



Flavour-Changing Yukawa Coupling in the  
Standard Model and Muon Polarization  
in  $K_L \rightarrow \mu\bar{\mu}$

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Abstract

The muon longitudinal polarization  $P_L$  in  $K_L \rightarrow \mu\bar{\mu}$  decay is investigated in the standard model. The induced  $\bar{s}dH$  vertex responsible for  $P_L$  turns out to be unexpectedly "large". The measurement of  $P_L$  above  $O(10^{-3})$  will, therefore, be a signal of the existence of rather light Higgs ( $M_H \lesssim 10$  GeV). It is also shown that the mechanism of gauge dependence cancellation between Higgs exchange and box diagrams works even in the standard model.

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The properties of gauge interactions of low energies ( $\leq 10^2$  GeV) are now relatively well known. Nevertheless, the scalar sector of the theory and the CP-violation mechanism are far from being well understood. There is still no direct evidence of the Kobayashi-Maskawa (K-M)<sup>1)</sup> mechanism of CP violation, in fact, the observed CP-violation in the kaon system is consistent with a superweak description. In addition, our knowledge about the mass of the elusive Higgs is extremely poor; the most stringent lower limit quoted in the literature comes from  $K^+ \rightarrow \pi^+ H$ ,<sup>2)</sup> and is 325 MeV. Other theoretical lower bounds can be avoided in several ways.<sup>3)</sup>

In this letter we will show how the measurement of the muon longitudinal polarization ( $P_L$ ) in  $K_L \rightarrow \mu\bar{\mu}$  decay can give us interesting information about these two obscure topics in the framework of the standard model.

Long time ago Sehgal<sup>4)</sup> pointed out that  $P_L$  is different from zero provided there is P and CP violation in addition to "unitary phases". Later on Herczeg<sup>5)</sup> concluded that in the standard model the polarization gets an important contribution only from the CP violation in  $K^0 \leftrightarrow \bar{K}^0$  mixing, i.e.,  $|P_L| \approx 7 \times 10^{-4}$  and that any signal above this value would be evidence of new physics. More recently the CP-violating amplitude contribution to  $P_L$  has been estimated in various current candidates of gauge model of CP violation.<sup>6)</sup> The main and most interesting result stated there is that  $P_L$  can be as big as  $10^{-3} - 10^{-2}$  for a rather small ( $\sim 1$  TeV)  $W_R$  mass in the left-right symmetric  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  model, if the neutrinos have large right-handed majorana masses. According to Ref. 6 the CP-violating amplitude contribution to  $P_L$  in the standard model is outrageously small; the one-loop induced flavour-changing Yukawa coupling  $\bar{s}dH$ , which may give potentially "large" contributions to the polarization through the Higgs-exchange diagram, is strongly suppressed by high powers

of the external quark masses,  $m_s$  and  $m_d$ . Their estimate of the  $\bar{s}dH$  vertex in the standard model seems to agree with the previously quoted result; the  $\bar{s}dH$  vertex in the standard model is at least proportional to the third power of the external quark masses.<sup>7)</sup>

In this paper, however, it is shown, by explicit calculation, that terms linear in the external quark masses survive in the  $\bar{s}dH$  vertex. Our result partially agrees with Ref. 6 and 7: the linear term accompanied by  $\ln(m_i^2/M_W^2)$  vanishes,  $m_i$  being internal quark masses. Willey and Yu<sup>2,8)</sup> also have obtained the result that the terms linear in external quark masses remain in the induced Higgs vertex coupled to flavour changing neutral currents in the standard model. Their argument concerns about Higgs production in  $K^+ \rightarrow \pi^+ H$  and  $b \rightarrow sH$ . In  $K_L \rightarrow \mu\bar{\mu}$ , which we are interested in, the Higgs boson appears only as an intermediate state generally off mass shell. Thus, we cannot immediately use their result for our purpose. We can check their result as a specific case of our general (off-shell Higgs) formula.

It is precisely the fact that the Higgs is off-shell in  $K_L \rightarrow \mu\bar{\mu}$  what makes the relevant  $\bar{s}dH$  vertex gauge dependent. So we advocate a technique to extract the gauge invariant piece without ambiguity and we show that the remaining gauge dependent term is cancelled out exactly in the  $d\bar{s} \rightarrow \mu\bar{\mu}$  amplitude by the gauge dependent contribution coming from the box diagram once we do not neglect external masses. The remaining next to leading order gauge invariant box contribution is of order  $M_H^2/M_W^2$  compared to the Higgs exchange, so it will be neglected throughout this paper where we are interested only in rather light Higgses.

As a consequence of the results mentioned above we find that the Higgs exchange contribution, in the standard model, to the muon longitudinal polariza-

tion can be bigger than the superweak value for Higgs masses less than say 4-5 GeV. In fact there are regions where the polarization can be large enough to be measured in the first generations of experiments.<sup>9)</sup> If in addition we consider the extension of the K-M model to four generations,<sup>10)</sup> there is a much more big window to surpass the superweak value so that the new physics pointed out by Herczeg could be the existence of a light Higgs (and the existence of a fourth generation).

In the course of the one-loop calculation of the  $\bar{s}dH$  vertex the contribution of the renormalization counterterms in the Lagrangian is also discussed. The details of our calculations, including exact formulas of the  $\bar{s}dH$  vertex, and a much more elaborated discussion about gauge dependence cancellations, etc., will be given in a separate publication.<sup>11)</sup>

The  $K_L \rightarrow \mu\bar{\mu}$  amplitude in the standard model can be written as

$$\begin{aligned}
 M(K_L \rightarrow \mu\bar{\mu}) &= \bar{u}(p_-)(a\gamma_5 + ib)v(p_+) \\
 a &= 2N_{\bar{\epsilon}} \sum_i \left( \text{Re}(\Lambda_i) + i\bar{\epsilon}\text{Im}(\Lambda_i) \right) a_i \\
 b &= 2N_{\bar{\epsilon}} \sum_i \left( \text{Im}(\Lambda_i) - i\bar{\epsilon}\text{Re}(\Lambda_i) \right) b_i
 \end{aligned} \tag{1}$$

where  $\bar{\epsilon}$  is the CP impurity parameter,  $N_{\bar{\epsilon}} = (2(1+|\bar{\epsilon}|^2))^{-1/2}$ ,  $\Lambda_i = V_{is}^* V_{id}$ ,  $V_{ij}$  being the K-M matrix and  $a_i$  and  $b_i$  are the corresponding amplitudes (the  $i$ -th quark contribution) in  $K^0 \rightarrow \mu\bar{\mu}$  without including the K-M mixing. For example, the standard (V-A) x (V-A) coupling arising from the ordinary box and Z-exchange graphs at leading order<sup>12)</sup> reduces to a term in the "a" amplitude. The muon polarization is proportional to  $\text{Im}(a^* b)$  divided by the decay rate.<sup>5)</sup> The  $\bar{\epsilon}$  contribution has been estimated by Herczeg so we will concentrate on the CP-violating amplitude effect ( $\bar{\epsilon} = 0$ ). Fortunately, the imaginary part of  $a$  ( $\text{Im} a_i$ ) can be calculated reliably, since it is dominated by the absorptive

part of  $2\gamma$ -exchange process; it provides the "unitary phases" of Sehgal. The Higgs exchange graph only contributes to the real part of  $b$  (dispersive part of  $b_i$ ).  $2\gamma$ -exchange contributions to  $b$  has been shown to be managed by  $\epsilon'$  in Ref. 5, and are negligible. Thus we arrive at the formula<sup>5)</sup>

$$|P_L| \approx (5.7 \times 10^{11}) |\text{Re}(b)| \quad (2)$$

where the numerical value comes from the decay rate and from the careful estimate by Martin, de Rafael and Smith of the absorptive part of the  $2\gamma$ -exchange graph.<sup>13)</sup> In Eq. (2)  $b$  gets its contribution from the Higgs exchange diagram shown in Fig. 1 and from the next to leading order of the box graph just to restore gauge invariance as we will see.

Let us now calculate the one-loop induced  $\bar{s}dH$  vertex in an arbitrary  $R_\xi$  gauge. It can be shown that the gauge dependent piece ( $\xi$ -term) in the  $\bar{s}dH$  vertex, is cancelled out in the  $d\bar{s} \rightarrow \mu\bar{\mu}$  amplitude by the  $\xi$ -term from the box graph with external mass effects included. At the first sight this type of gauge dependence cancellation seems to be curious, since Higgs exchange diagram has generally an  $M_H$  dependence, while the box graph is  $M_H$  independent. However, as far as the gauge dependent term of the Higgs-exchange diagram is concerned, it is not actually the case. This is simply because the gauge dependence of the  $\bar{s}dH$  vertex has to disappear for Higgs on-shell. Therefore, the  $\xi$ -term in the  $\bar{s}dH$  vertex is expected to be proportional to a factor  $q^2 - M_H^2$ , and such a factor is cancelled out by the Higgs propagator in the  $d\bar{s} \rightarrow \mu\bar{\mu}$  process. This type of mechanism of gauge-dependence cancellation was first pointed out in  $\mu \rightarrow e\bar{e}$  decay,<sup>14)</sup> and more recently has been studied in  $K_L - K_S$  mass difference,<sup>15)</sup> but only in the context of left-right symmetric models. We want to stress here that

such a mechanism is a general feature of gauge models where fermion masses are generated after the spontaneous breaking of symmetries through Yukawa couplings.

We further point out that the gauge-independent pieces and the  $\xi$ -terms can be separated without ambiguity. The procedure starts with dividing the  $W^\pm$  propagator into two pieces:  $-i(g_{\mu\nu} - k_\mu k_\nu / M_W^2) / (k^2 - M_W^2) - ik_\mu k_\nu / M_W^2 (k^2 - \xi M_W^2)^{-1}$ , the first part being the "unitary gauge" propagator corresponding to  $\xi \rightarrow \infty$ . The gauge-independent amplitude is obtained summing up all  $W^\pm$ -exchange diagrams, in which only the unitary part is used as  $W^\pm$  propagator. The  $\xi$ -term is obtained by summing up all other remaining contributions of  $W^\pm$ -exchange graphs together with those diagrams including unphysical scalars. The calculated  $\xi$ -term of the  $\bar{s}dH$  vertex turns out to be proportional to  $q^2 - M_H^2$ , as expected, and the result is shown in a diagrammatical way in Fig. 2. The corresponding gauge-dependent part of the box diagram has been confirmed to cancel exactly the contribution of Fig. 2 to the  $d\bar{s} \rightarrow \mu\bar{\mu}$  amplitude.

The exact calculation<sup>11</sup> of the gauge invariant part shows that it contains three types of couplings  $\bar{s}(m_s L + m_d R)d$ ,  $\bar{s}(m_s R + m_d L)d$  and  $\bar{s}(m_s L - m_d R)d$  with  $R, L \equiv (1 \pm \gamma_5)/2$ . In the real world ( $m_s, m_d, \sqrt{q^2} = M_K \ll M_W$ ), only the  $\bar{s}(m_s L + m_d R)d$  coupling turns out to be important. The gauge-independent effective  $\bar{s}dH$  vertex is thus given by

$$\Gamma(m_s, m_d, q^2) \approx \frac{3}{8} \frac{g^3}{(4\pi)^2} \sum_i \frac{m_i^2}{M_W^3} \bar{s}(m_s L + m_d R)d \cdot H \quad (3)$$

$g$  is the gauge coupling constant and  $m_i = m_u, m_c, \dots$ . The result (3) is exactly the same as the one discussed in Ref. 8 because in both calculations  $\sqrt{q^2} \ll M_W$  has been used. Result (3) is exact as far as internal masses are concerned. Equation (3) has also been calculated separately by summing up all the diagrams in the Feynman gauge and ignoring a piece corresponding to Fig. 2 with  $\xi = 1$ .

We want to point out that in this gauge the contribution to Eq. (3) coming from diagrams containing only W boson cancels exactly, so in the Feynman gauge Eq. (3) comes from diagrams containing at least one unphysical scalar(!). In general one must consider also the effects of the flavour changing renormalization counterterms (Fig. 3). Nevertheless on the basis of the Kabir, Feinberg and Weinberg theorem<sup>16)</sup> it is well known that the contribution of the relevant counterterms, for  $\bar{d}sZ$  or  $\bar{d}s\gamma$  coupling, vanishes for external quarks on-shell. The same result holds for the Higgs sector. The counterterms relevant to the effective  $\bar{s}dH$  vertex are<sup>17)</sup>

$$L_c = \bar{s}(i\cancel{\partial}(\alpha L + \beta R) + (\gamma L + \delta R))d + \frac{g}{2M_W} \bar{s}(\gamma L + \delta R)dH + \text{h.c.} \quad (4)$$

where  $\alpha, \beta, \gamma$  and  $\delta$  are momentum independent constants. Let us note that the mass counterterm and the counterterm for Yukawa coupling are not independent, since we are considering the flavour changing part. It can be checked that the contributions from  $(\alpha L + \beta R)$  cancel each other between diagrams (a) and (b) of Fig. 3, while the  $\gamma L + \delta R$  term cancels out when all diagrams are summed up. This argument is valid for arbitrary  $\alpha, \beta, \gamma$  and  $\delta$ , and therefore is independent of the renormalization scheme.

Using now Eq. (2) and (3) it is straightforward to get

$$|P_L| = 1.3 \times 10^{-2} \left| \frac{\sum_i \text{Im}(\Lambda_i) m_i^2}{M_K^2 - M_H^2} \right| \quad (5)$$

where the numerical factor comes from Eq. (2) and the well-established parameters  $G_F, m_\mu, f_K$  and  $M_K$ ; note that after evaluating the matrix element  $\langle 0 | \bar{s}\gamma_5 d | K^0 \rangle$  the external quark mass dependence disappear in the factor  $(m_s - m_d)/(m_s + m_d) \approx 1$ .

Equation (5) is our main result. Although the values of  $m_t$ ,  $s_3$  and  $\delta$  have not been settled yet, we can avoid such uncertainty in the evaluation of the dominant  $t$  quark contribution to the numerator of Eq. (5), utilizing the knowledge about  $\epsilon$ . Namely, adopting a very reasonable assumption that the  $t$  quark contribution in box diagram saturates the observed  $\epsilon$ ,<sup>18)</sup> we get  $\text{Im}(\Lambda_t)m_t^2 \approx 2.9 \times 10^{-3} / B s_2^2 f_t(\text{GeV}^2)$  where  $f_t$  is a smooth function of  $m_t^2/M_W^2$  changing from 1, for light top quark, to the asymptotic value  $1/4$ .<sup>12)</sup> Taking the conservative values  $s_2 = 0.055$  and  $B f_t = 0.66$  we get  $\text{Im}(\Lambda_t)m_t^2 \approx 1.45$ . We thus obtain  $|P_L| \approx 1.88 \times 10^{-2} (\text{GeV}^2) / |M_K^2 - M_H^2|$ . This result can be taken as an illustrative example, since the numerator can change by a factor of 3 up or down due to the uncertainty of  $s_2$  and  $B f_t$ . The formula tells us that  $|P_L|$  grows from 13% to 96% in the range  $M_H = 325-477 \text{ MeV}$ <sup>19)</sup> and decreases from 96% to 0.1% for  $M_H = 0.517 - 4.36 \text{ GeV}$ . The region  $M_H = 477-517 \text{ MeV}$  is forbidden by the rate  $K_L \rightarrow \mu\bar{\mu}$ .<sup>5)</sup>

If in addition one keep in mind the interesting possibility of the existence of a fourth generation<sup>10)</sup> the numerator in Eq. (5) can be much more bigger. Taking for example the first solution given by He and Pakvasa<sup>10)</sup> we get  $\sum_i \text{Im}(\Lambda_i)m_i^2 \approx 10.3 \text{ GeV}^2$ , which implies that for a Higgs mass bellow 11.5 GeV the longitudinal muon polarization would be in the range 96-0.1%, i.e., bigger than the superweak value. As the conclusion, the planned experiment at BNL can be a very useful tool to look for the existence of a relatively light Higgs and will also provide useful information on the existence of a fourth generation. In addition to the  $\underline{T} \rightarrow \gamma H$  process,<sup>21)</sup> future kaon factories, where polarization smaller than a 10% could be measured, would be probably the best place to search for the range of  $M_H \lesssim 10 \text{ GeV}$ . Finally, we will emphasize that our analysis about the Higgs exchange contribution to  $P_L$  applies for other present candidates of gauge models of CP violation, including left-right model and two doublet Higgs



model with  $SU(2) \times U(1)$ . Especially a contribution to  $P_L$  above  $O(10^{-3})$  is always there, as long as the Higgs boson is rather light.

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19. Strictly speaking, the range  $M_H = 18-140$  MeV should be also included as a possibility; the range is not excluded by  $K^+ \rightarrow \pi^+ H \rightarrow \pi^+ e^+ e^-$  because the experiment<sup>20)</sup> used a cut on the  $e^+ e^-$  invariant mass of 140 MeV. We thank W. Morse for making us aware of the point.
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### Figure Captions

- Fig. 1 The Higgs exchange diagram relevant to the  $b$  amplitude in  $K_L \rightarrow \mu\bar{\mu}$  decay. The blob denotes induced  $\bar{s}dH$  vertex.
- Fig. 2 The diagram giving effectively the gauge dependent  $\xi$ -term in  $\bar{s}dH$  vertex. The factor  $igM_H^2/(2M_W)$ , assigned in ordinary Feynman rule should be changed to the factor shown in the figure.
- Fig. 3 The contributions of the counterterms, indicated by  $x$ , to the  $\bar{s}dH$  vertex.

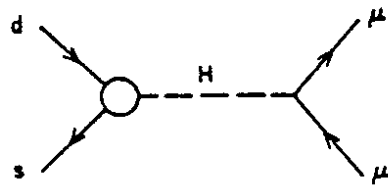


Fig. 1

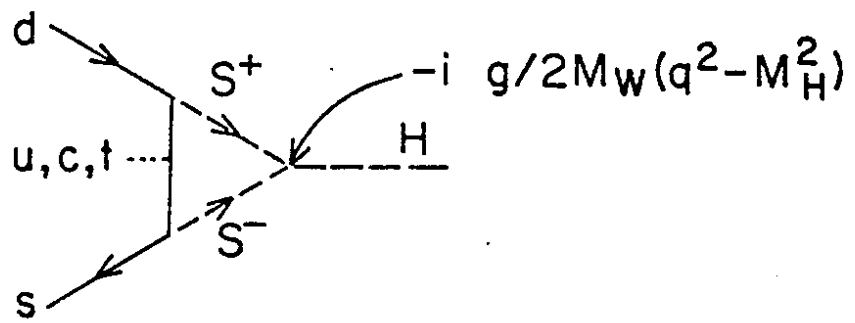


Fig. 2

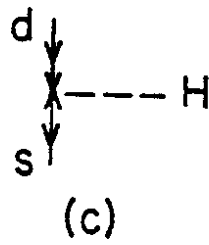
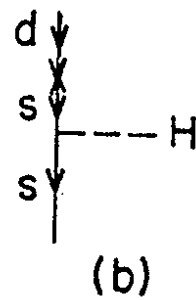
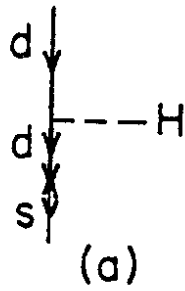


Fig. 3