Minimal Flavour Violation and Multi-Higgs Models

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Abstract

We propose an extension of the hypothesis of Minimal Flavour Violation (MFV) to general multi-Higgs Models without the assumption of Natural Flavour Conservation in the Higgs sector. We study in detail under what conditions the neutral Higgs couplings are only functions of V_{CKM} and propose a MFV expansion for the neutral Higgs couplings to fermions.

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1 Introduction

The Standard Model (SM) of the electroweak and strong interactions has had an impressive success in accounting for most of the presently available experimental data. The discovery of non-vanishing neutrino masses provided a notable exception [1], pointing towards New Physics (NP), since in the SM neutrinos are strictly massless.

In spite of its great success, there is a general consensus that the SM including its simple extension incorporating neutrino masses, cannot be the “final theory”. One of the reasons for this, has to do with the large number of free parameters, most of them arising from the flavour sector of the SM. This proliferation of free parameters reflects the fact that the flavour structure of Yukawa couplings is not constrained by gauge invariance. In the SM, flavour changing neutral currents (FCNC) are forbidden at tree level both in the gauge and the Higgs sectors. From the early stages of gauge theories, some principles of flavour conservation by neutral currents have been introduced both in the gauge sector through a generalization of the GIM mechanism [2], as well as in the scalar sector through the principle of Natural Flavour Conservation (NFC) proposed by Glashow and Weinberg [3]. It is interesting to note that one may have non-zero but naturally suppressed FCNC in the gauge sector in models where vector-like quarks [4] [5], [6] are added to the SM. In this case, gauge mediated FCNC arise at tree level, suppressed by the small ratio m^2/M^2 where m and M denote standard quark masses and vector-like quark masses, respectively. Vector-like quarks arise in various extensions of the SM, including E_6 grand-unified theories and extra-dimension models. Other motivations for considering vector-like quarks include the possibility of finding a solution to the strong CP problem [7], [5] and accounting for [6] the potentially large CP asymmetry recently observed in $B_s \to J/\Psi\phi$ decays [8], [9]. Recently, a different possibility was considered [10] to avoid tree-level FCNC processes in the framework of two Higgs doublet models, allowing for new sources of CP violation.

All the flavour changing transitions in the SM are mediated by charged weak currents with the flavour mixing controlled by the Cabibbo-Kobayashi-Maskawa (CKM) matrix, V_{CKM} [11]. Any extension of the SM which attempts at solving the flavour puzzle has to confront the strict limits on FCNC processes as well as limits on CP violating transitions leading, for example, to electric dipole moments of quarks and leptons [12].

In the scalar sector, it has been considered the possibility of allowing for deviations of strict NFC by invoking the presence of suppression factors [13] [14] involving small off-diagonal elements of the quark mixing matrix V_{CKM}. The first models of this type were proposed by Branco, Grimus and Lavoura (BGL) [15] who have shown that there are extensions of the SM with two Higgs doublets and an additional discrete symmetry, where there are FCNC at tree level, with couplings entirely determined in terms of the CKM matrix elements, with no other free parameters. In some variants of these models [15] the Higgs particles can be relatively light, without entering in conflict with the stringent limits on FCNC processes.

The success of the SM and its CKM mechanism of mixing and CP violation shows that if there are New Physics contributions to flavour changing interactions at the Tev scale its couplings should occur at a much higher scale or else should be strongly non-generic. This is natural and in a certain sense to be expected if one takes into account that flavour changing transitions in the SM have a special flavour structure, not predicted within its framework. For example, in the SM there is no explanation for the pattern of flavour mixings and in particular why $(V_{CKM})_{12} \sim (m_d/m_s)^{1/2}$ while $(V_{CKM})_{23} \sim (m_s/m_b)$.

One of the suggestions for the flavour structure of New Physics is the proposal of Minimal Flavour Violation (MFV) [16], [17] where all new flavour changing transitions are controlled by the CKM matrix. The gauge sector of the Standard Model (SM) with three generations of quarks and leptons has a large $G_F = U(3)^5$ flavour symmetry which is only broken by Yukawa couplings. One may formally recover [17] this flavour symmetry by promoting Yukawa couplings to auxiliary fields Y, transforming under G_F in such a way that Yukawa interactions become G_F invariant. Then an effective theory arising from New Physics is of MFV type if all higher order operators, constructed from SM fields and Y fields are formally invariant under G_F. This hypothesis, together with the realization that in the SM Yukawa couplings for all fermions, except the top, are small, leads to specific predictions [18].

If one regards the SM as an effective theory, valid up to some energy scale Λ, then in order to have a solution of the hierarchy problem, one expects the scale Λ of New Physics to be of the order of a few TeV. The above considerations have motivated the idea of Minimal Flavour Violation (MFV) both in the quark [16], [17] and lepton sectors [19], [20]. The MFV hypothesis requires that all flavour and CP violating interactions be related to the structure of Yukawa couplings and controlled by V_{CKM}.

The MFV idea has been applied to two Higgs doublet extensions of the SM where there is Natural Flavour Conservation (NFC) in the Higgs sector at tree level, as it is the case in the minimal supersymmetric extension of the Standard Model (MSSM).

In this paper we examine how to extend the ideia of Minimal Flavour Violation to the scalar sector with two and three Higgs doublets, without the assumption of Natural Flavour Conservation in the Higgs sector. This paper is organized as follows: in section 2 we recall the important requirement of rephasing invariance, and in section 3 we analyse in detail how the requirement of MFV can be fulfilled in the context of an extension of the SM where two Higgs doublets are introduced. In section 4 we propose a general MFV expansion of the neutral Higgs couplings to quarks and we stress the important role of discrete symmetries in fixing the parameters of this expansion. The case of three Higgs doublets in the context of MFV is analysed in section 5 and our conclusions are presented in the last section.

2 The Requirement of Rephasing Invariance

As we have seen, the definition of MFV includes the requirement that all flavour transitions are controlled by the CKM matrix. Let us consider a FCNC transition connecting, for definiteness, a $Q = -1/3$ quark d_j to a different quark of the same charge d_k. The transition could be mediated by a scalar or a vector boson:

$$\mathcal{L}_{scalar} = \overline{d_{Lj}} \, {\Gamma^S}_{jk} \; d_{Rk} \, S \tag{1}$$

$$\mathcal{L}_{vector} = \overline{d_{Lj}} \, {\Gamma^V}_{jk} \, \gamma_\mu \, d_{Lk} \, V^\mu \tag{2}$$

Note that the couplings Γ^S, Γ^V may arise at tree level or in higher orders. Let us assume that the quark mass matrices have been diagonalized, so that d_j denote quark mass eigenstates. Under rephasing of the quark fields:

$$d_j \to d'_j = \exp\left(-i\beta_j\right) \, d_j \tag{3}$$

the couplings ${\Gamma^S}_{jk}$ and ${\Gamma^V}_{jk}$ have to transform in such a way that the interactions of Eqs. (2) and (2) remain rephasing invariant. This implies that under rephasing

$$\Gamma_{jk} \to \Gamma'_{jk} = \exp\left[i(\beta_k - \beta_j)\right] \, \Gamma_{jk} \tag{4}$$

The fact that in MFV theories, the flavour dependence of Γ_{jk} is completely controlled by the CKM matrix, severely restricts the functional dependence of Γ_{jk} on V_{CKM}. The simplest forms allowed by rephasing invariance are:

$$\Gamma_{jk} = \sum_\alpha c_\alpha V_{\alpha j} V^*_{\alpha k} \tag{5}$$

where c_α are rephasing invariant coefficients. In the sequel, we shall see that the simplest two Higgs doublet (2HD) models which conform to the MFV requirement do have FCNC couplings with such functional dependence on V_{CKM}.

3 The case of Two Higgs Doublets

In this section, we analyse in detail how the requirement of MFV can be fulfilled in the context of an extension of the SM, where two Higgs doublets are introduced. In order to fix our notation, we explicitly write the Yukawa interactions:

$$L_Y = -\overline{Q^0_L} \; \Gamma_1 \Phi_1 d^0_R - \overline{Q^0_L} \; \Gamma_2 \Phi_2 d^0_R - \overline{Q^0_L} \; \Delta_1 \tilde{\Phi}_1 u^0_R - \overline{Q^0_L} \; \Delta_2 \tilde{\Phi}_2 u^0_R \; + \text{h. c.} \tag{6}$$

where Γ_i and Δ_i denote the Yukawa couplings of the lefthanded quark doublets Q^0_L to the righthanded quarks d^0_R, u^0_R and the Higgs doublets Φ_j. The quark mass matrices generated after spontaneous gauge symmetry breaking are given by:

$$M_d = \frac{1}{\sqrt{2}}(v_1 \Gamma_1 + v_2 e^{i\alpha} \Gamma_2), \quad M_u = \frac{1}{\sqrt{2}}(v_1 \Delta_1 + v_2 e^{-i\alpha} \Delta_2), \tag{7}$$

where $v_i \equiv | < 0|\phi_i^0|0 > |$ and α denotes the relative phase of the vacuum expectation values (vevs) of the neutral components of Φ_i. The matrices M_d, M_u are diagonalized by the usual bi-unitary transformations:

$$U^\dagger_{dL} M_d U_{dR} = D_d \equiv \mathrm{d}iag\ (m_d, m_s, m_b) \tag{8}$$
$$U^\dagger_{uL} M_u U_{uR} = D_u \equiv \mathrm{d}iag\ (m_u, m_c, m_t) \tag{9}$$

In terms of the quark mass eigenstates u, d, the Yukawa couplings are:

$$\begin{aligned} L_Y &= \frac{\sqrt{2}H^+}{v}\bar{u}\left(VN_d\gamma_R + N_u^\dagger\ V\gamma_L\right)d + \text{h.c.} - \frac{H^0}{v}\left(\bar{u}D_u u + \bar{d}D_d\ d\right) - \\ &- \frac{R}{v}\left[\bar{u}(N_u\gamma_R + N_u^\dagger\gamma_L)u + \bar{d}(N_d\gamma_R + N_d^\dagger\gamma_L)\ d\right] + \\ &+ i\frac{I}{v}\left[\bar{u}(N_u\gamma_R - N_u^\dagger\gamma_L)u - \bar{d}(N_d\gamma_R - N_d^\dagger\gamma_L)\ d\right] \end{aligned} \tag{10}$$

where $v \equiv \sqrt{v_1^2 + v_2^2} = (\sqrt{2}G_F)^{-1/2} \approx 246$ GeV, G_F is the Fermi coupling constant, $\gamma_L = (1-\gamma_5)/2$, $\gamma_R = (1+\gamma_5)/2$, V stands for the V_{CKM} matrix and H^0, R are orthogonal combinations of the fields ρ_j, arising when one expands [21] the neutral scalar fields around their vevs, $\phi_j^0 = \frac{e^{i\alpha_j}}{\sqrt{2}}(v_j + \rho_j + i\eta_j)$. Similarly, I denotes the linear combination of η_j orthogonal to the neutral Goldstone boson. The physical neutral Higgs fields are combinations of H^0, R and I.

The Flavour Changing Neutral Yukawa Couplings (FCNYC) are controlled by the matrices N_d, N_u, given by:

$$N_d = \frac{1}{\sqrt{2}}U^\dagger_{dL}\ (v_2\Gamma_1 - v_1 e^{i\alpha}\Gamma_2)\ U_{dR}, \tag{11}$$
$$N_u = \frac{1}{\sqrt{2}}U^\dagger_{uL}(v_2\Delta_1 - v_1 e^{-i\alpha}\Delta_2)\ U_{uR} \tag{12}$$

For generic two Higgs doublet models, the coupling matrices N_d, N_u are non-diagonal and arbitrary. We are interested in analysing under what circunstances the flavour structure of N_d, N_u is entirely controlled by the CKM matrix, as required by the MFV paradigm.

For definiteness, let us consider N_d, which can be written [22] from Eqs.(7), (8)and (11) :

$$N_d = \frac{v_2}{v_1}D_d - \frac{v_2}{\sqrt{2}}\left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)U^\dagger_{dL}e^{i\alpha}\Gamma_2\ U_{dR} \tag{13}$$

From Eq. (13), one sees that there are two obstacles which one has to surmount in order to have N_d entirely controlled by V_{CKM}:

(i) It is U_{dL} rather than the combination $U^\dagger_{uL}U_{dL}$ corresponding to V_{CKM} that appears in N_d given by Eq. (13)

(ii) How to get rid of the dependence on U_{dR}?

The first difficulty can be solved by means of a flavour symmetry constraining U_{uL} to have mixing only among two generations, for example:

$$U_{uL} = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

In this case one has:

$$(V_{CKM})_{3j} = (U_{dL})_{3j} \tag{15}$$

In order to surmount obstacle (i) one has to further require that the above symmetry should also impose that the dependence of the second term of Eq. (13) on U_{dL} be only on elements of its third row, $(U_{dL})_{3j}$. We now turn to question (ii) namely, how to avoid the dependence on U_{dR}. Let us assume that the flavour structures o Γ_2, is such that:

$$\Gamma_2 \propto P M_d \tag{16}$$

Where P is a fixed matrix. In this case:

$$U^\dagger_{dL} \Gamma_2 \, U_{dR} \propto U^\dagger_{dL} P M_d \, U_{dR} \propto U^\dagger_{dL} P \, U_{dL} D_d \tag{17}$$

thus answering question (ii).

Let us now see what should be the flavour structure of Γ_1, Γ_2 so that a fixed matrix P exists, statisfying Eq. (16). One way of achieving this is by having

$$P\Gamma_2 = k\Gamma_2 \tag{18}$$

$$P\Gamma_1 = 0 \tag{19}$$

where k is a constant.

Branco, Grimus and Lavoura have shown [15] that it is possible to find a symmetry which, when imposed to a two Higgs doublet extension of the SM, leads to a structure for Γ_i and Δ_i such that there are scalar FCNC at tree level, with strength completely controlled by V_{CKM}. BGL have imposed the following symmetry S on the Lagrangian:

$$Q^0_{L3} \rightarrow \exp{(i\alpha)} \; Q^0_{L3}, \qquad u^0_{R3} \rightarrow \exp{(i2\alpha)} u^0_{R3}, \qquad \Phi_2 \rightarrow \exp{(i\alpha)} \Phi_2 \tag{20}$$

where $\alpha \neq 0, \pi$, with all other fields transforming trivially under S. The most general Yukawa couplings consistent with this symmetry have the following structure:

$$\Gamma_1 = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix}; \qquad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix} \tag{21}$$

$$\Delta_1 = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{bmatrix}; \qquad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix} \tag{22}$$

where $\times$ denotes an arbitrary entry while the zeros are imposed by the symmetry S.

It is clear that these Yukawa couplings guarantee that Eqs. (14) and (15) are satisfied. They also satisfy Eqs. (16), (18), (19) with

$$P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad \frac{v_2}{\sqrt{2}} e^{i\alpha} \Gamma_2 = P M_d; \quad k = 1 \tag{23}$$

It follows then that the Yukawa couplings of Eqs. (21) and (22) lead to FCNC at tree level, entirely determined by V_{CKM}. Notice that in this example there are no Higgs mediated FCNC in the up sector, which is due to the fact that the Δ_i matrices are block diagonal with each one of these matrices having non-zero entries in different blocks. This also automatically leads to a matrix U_{uL} which is block diagonal and therefore of the form given by Eq. (14). The structure of zeros in the matrix Γ_2 leads to the important relation:

$$\left(U_{dL}^\dagger \Gamma_2\right)_{ij} = (U_{dL}^\dagger)_{i3} (\Gamma_2)_{3j} = (V_{CKM}^\dagger)_{i3} (\Gamma_2)_{3j} \tag{24}$$

this result together with Eqs. (16),(18), (19) and (13) leads to N_d given by [15]

$$(N_d)_{ij} = \frac{v_2}{v_1} (D_d)_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) (V_{CKM}^\dagger)_{i3} (V_{CKM})_{3j} (D_d)_{jj} \tag{25}$$

whereas

$$N_u = -\frac{v_1}{v_2} \text{diag } (0, 0, m_t) + \frac{v_2}{v_1} \text{diag } (m_u, m_c, 0) \tag{26}$$

In this example, the Higgs mediated FCNC are suppressed by the third row of the matrix V_{CKM} and have the structure of Eq. (5) A crucial feature in this example is the fact that each row of M_d only receives contribution from a single Higgs field and the same applies to M_u.

The example given above corresponds to a class of six different models, as was emphasized in [15] . Three of these models have FCNC only in the down sector, and are obtained from the three different projection matrices of a form similar to P in Eq. (23), the other two cases with the diagonal entry in the other two possible entries. In these additional cases the suppression in the Higgs mediated FCNC is not as large as that of the example given above. Another three models are obtained by exchanging the the patterns of zeros of Γ_i matrices with Δ_i matrices, leading to FCNC in the up sector, and flavour conservation in the down sector.

4 MFV Expansion of Yukawa Couplings

The neutral Higgs interactions, beyond those present in the SM, i.e., couplings to R and I, are those that may introduce Higgs mediated FCNC and are given by Eqs. (12), where N_d and N_u are written in the quark mass eigenstate basis. In a weak basis these couplings are:

$$N_d^0 = U_{dL}\ N_d\ U_{dR}^\dagger = \frac{1}{\sqrt{2}}\ (v_2\Gamma_1 - v_1 e^{i\alpha}\Gamma_2), \tag{27}$$

$$N_u^0 = U_{uL}\ N_u\ U_{uR}^\dagger = \frac{1}{\sqrt{2}}\ (v_2\Delta_1 - v_1 e^{i\alpha}\Delta_2) \tag{28}$$

All other couplings involving neutral scalars are flavour conserving, therefore they are not relevant for our analysis. The question that we address in this section is how to find a general expansion of N_d^0, N_u^0 which conforms to the MFV requirements. It is clear that a necessary condition for N_d^0, N_u^0 to be of the MFV type is that they should be functions of M_d, M_u and no other flavour dependent couplings. The terms entering in the expansion of N_d^0, N_u^0 should have the right transformation properties under weak basis (WB) transformations, defined by:

$$Q_L^0 \to W_L\ Q_L^0, \qquad d_R^0 \to W_R^d\ d_R^0, \qquad u_R^0 \to W_R^u\ u_R^0 \tag{29}$$

Under a WB transformation defined by Eq. (29), the quark mass matrices M_d, M_u transform as:

$$M_d \to W_L^\dagger\ M_d\ W_R^d; \qquad M_u \to W_L^\dagger\ M_u\ W_R^u \tag{30}$$

The matrices U_{dL}, U_{dR}, U_{uL}, U_{uR} defined in Eqs. (8), (9) transform under a WB transformation in the following way:

$$U_{dL} \to W_L^\dagger\ U_{dL}; \quad U_{uL} \to W_L^\dagger\ U_{uL}; \quad U_{dR} \to W_R^{d\dagger}\ U_{dR}; \quad U_{uR} \to W_R^{u\dagger}\ U_{uR} \tag{31}$$

The Hermitian matrices H_d, H_u with $H_{d,u} \equiv (M_{d,u})(M_{d,u}^\dagger)$ transform under a WB transformation as:

$$H_d \to W_L^\dagger\ H_d\ W_L; \qquad H_u \to W_L^\dagger\ H_u\ W_L \tag{32}$$

From Eqs. (8), (9) it follows that:

$$U_{dL}^\dagger\ H_d U_{dL} = D_d^2 \tag{33}$$

with analogous expression for H_u. It is convenient to write H_d, H_u in terms of projection operators [23]:

$$H_d = \sum_i m_{di}^2 P_i^{dL} \tag{34}$$

where:

$$P_i^{dL} = U_{dL} P_i U_{dL}^\dagger \tag{35}$$

with

$$(P_i)_{jk} = \delta_{ij}\delta_{ik} \tag{36}$$

Obviously, analogous expressions hold for H_u. It is clear that under a WB transformation, N_d^0, N_u^0 transform as M_d, M_u. A MFV expansion for N_d^0, N_u^0 with proper transformation properties under a WB transformation can then be built with terms proportional to M_d (M_u) respectively, as well as products of terms transforming as H_d and H_u multiplying M_d (M_u) respectively:

$$N_d^0 = \lambda_1 \ M_d + \lambda_{2i} \ U_{dL} P_i U_{dL}^\dagger \ M_d + \lambda_{3i} \ U_{uL} P_i U_{uL}^\dagger \ M_d + ... \tag{37}$$

$$N_u^0 = \tau_1 \ M_u + \tau_{2i} \ U_{uL} P_i U_{uL}^\dagger \ M_u + \tau_{3i} \ U_{dL} P_i U_{dL}^\dagger \ M_u + ... \tag{38}$$

In the quark mass eigenstate basis N_d^0, N_u^0 become:

$$N_d = \lambda_1 \ D_d + \lambda_{2i} \ P_i \ D_d + \lambda_{3i} \ (V_{CKM})^\dagger \ P_i \ V_{CKM} \ D_d + ... \tag{39}$$

$$N_u = \tau_1 \ D_u + \tau_{2i} \ P_i \ D_u + \tau_{3i} \ V_{CKM} \ P_i \ (V_{CKM})^\dagger \ D_u + ... \tag{40}$$

which conforms explicitly to the MFV requirement. Terms of the form $U_{dL} P_i U_{dL}^\dagger \ M_d$ and $U_{uL} P_i U_{uL}^\dagger \ M_u$ do not lead to Higgs mediated FCNC, whereas terms of the form $U_{uL} P_i U_{uL}^\dagger \ M_d$ and $U_{dL} P_i U_{dL}^\dagger \ M_u$ do lead to FCNC. At this stage the lambda and tau coefficients of these expansions appear as free parameters. This was to be expected, since the expansions of Eqs. (39), (40), conform to the MFV requirement but have no further restriction. In theories where the MFV requirement results from the imposition of a symmetry on the Lagrangian, the coefficients lambda and tau are constrained.

Comparing Eqs. (25) and (26) to Eqs. (39) and (40) one realizes that the BGL example presented in the previous section corresponds to the following truncation of our MFV expansion:

$$N_d^0 = \frac{v_2}{v_1} M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_{uL} P_3 U_{uL}^\dagger \ M_d \tag{41}$$

$$N_u^0 = \frac{v_2}{v_1} M_u - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_{uL} P_3 U_{uL}^\dagger \ M_u \tag{42}$$

This result, together with equations:

$$N_d^0 = \frac{v_2}{v_1} M_d - \frac{v_2}{\sqrt{2}} \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) e^{i\alpha} \Gamma_2 \tag{43}$$

$$N_u^0 = \frac{v_2}{v_1} M_u - \frac{v_2}{\sqrt{2}} \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) e^{-i\alpha} \Delta_2 \tag{44}$$

implies that the BGL model is fully defined in a covariant way under WB transformations by:

$$\frac{v_2}{\sqrt{2}} e^{i\alpha} \Gamma_2 = U_{uL} P_3 U_{uL}^\dagger \ M_d \tag{45}$$

$$\frac{v_2}{\sqrt{2}} e^{-i\alpha} \Delta_2 = U_{uL} P_3 U_{uL}^\dagger \ M_u \tag{46}$$

The factors multiplying Γ_2 and Δ_2 coincide with the coefficients for these matrices in the expressions of M_d and M_u. Replacing in these equations the mass matrices written in terms of the Yukawa couplings one obtains:

$$U_{uL} P_3 U_{uL}^\dagger \Gamma_2 = \Gamma_2; \qquad U_{uL} P_3 U_{uL}^\dagger \Gamma_1 = 0 \tag{47}$$

$$U_{uL} P_3 U_{uL}^\dagger \Delta_2 = \Delta_2; \qquad U_{uL} P_3 U_{uL}^\dagger \Delta_1 = 0 \tag{48}$$

These relations are the generalization to an arbitrary basis of the relations satisfied by the BGL model, namely $P_3 \Gamma_2 = \Gamma_2$, $P_3 \Gamma_1 = 0$, $P_3 \Delta_2 = \Delta_2$ and $P_3 \Delta_1 = 0$ which result from the imposed symmetry. Now, we show, that in fact, in this case there is a WB where the matrices Γ_1, Γ_2, Δ_1 and Δ_2 have the forms given by Eqs. (21) and (22). Starting from a WB where M_u is real and diagonal, and therefore $U_{uL} = 1$, we may perform a WB transformation by choosing W_L and W_R^u block diagonal with mixing in the (12) block only. As a result, the matrix M_u will also be block diagonal, in this WB. Eq. (45) becomes:

$$\frac{v_2}{\sqrt{2}} e^{i\alpha} \Gamma_2 = W_L^\dagger(12)\ P_3 W_L(12)\ M_d = P_3\ M_d \tag{49}$$

which is exactly the form of Γ_2 given by Eq. (21). The condition $P_3\ \Gamma_1 = 0$ also leads to the Γ_1 of Eq. (21). For Δ_2 we have:

$$\frac{v_2}{\sqrt{2}} e^{-i\alpha} \Delta_2 = W_L^\dagger(12)\ P_3 W_L(12)\ M_u = P_3\ M_u \tag{50}$$

In this case, the projector P_3 picks the diagonal (33) entry of M_u, which together with $P_3\ \Delta_1 = 0$ leads to the matrix forms of Eq. (22). The two other models of the same class, with FCNC in the down sector are obtained by taking the two other projectors, P_1 and P_2, in each case. The three other cases with FCNC in the up sector only, correspond to:

$$N_d^0 = \frac{v_2}{v_1}\ M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right)\ U_{dL} P_i U_{dL}^\dagger\ M_d \tag{51}$$

$$N_u^0 = \frac{v_2}{v_1}\ M_u - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right)\ U_{dL} P_i U_{dL}^\dagger\ M_u \tag{52}$$

with $i = 1, 2, 3$ respectively. The special feature of these six different models is the fact that there are WB's where the Γ and the Δ matrices have sectors with zero textures that do not mix with each other and, as BGL have shown, these models can be implemented by S-type symmetries.

It is also possible to build simple models of MFV type with Higgs mediated FCNC in both sectors, like the one defined by the following equations:

$$N_d^0 = \frac{v_2}{v_1}\ M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right)\ U_{uL} P_i U_{uL}^\dagger\ M_d \tag{53}$$

$$N_u^0 = \frac{v_2}{v_1}\ M_u - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right)\ U_{dL} P_i U_{dL}^\dagger\ M_u \tag{54}$$

It is also possible to have MFV models beyond standard NFC [3] but without FCNC, like

$$N_d^0 = \frac{v_2}{v_1}\ M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)\ U_{dL} P_i U_{dL}^\dagger\ M_d \tag{55}$$

$$N_u^0 = \frac{v_2}{v_1}\ M_u - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)\ U_{uL} P_i U_{uL}^\dagger\ M_u \tag{56}$$

One can also construct more involved MFV models of the BGL type:

$$N_d^0 = \frac{v_2}{v_1}\ M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)\ U_{uL} P_i U_{uL}^\dagger\ M_d \tag{57}$$

$$N_u^0 = \frac{v_2}{v_1}\ M_u - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)\ U_{uL} P_j U_{uL}^\dagger\ M_u \tag{58}$$

with $i \neq j$. In all cases the Γ and Δ matrices obey relations of the same type as those written in Eqs (47) and (48). However, the zero texture structure of these models is more involved than in the BGL case and the question of assuring its loop stability, through the introduction of symmetries, is not obvious [24].

5 Models with Three Higgs Doublets

Let us now consider the case of three Higgs doublets in the context of MFV, where the analogous to Eq. (6) includes the Yukawa terms of the third Higgs doublet.

After spontaneous symmetry breakdown the Higgs doublets can be decomposed as:

$$\Phi_j = e^{i\alpha_j} \begin{pmatrix} \phi_j^+ \\ \frac{1}{\sqrt{2}}(v_j + \rho_j + i\eta_j) \end{pmatrix}, \qquad j = 1, 2, 3 \tag{59}$$

with real scalar fields ρ_j, η_j. The following transformation:

$$\begin{pmatrix} H^0 \\ R \\ R' \end{pmatrix} = O \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix}, \qquad \begin{pmatrix} G^0 \\ I \\ I' \end{pmatrix} = O \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} \tag{60}$$

with the matrix O given by:

$$O = \begin{bmatrix} \frac{v_1}{v} & \frac{v_2}{v} & \frac{v_3}{v} \\ \frac{v_2}{v'} & -\frac{v_1}{v'} & 0 \\ \frac{v_1}{v''} & \frac{v_2}{v''} & \frac{-(v_1^2+v_2^2)/v_3}{v''} \end{bmatrix} \tag{61}$$

where $v = \sqrt{v_1^2 + v_2^2 + v_3^2}$, $v' = \sqrt{v_1^2 + v_2^2}$ and $v'' = \sqrt{v_1^2 + v_2^2 + (v_1^2 + v_2^2)^2/v_3^2}$. The orthogonal matrix O singles out H^0 and the neutral pseudo-Goldstone boson G. H^0 has couplings to the quarks which are proportional to the mass matrices. In general, flavour changing neutral currents arise from the couplings to the remaining four neutral Higgs fields. The diagonalization of the quark mass matrices gives rise to the following neutral Higgs interactions of the physical quarks:

$$\begin{aligned} L_Y(neutal) &= -\frac{H^0}{v}\left(\bar{d_L}D_d\ d_R+\bar{u_L}D_u u_R\right)- \\ &- \ \bar{d_L}\frac{1}{v\prime}\ \mathcal{N}_d(R+iI)d_R-\bar{u_L}\frac{1}{v\prime}\mathcal{N}_u(R-iI)u_R- \\ &- \ \bar{d_L}\frac{1}{v''}\ \mathcal{N}'_d(R'+iI')d_R-\bar{u_L}\frac{1}{v''}\ \mathcal{N}'_u(R'-iI')u_R+\text{h.c.} \end{aligned} \tag{62}$$

with

$$\mathcal{N}_d = \frac{1}{\sqrt{2}}U^\dagger_{dL}\ (v_2 e^{i\alpha_1}\Gamma_1 - v_1 e^{i\alpha_2}\Gamma_2)\ U_{dR}, \tag{63}$$

$$\mathcal{N}_u = \frac{1}{\sqrt{2}}U^\dagger_{uL}(v_2 e^{-i\alpha_1}\Delta_1 - v_1 e^{-i\alpha_2}\Delta_2)\ U_{uR}, \tag{64}$$

$$\mathcal{N}'_d = \frac{1}{\sqrt{2}}U^\dagger_{dL}\ (v_1 e^{i\alpha_1}\Gamma_1 + v_2 e^{i\alpha_2}\Gamma_2 + x e^{i\alpha_3}\Gamma_3)\ U_{dR}, \tag{65}$$

$$\mathcal{N}'_u = \frac{1}{\sqrt{2}}U^\dagger_{uL}\ (v_1 e^{-i\alpha_1}\Delta_1 + v_2 e^{-i\alpha_2}\Delta_2 + x e^{-i\alpha_3}\Delta_3)\ U_{uR} \tag{66}$$

where $x=-(v_1^2+v_2^2)/v_3$.

For definiteness, let us consider $\mathcal{N}_d$ and $\mathcal{N}'_d$, which can be written:

$$\mathcal{N}_d = \frac{v_2}{v_1}D_d - \frac{v_2}{\sqrt{2}}\left(\frac{v_2}{v_1}+\frac{v_1}{v_2}\right)U^\dagger_{dL}e^{i\alpha_2}\Gamma_2\ U_{dR} - \frac{v_2\ v_3}{v_1\sqrt{2}}U^\dagger_{dL}e^{i\alpha_3}\Gamma_3 U_{dR} \tag{67}$$

$$\mathcal{N}'_d = D_d - \frac{v_3 - x}{\sqrt{2}}\ U^\dagger_{dL}e^{i\alpha_3}\Gamma_3\ U_{dR} \tag{68}$$

Imposing the following symmetry on the Lagrangian:

$$\begin{aligned} Q^0_{L1} &\to \omega\ Q^0_{L1}, \qquad Q^0_{L2}\to\omega^2\ Q^0_{L2}, \qquad Q^0_{L3}\to\omega^4\ Q^0_{L3}, \\ \Phi_1 &\to \omega\ \Phi_1, \qquad \Phi_2\to\omega^2\ \Phi_2, \qquad \Phi_3\to\omega^4\ \Phi_3, \\ u^0_{R1} &\to \omega^2\ u^0_{R1}, \qquad u^0_{R2}\to\omega^4\ u^0_{R2}, \qquad u^0_{R3}\to\omega^8\ u^0_{R3}, \\ d^0_{Rj} &\to d^0_{Rj} \end{aligned} \tag{69}$$

with $\omega = \exp i\pi/4$, restricts the Yukawa coupling matrices to have the following structure:

$$\Gamma_1 = \begin{bmatrix} \times & \times & \times \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix}; \quad \Gamma_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix} \tag{70}$$

$$\Delta_1 = \begin{bmatrix} \times & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \Delta_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix} \tag{71}$$

where × denotes an arbitrary entry while the zeros are imposed by the above symmetry.

It can be readily verified that in this case there are Higgs mediated FCNC only in the down sector, with $\mathcal{N}_d$ and $\mathcal{N}'_d$ given by:

$$(\mathcal{N}_d)_{ij} = \frac{v_2}{v_1}(D_d)_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)(V^\dagger_{CKM})_{i2}(V_{CKM})_{2j}(D_d)_{jj} - \\ - \frac{v_2}{v_1}(V^\dagger_{CKM})_{i3}(V_{CKM})_{3j}(D_d)_{jj} \quad (72)$$

$$(\mathcal{N}'_d)_{ij} = (D_d)_{jj} - \frac{v_3 - x}{v_3}(V^\dagger_{CKM})_{i3}(V_{CKM})_{3j}(D_d)_{jj} \quad (73)$$

In this case the couplings $\mathcal{N}_d$ include terms that violate flavour proportional to $(V^\dagger_{CKM})_{i2}(V_{CKM})_{2j}(D_d)_{jj}$ together with terms proportional to $(V^\dagger_{CKM})_{i3}(V_{CKM})_{3j}(D_d)_{jj}$. The couplings $\mathcal{N}'_d$ only include terms that violate flavour proportional to $(V^\dagger_{CKM})_{i3}(V_{CKM})_{3j}(D_d)_{jj}$. It is clear that all Higgs mediated neutral couplings are only function of V_{CKM} and terefore the symmetry of Eq. (69) leads to a MFV structure in the context of a three Higgs-doublet model. From a phenomenological point of view, there is an important difference between the scalar FCNC in this MFV three Higgs doublet model and those encountered in the MFV two Higgs doublet model considered in the previous chapters. In the case of two Higgs doublet models, there is one variant of the BGL models where the tree level Higgs mediated $\Delta S = 2$ amplitude is naturally suppressed by terms proportional to $(V^*_{td}V_{ts})^2$. This very strong suppression opens the possibility of having neutral Higgs relatively light of order 10^2 Gev, without entering in conflict with the size of the $K_L - K_S$ mass difference or the strength of CP violation in the kaon sector. In the case of the MFV three Higgs doublet model $\mathcal{N}_d$ includes FCNC terms where the suppression factor in $\Delta S = 2$ transitions is only $(V^*_{cd}V_{cs})^2$, which then requires quite heavy neutral Higgs, with mass of order Tev.

6 Conclusions

We have analysed how to extend the MFV concept to general multi-Higgs models without NFC in the Higgs sector. We have studied in special detail the case of two Higgs doublet models, analysing the requirements which have to be satisfied in order that the neutral Higgs couplings to quarks be only functions of V_{CKM}, with no other flavour dependent parameters. The Branco-Grimus-Lavoura (BGL) models proposed some time ago are an example where the MFV constraints are satisfied as the result of a symmetry of the Lagrangian. We have proposed a general MFV expansion of the neutral Higgs couplings to quarks and have shown that the BGL models correspond to specific values of the coefficients of the proposed MFV expansion and, in addition, we have shown that the values of these coefficients are fixed by the symmetry.

Multi-Higgs models with Higgs mediated FCNC have a rich phenomenology and some of its aspects have been recently analysed in the literature [25]. A detailed phenomenological analysis of multi-Higgs MFV models without NFC, is beyond the scope of this paper and will be left to a separate work [26]

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References

[1] For references and a review of the experimental situation See "NEUTRINO MASS, MIXING, AND FLAVOR CHANGE", Revised March 2008 by B. Kayser (Fermilab), in C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667** (2008) 1.

[2] S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D **2** (1970) 1285.

[3] S. L. Glashow and S. Weinberg, Phys. Rev. D **15** (1977) 1958.

[4] F. del Aguila and J. Cortes, Phys. Lett. B **156** (1985) 243; G. C. Branco and L. Lavoura, Nucl. Phys. B **278** (1986) 738; F. del Aguila, M. K. Chase and J. Cortes, Nucl. Phys. B **271** (1986) 61; Y. Nir and D. J. Silverman, Phys. Rev. D **42** (1990) 1477; D. Silverman, Phys. Rev. D **45** (1992) 1800; G. C. Branco, T. Morozumi, P. A. Parada and M. N. Rebelo, Phys. Rev. D **48** (1993) 1167; V. D. Barger, M. S. Berger and R. J. N. Phillips, Phys. Rev. D **52** (1995) 1663 [arXiv:hep-ph/9503204]; M. Gronau and D. London, Phys. Rev. D **55** (1997) 2845 [arXiv:hep-ph/9608430]; F. del Aguila, J. A. Aguilar-Saavedra and G. C. Branco, Nucl. Phys. B **510** (1998) 39 [arXiv:hep-ph/9703410]; G. Barenboim, F. J. Botella, G. C. Branco and O. Vives, Phys. Lett. B **422** (1998) 277 [arXiv:hep-ph/9709369]; G. Barenboim, F. J. Botella and O. Vives, Phys. Rev. D **64** (2001) 015007 [arXiv:hep-ph/0012197]; G. Barenboim, F. J. Botella and O. Vives, Nucl. Phys. B **613** (2001) 285 [arXiv:hep-ph/0105306]; K. Higuchi and K. Yamamoto, arXiv:0911.1175 [hep-ph].

[5] L. Bento, G. C. Branco and P. A. Parada, Phys. Lett. B **267** (1991) 95.

[6] F. J. Botella, G. C. Branco and M. Nebot, Phys. Rev. D **79** (2009) 096009 [arXiv:0805.3995 [hep-ph]].

[7] A. E. Nelson, Phys. Lett. B **136** (1984) 387; S. M. Barr, Phys. Rev. Lett. **53** (1984) 329; A. E. Nelson, Phys. Lett. B **143** (1984) 165.

[8] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **101** (2008) 241801 [arXiv:0802.2255 [hep-ex]].

[9] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **100** (2008) 161802 [arXiv:0712.2397 [hep-ex]].

[10] A. Pich and P. Tuzon, arXiv:0908.1554 [hep-ph].

[11] N. Cabibbo, Phys. Rev. Lett. **10** (1963) 531.
M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652.

[12] M. Raidal *et al.*, Eur. Phys. J. C **57** (2008) 13 [arXiv:0801.1826 [hep-ph]].

[13] A. Antaramian, L. J. Hall and A. Rasin, Phys. Rev. Lett. **69** (1992) 1871 [arXiv:hep-ph/9206205]; L. J. Hall and S. Weinberg, Phys. Rev. D **48** (1993) 979 [arXiv:hep-ph/9303241].

[14] A. S. Joshipura and S. D. Rindani, Phys. Lett. B **260** (1991) 149;

[15] G. C. Branco, W. Grimus and L. Lavoura, Phys. Lett. B **380** (1996) 119 [arXiv:hep-ph/9601383].

[16] A. J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini, Phys. Lett. B **500** (2001) 161 [arXiv:hep-ph/0007085].

[17] G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645** (2002) 155 [arXiv:hep-ph/0207036].

[18] C. Bobeth, M. Bona, A. J. Buras, T. Ewerth, M. Pierini, L. Silvestrini and A. Weiler, Nucl. Phys. B **726** (2005) 252 [arXiv:hep-ph/0505110]; M. Blanke, A. J. Buras, D. Guadagnoli and C. Tarantino, JHEP **0610** (2006) 003 [arXiv:hep-ph/0604057].

[19] V. Cirigliano, B. Grinstein, G. Isidori and M. B. Wise, Nucl. Phys. B **728** (2005) 121 [arXiv:hep-ph/0507001].

[20] G. C. Branco, A. J. Buras, S. Jager, S. Uhlig and A. Weiler, JHEP **0709** (2007) 004 [arXiv:hep-ph/0609067].

[21] T. D. Lee, Phys. Rev. D **8** (1973) 1226.

[22] L. Lavoura, Int. J. Mod. Phys. A **9** (1994) 1873.

[23] F. J. Botella, M. Nebot and O. Vives, JHEP **0601** (2006) 106 [arXiv:hep-ph/0407349].

[24] Work in progress.

[25] M. Misiak, S. Pokorski and J. Rosiek, Adv. Ser. Direct. High Energy Phys. **15** (1998) 795 [arXiv:hep-ph/9703442]; J. A. Aguilar-Saavedra and G. C. Branco, Phys. Lett. B **495** (2000) 347 [arXiv:hep-ph/0004190]; A. S. Joshipura and B. P. Kodrani, Phys. Rev. D **77** (2008) 096003 [arXiv:0710.3020 [hep-ph]]; R. Zwicky and T. Fischbacher, Phys. Rev. D **80** (2009) 076009 [arXiv:0908.4182 [hep-ph]]; A. S. Joshipura and B. P. Kodrani, arXiv:0909.0863 [hep-ph].

[26] To appear in a future joint publication.