

# NEUTRINO MAGNETIC MOMENT AND THE PROCESS $\nu e \rightarrow \nu e \gamma$

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## Abstract.

The contribution of a neutrino magnetic moment  $\mu_\nu$  to the cross section of the process  $\nu e^- \rightarrow \nu e^- \gamma$  has been calculated and compared with the Standard Electroweak one. The radiative process allows to reach low enough values of  $Q^2$  without the need to operate at very small energies of recoil electrons. Regions in the phase space which are more favourable to set bounds on  $\mu_\nu$  are suggested.

# 1 Introduction

One of the most important problems of modern neutrino physics is the investigation of neutrino properties [1]: neutrino masses and mixings, nature of massive neutrinos (Dirac or Majorana), electromagnetic properties of neutrinos, etc. In this paper we shall be interested in the existence of a neutrino magnetic moment and its manifestation.

In the last few years, the interest in a magnetic moment of neutrinos was connected in part with the solar neutrino problem. It has been argued from time to time that the solar neutrino flux detected in the Chlorine experiment [2] has shown some anticorrelation with sun spot activity. Its most reasonable explanation would involve [3] a neutrino magnetic moment. Results from Kamiokande III [4] however do not indicate any time variation of the neutrino signal. Nevertheless, the search for a neutrino magnetic moment continues to be one of the ways to look for effects beyond the standard model and efforts are worth to continue in this direction [5].

If the standard theory is extended to include the right-handed neutrino field, the resulting Dirac neutrino with mass  $m_\nu$  acquires a magnetic moment [6]

$$\mu_\nu^s = \frac{3}{4\sqrt{2}\pi^2} G_F m_\nu m \mu_B \simeq 3.2 \cdot 10^{-19} \left(\frac{m_\nu}{eV}\right) \mu_B \quad (1)$$

where  $\mu_B = e/2m$  is the Bohr magneton and  $m$  is the electron mass. From the latest measurements of the electron spectrum in  ${}^3H$   $\beta$ -decay [7] the following upper limit of the electron neutrino mass was obtained

$$m_{\nu_e} < 7.2 eV (95\% C.L.) \quad (2)$$

It follows from (1) and (2) that the "standard" contribution (1) to the electron neutrino magnetic moment is less than  $2 \cdot 10^{-18} \mu_B$ . Such a small upper bound cannot be reached in any present day experiment. However, there exist many models beyond the standard theory in which the induced magnetic moment of neutrinos could be many orders of magnitude bigger than  $\mu_\nu^s$  [8].

The lowest bounds on the neutrino magnetic moment come from astrophysical arguments. If neutrinos have magnetic moments, then their coupling with an off-shell photon  $\gamma^*$  in a star can cause  $\gamma^* \rightarrow \nu + \bar{\nu}$  to occur. Once the neutrinos are produced,

they will escape carrying away energy. From the absence of such an anomalous energy loss mechanism in red giants one finds

$$\mu_\nu < 7 \cdot 10^{-11} \mu_B$$

Using the neutrino data from Supernova 1987A, there are stringent bounds which apply for Dirac neutrinos, in order to allow the right-handed species to escape from the supernova. One gets [9]

$$\mu_\nu < 10^{-12} \mu_B$$

The bounds from the supernova have been questioned by Voloshin [10] if there are strong magnetic fields in the supernova. Another astrophysical constraint comes from consideration of the luminosity before and after stellar helium flash [11] in red giants

$$\mu_\nu < 3 \cdot 10^{-12} \mu_B$$

There are laboratory bounds from terrestrial neutrino experiments. From  $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$  in reactor experiments [12] the bound on the neutrino magnetic moment

$$\mu_\nu < 2.4 \cdot 10^{-10} \mu_B$$

has been set. This limit applies to electron antineutrinos. From beam stop LAMPF neutrino data it follows [13]

$$\mu_{\nu_e} < 1.1 \cdot 10^{-9} \mu_B, \quad \mu_{\nu_\mu} < 7.4 \cdot 10^{-10} \mu_B$$

Several new proposals [14] plan to reach a much better sensitivity in the investigation of the  $\bar{\nu}_e$  magnetic moment ( at the level of  $10^{-11} \mu_B$  ). In these experiments, the process under study to obtain information about  $\mu_\nu$  is that of elastic antineutrino-electron scattering at small energies. Its sensitivity to  $\mu_\nu$  is connected with the fact that at low enough values of  $Q^2$  the contribution of the electromagnetic amplitude to the cross section of the process becomes comparable to the contribution of the weak amplitude. This is the case for  $Q^2 \sim MeV^2$  at values  $\mu_\nu \simeq (10^{-10}, 10^{-11}) \mu_B$ .

The penetration in the region of such small  $Q^2$  requires, however, to measure small energies of recoil electrons ( $\leq MeV$ ).

Several other transitions [15] could be envisaged and have been proposed to obtain information about the neutrino magnetic moment. An appropriate selection of quantum numbers using nuclear transitions to enhance the electromagnetic amplitude looks, however, negative due to the presence of both vector and axial-vector components in the weak amplitude, so no general enhancement of the magnetic moment contribution relative to the weak one is found on these grounds. Coherent neutrino-nucleus scattering keeps, as in the electron case, the vector current contribution to both the magnetic and the weak amplitude, but the nuclear recoil is difficult to measure ( much more difficult than for electrons ). The strategy to get a relative enhancement of the magnetic moment amplitude on this dynamical basis is satisfied exceptionally around one point for electron antineutrino-electron elastic scattering, for which there is a dynamical zero for the weak cross section [16] at leading order for  $E_\nu = m_e/(4\sin^2\theta_w)$  and forward electrons.

In this paper we will consider the process

$$\nu(\bar{\nu}) + e \rightarrow \nu(\bar{\nu}) + e + \gamma \quad (3)$$

for which there are contributions to the cross section from the weak interaction as well as from the neutrino magnetic moment. This reaction has been considered before in a different context [17]. Even if the process (3) has an additional power of  $\alpha$  in the cross section relative to the elastic case, the restriction to low recoil energies in order to reach down low values of  $Q^2$  is a priori not necessary. As we will see, the limit  $Q^2 = 0$  at fixed values of the recoil energies is precisely obtained at the favourable situation of the maximal opening angle between electron and photon in the final state. Whatever the experimental limit on the total recoil energies  $\nu$  could be, the inelastic process (3) is able to lead to lower values of  $Q^2$  than the elastic one, as shown by the ratio  $x = Q^2/(2m\nu)$  varying from 1 to 0. The argument of using low incident neutrino energies to lower the effective contact interaction cross section of the standard theory relative to the smoother energy dependent magnetic cross section comes here as for the elastic process.

The paper is organized as follows. In Section 2 we present the calculation of the

amplitudes for the process (3) and the observables. Section 3 discusses the kinematics and the phase space details in different variables appropriate to their experimental accessibility. In Section 4 we analyze the behaviour of both weak and magnetic cross sections at low  $Q^2$  for different limiting cases: either at fixed  $\nu$  or at fixed  $x$ -values, by performing an analytic calculation in these limits. General results are given in Section 5 with special emphasis in its presentation for the inclusive distribution  $d^2\sigma/dx d\nu$ . Some conclusions are given in Section 6.

## 2 Weak and electromagnetic amplitudes

In this Section we present shortly the results of the calculation of the cross section for the process

$$\nu(l) + e(p) \rightarrow \nu(l') + e(p') + \gamma(k) \quad (4)$$

The standard effective Hamiltonian of the weak interaction of neutrinos and electrons has the form

$$H_W = \frac{G_F}{\sqrt{2}} \sum_l \bar{\nu}_l \gamma^\mu (1 - \gamma_5) \nu_l \bar{e} \Gamma_\mu e + h.c. \quad (5)$$

Here

$$\Gamma_\mu = \gamma_\mu \left[ g_L^{(l)} \frac{1 - \gamma_5}{2} + g_R^{(l)} \frac{1 + \gamma_5}{2} \right] \quad (6)$$

where

$$\begin{aligned} g_L^{(l)} &= -1 + 2\sin^2\theta_W + 2\delta_{le} \\ g_R^{(l)} &= 2\sin^2\theta_W \end{aligned} \quad (7)$$

and  $\theta_W$  is the electroweak mixing angle. The term  $\delta_{le}$  in Eq. (7) takes into account the charged current contribution to the effective Hamiltonian in its charge-retention-form.

The invariant T-matrix element generated by the Hamiltonian (5) for the radiative process (3) is obtained by adding the two amplitudes associated with the insertion of the photon in the incoming or outgoing electron leg:

$$\begin{aligned}
T_W = & \frac{G_F}{\sqrt{2}} e \bar{u}(l') \gamma^\mu (1 - \gamma_5) u(l) \\
& \times \bar{u}(p') \left\{ \Gamma_\mu(\mathbf{a}\varepsilon^*) + \left[ \Gamma_\mu \frac{\mathbf{k} \cdot \boldsymbol{\varepsilon}^*}{2(p\mathbf{k})} + \frac{\boldsymbol{\varepsilon}^* \cdot \mathbf{k}}{2(p'\mathbf{k})} \Gamma_\mu \right] \right\} u(p)
\end{aligned} \tag{8}$$

where  $\mathbf{a}$  is the four-vector

$$\mathbf{a}^\alpha = \frac{p'^\alpha}{(p'\mathbf{k})} - \frac{p^\alpha}{(p\mathbf{k})} \tag{9}$$

and  $\varepsilon$  is the polarization vector of the photon. Let us notice that the use of the Dirac equation has allowed to rewrite the matrix element of the process in such a way that the first term of Eq. (8) corresponds to  $\gamma$ - emission by the electron charge whereas the second term is induced by the electron magnetic moment. Such a decomposition simplifies considerably the calculation of the cross section.

We will not enter into the details of the rather cumbersome calculations for the cross section. Taking the appropriate sum for the neutrino spin states (only left-handed components contribute) as well as the sum and average for the electron spin states, one obtains from Eq. (8) the following.

$$\begin{aligned}
\sum |T_W|^2 = & 32G_F^2 e^2 \{ [-g_L^2(lp)(l'p') - g_R^2(l'p)(lp') + g_L g_R m^2(ll')] \mathbf{a}^2 \\
& + [g_L^2(l'p') \{ (\mathbf{a}l)(p\mathbf{k}) - [(\mathbf{a}p) - 1](lk) \} \\
& + g_R^2(lp') \{ (\mathbf{a}l')(p\mathbf{k}) - [(\mathbf{a}p) - 1](l'k) \}] \frac{1}{(kp)} \\
& + [g_L^2(lp) \{ (\mathbf{a}l')(p'\mathbf{k}) - [(\mathbf{a}p') - 1](l'k) \} \\
& + g_R^2(l'p) \{ (\mathbf{a}l)(p'\mathbf{k}) - [(\mathbf{a}p') - 1](lk) \}] \frac{1}{(kp')} \\
& - 2g_L g_R m^2 \frac{(lk)(l'k)}{(p\mathbf{k})(p'\mathbf{k})} \}
\end{aligned} \tag{10}$$

where  $m$  is the electron mass.

We are going to take also into account the contribution to the cross section of the process from the diagrams with  $\gamma$ -exchange between neutrino and electron vertices, due to a possible neutrino magnetic moment. The matrix element of the electromagnetic current between initial and final neutrino states has the form

$$i f_M \sigma_{\mu\nu} q^\nu \tag{11}$$

where  $q = l - l' = p' + k - p$  is the momentum transfer. The coupling  $f_M$  at  $q^2 = 0$  is the neutrino magnetic moment  $\mu_\nu$ . We are not going to consider a possible neutrino electric dipole moment, which is both P- and CP-odd.

The corresponding invariant T-matrix element is given now by the amplitudes associated to the two diagrams of Fig. 1.

$$\begin{aligned}
T_M = & \frac{e^2}{q^2} f_M \bar{u}(l') \sigma^{\mu\nu} q_\nu u(l) \\
& \times \bar{u}(p) \left\{ \gamma_\mu (\mathbf{a} \varepsilon^*) + \left[ \gamma_\mu \frac{\mathbf{k} \cdot \boldsymbol{\varepsilon}^*}{2(pk)} + \frac{\boldsymbol{\varepsilon}^* \cdot \mathbf{k}}{2(p'k)} \gamma_\mu \right] \right\} u(p)
\end{aligned} \tag{12}$$

with  $\mathbf{a}$  as given by Eq. (9). Again the two terms of Eq. (12) correspond to  $\gamma$ -emission by the electron charge and magnetic moment, respectively. In this case the neutrino vertex changes its chirality, so for massless left handed incoming neutrinos one can obtain the corresponding transition probability by averaging over initial neutrino spin states and summing over final ones. With this recipe, it is straightforward to obtain

$$\begin{aligned}
\sum |T_M|^2 = & \frac{32e^4 f_M^2}{q^2} \{ (lp)(lp') \mathbf{a}^2 \\
& + [(\mathbf{a}p)(lk) - (\mathbf{a}l)(pk)] \frac{(lp')}{(kp)} \\
& + [(\mathbf{a}p')(lk) - (\mathbf{a}l)(p'k)] \frac{(lp)}{(kp')} \\
& - \frac{(lk)(lp')}{(kp)} - \frac{(lk)(lp)}{(kp')} + \frac{m^2(kl)^2}{(kp)(kp')} \}
\end{aligned} \tag{13}$$

This neutrino magnetic moment contribution (13) adds incoherently to the weak interaction result (10) as a consequence of the opposite final neutrino helicity induced by  $T_W$  and  $T_M$  for massless neutrinos.

The three-body final state cross section is given, with the normalization used for the invariant amplitudes, by

$$\begin{aligned}
d\sigma = & \frac{1}{8(lp)} \frac{1}{(2\pi)^5} \delta^4(l + p - l' - p' - k) \frac{d^3 l'}{2E'_\nu} \frac{d^3 p'}{2E'} \frac{d^3 k}{2E_\gamma} \\
& \times \sum [ |T_w|^2 + |T_M|^2 ]
\end{aligned} \tag{14}$$

The observables of interest in terms of momenta of the recoil electron and the emitted photon are studied in the next section.

### 3 Kinematics

The differential cross section of the process (3) depends on 5 independent variables. It is convenient to choose the following invariant variables

$$\begin{aligned}
s &= (l + p)^2 \\
t_1 &= (l' - l)^2 \\
s_1 &= (l' + p')^2 \\
t_2 &= (p - k)^2 \\
s_2 &= (p' + k)^2
\end{aligned} \tag{15}$$

for which the phase space integral can be written as

$$\begin{aligned}
&\int \frac{d^3p'}{2E'} \frac{d^3l'}{2E'_\nu} \frac{d^3k}{2E_\gamma} \delta(p + l - p' - l' - k) = \\
&= \frac{\pi}{16\lambda^{1/2}(s, m^2, 0)} \int \frac{dt_1 ds_1 dt_2 ds_2}{(-\Delta_4)^{1/2}}
\end{aligned} \tag{16}$$

where  $\Delta_4$  is the 4 x 4 symmetric Gram determinant [18]. The integration domain is given by the condition  $\Delta_4 \leq 0$ .

The weak and electromagnetic squared amplitudes, as obtained in Section 2, can be written in the form

$$\begin{aligned}
|T_W|^2 &= \frac{f(s, t_1, s_1, t_2, s_2)}{(s_2 - m^2)^2 (t_2 - m^2)^2} \\
|T_M|^2 &= \frac{g(s, t_1, s_1, t_2, s_2)}{t_1 (s_2 - m^2)^2 (t_2 - m^2)^2}
\end{aligned} \tag{17}$$

where  $f$  and  $g$  are third degree (or lower) polynomials of the invariants. Fixing the other invariants, the variable  $s_1$  corresponds to the ( unobservable ) angle between the outgoing neutrino and electron momenta. The integration over  $s_1$  can be performed analytically, being  $f$  and  $g$  second degree polynomials in  $s_1$ . In the Appendix we give the exact results for the triple differential cross section once  $s_1$  has been integrated over, in terms of appropriate variables (see below).

The remaining variables  $t_1, s_2, t_2$  are observable quantities, for which the physical region is given by the following invariant conditions:



$$\begin{aligned}
& 1) t_1 \leq 0; m^2 \leq s_2 \leq s, t_2 \leq m^2 \\
& 2) G(s, t_1, s_2, 0, m^2, 0) \leq 0 \Rightarrow (s - s_2)(s - m^2) + st_1 \geq 0 \\
& 3) G(s_2, t_2, 0, t_1, m^2, m^2) \leq 0 \Rightarrow \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & m^2 & s_2 & 0 \\ 1 & m^2 & 0 & t_1 & t_2 \\ 1 & s_2 & t_1 & 0 & m^2 \\ 1 & 0 & t_2 & m^2 & 0 \end{vmatrix} \geq 0
\end{aligned} \tag{18}$$

with the  $G$ -function as defined in reference [18].

The integration over  $t_2$  which is associated with the photon energy  $E_\gamma$  in the LAB frame,  $t_2 = m^2 - 2mE_\gamma$ , can still be performed on analytic grounds in some cases.

Our next discussion is the translation of the physical region (18) of the invariant variables into that for the geometrical variables in the LAB frame: electron-photon opening angle  $\theta$ , electron recoil energy  $T$  and photon energy  $E_\gamma$ , or in terms of the dimensionless variables  $x, y, \omega$  to be defined below.

The relation is given by

$$\begin{cases} t_1 & \equiv -Q^2 = 2T \left[ (m - E_\gamma) + E_\gamma \sqrt{1 + \frac{2m}{T}} \cos\theta \right] \\ s_2 & = t_1 + m^2 + 2m(T + E_\gamma) \\ t_2 & = m^2 - 2mE_\gamma \end{cases} \tag{19}$$

Then eqs. (18) lead to  $-4E_\nu(E_\nu - T - E_\gamma) \leq t_1 \leq 0$  and  $E_\nu \geq T + E_\gamma$ . For a given recoil energy  $T$  of the electron, the physical region in the plane  $(E_\gamma, \theta)$  is given by Fig. 2

There are many interesting features in Fig. 2. The line at which  $Q^2 = 0$  corresponds to the maximal opening angle

$$Q^2 = 0 \longleftrightarrow \cos\theta = \frac{1}{\sqrt{1 + \frac{2m}{T}}} \left( 1 - \frac{m}{E_\gamma} \right) \tag{20}$$

allowed for photon energies

$$E_\gamma^0 = \frac{m}{1 + \sqrt{1 + \frac{2m}{T}}} \leq E_\gamma \leq E_\gamma^m = E_\nu - T \tag{21}$$

For lower photon energies  $0 < E_\gamma < E_\gamma^0$ , the maximum opening angle is  $180^\circ$  and  $Q^2$  decreases from its elastic scattering value  $Q_{el}^2 = 2mT$  (at  $E_\gamma = 0$ ) to reach  $Q^2 = 0$

(at  $E_\gamma = E_\gamma^0$ ). We see, therefore, that for any values of  $T$  and  $E_\gamma$  ( $T + E_\gamma \leq E_\nu$ ) there always exists a region of opening angles for which  $Q^2$  is lower than the corresponding  $Q_{el}^2$ . Furthermore, this region is found at the highest allowed values of  $\theta$ .

Other interesting points and boundaries in Fig. 2 are the following:

-  $\theta_1$  is the opening angle in the inelastic process for which one obtains  $Q^2 = Q_{el}^2$ .

It is given by

$$Q^2 = Q_{el}^2 \longleftrightarrow \cos\theta_1 = \frac{1}{\sqrt{1 + \frac{2m}{T}}} \quad (22)$$

-  $\theta_0$  is the minimum opening angle for which  $Q^2 = 0$  is reachable. It is given by

$$\cos\theta_0 = \left[1 - \frac{m}{E_\nu - T}\right]\cos\theta_1 \implies \theta_0 > \theta_1 \quad (23)$$

- For the domain of the high energy photons

$$E_\gamma^1 = E_\nu - \frac{T}{2}\left(1 + \sqrt{1 + \frac{2m}{T}}\right) \leq E_\gamma \leq E_\gamma^m \quad (24)$$

the maximum  $Q^2 = 4E_\nu(E_\nu - T - E_\gamma)$  corresponds to a minimum opening angle

$$\cos\theta = \frac{4E_\nu(E_\nu - T - E_\gamma) - 2T(m - E_\gamma)}{2TE_\gamma\sqrt{1 + \frac{2m}{T}}} \quad (25)$$

It is now of interest to introduce the dimensionless variables

$$x = Q^2/(2m\nu), \quad y = \frac{\nu}{E_\nu}, \quad \omega = \frac{E_\gamma}{E_\nu} \quad (26)$$

with  $\nu = T + E_\gamma$  the total energy release of the process in the laboratory system. For fixed  $x$  and  $y$ , the  $\omega$ -integration in the cross section, although cumbersome, is straightforwardly made in an analytic way. We discuss some interesting limits for the inclusive cross section in  $x$  and  $y$  in the next section, in particular for its low  $Q^2$ -behaviour as a consequence of CVC and PCAC. First we determine the physical region in terms of these variables, following eqs. (18):

1)

$$0 \leq x \leq 1 \quad (27)$$

where  $Q^2 = 0$  for  $x = 0$  at fixed  $y$ , whereas  $E_\gamma = 0$  for  $x = 1$

2)

$$Q^2 \leq 4E_\nu(E_\nu - \nu) \Rightarrow 0 \leq y \leq \left(1 + \frac{mx}{2E_\nu}\right)^{-1} \quad (28)$$

with no threshold for the inelastic process.

3)

$$\left\{ \begin{array}{l} \omega^- \leq \omega \leq \omega^+ \\ \omega^\pm = y(1-x) \frac{1 + \frac{E_\nu}{m}y[1 \pm \sqrt{1 + \frac{2mx}{E_\nu y}}]}{1 + 2\frac{E_\nu}{m}y(1-x)} \end{array} \right. \quad (29)$$

One notices in Eq. (29) the soft-photon limit  $E_\gamma \rightarrow 0$  at the elastic scattering kinematics  $x \rightarrow 1$ . We represent in Fig. 3, the photon energy limits  $E_\nu\omega^\pm/m$  as functions of  $x$  and  $\frac{E_\nu}{m}y$ ; in this form these results are universal, independent of the incoming neutrino energy  $E_\nu$  except for the maximum allowed value for  $\frac{E_\nu}{m}y$  ( see Eq. (28) ).

In Fig. 4 we give the allowed domain of the variables  $(Q^2, \nu)$ , where the constraint of  $x$  fixed represents an straight-line and  $\nu_0 = 2E_\nu^2/(2E_\nu + m)$ . For a given  $y$ , associated for example with an experimental cut in energy release, it is possible now to reach  $Q^2$ -values lower than  $Q_{el}^2$  for  $x < 1$ . This is nothing but a manifestation of the features discussed in Fig. 2 for the geometrical variables. Furthermore, at fixed  $x$ , one can also approach  $Q^2 \rightarrow 0$  taking  $\nu \rightarrow 0$ .

## 4 Low- $Q^2$ behaviour

We are interested in the behaviour of both the weak and the electromagnetic cross sections at low  $Q^2$ , with a view to enhance the second contribution with respect to the first one. As emphasized before, it is an straightforward, though cumbersome, matter to obtain the triple differential cross section in the variables,  $x, y, \omega$ , as given in the Appendix; in order to check the results and discuss the physics of the process some

limits will be illuminating. First we consider, at  $y, \omega$  fixed, the expansion around  $x \rightarrow 0$ . The weak cross section is

$$\begin{aligned} \frac{d\sigma_W}{dx dy d\omega}_{x \ll 1} \simeq & \frac{G^2 m^2}{\pi^2} \alpha \frac{1}{y^3 \omega} \left\{ W(y, \omega) g_A^2 \right. \\ & \left. + \frac{E_\nu x y}{2m} [V(y, \omega) g_V^2 + A(y, \omega) g_A^2 + I(y, \omega) g_V g_A] \right\} \end{aligned} \quad (30)$$

where

$$\begin{aligned} W(y, \omega) &= (1 - y)(y - \omega)^2 \\ V(y, \omega) &= (1 - y + \frac{y^2}{2})[y^2 + \omega^2 - \frac{2m}{E_\nu}(y - \omega) + \frac{m^2}{E_\nu^2 y \omega}(y - \omega)^2] \\ A(y, \omega) &= (1 - y + \frac{y^2}{2})(y^2 + \omega^2) \\ &\quad - \frac{2m}{E_\nu}(y - \omega)[(1 - y)\frac{2y - 5\omega}{y} - \frac{y}{2}(y + 2\omega)] \\ &\quad + \frac{m^2}{E_\nu^2 y \omega}(y - \omega)^2[(1 - y)\frac{y - 12\omega}{y} - \frac{y}{2}(y + 4\omega)] \\ I(y, \omega) &= y(2 - y)(y^2 - \omega^2) \end{aligned} \quad (31)$$

and the couplings are

$$g_V = \frac{g_L + g_R}{2}, \quad g_A = \frac{g_L - g_R}{2} \quad (32)$$

in terms of the chiral couplings of Eq. (7).

There are interesting features associated with this result. At  $x = 0$  the only survival term in the cross section goes like  $g_A^2$ . By the use of CVC and a leptonic analogue of PCAC, Sehgal and Weber [19] reproduced this term as the analogue of Adler's theorem for hadronic reactions. It is well known that, due to CVC, the structure function associated with inelastic excitations mediated by the vector current goes like  $Q^2$  at fixed  $\nu$ . So only the  $g_A^2$ -term can survive at  $x = 0$ . This term has a contribution for the leptonic current proportional to the electron mass, hence the global scale  $m^2$  appearing in Eq. (30) is now understood.

Nevertheless, it is important to stress that  $W(y, \omega)$  will be the dominant term only in a very restricted range around  $x = 0$ . So, for example, this term gives a good approximation provided  $\nu \gg m$  (high incoming energies) but within the restricted

range  $Q^2 \ll 4m^2$ . This is so because the linear term in  $x$ , in fact, goes as  $Q^2/4m^2$ . Furthermore, the  $W(y, \omega)$  dependence goes like the square of the recoil energy of the electron. If  $\nu \ll m$  there are high cancellations in this term, seen for example when one integrates over  $\omega$  at fixed  $y$ . We conclude that the  $x = 0$  term is only important at high incoming energies with  $\nu \gg m$ , but with  $Q^2 \ll 4m^2$ . Our strategy will be just the contrary, i.e., have  $\nu < m$  with low  $Q^2$ , in order to suppress the  $x = 0$   $g_A^2$ -term in the weak cross section.

The linear term in  $Q^2$ , within the bracket of Eq. (30), contains contributions from the vector and axial couplings to electrons and their interference. The purely vector contribution can be understood from the Compton scattering cross section, where  $y$  would be the energy of the incoming photon and  $\omega$  the energy of the outgoing photon, both normalized to  $E_\nu$ . The Klein-Nishina formula [20] for the cross section distribution, when written with the appropriate variable  $\omega$  instead of the scattering angle, reads

$$\frac{d\sigma^{\gamma\gamma}}{d\omega} = \frac{\pi\alpha^2}{mE_\nu} \frac{1}{y^3\omega} [y^2 + \omega^2 - 2\frac{m}{E_\nu}(y - \omega) + \frac{m^2}{E_\nu^2 y \omega} (y - \omega)^2] \quad (33)$$

which is immediately identified with the  $V(y, \omega)$  term of Eq. (30). The axial term  $A(y, \omega)$  has a different behaviour and it will tend to  $V(y, \omega)$  only in the limit  $m/E_\nu \rightarrow 0$ .

The cross section induced by a neutrino magnetic moment  $\mu_\nu \neq 0$  gives, in the limit  $x \rightarrow 0$ .

$$\frac{d\sigma_M}{dx dy d\omega}_{x \ll 1} \simeq \frac{\alpha^3}{2m^2} \left(\frac{\mu_\nu}{\mu_B}\right)^2 \frac{1}{y^3\omega} \left\{ M(y, \omega) + \frac{x}{2y^2\omega} N(y, \omega) \right\} \quad (34)$$

where the  $(y, \omega)$ -functions are

$$\begin{aligned}
M(y, \omega) &= (1 - y)[(y^2 + \omega^2) - 2\frac{m}{E_\nu}(y - \omega) + \frac{m^2}{E_\nu^2 y \omega}(y - \omega)^2] \\
N(y, \omega) &= 2y^2\omega(y - \omega)[(1 - y)(y + 5\omega) + y^2\omega] \\
&\quad - \frac{2m}{E_\nu}y[(1 - y)(y^3 - 6y^2\omega + 5y\omega^2 + 6\omega^3) - y^2\omega(y^2 - y\omega - \omega^2)] \\
&\quad - \frac{2m^2}{E_\nu^2}y(y - \omega)[2(1 - y)(3y - 11\omega) + y^2(y - 4\omega)] \\
&\quad - \frac{m^3}{E_\nu^3}(y - \omega)^2[8(1 - y) + 3y^2]
\end{aligned} \tag{35}$$

The first point to be noticed in Eq. (34) is the absence of the  $1/x$  singularity associated with the photon propagator in the magnetic contribution present in the elastic scattering cross section. This is again due to the conservation of the electromagnetic current in the electron vertex, implying a linear  $Q^2$ -behaviour of the structure function, at  $\nu$  fixed, for inelastic excitations. The leading  $M(y, \omega)$  term is again, like  $V(y, \omega)$ , obtainable from the Compton scattering cross section. In fact, one can write

$$\left. \frac{d\sigma_M}{dx dy d\omega} \right|_{x=0} = \frac{\alpha}{2\pi} \frac{E_\nu}{m} \left( \frac{\mu_\nu}{\mu_B} \right)^2 (1 - y) \frac{d\sigma^{\gamma\gamma}}{d\omega} \tag{36}$$

with  $\sigma^{\gamma\gamma}$  given by Eq. (33). Contrary to the behaviour that we have discussed for  $W(y, \omega)$  in the weak cross section, the term  $M(y, \omega)$  is not here suppressed with respect to the linear term in  $x$ ,  $N(y, \omega)$ , so Eq. (36) is a very good approximation to the magnetic cross section at low energies and low values of  $Q^2$ . Taking the ratio of cross sections at  $Q^2 = 0$ , we have

$$\begin{aligned}
\left. \frac{d\sigma_M}{d\sigma_W} \right|_{x=0} &= \left( \frac{\mu_\nu}{\mu_B} \right)^2 \frac{\pi^2 \alpha^2}{G^2 m^2} \frac{1}{2mg_A^2 T} \\
&\quad \times \left\{ \frac{2E_\gamma(E_\gamma + T) + T^2}{mT} + \frac{mT - 2E_\gamma(E_\gamma + T)}{E_\gamma(E_\gamma + T)} \right\}
\end{aligned} \tag{37}$$

where the global factor in front of the bracket is a typical measure of this ratio for the elastic scattering process at the same value of  $T$ . We remind the reader that  $x = 0$  is then obtained by the suitable choice of the maximal opening angle between electron

and photon. A glance at Eq. (37) would say that the highest cross section ratios are obtained for the hardest photon limit  $E_\gamma \gg T$ , with values higher than the elastic ones at will. Even more, one would say that higher neutrino energies are favoured in order to have hard photons but the discussion after Eq. (30) should have clarified that a little departure from  $x = 0$  under these conditions is enough to enhance the next linear term in  $x$  so that the ratio (37) becomes diluted. To conclude, the strategy to reach low enough  $Q^2$ -values, approaching  $\theta_{max}$  at fixed  $(y, \omega)$ , works only in a very limited angular range around  $\theta \simeq \theta_{max}$ . Whenever the results are integrated over a wider region of  $\theta$ , the ratio  $d\sigma_M/d\sigma_W$  will be diluted.

We can consider the approach to  $Q^2 \rightarrow 0$  following the lines of fixed  $x$  of Fig. 4. The vector contribution is in this case not penalized due to CVC with respect to the axial contribution, as it was the case for  $x \rightarrow 0$ : the structure function goes like  $Q^2/\nu$  and the limit  $\nu \rightarrow 0$  is not physically forbidden for our process. It is thus of interest to study the inclusive cross sections  $d\sigma/dxdy$  and explore their behaviour when  $y \rightarrow 0$  at fixed  $x$ . We can use the results of the triple differential cross sections given in the Appendix for the integration in  $\omega$ , with the condition  $\nu \ll m$ , and obtain

$$\begin{aligned} \frac{d^2\sigma_W}{dxd\nu} \simeq & \frac{4}{3} \frac{G^2\alpha}{\pi^2} \frac{1}{1-x} \nu \left\{ x[(g_V^2 + g_A^2) - \frac{\nu}{E_\nu}(g_V^2 + g_A^2 - 2xg_Vg_A) - \frac{x}{2} \frac{m\nu}{E_\nu^2}(g_V^2 - g_A^2)] \right. \\ & \left. + \frac{\nu}{m} [(\frac{17}{10} - 2)xg_V^2 + \frac{1}{10}(37x^2 - 60x + 20)g_A^2] + O(\nu^2) \right\} \end{aligned} \quad (38)$$

for the weak cross section, whereas

$$\frac{d^2\sigma_M}{dxd\nu} \simeq \frac{4\alpha^3}{3m^3} \left(\frac{\mu_\nu}{\mu_B}\right)^2 \frac{1}{1-x} \left\{ 1 - \frac{\nu}{E_\nu} + \left(\frac{17}{10}x - 2\right) \frac{\nu}{m} + O(\nu^2) \right\} \quad (39)$$

gives the magnetic moment cross section, which is much less sensitive to low  $x$  values. There is no need of an infrared cutoff in  $\omega$  as far as  $x \neq 1$  and  $\nu \neq 0$  ( see eq. (29) ); if needed experimentally, it must be included in this integration.

The ratio of (39) to (38) shows a very essential feature: the most favourable sensitivity to a neutrino magnetic moment in the inelastic process comes from the region of low excitation energy  $\nu$  and, subsequently, from low  $x$  values. As seen in Fig.4, lowering  $\nu$  automatically lowers  $Q^2$  and the behaviour of the structure functions are then more favourable than for low  $Q^2$  at fixed  $\nu$ .

The equations (38) and (39), valid for  $x$  fixed, show the soft photon factorization in the limit  $x \rightarrow 1$ . The factor in the first square bracket in the r.h.s. of eq. (38) is, at  $x = 1$ , proportional to the weak elastic cross section up to  $O(\nu^2)$ ; so it is the second square bracket in front of  $\nu/m$ , once more at leading order in  $\nu$ . Note that this factor becomes  $-\frac{3}{10}(g_V^2 + g_A^2)$  at  $x = 1$ . In eq. (39),  $1 - \nu/E_\nu$  is proportional to the elastic magnetic moment cross section and the remaining  $-3/10$  factor of the  $\nu/m$  term  $x = 1$  is the same signal of soft photon factorization as before. Finally, note that in eq. (38) only the last  $g_A^2$  term survives at  $x = 0$ , with a  $\nu^2$  suppression due to the strong cancellations in the  $\omega$ -integration of  $W(y, \omega)$ .

## 5 Results.

In this Section we present detailed numerical results of the weak and electromagnetic cross section for the inelastic process (3) both for the triple differential cross sections  $d^3\sigma_{W,M}/dTdE_\gamma d(\cos\theta)$  and for the inclusive cross sections  $d^2\sigma_{W,M}/d\nu dx$ .

We have made an analysis of the ratio  $d\sigma_M/d\sigma_W$  as illustrated in figs. 5 and 6 for incoming energies  $E_\nu = 1MeV$  for electron antineutrinos and  $E_\nu = 29.79 MeV$  for muon neutrinos from  $\pi$ -decay at rest, respectively, using the complete expressions without any approximations ( $T = 0.2MeV$ ). We give the regions in the plane  $(\theta, E_\gamma)$  for which the cross section, when integrated from  $\theta$  to  $\theta_{\max}$  ( $Q^2 = 0$ ) at each  $E_\gamma$ , satisfies the following requirement: the ratio  $d\sigma_M/d\sigma_W$  is 5, 4, 3 or 2 times larger than the elastic scattering ratio for the same  $T$ . Even if the ratio increases with  $E_\gamma$  on the  $Q^2 = 0$  line, the angular width becomes more and more narrow, as we expected from the analysis of the previous section.

Fig. 7 gives the inclusive cross section  $d^2\sigma/d\nu dx$  for electron-antineutrino scattering at  $E_\nu = 1MeV$ , separating (a) the weak contribution, (b) the magnetic moment contribution for  $\mu_\nu = 10^{-10}\mu_B$ , and (c) their ratio. The conclusion obtained in the last section by the use of analytic limits is dramatically confirmed by these results: the highest sensitivity is obtained for the lowest values of  $\nu$  and, by going down to low values of  $x$ , the sensitivity is higher than for the elastic scattering case with  $x = 1$ . On the contrary, once  $\nu$  is high enough, the sensitivity is not improved when lowering the value of  $x$ . At  $x = 1$  and  $\nu = 2E_\nu^2/(2E_\nu + m)$  one can still see at  $E_\nu = 1MeV$  the residual effect of the elastic zero present [16] at  $E_\nu = m/(4\sin^2\theta_W) \simeq 0.51MeV$ .



Experimentally one should consider cuts both in  $E_\gamma$  and  $T$  which could modify our results for the inclusive cross sections which are sensitive to the cut in  $T$  ( $T^{th}$ ) for small  $x$  and to the experimental threshold in  $E_\gamma$  ( $E_\gamma^{th}$ ) for  $x \simeq 1$ . However the main features remain the same as shown in figure 7(d), where the ratio  $d\sigma_M/d\sigma_W$  has been plotted for  $E_\nu = 1MeV$  taking as experimental thresholds  $T \geq T^{th} = 100KeV$ ,  $E_\gamma \geq E_\gamma^{th} = 100KeV$ . Note that there is still a high sensitivity at small  $\nu$  values, higher when we subsequently consider low  $x$  values.

The general features are not highly sensitive to the incoming neutrino energy within the range of the reactor antineutrino spectrum. In Fig. 8 we present a similar analysis to that of Fig. 7(c), but the cross sections have been averaged over a realistic [21] antineutrino spectrum. This result shows similar features as those described in the monoenergetic case. The only difference is the disappearance of the remanent of the elastic zero at maximum electron recoil energy, due to the average over incoming neutrino energy

## 6 Outlook.

New neutrino physics can be introduced to generate a neutrino magnetic moment as large as  $10^{-10}\mu_B$ , a magnitude which can be tested in planned laboratory experiments on electron-antineutrino scattering by electrons. Furthermore this value corresponds roughly to the scale needed to play a role in solar neutrino physics. The laboratory tests look for a high enough sensitivity to the neutrino magnetic moment by lowering the accessible  $Q^2$  to enhance this contribution relative to the standard weak interaction cross section. The method based on the elastic scattering has the limitation associated with the cut in recoil energy needed to observe the process.

With a view to be able to reach, for a given recoil energy, lower values of  $Q^2$  than for the elastic process, we have studied in this paper the weak and neutrino magnetic moment contributions to the cross section for the inelastic radiative process  $\bar{\nu} + e \rightarrow \bar{\nu} + e + \gamma$ .

We have analyzed the inelastic process in its kinematic and dynamic behaviour in order to find the regions of higher sensitivity. For given recoil energies of both electron and photon, the value  $Q^2 = 0$  is reachable for the highest possible values of the opening angle between the two outgoing particles: if  $E_\gamma < m$ , this configuration

corresponds to  $\theta > 90^\circ$ . The  $Q^2 = 0$  kinematic configuration is always very favourable to enhance the neutrino magnetic moment contribution, even if at high values of the inelastic excitation energy  $\nu$  the beneficial effect of the  $1/Q^2$  -photon propagator is lost for inelastic scattering due to CVC. The integration of events around an angular region below the maximum  $\theta$  dilutes, however, this enhancement: the angular region of interest is more limited with increasing energy of the photon. We conclude that the most interesting situation corresponds to the inelastic configurations  $x < 1$  for low values of the excitation energy  $\nu$ . Even for the inclusive cross section  $d^2\sigma/dx d\nu$ , this effect is clearly manifested in our results of Figs. 7 and 8. It is understood as a suppression of the weak cross section, Eq. (38), whereas the magnetic moment contribution has a smooth behaviour, Eq. (39). Although absolute cross sections are small ( for instance,  $\sigma_M/\sigma_W = 4.4$ ,  $\sigma_M = 2.7 \cdot 10^{-47} \text{cm}^2$  for  $\mu_\nu = 10^{-10} \mu_B$  at  $E_\nu = 1 \text{MeV}$  integrating over  $\nu < 0.5 \text{MeV}$ ,  $x < 0.5$  ), the standard model contribution is suppressed in these circumstances more strongly than in the elastic scattering case.

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# A Appendix

## A.1 Triple differential cross sections

We now present the exact results for the triple differential cross sections in terms of the dimensionless variables (26).

The magnetic moment cross section can be written as

$$\begin{aligned} \frac{d\sigma_M}{dx dy d\omega} = & \frac{\alpha^3}{4m^2} \left(\frac{\mu_\nu}{\mu_B}\right)^2 \frac{1}{(1-x)^2} \frac{1}{(2mx + E_\nu y)^2} \sqrt{\frac{E_\nu y}{E_\nu y + 2mx}} \\ & \times \frac{1}{y^3 \omega^2} [M_0 + mM_1 + m^2 M_2 + m^3 M_3] \end{aligned} \quad (40)$$

where

$$\begin{aligned} M_0 = & 2E_\nu^2 y^2 (1-x) \{[-x(1-x)y^2 + (6x^2 - 6x + 1)(1-y)]\omega^3 \\ & + xy[(1-x)y^2 - 2(3x-2)(1-y)]\omega^2 \\ & + (x^2 + 1)y^2(1-y)\omega\} \\ M_1 = & 2yE_\nu \{-x(1-x)[(x^2 - x + 1)y^2 + (1-y)]\omega^3 \\ & + y[-x(x-1)^2(x+1)y + (4x^3 + 2x^2 - 9x + 2)(1-y)]\omega^2 \\ & - (1-x)y^2[x(x^2 - 1)y^2 + (x^3 - 13x + 2)(1-y)]\omega \\ & - x(x-1)^2 y^3(1-y)\} \\ M_2 = & y \{-x^2(1-x)y\omega^3 \\ & + 2[x(1-x)(2x^2 + x - 4)y^2 + (9x^2 - 14x + 1)(1-y)]\omega^2 \\ & + (1-x)y[x(x^2 + x + 10)(1-x)y^2 - 4(x^2 - 10x + 1)(1-y)]\omega \\ & + 2y^2(x-1)^2[-x(1-x)y^2 + (x^2 - 6x + 1)(1-y)]\} \\ M_3 = & \frac{x}{E_\nu} [-(1-x)(x+3)y^2 - 8(1-y)](\omega - y(1-x))^2 \end{aligned} \quad (41)$$

The weak cross section reads

$$\begin{aligned} \frac{d\sigma_W}{dx dy d\omega} = & \frac{G^2 m}{4\pi^2} \alpha \frac{1}{(1-x)^2} \frac{1}{(2mx + E_\nu y)^2} \sqrt{\frac{E_\nu y}{E_\nu y + 2mx}} \\ & \times \frac{1}{y^2 \omega^2} [W_0 + mW_1 + m^2W_2 + m^3W_3 + m^4W_4] \end{aligned} \quad (42)$$

where

$$\begin{aligned} W_0 = & x(1-x)E_\nu^3 y^2 \{y^2(x^2+1)[-y(2-y)(g_V - g_A)^2 + 2(g_V^2 + g_A^2)]\omega \\ & - 2xy[(xy^2 + 2(3x-2)(1-y))(g_V^2 + g_A^2) + 2x(2-y)g_V g_A]\omega^2 \\ & + [((2x^2 - 2x + 1)y^2 + 2(6x^2 - 6x + 1)(1-y))(g_V^2 + g_A^2) \\ & + 2(2x-1)y(2-y)g_V g_A]\omega^3\} \\ W_1 = & E_\nu^2 y \{(x-1)^2 x^2 y^3 [y(2-y)(g_V - g_A)^2 - 2(g_V^2 + g_A^2)] \\ & + 2(1-x)y^2 [xy^2((x^3 + 4x - 1)g_V^2 + (x^3 + 4x + 1)g_A^2) \\ & - (1-y)(x(x^3 - 13x + 2)g_V^2 + (x^4 - 15x^2 + 6x - 2)g_A^2) \\ & + 2x^2(x^2 + 4)y(2-y)g_V g_A]\omega \\ & - y[xy^2((2x^4 - 10x^3 + 6x^2 + 5x - 2)g_V^2 \\ & + (2x^4 - 10x^3 + 2x^2 + 9x - 2)g_A^2) \\ & - 2(1-y)(x(4x^3 + 2x^2 - 9x + 2)g_V^2 \\ & + (4x^4 + 14x^3 - 37x^2 + 22x - 4)g_A^2) \\ & - 2(4x^3 - 4x^2 + 4x - 5)x^2 y(2-y)g_V g_A]\omega^2 \\ & + 2(1-x)[-xy^2(x(x^2 - x - 1)g_V^2 + (x^3 - x^2 - 3x + 2)g_A^2) \\ & - (1-y)(x^2 g_V^2 + (-11x^2 + 12x - 2)g_A^2)]\omega^3\} \end{aligned}$$

$$\begin{aligned}
W_2 = & E_\nu x \{ (x-1)^2 y^3 [-y^2((2x^2+2x-1)g_V^2 + (2x^2+2x+1)g_A^2) \\
& + 2(1-y)(x^2-6x+1)(g_V^2 + g_A^2) - 4y(2-y)(x+1)xg_V g_A] \\
& + (1-x)y^2[y^2((3x^4+3x^2+2x-2)g_V^2 + (3x^4-x^2+26x-2)g_A^2) \\
& - 4(1-y)((x^2-10x+1)g_V^2 + 2(x^2-6x+1)g_A^2) \\
& + 8y(2-y)(3x+1)xg_V g_A] \omega \\
& + y[y^2((4x^4-6x^3-2x^2+1)g_V^2 + (4x^4-10x^3+30x^2-36x+7)g_A^2) \\
& + 2(1-y)((9x^2-14x+1)g_V^2 + (13x^2-22x+5)g_A^2) \\
& + 4y(2-y)(4x^2-5x-1)xg_V g_A] \omega^2 \\
& + (1-x)[y^2(3x^2g_V^2 + (-x^2+4x-4)g_A^2) - 4(1-y)g_A^2] \omega^3 \} \tag{43}
\end{aligned}$$

$$\begin{aligned}
W_3 = & x^2 \{ (1-x)^2 y^2 [y^2((1-x)(3x+1)g_V^2 - (3x^2-2x+7)g_A^2) \\
& - 8(1-y)(g_V^2 + g_A^2) - 8y(2-y)xg_V g_A] \\
& + 2(1-x)y[y^2((3x^2-6x-1)g_V^2 + (2x^2+4x+6)g_A^2) \\
& + 8(1-y)(g_V^2 + g_A^2) + 8y(2-y)xg_V g_A] \omega \\
& + [-y^2((11x^2-10x-1)g_V^2 + (-9x^2+14x+3)g_A^2) \\
& - 8(1-y)(g_V^2 + g_A^2) - 8y(2-y)xg_V g_A] \omega^2 \\
& + 2(1-x)y g_A^2 \omega^3 \}
\end{aligned}$$

$$W_4 = \frac{4x^3 y}{E_\nu} [\omega - y(1-x)]^2 (g_V^2 - g_A^2)$$

## A.2 Soft-photon limit

From the expressions for the triple differential cross sections given above we can now perform the integration over  $\omega$  around  $x \simeq 1$ ; the result both for the weak and the magnetic cross section reads

$$\frac{d^2\sigma}{dx dy} = \frac{\alpha}{\pi} \left[ \frac{d\sigma}{dy} \right]_{el} \frac{1}{1-x} [F(z) + O(1-x)] \tag{44}$$

where  $\left[\frac{d\sigma}{dy}\right]_{el}$  is the corresponding elastic scattering cross section, taking  $y = T/E_\nu$ , and

$$F(z) = \left[-2 + \frac{1}{z} \ln \left(\frac{1+z}{1-z}\right)\right] = 2 \sum_{n=1}^{\infty} \frac{z^{2n}}{2n+1} \quad (45)$$

where

$$z = \frac{\sqrt{\nu^2 + 2m\nu}}{m + \nu} < 1 \quad (46)$$

From this result it is easy to check the  $\nu/m \rightarrow 0$  limit in eqs. (38) and (39) for  $x \rightarrow 1$ . Notice the logarithmic infrared divergence of  $d\sigma/dy$ .

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## Figure Captions

- **Fig. 1)** Electromagnetic interaction Feynman diagrams for the process  $\nu e^- \rightarrow \nu e^- \gamma$ .
- **Fig. 2)** Physical region in the plane  $(E_\gamma, \theta)$  for a fixed  $T$ -value.
- **Fig. 3)** Photon energy limits  $E_\gamma^\pm/m$  as functions of  $x$  and  $\nu/m$ ; these limits are independent of incoming neutrino energy except for the maximum value of  $\nu$ ,  $\nu \leq 2E_\nu^2/(2E_\nu^2 + mx)$ .
- **Fig. 4)** Allowed domain for the variables  $(Q^2, \nu)$ . The straight lines passing through the origin represent fixed  $x$  values.  $x = 1$  ( upper line ) corresponds to the soft photon limit.
- **Fig. 5)** Regions in the plane  $(\theta, E_\gamma)$  where the  $\bar{\nu}_e$  radiative cross sections, when integrated from  $\theta$  to  $\theta_{max}$  ( $Q^2 = 0$ ), satisfy that the ratio  $d\sigma_M/d\sigma_W$  is 5,4,3 or 2 times larger than the elastic ratio at the same  $T$ -value ( $T = 0.2MeV$ ). The solid line represents the  $Q^2 = 0$  curve. In this figure  $E_\nu = 1MeV$ .
- **Fig. 6)** Same as figure 5 but for muon antineutrinos from pion decay at rest ( $E_\nu = 29.79MeV$ ). In this figure  $T = 0.2MeV$ .
- **Fig. 7(a)** Inclusive weak cross section  $d^2\sigma_W/dxd\nu$  for electron antineutrinos. The physical region is bounded by  $0 \leq x \leq 1$  and  $0 \leq \nu \leq 2E_\nu^2/(2E_\nu^2 + mx)$ ; the flat region on the right is unphysical. The decimal logarithm of the cross section in  $10^{-45}cm^2/MeV$  units is represented.
- **Fig. 7(b)** Same as 7(a) but for the magnetic moment cross section with  $\mu_\nu = 10^{-10}\mu_B$ .
- **Fig. 7(c)** Same as 7(a) for the ratio  $d^2\sigma_M/d^2\sigma_W$ .
- **Fig. 7(d)** Same as 7(c) including thresholds both for  $E_\gamma$  and  $T$  ( $E_\gamma^{th} = 100KeV$  and  $T^{th} = 100KeV$ ). The flat regions ( $d^2\sigma_M/d^2\sigma_W = 0$ ) are unphysical.
- **Fig. 8)** Same as figure 7(c) but averaged over a realistic reactor antineutrino spectrum.

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