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THE ε -TYPE CONTRIBUTION TO BARYON ASYMMETRY
FROM COLOURED HIGGS TRIPLETS

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ABSTRACT

In scenarios where the cosmological baryon asymmetry is generated in the decay of heavy coloured Higgs triplets at the one loop level (at least two triplets are needed), it is shown that in addition to the conventional triangle loop (ε' -type effect), a new kind of diagrams must be considered. This new type of diagrams have its origin in the mixing of the two Higgs fields (ε -type effect). The mixing of the two Higgs fields violates CP provided they have at least two common decay channels. For small mixing angles, the new contribution can be bigger than the conventional one.

Baryon number (B) violation, C and CP violation and lack of thermal equilibrium are the minimal ingredients to explain the observed matter-antimatter asymmetry¹⁾. With the discovery of Grand Unified theories (GUT), it was soon realized that all these ingredients are present in GUT's implemented with the standard Big-Bang cosmology²⁾. So GUT's are the first type of theory that offers the possibility of understanding the observed baryon asymmetry.

One of the most important and promising scenarios is the following: The heavy Higgs boson \mathcal{H} has an equilibrium distribution at $kT \gg M_{\mathcal{H}}$. when kT drops below $M_{\mathcal{H}}$ the equilibrium is lost provided the decay rates of \mathcal{H} and $\bar{\mathcal{H}}$ are at least of the order of the expansion rate of the universe at that temperature.

When \mathcal{H} and $\bar{\mathcal{H}}$ decay, the ratio of baryon number to entropy is³⁾:

$$\frac{k n_B}{s} = 45 \zeta(3) (N_{\mathcal{H}}/N) \frac{\Delta B}{4\pi^4} \quad (1)$$

where ΔB is the mean net baryon number produced in a pair of \mathcal{H} and $\bar{\mathcal{H}}$ decay. $N_{\mathcal{H}}$ is the number of \mathcal{H} and $\bar{\mathcal{H}}$ spin states, and N is related to the number of light boson and fermion spin states at $T \approx M_{\mathcal{H}}$ ³⁾.

More recently it has been suggested the generation of the baryon asymmetry in the framework of the standard model at the time of the $SU(2) \times U(1)$ phase transition. B violation associated with weak instantons is negligible at $T=0$. But at $T \approx M_W/\alpha \sim 10 \text{ TeV}$ it could be unsuppressed.⁴⁾

In this letter we are interested in the first scenario where ΔB is generated at high temperatures. ΔB is the microscopic quantity we are interested in and will be different from zero provided there is B , C and CP violation in the Higgs decay. In order to calculate ΔB we need to specify a particular GUT. Let us concentrate for illustrative purpose in $SU(5)$ with two five Higgs representations. In this theory, the relevant Lagrangian for our purpose is the Yukawa sector :

$$\begin{aligned} \mathcal{L}_Y = & \bar{\Psi}_m [(f_1)_{mn} \varphi_1 + (f_2)_{mn} \varphi_2] \chi_n + \text{h.c.} \\ & + \frac{1}{2} \bar{\Psi}_m^c [(g_1)_{mn} \varphi_1 + (g_2)_{mn} \varphi_2] \psi_n + \text{h.c.} \end{aligned} \quad (2)$$

where

$$\bar{\Psi} \chi \varphi \equiv \bar{\Psi}_{\mu\nu} \chi_{\mu} \varphi_{\nu} \quad \bar{\Psi}^c \psi \varphi \equiv -(1/4) \varepsilon_{\mu\nu\sigma\rho\lambda} \bar{\Psi}^{c\mu\nu} \psi^{\sigma\rho} \varphi^{\lambda}$$

The fermionic fields ψ and χ transform as the 10 and 5 representations respectively of $SU(5)$, the Higgs ϕ_k as a 5 and the matrices f_k, g_k are matrices in the flavour space and must contain some imaginary piece in order to introduce CP violation. It is well-known that the coloured triplet piece \mathcal{H}_k of ϕ_k must be superheavy and is coupled to the channels q_l and $\bar{q} \bar{q}$ thus violating baryon number. If we define

$$\bar{r} = \Gamma(\mathcal{H} \rightarrow \bar{q} \bar{q}) / \Gamma(\mathcal{H} \rightarrow \text{all}) \quad (3)$$

we will have

$$1 - \bar{r} = \Gamma(\mathcal{H} \rightarrow \bar{q} q) / \Gamma(\mathcal{H} \rightarrow \text{all}) \quad (4)$$

so that ΔB for one kind of \mathcal{H} boson will be⁵⁾:

$$\Delta B = \frac{1}{2} \left[\frac{1}{3} r - \frac{2}{3} (1-r) - \frac{1}{3} \bar{r} + \frac{2}{3} (1-\bar{r}) \right] = \frac{1}{2} (r - \bar{r}) \quad (5)$$

Following a theorem due to D.V.Nanopoulos and S.Weinberg⁵⁾, eq(4) is zero to first order in baryon violating interactions and to all orders in baryon conserving interactions. To get $\Delta B \neq 0$ it is necessary to calculate r and \bar{r} at least to second order in \mathcal{H} couplings of eq(2). So we expect $\Delta B \neq 0$, for example, from the interference of the tree and one loop diagrams of figure 1.

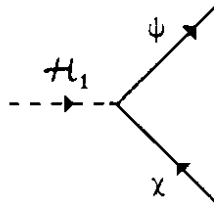


Fig. 1.a

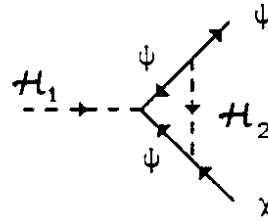


Fig. 1.b

A well-known result from CP violation tells us that for a particular fermionic channel, $r - \bar{r}$ is proportional to the imaginary part of a product of coupling constants times the imaginary part of the relative dynamical phase of the two amplitudes: in the diagrams of figure 1 this dynamical phase comes from the absorptive part of diagram 1b.

It is now quite evident that ΔB can be obtained from the graph of figure 2:

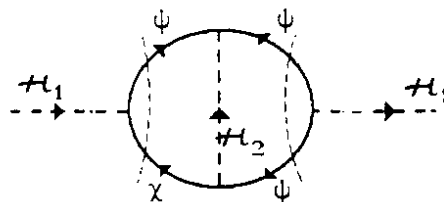


Fig. 2

This is a twice cutted graph, the second cut reflects the contribution of the absorptive part of the diagram 1b . In this graph we must take the imaginary part of the product of different flavour couplings summed over all flavours, that is to say, the Yukawa coupling contribution to ΔB in the diagram of figure 2 will be $\text{Im}\{\text{Tr}[g_1 f_2 f_1^\dagger g_2^\dagger]\}$, it is this imaginary trace the track of CP violation. In addition we must sum also over all kind of fermions in the first cut ($SU(5)$ indices) with the baryon number of the decay channel considered ($q|$) : C violation is needed to avoid cancellations when we performe this last sum.

A very simple way of finding out the topology of the diagrams that potentially can contribute to ΔB is based on the following simple remark. In the approximation that $SU(3)_C \otimes SU(2)_L \otimes U(1)$ is an unbroken symmetry (massless fermions) the Lagrangian (2) has the following invariance:

$$\begin{aligned} \psi &\rightarrow \psi' = V_{10} \psi \\ \chi &\rightarrow \chi' = V_5 \chi \end{aligned} \tag{6}$$

where V_{10} and V_5 are arbitrary unitary matrices in flavour space. This means that phisical observables must be invariant under

$$\begin{aligned} f_k &\rightarrow V_{10} f_k V_5^\dagger \\ g_k &\rightarrow V_{10}^* g_k V_{10}^\dagger \end{aligned} \tag{7}$$

so the graphs that will contribute to ΔB must be proportional to the imaginary part of an invariant trace made up of products of f_k and g_k . Thus, in order to classify all possible contribution to ΔB , *one must find out all possible traces invariant under (7) with non vanishing imaginary part.*

In the minimal $SU(5)$ there is only one five-plet φ so we will take in eq (2) $f_1 = f$, $g_1 = g$, $f_2 = g_2 = 0$. After a tedious but straightforward procedure it is easy to realize, following the method previously explained, that the leading contribution to ΔB is of the form $\text{Im}\{\text{Tr}[f^\dagger g^\dagger g g^\dagger f^* f^\dagger g f]\}^6$. So one of the diagrams giving contribution to ΔB at leading order in the minimal $SU(5)$ model will be the twice cutted four loops diagram of figure 3:

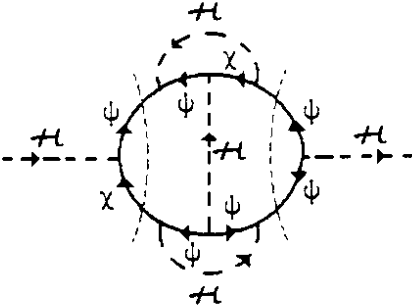


Fig. 3

Applying the same idea to the two Higgs model, the first surprise that immediately arises is that at order four in the Yukawa couplings in addition to the structure $\text{Im}\{\text{Tr}[g_1 f_2 f_1^\dagger g_2^\dagger]\}$ we also get $\text{Im}\{\text{Tr}(f_2 f_1^\dagger) \text{Tr}(g_2^\dagger g_1)\}$. In general these two structures will be linearly independent, the new one corresponds to the interference of the graph 1.a with an off-diagonal Higgs self-energy inserted in $\mathcal{H}_1 \rightarrow \psi \bar{\chi}$ giving rise to the new piece defined by the graph of figure 4:

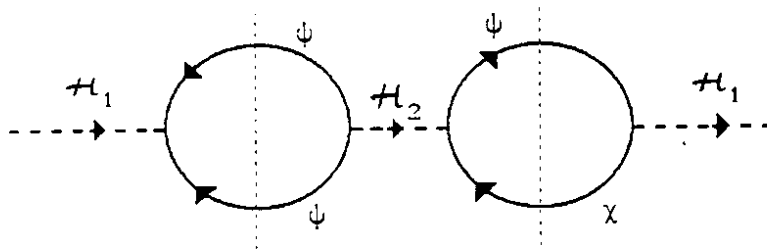


Fig. 4

To our knowledge, this structure has never before been considered in the literature, so let us give two very simple arguments in favour of graph 4 :

- i) In order to calculate the widths of \mathcal{H}_1 and \mathcal{H}_2 at the one loop level, the inverse propagator matrix including off-diagonal \mathcal{H}_1 - \mathcal{H}_2 transitions must be diagonalized. The diagram of figure 5 is generated from one-particle irreducible diagrams when we take into account in a perturbative way the diagonalization of this matrix.
- ii) When calculating the absorptive part of $\mathcal{H}_\varepsilon \rightarrow \psi \bar{\chi}$ using unitarity it is quite evident that after the insertion of $\bar{\psi} \psi$ as an intermediate on-shell state, one needs the scattering amplitude $\bar{\psi} \psi \rightarrow \psi \bar{\chi}$. This scattering can take place through an exchange of \mathcal{H} in the t-channel (diagram 1b) but also through an exchange in the s-channel, thus generating the diagram of figure 5.

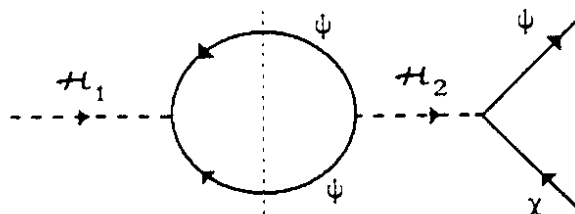


Fig. 5

In order to show explicit results and analyze general implications of the appearance of this new type of graphs, let us consider for simplicity the Lagrangian (2) and study the process $\mathcal{H}_\xi \rightarrow \psi \bar{\chi}$ in the case that $M_1 \gg M_2$ ⁷⁾. Now the relevant amplitude is given by the sum of diagrams 1.a, 1.b and 5. Taking out a common Dirac structure we get

$$A(\mathcal{H}_\xi \rightarrow \psi_m \bar{\chi}_n) = (f_\xi)_{mn} + (g_\xi^\dagger \cdot g_\xi f_\xi)_{mn} I_t(\rho) \\ + \frac{I_s(\rho)}{(1-\rho^2)} (f_\xi)_{mn} [\text{Tr}(f_\xi f_\xi^\dagger) + \text{Tr}(g_\xi g_\xi^\dagger)] \quad (8)$$

where $\rho \equiv M_{\xi'}/M_\xi$ and $I_t(\rho)$ and $I_s(\rho)$ are the corresponding Feynman integrals.

Note that $\xi' \neq \xi$ both in diagram 1.b and 5. In the second by definition and in the first because $\xi' = \xi$ will not contribute to ΔB .

Thus, we have :

$$\Delta_\xi(\psi \bar{\chi}) \equiv \sum_{m,n} \{ |A(\mathcal{H}_\xi \rightarrow \psi_m \bar{\chi}_n)|^2 - |A(\bar{\mathcal{H}}_\xi \rightarrow \bar{\psi}_m \chi_n)|^2 \} \\ = 4 \{ \text{Im Tr}[f_\xi \cdot f_\xi^\dagger g_\xi^\dagger \cdot g_\xi] \text{Im } I_t(\rho) + \\ + \frac{1}{(1-\rho^2)} \text{Im}[\text{Tr}(f_\xi \cdot f_\xi^\dagger) \text{Tr}(g_\xi g_\xi^\dagger)] \text{Im } I_s(\rho) \} \quad (9)$$

From equation (9) it is now apparent that $\Delta_\xi(\psi \bar{\chi})$ will receive contributions only from diagrams where $\xi' \neq \xi$.

The imaginary parts of $I_t(\rho)$ and $I_s(\rho)$ are given by :

$$\text{Im } I_t(\rho) = \frac{1}{16\pi} [1 - \rho^2 \ln(1+1/\rho^2)] \\ \text{Im } I_s = \frac{1}{16\pi} \quad (10)$$

If we consider the other channel $\mathcal{H}_\xi \rightarrow \psi^c \psi^c$ we get $\Delta_\xi(\psi^c \psi^c) = -\Delta_\xi(\psi \bar{\chi})$ as it should be by *CPT*.

It is interesting to note that for $\rho \rightarrow 0$ both contributions in (9) are bigger than in the opposite case $\rho \rightarrow \infty$ where they fall down as $1/\rho^2$. This in turn means that this behaviour goes in the same direction that the out of equilibrium decay. In other words, provided $\Gamma_1 \approx \Gamma_2$, the baryon asymmetry will be produced essentially by the heaviest Higgs, in our case M_1 .

Now we can give the mean net baryon number produced per decay of \mathcal{H} and $\bar{\mathcal{H}}$. In fact what we need is $\Delta_1(q/l)$ that is not exactly equation (9). Taking into account the different channels contributions to this piece we get

$$\Delta B \approx 1/2 (r - \bar{r})_1 \approx \frac{\text{Im}\{\text{Tr}[g_1 f_2 f_1^\dagger g_2^\dagger]\} + 2 \text{Im}\{\text{Tr}(f_2 f_1^\dagger) \text{Tr}(g_2^\dagger g_1)\}}{8\pi [\text{Tr}(f_1^\dagger f_1) + \text{Tr}(g_1^\dagger g_1)]} \quad (11)$$

The first piece of the numerator is the ε' -type contribution (direct CP violation) that reproduces the result of Weinberg and Nanopoulos. We call the second one the ε -type contribution because its origin is in the CP violation of the mixing matrix of the Higgs fields. It seems quite evident that both contributions are independent. In order to clarify this point and to get an idea of the importance of the new piece we will estimate both pieces in a simple case.

Let us consider the case where f_1 and g_1 are much bigger than f_2 and g_2 in such a way that the quark masses comes essentially from f_1 and g_1 times the vacuum expectation value (v_1) of the doublet component of the φ_1 field. In spite that naturally $v_2 \gg v_1$, provided f_2 and g_2 are very small the scenario we are presenting can be fulfilled.⁸⁾

So after the spontaneous breakdown of the $SU(2) \otimes U(1)$ gauge group and doing some unitary transformation we have for the two heavier generations^{9,10)}

$$\begin{aligned} f_1 &\approx \frac{D^d}{v_1} \\ g_1 &\approx \frac{1}{v_1} U^T R D^u U \end{aligned} \quad (12)$$

where $D^d = \text{diag}(m_s, m_b)$, $D^u = \text{diag}(m_c, m_t)$ are the diagonal quark mass matrices. U is the Cabibbo matrix and R contains the CP -violating phase of $SU(5)$ not related to the number of generations in the way of the Standard model :

$$\mathbf{U} = \begin{bmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} e^{i\delta} & 0 \\ 0 & 1 \end{bmatrix} \quad (13)$$

In order to simplify at maximum we will take $f_2 = \mu I$ and $g_2 = \mu' I$ where the essential point is to take $\mu, \mu' \ll 1$ and real.

In this case we obtain :

$$\text{Im}\{\text{Tr}(f_2 f_1^\dagger) \text{Tr}(g_2^\dagger g_1)\} \simeq \frac{-\mu\mu'}{v_1^2} m_b m_c \sin \gamma \quad (14)$$

and for a realistic mixing ($\sin \vartheta$ small) :

$$\left| \frac{\text{Im}\{\text{Tr}[g_1 f_2 f_1^\dagger g_2^\dagger]\}}{\text{Im}\{\text{Tr}(f_2 f_1^\dagger) \text{Tr}(g_2^\dagger g_1)\}} \right| \sim \frac{\cos^2 \vartheta m_s + \sin^2 \vartheta m_b}{m_b} \ll 1 \quad (15)$$

These result tell us that the ε -type CP -violating contributions to the baryon asymmetry in this particular case is much more important than the more conventional ε' -type contribution coming from the triangle graph.

In conclusion, we have shown that in scenarios where the cosmological baryon asymmetry is generated in the decay of heavy coloured Higgs triplets at the one loop level (thus we have at least two of them), provided the two Higgses have two common decay channels, there is a new contribution to baryon asymmetry coming from the mixing among the two Higgses. This new contribution that we call ε -type can be bigger than the conventional one in the natural case of small mixing.

$SU(5)$ with two coloured Higgs triplets and $SU(3)^3 \otimes Z_3$ are some examples (see ref. 11) where the new contribution can be relevant. Others models with several Higgs triplets that do not have this new contribution can be found in ref. (12)

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 It is amusing to find out that there is an apparent difference between our trace and the one quoted in this reference. It is apparent because we differ by a substitution $f^*f^\dagger \leftrightarrow ff^\dagger$, but by eq.(6) f can be brought to a diagonal form. Nevertheless it must be stressed that the invariant traces must be constructed out of $f, f^\dagger, f^*f^\dagger, f^*g$ and g^\dagger (g is symmetric) and a mistake in the routine of the trace could be crucial in other models.
- 7) The case $M_1 \approx M_2$ is rather difficult to analyze because we cannot use perturbative diagonalization. The case $M_1 = M_2$ considered by Yildiz and Cox in Phys. Rev. D21 (1980) 906, will be study in a future work.
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