decuplet-octet mass splitting increases, one expects that the decuplet contributions decrease and eventually vanish as the splitting goes to infinity. This is indeed the case, as can be clearly seen from Fig. 5, where the $\mathcal{O}(p^3)$ decuplet contributions are plotted as a function of the decuplet-octet mass splitting.

C. Full results and comparison with other approaches

Summing the octet and the decuplet contributions, we obtain the numbers shown in Table IV. Two things are noteworthy. First, the convergence is slow, even taking into account the scale dependence of the $\delta^{(3)}$ corrections. Second, for three of the four transitions, the $\delta^{(3)}$ corrections have a different sign than the $\delta^{(2)}$ ones.

[5] The HB results are obtained in a slightly different way than the relativistic ones. To obtain the $\mathcal{O}(p^3)$ numbers, we have used $M_B = 1.151$ GeV and $M_D = 1.382$ GeV and have performed an expansion in terms of the decuplet-octet mass splitting, $M_D - M_B$. To obtain the $\mathcal{O}(p^4)$ ones, we have used physical masses for both the octet and the decuplet baryons and have performed an additional expansion keeping only the terms linear in the octet and the decuplet baryon mass splittings. Although this procedure is the same as that of Ref. [17], we get different results. We find that the discrepancy comes from the octet mass-splitting corrections to the meson-pole diagram of Fig. 3. If we had mistakenly exchanged the masses of the mesons in the loop, we would have obtained the same results as those of Ref. [17].

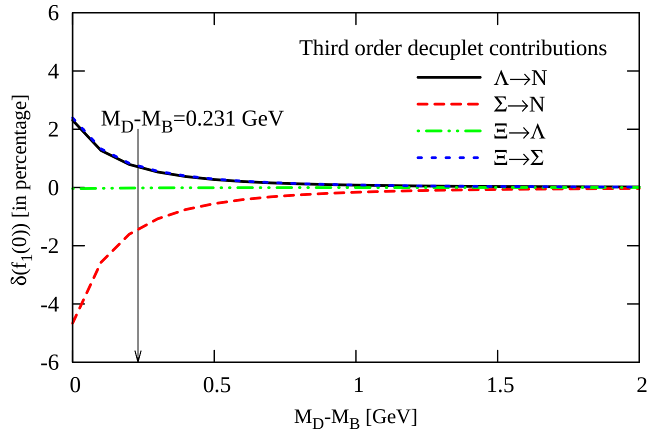

FIG. 5: Decuplet $\mathcal{O}(p^3)$ contributions to the SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$ as a function of the decuplet-octet mass splitting $M_D - M_B$.

In Table V, we compare our results with those obtained from other approaches, including large N_c fits [3], quark models [11, 12, 13], and two quenched LQCD calculations [19, 20]. The large N_c results in general favor positive corrections, which are consistent with our central values. Two of the quark models predict negative corrections, while that of Ref. [13] favors positive corrections. It is interesting to note that in Ref. [13] the valence quark effects give negative contributions, as in the other two quark models, while the chiral effects provide positive contributions, resulting in net positive corrections. Our numbers also agree, within uncertainties, with the quenched LQCD ones. In principle, LQCD calculations provide another model-independent way to obtain the SU(3)-breaking corrections to $f_1(0)$. At present, however, the quenched LQCD calculations are not yet accurate enough to determine these numbers, due to the large quark masses used in the simulation and other systematic uncertainties.

Finally, we will briefly discuss the implications of our results for the estimation of V_{us}. There have been several previous attempts to extract this parameter using hyperon semileptonic decays [1, 2, 3, 4]. As discussed in Ref. [4] a rather clean determination of $f_1 V_{us}$ can be done by using g_1/f_1 and the decay rates from experiment and taking for g_2 and f_2 their SU(3) values. This latter approximation is supported by the fact that their contributions to the decay rate are reduced by kinematic factors (See, for instance, Eq. (10) of Ref. [3]). Using the values of $f_1 V_{us}$ compiled in Table 3 of Ref. [4] and our results for f_1 we get

$$V_{us} = 0.2177 \pm 0.0030\,. \tag{16}$$

This value is consistent with the large N_c fits of Ref. [3] and with the result obtained from τ decays[8], and lower than the results of kaon decays [7] and the fits to hyperon decays from Refs. [1, 2]. Although the quoted error seems competitive with calculations using other processes, we must remark that the error estimation for Eq. (16) includes only the experimental errors and the uncertainties related to the scale dependence. Other systematic uncertainty sources, like the effect of higher order SU(3)-breaking corrections are hard to estimate and have not been included. Even with these limitations, our results clearly point to positive values for the SU(3)-breaking corrections to f_1 and therefore towards relatively small values of V_{us}.

TABLE V: SU(3)-breaking corrections (in percentage) to $f_1(0)$ obtained in different approaches.

	Present work	Large N_c	Quark model			quenched LQCD
		Ref. [3]	Ref. [11]	Ref. [12]	Ref. [13]	
$\Lambda \to N$	$0.1^{+1.3}_{-1.0}$	2 ± 2	-1.3	-2.4	0.1	
$\Sigma \to N$	$8.7^{+4.2}_{-3.1}$	4 ± 3	-1.3	-2.4	0.9	$-1.2 \pm 2.9 \pm 4.0$ [19]
$\Xi \to \Lambda$	$4.0^{+2.8}_{-2.1}$	4 ± 4	-1.3	-2.4	2.2	
$\Xi \to \Sigma$	$1.7^{+2.2}_{-1.6}$	8 ± 5	-1.3	-2.4	4.2	-1.3 ± 1.9 [20]

IV. SUMMARY AND CONCLUSIONS

We have performed a study of the SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$ in covariant baryon chiral perturbation theory including both the octet and the decuplet contributions. We confirm earlier findings in HBChPT and IRChPT that the convergence of the chiral series is slow in the case with only dynamical octet baryons. Our study of the decuplet contributions shows that at $\mathcal{O}(p^3)$ they are in general smaller than those of their octet counterparts, while at $\mathcal{O}(p^4)$ they are sizable. Combining both octet and decuplet contributions, we found positive SU(3)-breaking corrections to all the four independent $f_1(0)$'s, which compare favorably with the large N_c fits and those of the quark model taking into account chiral effects.

The fact that the $\mathcal{O}(p^4)$ chiral contributions are comparable to the $\mathcal{O}(p^3)$ ones suggests that the $\mathcal{O}(p^5)$ chiral effects may not be negligible. We have estimated their size by varying μ from 0.7 to 1.3 GeV. Taking into account these higher-order uncertainties, our results still favor positive SU(3)-breaking corrections to the four $f_1(0)$'s.

An accurate determination of V_{us} from hyperon semileptonic decays depends largely on our knowledge on the value of $f_1(0)$. While the SU(3)-symmetric values have been used in some fits to extract V_{us}, most studies have taken into account SU(3)-breaking corrections to $f_1(0)$. We have provided the first covariant baryon ChPT predictions for $f_1(0)$ up to $\mathcal{O}(p^4)$ including both the octet and the decuplet contributions. We encourage their uses in new attempts to extract V_{us} from hyperon decay data.

V. ACKNOWLEDGMENTS

This work was partially supported by the MEC grant FIS2006-03438 and the European Community-Research Infrastructure Integrating Activity Study of Strongly Interacting Matter (Hadron-Physics2, Grant Agreement 227431) under the Seventh Framework Programme of EU. L.S.G. acknowledges support from the MICINN in the Program "Juan de la Cierva." J.M.C. acknowledges the same institution for a FPU grant.

[1] N. Cabibbo, E. C. Swallow and R. Winston, Ann. Rev. Nucl. Part. Sci. **53**, 39 (2003).
[2] N. Cabibbo, E. C. Swallow and R. Winston, Phys. Rev. Lett. **92**, 251803 (2004).
[3] R. Flores-Mendieta, Phys. Rev. D **70**, 114036 (2004).
[4] V. Mateu and A. Pich, JHEP **0510**, 041 (2005).
[5] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
[6] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[7] E. Blucher *et al.*, arXiv:hep-ph/0512039.
[8] E. Gamiz, M. Jamin, A. Pich, J. Prades and F. Schwab, PoS **KAON**, 008 (2008) [arXiv:0709.0282 [hep-ph]].
[9] W. J. Marciano, Phys. Rev. Lett. **93**, 231803 (2004).
[10] M. Ademollo and R. Gatto, Phys. Rev. Lett. **13**, 264 (1964).
[11] J. F. Donoghue, B. R. Holstein and S. W. Klimt, Phys. Rev. D **35**, 934 (1987).
[12] F. Schlumpf, Phys. Rev. D **51**, 2262 (1995).
[13] A. Faessler, T. Gutsche, B. R. Holstein, M. A. Ivanov, J. G. Korner and V. E. Lyubovitskij, Phys. Rev. D **78**, 094005 (2008).
[14] A. Krause, Helv. Phys. Acta **63**, 3 (1990).
[15] J. Anderson and M. A. Luty, Phys. Rev. D **47**, 4975 (1993).
[16] N. Kaiser, Phys. Rev. C **64**, 028201 (2001).
[17] G. Villadoro, Phys. Rev. D **74**, 014018 (2006).
[18] A. Lacour, B. Kubis and U. G. Meissner, JHEP **0710**, 083 (2007).
[19] D. Guadagnoli, V. Lubicz, M. Papinutto and S. Simula, Nucl. Phys. B **761**, 63 (2007).
[20] S. Sasaki and T. Yamazaki, arXiv:0811.1406 [hep-ph].
[21] L. S. Geng, J. M. Camalich, L. Alvarez-Ruso and M. J. V. Vacas, Phys. Rev. Lett. **101**, 222002 (2008).
[22] T. Becher and H. Leutwyler, Eur. Phys. J. C **9** (1999) 643.
[23] V. Pascalutsa, Phys. Lett. B **503**, 85 (2001).
[24] S. Scherer, Adv. Nucl. Phys. **27** (2003) 277.
[25] C. Hacker, N. Wies, J. Gegelia and S. Scherer, Phys. Rev. C **72**, 055203 (2005).
[26] L.S. Geng, J. Martin Camalich and M.J. Vicente Vacas, arXiv:0903.0779 [hep-ph].
[27] M. N. Butler, M. J. Savage and R. P. Springer, Nucl. Phys. B **399**, 69 (1993).

VI. APPENDIX

A. Octet $\mathcal{O}(p^3)$ contributions

In this subsection, we present the coefficients and loop functions appearing in the calculation of the $\mathcal{O}(p^3)$ octet contributions, i.e., Eq. (14).

$$H_{\mathrm{BP}} = \frac{1}{(4\pi F_0)^2}\left[-3m^2 - \frac{4\cos^{-1}(\frac{m}{2M_B})m^3}{\sqrt{4M_B^2-m^2}}\left(\frac{m^2}{M_B^2}-3\right) + \frac{2\log\left(\frac{m^2}{M_B^2}\right)m^4}{M_B^2} + 2\log\left(\frac{M_B^2\mu}{m^3}\right)m^2\right], \tag{17}$$

$$\begin{aligned} H_{\mathrm{MP}} = & \frac{1}{(4\pi F_0)^2}\frac{1}{4\,(m_1^2-m_2^2)}\times \\ & \left[\frac{8\left(\frac{m_1^2}{M_B^2}-4\right)\cos^{-1}\left(\frac{m_1}{2M_B}\right)m_1^6}{\sqrt{4m_1^2M_B^2-m_1^4}} - \frac{4\log\left(\frac{m_1^2}{M_B^2}\right)m_1^6}{M_B^2} + \left(6\log\left(\frac{m_1^2}{M_B^2}\right)+2\log\left(\frac{\mu^2}{M_B^2}\right)+11\right)m_1^4\right. \\ & -\frac{8\left(\frac{m_2^2}{M_B^2}-4\right)\cos^{-1}\left(\frac{m_2}{2M_B}\right)m_2^6}{\sqrt{4m_2^2M_B^2-m_2^4}} + \frac{4\log\left(\frac{m_2^2}{M_B^2}\right)m_2^6}{M_B^2} - (6\log\left(\frac{m_2^2}{M_B^2}\right)+2\log\left(\frac{\mu^2}{M_B^2}\right)+11\Big)m_2^4 \\ & \left. +8M_B^2(1+\log\left(\frac{\mu^2}{M_B^2}\right))(m_1^2-m_2^2)\right], \end{aligned} \tag{18}$$

$$H_{\mathrm{KR}} = \frac{1}{(4\pi F_0)^2}\left[\frac{\log\left(\frac{M_B}{m}\right)m^4}{M_B^2} - \frac{\sqrt{4M_B^2-m^2}\cos^{-1}\left(\frac{m}{2M_B}\right)m^3}{M_B^2} + m^2(\log\left(\frac{\mu^2}{M_B^2}\right)+2) + M_B^2(1+\log\left(\frac{\mu^2}{M_B^2}\right))\right], \tag{19}$$

$$H_{\mathrm{TD1}} = \frac{1}{(4\pi F_0)^2}\left[\frac{\log\left(\frac{\mu^2}{m_1^2}\right)m_1^4 - \log\left(\frac{\mu^2}{m_2^2}\right)m_2^4}{m_1^2-m_2^2} + \frac{3}{2}(m_1^2+m_2^2)\right], \tag{20}$$

$$H_{\mathrm{TD2}} = \frac{1}{(4\pi F_0)^2}\left[(\log\left(\frac{\mu^2}{m^2}\right)+1)m^2\right], \tag{21}$$

$$H_{\mathrm{WF}} = \frac{1}{(4\pi F_0)^2}\left[\frac{2\log\left(\frac{m^2}{M_B^2}\right)m^4}{M_B^2} + \frac{4\left(3-\frac{m^2}{M_B^2}\right)\cos^{-1}\left(\frac{m}{2M_B}\right)m^3}{\sqrt{4M_B^2-m^2}} + \left(\log\left(\frac{\mu^2}{m^2}\right)-2\log\left(\frac{m^2}{M_B^2}\right)-3\right)m^2\right]. \tag{22}$$

TABLE VI: Coefficients β^{BP} appearing in Eq. (14).

Channel	π loop	η loop	K loop
$\Lambda \to N$	$-\frac{1}{2}D(D+F)$	$-\frac{1}{6}D(D-3F)$	$-\frac{1}{6}(D-3F)^2$
$\Sigma \to N$	$-\frac{1}{2}\left(D^2+3FD+2F^2\right)$	$\frac{1}{6}D(D-3F)$	$-\frac{1}{2}(D+F)^2$
$\Xi \to \Lambda$	$\frac{1}{2}D(F-D)$	$-\frac{1}{6}D(D+3F)$	$-\frac{1}{6}(D+3F)^2$
$\Xi \to \Sigma$	$-\frac{1}{2}\left(D^2-3FD+2F^2\right)$	$\frac{1}{6}D(D+3F)$	$-\frac{1}{2}(D-F)^2$

TABLE VII: Coefficients β^{MP} appearing in Eq. (14).

Channel	πK loop	ηK loop
$\Lambda \to N$	$\frac{1}{4}\left(3D^2+2FD+3F^2\right)$	$\frac{1}{12}(D+3F)^2$
$\Sigma \to N$	$\frac{1}{12}\left(D^2-18FD+9F^2\right)$	$\frac{3}{4}(D-F)^2$
$\Xi \to \Lambda$	$\frac{1}{4}\left(3D^2-2FD+3F^2\right)$	$\frac{1}{12}(D-3F)^2$
$\Xi \to \Sigma$	$\frac{1}{12}\left(D^2+18FD+9F^2\right)$	$\frac{3}{4}(D+F)^2$

TABLE VIII: Coefficients β^{KR} appearing in Eq. (14).

Channel	π loop	η loop	K loop
$\Lambda \to N$	$-\frac{1}{4}\left(3D^2+2FD+3F^2\right)$	$-\frac{1}{12}(D+3F)^2$	$-\frac{1}{6}\left(5D^2+6FD+9F^2\right)$
$\Sigma \to N$	$-\frac{1}{12}\left(D^2-18FD+9F^2\right)$	$-\frac{3}{4}(D-F)^2$	$-\frac{1}{6}(D-3F)(5D-3F)$
$\Xi \to \Lambda$	$-\frac{1}{4}\left(3D^2-2FD+3F^2\right)$	$-\frac{1}{12}(D-3F)^2$	$-\frac{1}{6}(5D^2-6FD+9F^2)$
$\Xi \to \Sigma$	$-\frac{1}{12}\left(D^2+18FD+9F^2\right)$	$-\frac{3}{4}(D+F)^2$	$-\frac{1}{6}(D+3F)(5D+3F)$

TABLE IX: Coefficients β^{WF} appearing in Eq. (14).

	π loop	η loop	K loop
Λ	D^2	$\frac{D^2}{3}$	$\frac{1}{3}\left(D^2+9F^2\right)$
Σ	$\frac{1}{3}\left(D^2+6F^2\right)$	$\frac{D^2}{3}$	D^2+F^2
N	$\frac{3}{4}(D+F)^2$	$\frac{1}{12}(D-3F)^2$	$\frac{1}{6}\left(5D^2-6FD+9F^2\right)$
Ξ	$\frac{3}{4}(D-F)^2$	$\frac{1}{12}(D+3F)^2$	$\frac{1}{6}(5D^2+6FD+9F^2)$

B. Decuplet $\mathcal{O}(p^3)$ contributions

In this subsection, we provide the coefficients and loop functions appearing in the calculation of the $\mathcal{O}(p^3)$ decuplet contributions, i.e., Eq. (15).

$$D_{\rm BP} = -\frac{\mathcal{C}^2}{(4\pi F_0)^2 M_D^2}\int_0^1 dx\, M_B^2(1-x)\left(\left((2x-1)M_D^2+2M_B x M_D-\left((x-2)M_B^2+2m^2\right)x\right)\times \log\left(\frac{\mu^2}{\left((x-1)M_B^2+m^2\right)x-M_D^2(x-1)}\right)-\left(M_B^2+2M_D M_B+M_D^2-m^2\right)x\right), \tag{23}$$

$$D_{\rm MP} = -\frac{\mathcal{C}^2}{(4\pi F_0)^2 M_D^2}\int_0^1 dx \int_0^{1-x} dy\, M_B^2\left(2M_B x(xM_B-M_B-M_D)+\left(-3(x-1)xM_B^2+2M_D x M_B-M_D^2 x-m_1^2 y +m_2^2(x+y-1)\right)\log\left(\frac{\mu^2}{(x-1)xM_B^2+M_D^2 x+m_1^2 y-m_2^2(x+y-1)}\right)\right), \tag{24}$$

$$D_{KR} = -\frac{\mathcal{C}^2}{(4\pi F_0)^2 M_D^2}\int_0^1 dx\, M_B(M_D+M_B x)\left(\left((x-1)M_B^2+m^2\right)x-M_D^2(x-1)\right)\times \log\left(-\frac{\mu^2}{M_D^2(x-1)-\left((x-1)M_B^2+m^2\right)x}\right), \tag{25}$$

$$D_{\rm WF} = -\frac{\mathcal{C}^2}{(4\pi F_0)^2 M_D^2}\int_0^1 dx\, M_B\left(2M_B^2(x-1)x(M_B x-M_B-M_D)+\left(-5M_B^3(x-1)^2x+4M_B^2M_D(x-1)x+ 3M_B(x-1)\left(m^2(x-1)-M_D^2x\right)+2M_D\left(M_D^2x-m^2(x-1)\right)\right)\log\left(-\frac{\mu^2}{m^2(x-1)-x\left(M_B^2(x-1)+M_D^2\right)}\right)\right). \tag{26}$$

TABLE X: Coefficients $\gamma^{\rm BP}$ appearing in Eq. (15).

Channel	π loop	η loop	K loop
$\Lambda\to N$	-4	0	-2
$\Sigma\to N$	$-\frac{4}{3}$	0	$-\frac{2}{3}$
$\Xi\to\Lambda$	-2	0	-2
$\Xi\to\Sigma$	$-\frac{4}{3}$	-2	$-\frac{14}{3}$

TABLE XI: Coefficients $\gamma^{\rm MP}$ appearing in Eq. (15).

Channel	πK loop	ηK loop
$\Lambda \to N$	1	0
$\Sigma \to N$	-2	-1
$\Xi \to \Lambda$	0	-1
$\Xi \to \Sigma$	1	2

TABLE XII: Coefficients $\gamma^{\rm KR}$ appearing in Eq. (15).

Channel	π loop	η loop	K loop
$\Lambda \to N$	7	0	3
$\Sigma \to N$	$\frac{14}{3}$	1	$\frac{13}{3}$
$\Xi \to \Sigma$	4	1	5
$\Xi \to \Sigma$	$\frac{5}{3}$	2	$\frac{19}{3}$

TABLE XIII: Coefficients $\gamma^{\rm WF}$ appearing in Eq. (15).

	π loop	η loop	K loop
Λ	-3	0	-2
Σ	$-\frac{2}{3}$	-1	$-\frac{10}{3}$
N	-4	0	-1
Ξ	-1	-1	-3