FTUV-02-0614

Unitarity Triangles and the Search for New Physics

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Abstract

Assuming that the Kobayashi-Maskawa mechanism gives the dominant contribution to CP violation at low energies, we propose a novel way of testing the flavour sector of the Standard Model which has the potencial for discovering New Physics. Using 3×3 unitarity of the V_{CKM} matrix and choosing a complete set of rephasing invariant phases, we derive a set of exact relations in terms of measurable quantities, namely moduli of V_{CKM} and arguments of rephasing invariant quartets. These tests complement the usual analysis in the ρ, η plane and, if there is New Physics, may reveal its source.

1 Introduction

The advent of various B-factories has triggered an important development in the study of CP violation, with both BaBar (SLAC) [1] and Belle (KEK) [2] providing for the first time evidence for CP violation outside the Kaon system. This new data and its expected improvement in the near future [3], [4], [5], [6], will provide a stringent test of one of the experimentally least constrained aspects of the Standard Model (SM), namely the Kobayashi-Maskawa (KM) mechanism of CP violation.

So far, all experimental data on flavour physics and CP violation [7], [8] are in agreement with the SM and its KM mechanism. This agreement is remarkable, since one has to account for a large number of data with a small number of parameters. The Cabibbo, Kobayashi and Maskawa (CKM) matrix is characterized by four parameters which one can choose to be three

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angles θ_i and the phase δ of the standard parametrization [9]. The values of s_1, s_2 and s_3 ($s_i = \sin\theta_i$) can be determined by the experimental value of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$. Once these parameters are fixed, one has to fit, using only the phase δ, a large amount of data, including ε_K, ε'/ε, $\sin(2\beta)$, ΔM_{B_d}, ΔM_{B_s}. It is remarkable that these five experimental quantities can be fitted with only one parameter [10], namely the KM phase δ.

In this paper, we address the question of finding the best strategy to perform precision tests of the SM mechanism of flavour mixing and CP violation, while at the same time searching for the presence of New Physics.

In view of the impressive success of the SM, one may wonder what is the motivation to look for Physics Beyond the SM. In what concerns CP violation, there are in our opinion, two main motivations to look for New Physics and in particular new sources of CP violation:

(i) By now, it has been established that the strength of CP violation in the SM is not sufficient to generate the observed Baryon Asymmetry in the Universe (BAU), thus suggesting the need for new sources of CP violation.

(ii) Almost all extensions of the SM, including supersymmetric extensions, have new sources of CP violation which can in principle be detected at B-factories.

Throughout the paper, we will assume that the tree level weak decays are dominated by the SM W-exchange diagrams, thus implying that the extraction of $|V_{us}|$, $|V_{ub}|$ and $|V_{cb}|$ from experiment continues to be valid even in the presence of New Physics (NP). We will allow for contributions from NP in processes like $B_d^0 - \overline{B}_d^0$ mixing and $B_s^0 - \overline{B}_s^0$ mixing, as well as in penguin diagrams. Since the SM contributes to these processes only at loop level, the effects of NP are more likely to be detectable. Examples of processes which are sensitive to NP, are the CP asymmetries corresponding to the decays ${B^0}_d \to J/\Psi K_s$ and ${B^0}_d \to \pi^+\pi^-$ which are affected by NP contributions to $B_d^0 - \overline{B}_d^0$ mixing. Significant contributions to $B_d^0 - \overline{B}_d^0$ and $B_s^0 - \overline{B}_s^0$ mixing can arise in many of the extensions of the SM, such as models with vector-like quarks [11], [12] and supersymmetric extensions of the SM [13]. Vector-like quarks naturally arise in theories with large extra-dimensions [14], as well as in some grand-unified theories like E_6. The presence of vector-like quarks leads to a small deviation of 3×3 unitarity of V_{CKM} which in turn leads to Z-mediated new contributions to $B_d^0 - \overline{B}_d^0$ and $B_s^0 - \overline{B}_s^0$ mixings. In the minimal Supersymmetric Standard Model (MSSM) the size of SUSY contributions to $B_d^0 - \overline{B}_d^0$ and $B_s^0 - \overline{B}_s^0$ mixing crucially depends on the choice of soft-breaking terms, but there is a wide range of the parameter space where SUSY contributions can be significant. Recently, it has been pointed out [15] that in the context of SUSY SO(10), there is an interesting connection between the observed large mixing in atmospheric neutrinos and the size of the SUSY contribution to $B_s^0 - \overline{B}_s^0$ mixing, which is expected to be large in this class of models.

The standard way of testing the compatibility of the SM with the existing data consists of adopting the Wolfenstein parametrization and plotting in the ρ, η plane the constraints derived from various experimental inputs, like the value of ε_K, the size of $|V_{ub}| / |V_{cb}|$, the value of $a_{J/\psi K_s}$, as well as the strength of $B_d^0 - \overline{B}_d^0$ and $B_s^0 - \overline{B}_s^0$ mixings. The challenge for the SM is then to find a region in the ρ, η plane where all the constraints are simultaneously satisfied.

In this paper, we will choose a complete set of rephasing invariant phases and use 3×3 unitarity of V_{CKM} to derive a set of exact relations written in terms of measurable quantities, namely moduli of V_{CKM} and arguments of rephasing invariant quartets. We will point out that these exact relations can play an important rôle in complementing the standard analysis in the ρ, η plane. Since all relations are exact and written in terms of measurable quantities, they are particularly suited to perform precise tests of the SM. Apart from providing stringent tests of the SM, these exact relations can be useful in finding the nature of NP. Let us assume that the presence of NP is established by the impossibility of finding a region in the ρ, η plane where all experimental data can be fitted, within the SM. This will not indicate which one of the measurements of the angles or the sides of the unitarity triangle were affected by the presence of NP. The knowledge of which of the exact relations are violated by the data may be useful for discovering the source of NP. By assuming a certain level of precision in future data arising from the various B-factories, we estimate the power of these exact relations in either putting bounds on the the strength of NP or in revealing its presence.

2 Choice of Rephasing Invariant phases

By using the freedom to rephase quark fields, one can readily show that in the 3×3 sector of a CKM matrix of arbitrary size there are only four independent rephasing invariant phases. Note that this result is completely general, it does not depend on the number of generations and holds true even if the CKM matrix is not unitary [7]. The number four, is obtained by observing that in the 3×3 sector of a CKM matrix there are, obviously, nine phases, and five of them can be removed by rephasing quark fields. We will choose the following rephasing invariant phases:

$$\begin{array}{c}\gamma \equiv \arg(-V_{ud}V_{cb}V_{ub}^{*}V_{cd}^{*}) = \arg\left(-\frac{V_{ud}V_{ub}^{*}}{V_{cd}V_{cb}^{*}}\right)\\ \beta \equiv \arg(-V_{cd}V_{tb}V_{cb}^{*}V_{td}^{*}) = \arg\left(-\frac{V_{cd}V_{cb}^{*}}{V_{td}V_{tb}^{*}}\right)\\ \chi \equiv \arg(-V_{cb}V_{ts}V_{cs}^{*}V_{tb}^{*}) = \arg\left(-\frac{V_{cb}V_{cs}^{*}}{V_{tb}V_{ts}^{*}}\right)\\ \chi' \equiv \arg(-V_{us}V_{cd}V_{ud}^{*}V_{cs}^{*}) = \arg\left(-\frac{V_{us}V_{ud}^{*}}{V_{cs}V_{cd}^{*}}\right)\end{array} \tag{1}$$

Notice that χ is frequently denoted in the literature as β_s and χ' as β_K. It is important to stress that since these phases are rephasing invariant quantities, they correspond to physical observables. If one assumes 3×3 unitarity of V_{CKM}, it has been shown that one can reconstruct the full CKM matrix, using the above four rephasing invariant phases as input [16]. At this stage, the following comment is in order. Having in mind that it has also been shown [17] that one can reconstruct the full CKM matrix from four independent moduli, one may test the SM by comparing the unitarity triangles obtained from the measurement of the four independent phases γ, β, χ, χ' with the unitary triangles obtained from the knowledge of four independent moduli, which can be chosen to be $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, $|V_{td}|$. Although such a test is possible in theory, its practical interest is limited by the fact that at least one of the phases, namely χ', is probably too small to be measurable through B decays and furthermore the extraction of $|V_{td}|$ and $|V_{ub}|$ from experiment is plagued by hadronic uncertainties. Obviously, within the framework of the SM, χ' can be obtained from the experimental value of ε_K, but its extraction also suffers from hadronic uncertainties.

In order to fix the invariant phases entering in B^0 CP asymmetries, it is useful to adopt the following phase convention, [7]:

$$\arg(V) = \begin{pmatrix} 0 & \chi' & -\gamma \\ \pi & 0 & 0 \\ -\beta & \pi+\chi & 0 \end{pmatrix} \tag{2}$$

Through the measurement of CP asymmetries, one can obtain the phases of the rephasing invariant quantities:

$$\lambda_f^{(q)} = \left(\frac{q_{B_q}}{p_{B_q}}\right)\left(\frac{A\left(\overline{B}_q^0 \to f\right)}{A\left(B_q^0 \to f\right)}\right) \; ; \; \lambda_{\overline{f}}^{(q)} = \left(\frac{q_{B_q}}{p_{B_q}}\right)\left(\frac{A\left(\overline{B}_q^0 \to \overline{f}\right)}{A\left(B_q^0 \to \overline{f}\right)}\right) \tag{3}$$

The first factor in $\lambda_f^{(q)}$ is due to mixing and its phase equals (-2β) and 2χ for B_d and B_s, respectively. Later on, we will assume that NP can give contributions to mixing. Then, it is useful to parametrize these NP contributions in the following way

$$M_{12}^{(q)} = \left(M_{12}^{(q)}\right)^{SM} r_q^2 e^{-2i\phi_q} \Rightarrow \Delta M_{B_q} = \left(\Delta M_{B_q}\right)^{SM} r_q^2 \quad (4)$$

$$\frac{q_{B_q}}{p_{B_q}} = \exp\left(i \arg\left(M_{12}^{(q)}\right)^*\right) = \left(\frac{q_{B_q}}{p_{B_q}}\right)^{SM} e^{2i\phi_q} \quad (5)$$

In the presence of NP, the phases from mixing become $2(-\beta+\phi_d) \equiv -2\overline{\beta}$ and $2(\chi+\phi_s) \equiv 2\overline{\chi}$ for B_d and B_s decays, respectively. It is clear that $r_q \neq 1$ and/or $\phi_q \neq 0$ would signal the presence of NP. How could one measure β without contamination of NP? One possibility would be to measure β through the t-quark contribution to the $b \longrightarrow d$ penguin, since this contribution is proportional to $V_{tb}V_{td}^*$, which in our phase convention is β. Unfortunately, this is not possible [20] without theoretical input about hadronic parameters which involve large uncertainties. This is essentially due to the fact that the $b \longrightarrow d$ penguin receives significant u and c quark contributions. Concerning χ, in general, the existence of NP contributions to mixing in the B_s system would no longer lead to a zero asymmetry, as predicted in the framework of the SM, in the B_s decay dominated by the $b \longrightarrow s$ penguin such as $B_s \rightarrow \phi\eta'$. In this case, a deviation from zero in this asymmetry would be a clear indication of NP also implying that the CP asymmetry in the channel $B_s \rightarrow D_s\overline{D_s}$ will measure directly a $\overline{\chi}$ which may differ from χ. The difficulty of separating β from a possible NP contribution ϕ_d in ${B^0}_d$ decays like ${B^0}_d \rightarrow J/\Psi K_s$, renders specially important the measurement of γ, which does not suffer from contamination of NP in the mixing. Note that γ can be either directly measured [18] or obtained through the knowledge of the asymmetries $a_{J/\Psi K_s} = \text{Im}\left(\lambda_{J/\Psi K_s}^{(d)}\right)$, $a_{\pi^+\pi^-} = \text{Im}\left(\lambda_{\pi^+\pi^-}^{(q)}\right)$. Indeed the phase ϕ_d cancels in the sum $\overline{\alpha}+\overline{\beta} = (\pi-\gamma-\beta+\phi_d)+(\beta-\phi_d)$ and one has:

$$\gamma = \pi - \frac{1}{2}\left[\arcsin a_{J/\Psi K_s} + \arcsin a_{\pi^+\pi^-}\right] \quad (6)$$

Note that we are using $a_{\pi^+\pi^-} = \sin(2\overline{\alpha})$ that can be extracted from the experimental asymmetry through various different approaches [19]. Once γ is known, β can be readily obtained, using unitarity and the knowledge of $|V_{ub}|$, $|V_{us}|$, $|V_{cb}|$. The knowledge of β, together with $a_{J/\Psi K_s}$ leads then to the determination of ϕ_d. Of course, this evaluation of ϕ_d will be restricted by the precision on $|V_{ub}|$, since $|V_{us}|$, $|V_{cb}|$ are extracted from experiment with good accuracy. Similar considerations apply to the extraction of r_d, r_s or r_d/r_s from ΔM_{B_d} and ΔM_{B_s} where $|V_{td}^*V_{tb}|$, $|V_{ts}^*V_{tb}|$ or its ratio, have to be reconstructed previously using unitarity.

K-M	β	γ	χ	χ'	$\alpha = \pi - \beta - \gamma$
N-P	$\overline{\beta} = \beta - \phi_d$	γ	$\overline{\chi} = \chi + \phi_s$	χ'	$\overline{\alpha} = \alpha + \phi_d$

Table 1: Phases that can be measured in different scenarios

In Table 1 we summarize the phases that can be measured from CP asymmetries both in the KM scheme or in the case of New Physics in the mixing. For example, $\overline{\beta}$ is the phase measured through $a_{J/\Psi K_s}$, i.e., $\overline{\beta} = (1/2)\arcsin a_{J/\Psi K_s}$.

3 Precision Tests of the SM and search for New Physics

In this section, we derive a complete set of exact relations involving moduli of V_{CKM} and the four rephasing invariant phases of Eq.(1) adopting the phase convention of Eq.(2). From the six unitarity relations corresponding to orthogonality of different rows and of different columns of V_{CKM} one obtains:

$$(uc) \qquad \sin\chi' = \frac{|V_{ub}|\,|V_{cb}|}{|V_{us}|\,|V_{cs}|}\sin\gamma \tag{7}$$

$$(ut) \qquad |V_{ud}|\,|V_{td}|\sin\beta - |V_{us}|\,|V_{ts}|\sin(\chi'-\chi) - |V_{ub}|\,|V_{tb}|\sin\gamma = 0 \tag{8}$$

$$(ct) \qquad \sin\chi = \frac{|V_{td}|\,|V_{cd}|}{|V_{ts}|\,|V_{cs}|}\sin\beta \tag{9}$$

$$(db) \qquad \frac{|V_{ub}|}{|V_{td}|} = \frac{\sin\beta}{\sin\gamma}\frac{|V_{tb}|}{|V_{ud}|} \tag{10}$$

$$(ds) \qquad \sin\chi' = \frac{|V_{td}|\,|V_{ts}|}{|V_{ud}|\,|V_{us}|}\sin(\beta+\chi) \tag{11}$$

$$(sb) \qquad \frac{\sin\chi}{\sin(\gamma+\chi')} = \frac{|V_{us}|\,|V_{ub}|}{|V_{ts}|\,|V_{tb}|} \tag{12}$$

where, in parenthesis, we have indicated the corresponding rows and columns. There are additional relations which can be readily obtained either by orthogonality or by applying the law of sines to the corresponding unitarity triangles, such as:

$$(db) \qquad |V_{ub}| = \frac{|V_{cd}|\,|V_{cb}|}{|V_{ud}|}\frac{\sin\beta}{\sin(\gamma+\beta)} \tag{13}$$

$$(db) \qquad |V_{td}| = \frac{|V_{cd}|\,|V_{cb}|}{|V_{tb}|}\frac{\sin\gamma}{\sin(\gamma+\beta)} \tag{14}$$

$$(sb) \qquad \sin\chi = \frac{|V_{us}|\,|V_{ub}|}{|V_{cs}|\,|V_{cb}|}\sin(-\chi+\chi'+\gamma) \tag{15}$$

Furthermore, by dividing Eq.(14) by $|V_{ts}|$ and using normalization of rows and columns one obtains

$$r = \frac{\sin\gamma}{\sin(\gamma+\beta)}\frac{|V_{cd}|}{|V_{tb}|}\left[1+r^2-r^2\frac{\sin^2\beta}{\sin^2\gamma}\frac{|V_{tb}|^2}{|V_{ud}|^2}\right]^{\frac{1}{2}} \tag{16}$$

where $r \equiv |V_{td}|\,/\,|V_{ts}|$. Using Eqs.(8) and (10), together with normalization conditions, one obtains:

$$\sin(\chi-\chi') = r\sin\beta\frac{|V_{us}|}{|V_{ud}|}\left[1-\frac{|V_{cb}|^2}{|V_{us}|^2}\right] \tag{17}$$

Another interesting relation is obtained by combining Eq.(12) with Eq.(13), leading to:

$$\sin\chi = \frac{|V_{us}|\,|V_{cd}|\,|V_{cb}|}{|V_{ts}|\,|V_{tb}|\,|V_{ud}|}\frac{\sin\beta\sin(\gamma+\chi')}{\sin(\gamma+\beta)} \tag{18}$$

Since the above formulae have the potential of providing precise tests of the SM, we have opted for writing exact relations. However, it is obvious that given the experimental knowledge on the size of the various moduli of the CKM matrix elements, some of the above relations can be, to an excellent approximation, substituted by simpler ones. For example, Eq.(18) is the exact version of the Aleksan-London-Kayser relation [16], the importance of which has been emphasized by Silva and Wolfenstein [21] :

$$\sin\chi \simeq \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{\sin\beta\sin\gamma}{\sin(\gamma+\beta)} \tag{19}$$

Similarly Eq.(9) can be well approximated by:

$$\sin\chi \simeq r\frac{|V_{us}|}{|V_{ud}|}\sin\beta \tag{20}$$

and Eqs.(14) and (12) lead, respectively to:

$$r \simeq |V_{us}| \frac{\sin\gamma}{\sin(\gamma+\beta)} \tag{21}$$

$$\sin\chi \simeq \frac{|V_{us}|\,|V_{ub}|}{|V_{cb}|} \sin\gamma \tag{22}$$

At this stage, the following comments are in order:

(i) Eq.(7) would provide an excellent test of the SM if the phase χ' could be measured without hadronic uncertainties. As previously mentioned, χ' can be obtained from the knowledge of ε_K, but its extraction suffers from hadronic uncertainties in ${f_K}^2 B_K$. Note that all quantities in Eq.(7) are immune to the presence of New Physics in $B_d^0 - \overline{B}_d^0$ and $B_s^0 - \overline{B}_s^0$ mixings.

(ii) Eq.(9) and its approximate form Eq.(20) would provide an excellent test of the SM, once χ, r and β are measured. Note that the theoretical errors in extracting $r \equiv |V_{td}| \,/\, |V_{ts}|$ from $B_d^0 - \overline{B}_d^0$ and $B_s^0 - \overline{B}_s^0$ mixings are much smaller than those present in the extraction of $|V_{td}|$, $|V_{ts}|$.

(iii) Eq.(22) has the important feature of only involving quantities which are not sensitive to the possible presence of New Physics in $B_d^0 - \overline{B}_d^0$ mixing. It has, of course, the disadvantage of requiring the knowledge of $|V_{ub}|$ with significant precision, in order to be a precise test of the SM.

(iv) Eq.(16) in an exact way and Eq.(21) in an excellent approximation, give r in terms of γ and β. This relation will provide an important test of the SM once r, γ and β are measured. Note that in the SM, one knows that r is of order $|V_{us}|$, the importance of Eq.(21) is that it provides the constant of proportionality.

In the context of the SM the above formulae can also be very useful for a precise determination of V_{CKM} from input data: for example, if β and γ are measured with sufficient accuracy, one can use Eqs.(13), (14) to determine $|V_{ub}|$, $|V_{td}|$. One can thus reconstruct the full CKM matrix, using $|V_{us}|$, $|V_{cb}|$, β and γ as input parameters. Furthermore we can also predict the SM value for $\sin 2\chi$ and $\sin\chi'$.

From Eq.(9) we can write

$$\sin\chi \simeq R_t\, |V_{us}|^2 \sin\beta \quad ; \quad R_t = \frac{|V_{td}||V_{tb}|}{|V_{cd}||V_{cb}|} \tag{23}$$

leading to

$$0.030 \leq \sin 2\chi \equiv \sin 2\beta_s \leq 0.045 \tag{24}$$

using the present knowledge of the parameters [22] , $R_t = 0.84 \pm 0.08$, $|V_{us}| = 0.221 \pm 0.002$ and $\beta = (26.9 \pm 5.0)^o$. The SM interval for $\sin\chi'$ can be obtained from either Eq.(7) or Eq.(11), its central value is predicted to be much smaller than the one of $\sin\chi$. Although γ has not yet been measured, the allowed interval in the SM is [22]

$$\gamma = (55.4 \pm 11.9)^o \tag{25}$$

Using Eq.(25)together with the present values [22] $|V_{cb}| = 0.0417 \pm 0.0010$, $|V_{ub}| = (4.05 \pm 0.42) \times 10^{-3}$, we obtain

$$5.4 \times 10^{-4} \leq \sin\chi' \leq 8.2 \times 10^{-4} \tag{26}$$

So far our discussion has been done in the framework of the SM. Let us now consider Physics Beyond the SM.There is a large class of extensions of the SM in which the CKM matrix remains unitary and therefore the unitarity relations previously written in this section are still valid. An important example is , of course , the case of supersymmetric extensions of the SM. We will assume that New Physics only contributes to one loop processes such as $B_d^0 - \overline{B}_d^0$ and $B_s^0 - \overline{B}_s^0$ mixing, so that the measurement of ΔM_{B_d} and ΔM_{B_s} will not allow us to extract directly the values of $|V_{td}V_{tb}|$ and $|V_{ts}V_{tb}|$. Furthermore, CP asymmetries will no longer measure the same angles as in the framework of the SM. As a result, a naive extraction of the values of the sides and angles of the unitarity triangles from input experimental data (i.e. assuming the validity of the SM) will not lead to the correct values [5]. This implies that although the unitarity relations continue to be valid, the presence of NP will simulate violations of the above relations. In the presence of NP one may wonder how large the deviations from the SM should be in order to be possible to detect them using these unitarity relations. This NP scenario is parametrized by Eq.(4) and the phases entering in each process are defined in Table 1. The magnitudes that will signal NP are $\phi_d, \phi_s \neq 0$ and/or $r_d, r_s \neq 1$.

Extraction of ϕ_d

From Eq.(13), we see that this unitarity relation can only be affected by the presence of ϕ_d, therefore this equation allows for a clean extraction of ϕ_d.

[5] It is interesting to note that the naive extraction of the same parameter through different processes affected in a different way by NP may not lead to a unique value, thus signalling NP

By writing Eq.(13) in terms of $\overline{\beta}$ and ϕ_d (note that $Im\left(\lambda^{(d)}_{J/\Psi K_s}\right) = \sin\left(2\overline{\beta}\right)$) we get

$$\tan\left(\phi_d\right) = \frac{R_u \sin\left(\gamma + \overline{\beta}\right) - \sin\left(\overline{\beta}\right)}{\cos\left(\overline{\beta}\right) - R_u \cos\left(\gamma + \overline{\beta}\right)} \tag{27}$$

with

$$R_u = \frac{|V_{ud}|\,|V_{ub}|}{|V_{cd}|\,|V_{cb}|} \tag{28}$$

From Eq.(27), we can find out the bounds that can be reached for ϕ_d, once we have a direct measurement of γ. To illustrate the usefulness of Eqs.(13) and (27), we will give examples of different sets of assumed data which hopefully will be available in the near future. We will assume that apart from $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$ and $\overline{\beta}$ also γ has been measured. We will consider three different cases. In the first case (Example 1), we assume data with relatively large errors for $\overline{\beta}$, γ and $|V_{ub}|$, leading to a value ϕ_d with relatively large errors and consistent with zero. In the second case (Example 2), we assume data with smaller errors, leading to a more precise determination of ϕ_d, but still consistent with zero. This is, of course, the scenario where no NP is discovered and strict bounds are set on the strength of NP. Finally in the third case (Example 3) we consider again data with small errors but leading to a value of ϕ_d which is not consistent with zero. This is the most optimistic scenario, where NP is discovered. For each set of input data we depict the corresponding ϕ_d distribution, generated by means of a toy Monte Carlo, with the assumption of Gaussian errors.

EXAMPLE 1: In our first example we use as input values

$$\begin{array}{lll} |V_{us}| = 0.221 \pm 0.002 & |V_{cb}| = 0.0417 \pm 0.0010 & |V_{ub}| = (4.05 \pm 0.42) \times 10^{-3} \\ \overline{\beta} = (26.9 \pm 5.0)^{\,o} & \gamma = (55.4 \pm 11.9)^{\,o} & \end{array} \tag{29}$$

The corresponding distribution is given by Fig.1. We conclude that if we assume the present experimental numbers and a poor determination of γ ($\sim 20\%$ error), one obtains $\phi_d = (-2.6 \pm 6.0)^{\,o}$, consistent with zero.

EXAMPLE 2: In our second example we assume a knowledge of $\overline{\beta}$, γ and $|V_{ub}|$ at the level of (1%), (10%) and (5%), respectively. This level of precision is expected in future B factories. In this case we use as input values

$$\begin{array}{lll} |V_{us}| = 0.221 \pm 0.002 & |V_{cb}| = 0.0417 \pm 0.0010 & |V_{ub}| = (4.05 \pm 0.21) \times 10^{-3} \\ \overline{\beta} = (25.1 \pm 0.25)^{\,o} & \gamma = (56.6 \pm 5.6)^{\,o} & \end{array} \tag{30}$$

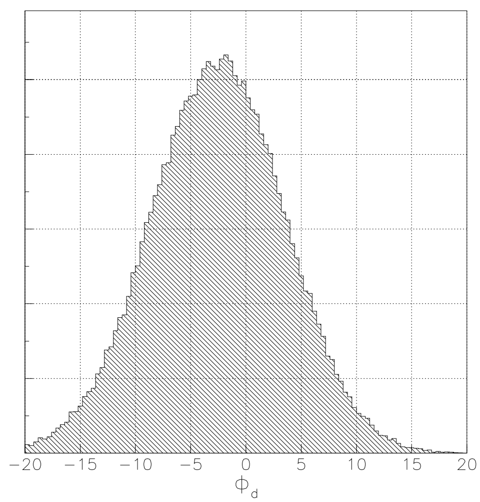

Figure 1: The ϕ_d distribution in degrees corresponding to EXAMPLE 1, consistent with $\phi_d = 0$.

and the corresponding distribution is given by Fig.2. This example leads to $\phi_d = (-0.1 \pm 1.7)^o$ which is, once again, consistent with zero. We conclude that discovering NP contributing to the phase of $B_d^0 - \overline{B}_d^0$ mixing will be possible for values of ϕ_d of a few degrees.

Note that for $\gamma + \overline{\beta}$ close to 90^o, as is the case in our two previous examples, the dependence of ϕ_d on the precise value of $\gamma + \overline{\beta}$ is rather weak so that, in these cases, it will be the precision of $|V_{ub}|$ that will control the final bound on ϕ_d. Obviously, this will no longer be true if $\gamma + \overline{\beta}$ differs significantly from 90^o.

EXAMPLE 3: So far we have only considered examples leading to ϕ_d consistent with zero. Let us now consider an example that would lead to a sizeable ϕ_d. If we replace the values of $\overline{\beta}$ and γ in Eq.(30) by:

$$\overline{\beta} = (30.0 \pm 0.3)^o \quad \gamma = (20 \pm 5)^o \tag{31}$$

the resulting ϕ_d distribution is presented in Fig.3 corresponding to $\phi_d = (-16.3 \pm 3.2)^o$ In this case one would have a clear indication of NP in the phase of $B_d^0 - \overline{B}_d^0$ mixing. Note that for this choice of γ the value of ε_K would not be saturated by the SM contribution. Therefore in this example one would conclude that NP also contributes to ε_K.

Extraction of ϕ_s

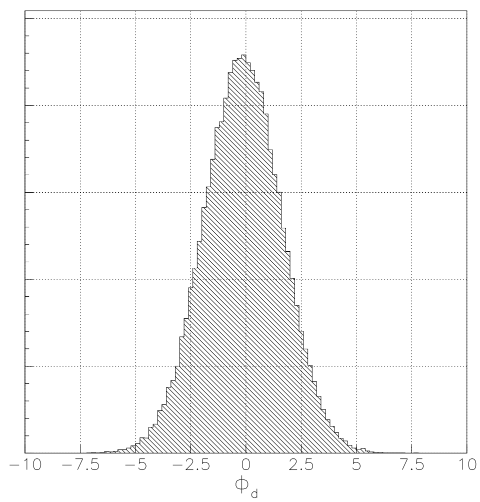

Figure 2: The ϕ_d distribution in degrees corresponding to EXAMPLE 2, consistent with $\phi_d = 0$.

New Physics in the phase of $B_s^0 - \overline{B}_s^0$ mixing would be seen through Eq.(15) where all variables except χ (and eventually χ' which is too small to play any rôle) can be extract from experiment independently of NP. This equation allows for a clean extraction of ϕ_s [6] through the measurement of an asymmetry sensitive to $\overline{\chi}$. Neglecting χ' and rewriting Eq.(15) in terms of $\overline{\chi}$ and ϕ_s we get

$$\tan(\phi_s) = \frac{\sin\overline{\chi} - C\sin(\gamma - \overline{\chi})}{C\cos(\gamma - \overline{\chi}) + \cos\overline{\chi}} \tag{32}$$

with

$$C = \frac{|V_{us}|\,|V_{ub}|}{|V_{cs}|\,|V_{cb}|} \tag{33}$$

Applying the previous procedure to Eq.(32) with the choice of assumed data given by $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$ in Eq.(29) together with

$$\gamma = (56.6 \pm 5.6)^o \quad \overline{\chi} = (1.06 \pm 0.50)^o \tag{34}$$

we are led to $\phi_s = (0.03 \pm 0.51)^o$, the corresponding Gaussian distribution is given by Fig.4. In this example we assumed γ and $\overline{\chi}$ to be known to a

[6] We are assuming that we are not trying to disentangle χ from ϕ_s with the measurement of an asymmetry directly sensitive just to ϕ_s

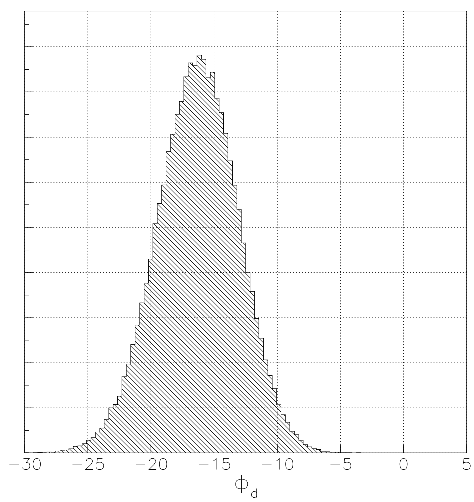

Figure 3: The ϕ_d distribution in degrees corresponding to EXAMPLE 3, pointing clearly to NP.

precision of 20% and 50% level respectively. We conclude that it will be possible to discover NP contributing to the phase of $B_s^0 - \overline{B}_s^0$ mixing for values of ϕ_s larger than about 1.5^o.

Extraction of r_d

Let us consider the possibility of having r_d and/or r_s different from 1, which would also be a sign of NP. From Eq.(10) we get

$$|V_{td}|\,|V_{tb}| = \frac{\sin\gamma}{\sin\beta}\,|V_{ud}|\,|V_{ub}| \tag{35}$$

With the measurement of $\overline{\beta}$, γ and R_u the value of ϕ_d can be determined from Eq.(27). As a result one can obtain $\beta = \overline{\beta} - \phi_d$ and use Eq.(35) to determine $|V_{td}|\,|V_{tb}|$ This determination of $|V_{td}|\,|V_{tb}|$ should be compared to the value of $|V_{td}|\,|V_{tb}|\,r_d$ extracted from the experimental value of ΔM_{B_d}, in order to obtain r_d.

From B-factories, CLEO and Tevatron one can expect, as mentioned before, significant improvements in the level of precision to which $\overline{\beta}$ and $|V_{ub}|$ are determined (to 1% and 5% respectively) together with a direct measurement of γ with about 10% precision. Therefore, we can expect to have a determination of the left hand side (lhs) of Eq.(35) at the 7% level if the central value of γ does not deviate significantly from the presently allowed

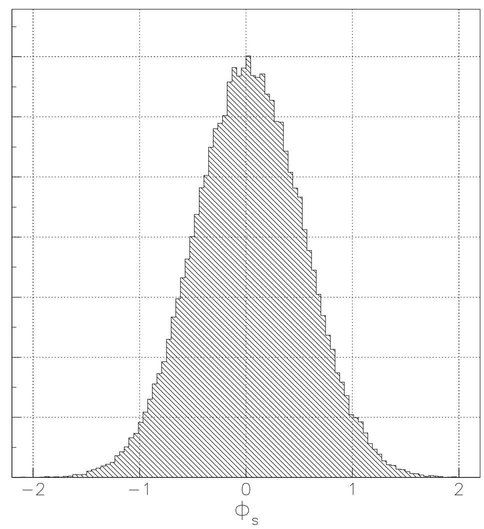

Figure 4: The ϕ_s distribution in degrees for the assumed values of $\gamma = (56.6 \pm 5.6)^o$ and $\overline{\chi} = (1.06 \pm 0.50)^o$. Other inputs are the values given in Eq.(29).

region within the SM. The largest uncertainty comes from the extraction of $|V_{td}|\,|V_{tb}|\,r_d$ from ΔM_{B_d}, which is theoretically limited by the presence of the non-perturbative parameter $f_{B_d} B_{B_d}^{1/2} = (230 \pm 34)\,MeV$. With the present theoretical knowledge one cannot expect a determination of r_d with a precision better than $15 - 20\%$. Future improvement depends on more precise lattice results. Similar arguments would apply to the determination of r_s from the experimental value of ΔM_{B_s}, which will be available in the near future. The CP asymmetry in semileptonic B decays also contains correlated information about r_d and ϕ_d [23], [24]. In fact the present world average $A_{SL} = (0.2 \pm 1.4) \times 10^{-2}$ [25] allows to put bounds in the r_d, ϕ_d plane. In spite of the large theoretical uncertainties, in future analysis one should also include A_{SL} to check the consistency of the extraction of ϕ_d and r_d.

Extraction of r_d/r_s

Once the B_s^0 mixing parameter $x_s \equiv \Delta M_{B_s} \tau_{B_s}$ is measured we can directly determine $|V_{td}|\,r_d/\,|V_{ts}|\,r_s$ from the ratio $\Delta M_{B_d}/\Delta M_{B_s}$ requiring the knowledge of $\xi \equiv f_{B_s} B_{B_s}^{1/2} / f_{B_d} B_{B_d}^{1/2} = 1.16 \pm 0.05$. The parameter ξ is determined from lattice calculations and suffers from a much smaller uncertainty than each of the terms in the ratio taken separately. In this case Eq.(14)

divided by $|V_{ts}|$ will become very useful in the determination of r_d/r_s. We can rewrite the resulting right hand side (rhs) of Eq.(14), to an excellent approximation, by replacing $|V_{cd}|$ by $|V_{us}|$, $|V_{tb}|$ by 1 and $|V_{ts}|$ by

$$|V_{ts}| \simeq |V_{cb}| \left\{ 1 + |V_{us}| \left[\frac{|V_{ub}|}{|V_{cb}|} \cos\gamma - \frac{1}{2} |V_{us}| \right] \right\} \tag{36}$$

leading to

$$r \equiv |V_{td}| \,/\, |V_{ts}| = \frac{|V_{us}|}{1 + |V_{us}| \left[\frac{|V_{ub}|}{|V_{cb}|} \cos\gamma - \frac{1}{2} |V_{us}| \right]} \frac{\sin\gamma}{\sin(\gamma+\beta)} + \mathcal{O}(\lambda^5) \tag{37}$$

From this equation together with

$$\frac{x_d}{x_s} = r^2 \left(\frac{r_d}{r_s} \right)^2 \left(\frac{M_{B_d} \tau_{B_d}}{M_{B_s} \tau_{B_s}} \right) \frac{1}{\xi^2} \tag{38}$$

we obtain

$$\begin{aligned} f(\phi_d) \times \left(\frac{r_d}{r_s} \right) &= \left(\xi^2 \frac{x_d M_{B_s} \tau_{B_s}}{x_s M_{B_d} \tau_{B_d}} \right)^{1/2} \left(\frac{\sin\left(\gamma + \overline{\beta}\right)}{|V_{us}| \sin\gamma} \right) \times \\ & \times \left(1 + |V_{us}| \left[\left| \frac{V_{ub}}{V_{cb}} \right| \cos\gamma - \frac{1}{2} |V_{us}| \right] \right) \end{aligned} \tag{39}$$

where $f(\phi_d)$ is given by

$$f(\phi_d) = \left(\cos\phi_d \left[1 + \frac{\tan\phi_d}{\tan\left(\gamma + \overline{\beta}\right)} \right] \right)^{-1} \tag{40}$$

Equation (39) is valid to a precision better than 1%. Obviously, $f(\phi_d)\, r_d/r_s = 1$ if there is no NP. The rhs of Eq.(39) will be evaluated from experimental data once we have a direct measurement of γ and x_s thus providing a test of NP in the ratio r_d/r_s. Note that ϕ_d can be computed from Eq.(27). New Physics affecting the moduli of $M_{12}^{(q)}$ will thus be seen here, provided that the relative contribution of NP to each sector is different.

By taking, as experimental input the same values as in Example 2, given by Eq.(30), together with[7] $x_d/x_s = 2.63 \times 10^{-2}$ and a Gaussian distribution for $\xi = 1.16 \pm 0.05$, we have generated the toy Monte Carlo distributions for $f(\phi_d)\, r_d/r_s$ and r_d/r_s presented in Figs.5 and 6, respectively. These

[7]For x_s we have taken the central value extracted from the other central values. We have not included any error for x_s/x_d consistent with BTeV design since it will be too small to play any rôle.

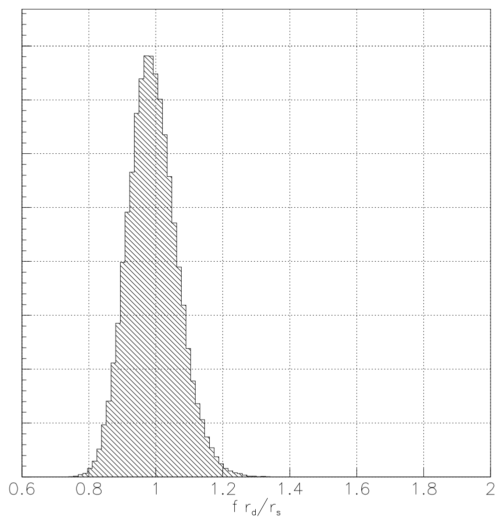

Figure 5: Distribution for $f\,(\phi_d)\, r_d/r_s$ for the input values of Eq.(30) and $x_d/x_s = 2.63 \times 10^{-2}$, together with a Gaussian distribution for $\xi = 1.16 \pm 0.05$

distributions are fitted by Gaussians corresponding to $f\,(\phi_d)\, r_d/r_s = 0.99 \pm 0.07$ and $r_d/r_s = 0.99 \pm 0.07$ and are almost identical. This is due to the fact that the $f\,(\phi_d)$ distribution resulting from Eq.(40) is quite narrow for the ϕ_d distribuition obtained in this example and represented in Fig.2. In this case the precision in the r_d/r_s determination is limited to 7%, due to the error of 10% assumed for γ rather than the error assumed for ξ, which is of the order of 5%. Once $\overline{\beta}$ and x_s are measured to the accuracy chosen in our example together with γ to a precision of a few per cent, the extraction of r_d/r_s will then be limited by the theoretical knowledge of ξ. In Fig. 7 we plot the corresponding r_d/r_s distribution for $\gamma = (56.6 \pm 2.6)^{\,o}$, together with the other input values assumed for Fig.6. In this case, r_d/r_s is very well fitted by a Gaussian corresponding to $r_d/r_s = 0.99 \pm 0.05$.

4 Discussion and Conclusions

We have presented a set of exact relations among specific moduli and rephasing invariant phases of V_{CKM}, which result from 3×3 unitarity of the quark mixing matrix. These exact relations provide a stringent precision test of the SM, with the potential for revealing New Physics. This is specially true if, on the one hand, γ, x_s and eventually χ are measured in the present or future B

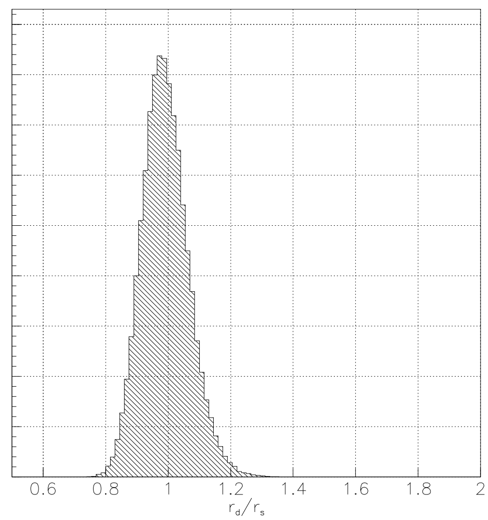

Figure 6: Distribution for r_d/r_s for the input values of Eq.(30) and $x_d/x_s = 2.63 \times 10^{-2}$, together with a Gaussian distribution for $\xi = 1.16 \pm 0.05$

factories and, on the other hand, there is significant decrease in the theoretical uncertainties in the evaluation of the relevant hadronic matrix elements. These tests may complement the standard analysis in the ρ, η plane, which consists of finding a region in that plane where all experimental data are accommodated by the SM. Discovering NP corresponds to having no region in the ρ, η plane where all data is accounted for. In this NP scenario, our exact relations may be useful in revealing the origin of NP. For example, if there are new contributions to ϕ_d in $B_d^0 - \overline{B}_d^0$ mixing, but no NP contributions to $B_s^0 - \overline{B}_s^0$ mixing, Eq.(13) will be violated, leading in general to $\phi_d \neq 0$ in Eq.(27), while, for example Eq.(22) will still be satisfied. Conversely, if there are NP contributions to $B_s^0 - \overline{B}_s^0$ mixing leading to $\phi_s \neq 0$, $r_s \neq 1$, but no NP contributing to $B_d^0 - \overline{B}_d^0$ mixing, Eq.(13) will hold but Eqs.(20),(21)and (22) will be violated.

In conclusion, the advent of B-factories offers the possibility of using exact relations among moduli and rephasing invariant phases of V_{CKM} to perform precision tests of the SM and hopefully to uncover the presence of New Physics.

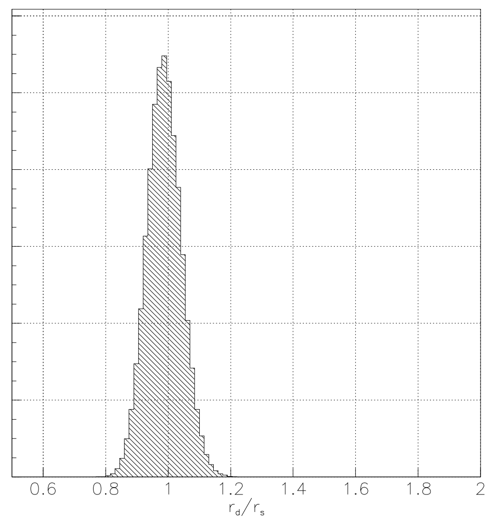

Figure 7: Distribution for r_d/r_s for $\gamma = (56.6 \pm 2.6)^o$, and the other input values equal to those of Eq.(30), together with $x_d/x_s = 2.63 \times 10^{-2}$ and a Gaussian distribution for $\xi = 1.16 \pm 0.05$

Acknowledgments

We would like to thank Helen Quinn and João Silva for useful conversations. FJB aknowledges the warm hospitality extended to him during his stay at IST, Lisbon where part of this work was done. FJB received partial support from CICYT AEN-99/0692 and from FCT through Project CERN/P/FIS/40134/2000. GCB and MNR also received partial support from this project as well as from FCT through Project CERN/FIS/43793/2001 and Project CERN/C/FIS/40139/2000 and from the EC under contract HPRN-CT-2000-00148. MN aknowledges MCED for a research fellowship.

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