Two-Higgs Leptonic Minimal Flavour Violation

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Abstract

We construct extensions of the Standard Model with two Higgs doublets, where there are flavour changing neutral currents both in the quark and leptonic sectors, with their strength fixed by the fermion mixing matrices V_{CKM} and V_{PMNS}. These models are an extension to the leptonic sector of the class of models previously considered by Branco, Grimus and Lavoura, for the quark sector. We consider both the cases of Dirac and Majorana neutrinos and identify the minimal discrete symmetry required in order to implement the models in a natural way.

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1 Introduction

Understanding the mechanism of gauge symmetry breaking is one of the fundamental open questions in Particle Physics. Indeed, even if elementary scalar doublets are responsible for the breaking of the gauge symmetry, one does not know whether the breaking is generated by one, two or more scalar doublets. This question is specially relevant in the beginning of the LHC era, with the prospects of experimentally probing the mechanism of gauge symmetry breaking.

In the Standard Model (SM) there is only one Higgs doublet and, as a result, scalar couplings are automatically flavour diagonal in the quark mass eigenstate basis. In multi-Higgs models, this is no longer true and one is confronted with dangerous flavour changing neutral currents (FCNC) at tree level, unless one introduces a symmetry leading to natural flavour conservation (NFC) [1], [2], [3], in the Higgs sector or some alternative mechanism to naturally suppress FCNC.

A possible alternative scenario for suppressing FCNC is through the assumption that all flavour violating neutral couplings be proportional [4], [5] to small entries of the Cabibbo-Kobayashi-Maskawa matrix (V_{CKM}) [6], [7]. This is one of the ingredients of the Minimal Flavour Violation (MFV) principle, introduced for the quark sector in Refs. [8], [9] and later on extended to the leptonic sector [10], [11], [12]. The first models of the MFV type, in the framework of two-Higgs doublets and without ad hoc assumptions, were proposed by Branco, Grimus and Lavoura (BGL) [13]. In the BGL models, the MFV character results from an exact discrete symmetry of the Lagrangian, spontaneously broken by the vacuum. Another proposal for the structure of the scalar couplings to fermions is the suggestion that the two Yukawa matrices are aligned in flavour space [14].

Recently, we have proposed an extension of the hypothesis of MFV to general multi-Higgs models with special emphasis on two Higgs doublets [15]. In that work there is a detailed analysis of the conditions for the neutral Higgs couplings to be only functions of V_{CKM} elements as well as a MFV expansion for the neutral Higgs couplings to fermions. This expansion is built by combining the most basic elements [16] that transform appropriately under weak basis transformations, with terms proportional to the fermion mass matrices.

In this paper, we study how our analysis can be extended to the leptonic sector, considering both the case where neutrinos are Dirac particles and the case where neutrinos are Majorana particles, acquiring naturally small masses through the seesaw mechanism. Note that this extension to the leptonic sector is crucial in order to study stability under renormalization as well as to do a full phenomenological analysis of BGL type models.

This paper is organized as follows: In section 2 we extend the BGL model to the leptonic sector with the imposition of lepton number conservation. Therefore, in this case, neutrinos are Dirac particles. Furthermore, we show that in this case the one loop renormalization equations for the corresponding Yukawa couplings, both in the quark and leptonic sector, obey the equations that guarantee the dependence of Higgs FCNC solely on functions of the mixing matices. In this section we also deal with the question of the uniqueness of BGL models. In section 3 we extend the BGL model to the leptonic sector without the imposition of lepton number conservation. We start by discussing the effective low energy scenario with Majorana neutrinos, and its stability. Next, we analyse the leptonic sector in the seesaw framework taking into consideration both the low and high energy couplings to the Higgs fields involving neutrinos. In section 4 we argue in favour of having the symmetry leading to MFV softly broken in the scalar potential. Finally, in section 5 we present our Conclusions.

2 Minimal Flavour Violation with Dirac Neutrinos

2.1 Framework

The extension of BGL models to the leptonic sector depends on the neutrino character. In this section, we analyse the leptonic sector of models that account for neutrino masses by enlarging the Standard Model (SM) through the introduction of three righthanded neutrinos ν_R^0 while at the same time imposing total lepton number conservation. As a result, only Dirac mass terms are generated and neutrinos are Dirac particles. We consider models with two Higgs doublets such that the flavour changing neutral currents are controlled by the V_{CKM} matrix in the quark sector and by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [17], [18], [19] in the leptonic sector.

The full Yukawa couplings are given by:

$$
\begin{aligned}
\mathcal{L}_Y &= -\overline{Q_L^0}\ \Gamma_1 \Phi_1 d_R^0 - \overline{Q_L^0}\ \Gamma_2 \Phi_2 d_R^0 - \overline{Q_L^0}\ \Delta_1 \tilde{\Phi}_1 u_R^0 - \overline{Q_L^0}\ \Delta_2 \tilde{\Phi}_2 u_R^0 \\
&\quad -\overline{L_L^0}\ \Pi_1 \Phi_1 l_R^0 - \overline{L_L^0}\ \Pi_2 \Phi_2 l_R^0 - \overline{L_L^0}\ \Sigma_1 \tilde{\Phi}_1 \nu_R^0 - \overline{L_L^0}\ \Sigma_2 \tilde{\Phi}_2 \nu_R^0 + \text{h.c.}\ , \qquad (1)
\end{aligned}
$$

where Γ_i, Δ_i denote the Yukawa couplings of the lefthanded quark doublets Q_L^0 to the righthanded quarks d_R^0, u_R^0 and to the Higgs doublets Φ_i; Π_i, Σ_i denote the couplings of the lefthanded leptonic doublets L_L^0 to the righthanded charged leptons l_R^0, neutrinos ν_R^0 and to the Higgs doublets. Lepton number conservation prevents the existence of invariant mass terms of Majorana type for righthanded neutrinos. These will appear in section 3 in the seesaw framework, where the requirement of lepton number conservation will be relaxed.

In order to obtain a structure for Γ_i, Δ_i such that there are FCNC at tree level with strength completely controlled by V_{CKM}, Branco, Grimus and Lavoura imposed the following symmetry on the quark and scalar sector of the Lagrangian [13]:

$$
Q_{Lj}^0 \to \exp{(i\alpha)}\ Q_{Lj}^0\ , \qquad u_{Rj}^0 \to \exp{(i2\alpha)} u_{Rj}^0\ , \qquad \Phi_2 \to \exp{(i\alpha)} \Phi_2\ , \quad (2)
$$

where $\alpha \neq 0, \pi$, with all other quark fields transforming trivially under the symmetry. The index j can be fixed as either 1, 2 or 3. Alternatively the symmetry may be chosen as:

$$
Q_{Lj}^0 \to \exp{(i\alpha)}\ Q_{Lj}^0\ , \qquad d_{Rj}^0 \to \exp{(i2\alpha)} d_{Rj}^0\ , \qquad \Phi_2 \to \exp{(-i\alpha)} \Phi_2\ . \quad (3)
$$

The symmetry given by Eq. (2) leads to Higgs FCNC in the down sector, whereas the symmetry specified by Eq. (3) leads to Higgs FCNC in the up sector. The neutral Higgs interactions with the fermions, obtained from the quark sector of Eq. (1) are given by

$$
\begin{aligned}
\mathcal{L}_Y(\text{neutral, quark}) &= -\overline{d_L^0} \frac{1}{v} \left[M_d H^0 + N_d^0 R + i N_d^0 I \right] d_R^0 + \\
&\quad - \overline{u_L^0} \frac{1}{v} \left[M_u H^0 + N_u^0 R + i N_u^0 I \right] u_R^0 + \text{h.c.}\ , \qquad (4)
\end{aligned}
$$

where $v \equiv \sqrt{v_1^2 + v_2^2} = (\sqrt{2} G_F)^{-1/2} \approx 246$ GeV, G_F is the Fermi coupling constant and H^0, R are orthogonal combinations of the fields ρ_j, arising when one expands [20] the neutral scalar fields around their vacuum expectation values (vevs), $\phi_j^0 = \frac{e^{i\theta_j}}{\sqrt{2}}(v_j + \rho_j + i\eta_j)$, choosing H^0 in such a way that it has couplings to the quarks which are proportional to the mass matrices, as can be seen from Eq. (4). Similarly, I denotes the linear combination of η_j orthogonal to the neutral Goldstone boson. The mass matrices M_d and M_u and the matrices N_d^0 and N_u^0 are given by:

$$M_d = \frac{1}{\sqrt{2}}(v_1\Gamma_1 + v_2 e^{i\theta}\Gamma_2) \ , \qquad M_u = \frac{1}{\sqrt{2}}(v_1\Delta_1 + v_2 e^{-i\theta}\Delta_2) \ , \tag{5}$$

$$N_d^0 = \frac{v_2}{\sqrt{2}}\Gamma_1 - \frac{v_1}{\sqrt{2}}e^{i\theta}\Gamma_2 \ , \qquad N_u^0 = \frac{v_2}{\sqrt{2}}\Delta_1 - \frac{v_1}{\sqrt{2}}e^{-i\theta}\Delta_2 \ , \tag{6}$$

here θ denotes the relative phase of the vevs of the neutral components of Φ_i. The matrices M_d, M_u are diagonalized by the usual bi-unitary transformations:

$$U_{dL}^\dagger M_d U_{dR} = D_d \equiv \text{diag } (m_d, m_s, m_b) \ , \tag{7}$$

$$U_{uL}^\dagger M_u U_{uR} = D_u \equiv \text{diag } (m_u, m_c, m_t) \ . \tag{8}$$

The flavour changing neutral currents are controlled by the matrices N_d and N_u related to N_d^0 and N_u^0 by the following transformations:

$$N_d = U_{dL}^\dagger N_d^0 U_{dR} \ , \qquad N_u = U_{uL}^\dagger N_u^0 U_{uR} \ . \tag{9}$$

In the case of the symmetry given by Eq. (2), for $j = 3$ there are FCNC in the down sector controlled by the matrix N_d given by [13]

$$(N_d)_{ij} \equiv \frac{v_2}{v_1}(D_d)_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)(V_{CKM}^\dagger)_{i3}(V_{CKM})_{3j}(D_d)_{jj} \ . \tag{10}$$

whereas, there are no FCNC in the up sector and the coupling matrix of the up quarks to the R and I fields is of the form:

$$N_u = -\frac{v_1}{v_2}\text{diag } (0, 0, m_t) + \frac{v_2}{v_1}\text{diag } (m_u, m_c, 0) \ . \tag{11}$$

In this example, the Higgs mediated FCNC are suppressed by the third row of the V_{CKM} matrix, therefore obeying to the additional constraint imposed on models designated as of the MFV type [21].

As shown in reference [15], the symmetries given by Eq. (2) or by Eq. (3) lead to

$$\mathcal{P}_j^\gamma \Gamma_2 = \Gamma_2 \ , \qquad \mathcal{P}_j^\gamma \Gamma_1 = 0 \ , \tag{12}$$

$$\mathcal{P}_j^\gamma \Delta_2 = \Delta_2 \ , \qquad \mathcal{P}_j^\gamma \Delta_1 = 0 \ , \tag{13}$$

where γ stands for u (up) or d (down) quarks, and $\mathcal{P}_j^\gamma$ are the projection operators defined [16] by

$$\mathcal{P}_j^u = U_{uL} P_j U_{uL}^\dagger \ , \qquad \mathcal{P}_j^d = U_{dL} P_j U_{dL}^\dagger \ , \tag{14}$$

and $(P_j)_{lk} = \delta_{jl}\delta_{jk}$. Note that Eqs (12) and (13), guarantee that the Higgs flavour changing neutral couplings can be written in terms of quark masses and V_{CKM} entries [15]. This is a crucial feature for BGL models to be considered as of Minimal Flavour Violation type (MFV).

In the leptonic sector, with Dirac type neutrinos, there is perfect analogy with the quark sector, consequently MFV is enforced by one of the following symmetries. Either

$$L_{Lk}^0 \to \exp{(i\alpha)}\ L_{Lk}^0 \ , \qquad \nu_{Rk}^0 \to \exp{(i2\alpha)}\nu_{Rk}^0 \ , \qquad \Phi_2 \to \exp{(i\alpha)}\Phi_2 \ , \tag{15}$$

or

$$L_{Lk}^0 \to \exp{(i\alpha)}\ L_{Lk}^0 \ , \qquad l_{Rk}^0 \to \exp{(i2\alpha)} l_{Rk}^0 \ , \qquad \Phi_2 \to \exp{(-i\alpha)}\Phi_2 \ , \tag{16}$$

where, once again, $\alpha \neq 0, \pi$, with all other leptonic fields transforming trivially under the symmetry. The index k can be fixed as either 1, 2 or 3.

Similarly, for the leptonic sector, these symmetries imply

$$\mathcal{P}_k^\beta \Pi_2 = \Pi_2 \ , \qquad \mathcal{P}_k^\beta \Pi_1 = 0 \ , \tag{17}$$

$$\mathcal{P}_k^\beta \Sigma_2 = \Sigma_2 \ , \qquad \mathcal{P}_k^\beta \Sigma_1 = 0 \ , \tag{18}$$

where β stands for charged lepton or neutrino. In this case

$$\mathcal{P}_k^l = U_{lL} P_k U_{lL}^\dagger \ , \qquad \mathcal{P}_k^\nu = U_{\nu L} P_k U_{\nu L}^\dagger \ , \tag{19}$$

where $U_{\nu L}$ and U_{lL} are the unitary matrices that diagonalize the corresponding square mass matrices

$$\begin{aligned} U_{lL}^\dagger M_l M_l^\dagger U_{lL} &= \text{diag}\left(m_e^2, m_\mu^2, m_\tau^2\right) \ , \\ U_{\nu L}^\dagger M_\nu M_\nu^\dagger U_{\nu L} &= \text{diag}\left(m_{\nu_1}^2, m_{\nu 2}^2, m_{\nu 3}^2\right) \ , \end{aligned} \tag{20}$$

with M_l and M_ν of the form

$$M_l = \frac{1}{\sqrt{2}}(v_1\Pi_1 + v_2 e^{i\theta}\Pi_2) \ , \quad M_\nu = \frac{1}{\sqrt{2}}(v_1\Sigma_1 + v_2 e^{-i\theta}\Sigma_2) \ . \qquad (21)$$

2.2 Renormalization Group Study

Equations (12) and (13) together with Eqs. (17) and (18) guarantee that the Higgs FCNC are functions of fermion masses and of the CKM and PMNS matrices. Therefore, it is crucial to guarantee the stability of these equations under renormalization.

The one loop renormalization group equations (RGE) for our Yukawa couplings can be generalized from reference [22] to

$$\begin{aligned} \mathcal{D}\Gamma_k &= a_\Gamma \Gamma_k + \\ &+ \sum_{l=1}^{2}\left[3\mathrm{Tr}\Big(\Gamma_k\Gamma_l^\dagger + \Delta_k^\dagger\Delta_l\Big) + \mathrm{Tr}\Big(\Pi_k\Pi_l^\dagger + \Sigma_k^\dagger\Sigma_l\Big)\right]\Gamma_l + \\ &+ \sum_{l=1}^{2}\left(-2\Delta_l\Delta_k^\dagger\Gamma_l + \Gamma_k\Gamma_l^\dagger\Gamma_l + \frac{1}{2}\Delta_l\Delta_l^\dagger\Gamma_k + \frac{1}{2}\Gamma_l\Gamma_l^\dagger\Gamma_k\right) \ , \end{aligned} \qquad (22)$$

$$\begin{aligned} \mathcal{D}\Delta_k &= a_\Delta \Delta_k + \\ &+ \sum_{l=1}^{2}\left[3\mathrm{Tr}\Big(\Delta_k\Delta_l^\dagger + \Gamma_k^\dagger\Gamma_l\Big) + \mathrm{Tr}\Big(\Sigma_k\Sigma_l^\dagger + \Pi_k^\dagger\Pi_l\Big)\right]\Delta_l + \\ &+ \sum_{l=1}^{2}\left(-2\Gamma_l\Gamma_k^\dagger\Delta_l + \Delta_k\Delta_l^\dagger\Delta_l + \frac{1}{2}\Gamma_l\Gamma_l^\dagger\Delta_k + \frac{1}{2}\Delta_l\Delta_l^\dagger\Delta_k\right) \ , \end{aligned} \qquad (23)$$

$$\begin{aligned} \mathcal{D}\Pi_k &= a_\Pi \Pi_k + \\ &+ \sum_{l=1}^{2}\left[3\mathrm{Tr}\Big(\Gamma_k\Gamma_l^\dagger + \Delta_k^\dagger\Delta_l\Big) + \mathrm{Tr}\Big(\Pi_k\Pi_l^\dagger + \Sigma_k^\dagger\Sigma_l\Big)\right]\Pi_l + \\ &+ \sum_{l=1}^{2}\left(-2\Sigma_l\Sigma_k^\dagger\Pi_l + \Pi_k\Pi_l^\dagger\Pi_l + \frac{1}{2}\Sigma_l\Sigma_l^\dagger\Pi_k + \frac{1}{2}\Pi_l\Pi_l^\dagger\Pi_k\right) \ , \end{aligned} \qquad (24)$$

$$
\begin{aligned}
\mathcal{D}\Sigma_k &= a_\Sigma \Sigma_k + \\
&+ \sum_{l=1}^{2} \left[3\mathrm{Tr}\Big(\Delta_k \Delta_l^\dagger + \Gamma_k^\dagger \Gamma_l\Big) + \mathrm{Tr}\Big(\Sigma_k \Sigma_l^\dagger + \Pi_k^\dagger \Pi_l\Big)\right] \Sigma_l + \\
&+ \sum_{l=1}^{2} \left(-2\Pi_l \Pi_k^\dagger \Sigma_l + \Sigma_k \Sigma_l^\dagger \Sigma_l + \frac{1}{2}\Pi_l \Pi_l^\dagger \Sigma_k + \frac{1}{2}\Sigma_l \Sigma_l^\dagger \Sigma_k \right) \ , \quad (25)
\end{aligned}
$$

where $\mathcal{D} \equiv 16\pi^2 \mu \, (d/d\mu)$ and μ is the renormalization scale. The coefficients a_Γ, a_Δ, a_Π and a_Σ are given by [23]:

$$
\begin{aligned}
a_\Gamma &= -8g_s^2 - \frac{9}{4}g^2 - \frac{5}{12}g'^2 \ , \qquad a_\Delta = -8g_s^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2 \ . \\
a_\Pi &= -\frac{9}{4}g^2 - \frac{15}{4}g'^2 \ , \qquad a_\Sigma = -\frac{9}{4}g^2 - \frac{3}{4}g'^2 \ , \qquad (26)
\end{aligned}
$$

where g_s, g and g' are the gauge coupling constants of $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ respectively. To show that Eqs. (12), (13), (17) and (18) are stable under RGE one has to show that

$$
\begin{aligned}
\mathcal{P}_j^\gamma \left(\mathcal{D}\Gamma_2\right) &= \left(\mathcal{D}\Gamma_2\right) \ , \qquad \mathcal{P}_j^\gamma \left(\mathcal{D}\Gamma_1\right) = 0 \ , \qquad (27) \\
\mathcal{P}_j^\gamma \left(\mathcal{D}\Delta_2\right) &= \left(\mathcal{D}\Delta_2\right) \ , \qquad \mathcal{P}_j^\gamma \left(\mathcal{D}\Delta_1\right) = 0 \ , \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}_k^\beta \left(\mathcal{D}\Pi_2\right) &= \left(\mathcal{D}\Pi_2\right) \ , \qquad \mathcal{P}_k^\beta \left(\mathcal{D}\Pi_1\right) = 0 \ , \qquad (29) \\
\mathcal{P}_k^\beta \left(\mathcal{D}\Sigma_2\right) &= \left(\mathcal{D}\Sigma_2\right) \ , \qquad \mathcal{P}_k^\beta \left(\mathcal{D}\Sigma_1\right) = 0 \ , \qquad (30)
\end{aligned}
$$

which guarantee that the Yukawa couplings at each different scale still verify equations of the same form.

It is interesting to notice that if one does not use the conditions for the leptonic sector given by Eqs. (17) and (18) one is lead, for example, to:

$$
\mathcal{P}_j^\gamma \left(\mathcal{D}\Gamma_1\right) = \mathrm{Tr}\Big(\Pi_1 \Pi_2^\dagger + \Sigma_1^\dagger \Sigma_2\Big)\, \Gamma_2 \ , \qquad (31)
$$

also, one must use the equality:

$$
\mathrm{Tr}\Big(\Pi_1 \Pi_2^\dagger + \Sigma_1^\dagger \Sigma_2\Big) = 0 \qquad (32)
$$

in order to show that $\mathcal{P}_i^\alpha \left(\mathcal{D}\Gamma_2\right) = \left(\mathcal{D}\Gamma_2\right)$. Clearly, Eq. (32) follows from Eqs. (17) and (18), since in this case we have:

$$
\begin{aligned}
\mathrm{Tr}\Big(\Pi_1 \Pi_2^\dagger + \Sigma_1^\dagger \Sigma_2\Big) &= \mathrm{Tr}\Big(\Pi_1 \Pi_2^\dagger \mathcal{P}_j^\beta + \Sigma_1^\dagger \mathcal{P}_j^\beta \Sigma_2\Big) = \\
&= \mathrm{Tr}\Big(\Big(\mathcal{P}_j^\beta \Pi_1\Big)\, \Pi_2^\dagger + \Big(\mathcal{P}_j^\beta \Sigma_1\Big)^\dagger \Sigma_2\Big) = 0 \ , \qquad (33)
\end{aligned}
$$

so that Eq. (32) is enforced by the MFV leptonic conditions. It is the entire set of equations both in the quark and in the leptonic sector that guarantee the stability of these models. This fact should not come as a surprise since the relations given by Eqs. (12), (13), (17) and (18) follow from the imposition of a symmetry on the full Lagrangian.

In the quark sector there were six possible different implementations of BGL type models. Three with FCNC in the down sector, each one corresponding to a different choice for the index j, and three with FCNC in the up sector also for the different choices of j. In the extension to the leptonic sector with Dirac neutrinos one has another set of six different leptonic implementations obtained in a similar fashion. In total, one may consider thirty six different MFV models of BGL type in the case of Dirac neutrinos.

In Ref. [15] we presented a MFV expansion for N_d^0 and N_u^0 built with terms proportional to M_d and M_u respectively, as well as products of terms which transform like H_d and H_u under weak basis transformations, multiplying M_d and M_u. We identified $\mathcal{P}_j^d$ and $\mathcal{P}_k^u$ as the simplest such elements with the appropriate transformation under changes of weak basis. In fact, H_d and H_u can be decomposed as [16]:

$$H_{d(u)} = \sum_i m^2_{d(u)_i} P_i^{d(u)} \ . \tag{34}$$

As a result we obtained, for example, simple models of MFV type with Higgs mediated FCNC in both sectors, like the one given by the following equations:

$$N_d^0 = \frac{v_2}{v_1}\ M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)\ U_{uL} P_i U_{uL}^\dagger\ M_d \ , \tag{35}$$

$$N_u^0 = \frac{v_2}{v_1}\ M_u - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)\ U_{dL} P_i U_{dL}^\dagger\ M_u \ . \tag{36}$$

Several different possible variations beyond BGL models were considered in Ref. [15], obtained from different combinations of $\mathcal{P}_j^d$ and $\mathcal{P}_k^u$ with M_d and M_u. We pointed out in Ref. [15] that the zero texture structure of these models is more involved than in the BGL case and that the question of assuring its loop stability, through the introduction of symmetries, was not obvious. We can now address this question with the help of the renormalization group equations for the Yukawa couplings given by Eqs. (22), (23),

(24) and (25). All these additional MFV models, as well as the BGL models, lead to relations of the following form:

$$\mathcal{P}_i^\alpha \Gamma_2 = \Gamma_2 \,, \qquad \mathcal{P}_i^\alpha \Gamma_1 = 0 \,, \tag{37}$$

$$\mathcal{P}_j^\beta \Delta_2 = \Delta_2 \,, \qquad \mathcal{P}_j^\beta \Delta_1 = 0 \,, \tag{38}$$

and similar equations for the leptonic sector. For $\alpha = \beta$ and $i = j$ we are in a BGL model in the quark sector. Models with $\alpha \neq \beta$ or $i \neq j$ correspond to additional cases presented in Ref. [15].

It can be readily verified that, in general

$$\mathcal{P}_i^\alpha \left(\mathcal{D}\Gamma_1\right) = -\frac{3}{2}\mathcal{P}_i^\alpha \Delta_1 \Delta_1^\dagger \Gamma_1 - 2\mathcal{P}_i^\alpha \mathcal{P}_j^\beta \Delta_2 \Delta_1^\dagger \mathcal{P}_i^\alpha \Gamma_2 + \frac{1}{2}\mathcal{P}_i^\alpha \mathcal{P}_j^\beta \Delta_2 \Delta_2^\dagger \mathcal{P}_j^\beta \Gamma_1 \,. \tag{39}$$

We have already shown that for BGL models we have

$$\mathcal{P}_i^\alpha \left(\mathcal{D}\Gamma_1\right) = 0 \,. \tag{40}$$

In the case $\alpha = \beta$, $i \neq j$ we have

$$\mathcal{P}_i^\alpha \mathcal{P}_j^\beta = 0 \,, \tag{41}$$

due to the fact that these are projection operators. So we are left with

$$\mathcal{P}_i^\alpha \left(\mathcal{D}\Gamma_1\right) = -\frac{3}{2}\mathcal{P}_i^\alpha \Delta_1 \Delta_1^\dagger \Gamma_1 \,, \tag{42}$$

which, in general, is different from zero. Therefore we conclude that this type of models cannot be enforced by symmetries. The consideration of equation $\mathcal{P}_i^\alpha \left(\mathcal{D}\Gamma_2\right) = \left(\mathcal{D}\Gamma_2\right)$ would lead to similar difficulties and therefore, would allow us to draw a similar conclusion. As a result we may conclude that out of the models described by Eqs. (37) and (38) and their generalization to the leptonic sector, only BGL type models can be enforced by some symmetry. The same question was recently addressed in Ref. [24] following a different approach. There it was shown that BGL models are the only ones that survive among a large set of models enforced by abelian symmetries.

3 Minimal Flavour Violation with Majorana Neutrinos

3.1 Low Energy Effective Theory and Stability

In the previous section, we assume that neutrinos are Dirac particles. An alternative possibility is to allow for lepton nonconservation leading to an effective Majorana mass term for the three light neutrinos of the form

$$\mathcal{L}_{\text{Majorana}} = \frac{1}{2}\nu_L^{0\,T} C^{-1} m_\nu \nu_L^0 + \text{h.c.} \ , \tag{43}$$

which violates lepton number. Such a mass term is generated after spontaneous gauge symmetry breaking from an effective dimension five operator $\mathcal{O}$ which, in the two Higgs doublet model can be written as:

$$\mathcal{O} = \sum_{i,j=1}^{2} \sum_{\alpha,\beta=e,\mu,\tau} \sum_{a,b,c,d=1}^{2} \left(L_{\text{L}\alpha a}^T \kappa_{\alpha\beta}^{(ij)} C^{-1} L_{\text{L}\beta c} \right) \left(\varepsilon^{ab} \phi_{ib} \right) \left(\varepsilon^{cd} \phi_{jd} \right) . \tag{44}$$

This operator contains two lefthanded lepton doublets and two Higgs doublets and can be viewed, for example, as arising from the seesaw mechanism after integrating out the heavy degrees of freedom. In the seesaw context the heavy degrees of freedom are the righthanded neutrinos. The seesaw framework will be analysed in the next subsection.

In this context we have, in the leptonic sector, the two flavour structures introduced before:

$$\mathcal{L}_{Y_l} = -\overline{L_L^0}\ \Pi_1 \Phi_1 l_R^0 - \overline{L_L^0}\ \Pi_2 \Phi_2 l_R^0 + \text{h.c.} \ , \tag{45}$$

together with the four new flavour structures given by the $\kappa^{(ij)}$ matrices. A priori, it looks more difficult to implement MFV in the case of Majorana neutrinos. However, this can be done by imposing the following Z_4 symmetry in the effective Lagrangian including the terms given by Eqs. (44) and (45):

$$L_{Lj}^0 \to \exp{(i\alpha)}\ L_{Lj}^0 \ , \qquad \Phi_2 \to \exp{(i\alpha)} \Phi_2 \ , \tag{46}$$

with $\alpha = \pi/2$. Imposing this Z_4 symmetry implies:

$$\kappa^{(12)} = \kappa^{(21)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \ , \tag{47}$$

and taking for definiteness $j = 3$ we get

$$\kappa^{(11)} = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{bmatrix} , \qquad \kappa^{(22)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix} , \tag{48}$$

fixing the angle α as $\pi/2$ ensures that $\kappa^{(22)}_{33} \neq 0$ so that the determinant of the resulting neutrino mass matrix does not vanish automatically. The Majorana mass matrix for the neutrinos is given by:

$$\frac{1}{2} m_\nu = \frac{1}{2} v_1^2 \kappa^{(11)} + \frac{1}{2} v_2^2 e^{2i\theta} \kappa^{(22)} \ . \tag{49}$$

This Z_4 symmetry also implies the following structure for Π_1 and Π_2:

$$\Pi_1 = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix} , \qquad \Pi_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix} . \tag{50}$$

The neutrino mass matrix m_ν is block diagonal with each block given by a different κ matrix. As a consequence, in the diagonalization of m_ν, the matrices $\kappa^{(11)}$ and $\kappa^{(22)}$ are diagonalized separately. Therefore, any linear combination of these two matrices will be simultaneously diagonalized. As a result the lepton number violating Weinberg operator [25] of Eq. (44) does not give rise to Higgs mediated FCNC in the neutrino sector. For the charged lepton sector the situation is similar to the one encountered in the previous section for the symmetry given by Eq. (15), leading to Higgs mediated FCNC in this sector.

The symmetry imposed by Eq. (46) in the effective low energy theory leads, for $j = 3$ for instance, to the following conditions:

$$\begin{aligned} \kappa^{(12)} = \kappa^{(21)} = 0 \ , \qquad \kappa^{(11)} \mathcal{P}^\nu_3 = 0 \ , \qquad \kappa^{(22)} \mathcal{P}^\nu_3 = \kappa^{(22)} \ , \\ \mathcal{P}^\nu_3 \Pi_1 = 0 \ , \qquad \mathcal{P}^\nu_3 \Pi_2 = \Pi_2 \ . \end{aligned} \tag{51}$$

It can be easily verified, as we have done in section 2.2, and following the RGE presented in Ref. [26] that these equations are indeed stable under renormalization, since they keep the same form at all scales.

3.2 Seesaw Framework

In this section, we analyse the leptonic sector in the seesaw framework [27], [28], [29], [30], [31]. We include one righthanded neutrino per generation and do not impose lepton number conservation. The leptonic part of Yukawa couplings and invariant mass terms can then be written:

$$\begin{aligned} \mathcal{L}_{Y+\text{mass}} &= -\overline{L_L^0}\ \Pi_1\Phi_1 l_R^0 - \overline{L_L^0}\ \Pi_2\Phi_2 l_R^0 - \overline{L_L^0}\ \Sigma_1\tilde{\Phi_1}\nu_R^0 - \overline{L_L^0}\ \Sigma_2\tilde{\Phi_2}\nu_R^0\ + \\ &+ \frac{1}{2}{\nu_R^0}^T C^{-1} M_R \nu_R^0 + \text{h.c.}\ . \end{aligned} \tag{52}$$

The matrix M_R stands for the righthanded neutrino Majorana mass matrix. The leptonic mass matrices generated after spontaneous gauge symmetry breaking are given by:

$$m_l = \frac{1}{\sqrt{2}}(v_1\Pi_1 + v_2 e^{i\theta}\Pi_2)\ , \quad m_D = \frac{1}{\sqrt{2}}(v_1\Sigma_1 + v_2 e^{-i\theta}\Sigma_2)\ . \tag{53}$$

Note that the notation has changed from the one in section 2, we now have $m_l \equiv M_l$ and m_D replaces M_ν in order to avoid confusion with light neutrino masses in the seesaw framework. The leptonic mass terms obtained from Eq. (52) can be written as:

$$\mathcal{L}_{\text{mass}} = -\overline{l_L^0}\, m_l\, l_R^0 + \frac{1}{2}(\nu_L^{0T}, (\nu_R^0)^{cT})\, C^{-1}\mathcal{M}^* \begin{pmatrix} \nu_L^0 \\ (\nu_R^0)^c \end{pmatrix} + \text{h.c.}\ , \tag{54}$$

with

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}\ . \tag{55}$$

We use the following convention:

$$(\psi_L)^c \equiv C\gamma_0^T(\psi_L)^*\ . \tag{56}$$

The charged current couplings are given by:

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} W_\mu^+\ \overline{l_L^0}\ \gamma^\mu\ \nu_L^0 + \text{h.c.}\ . \tag{57}$$

The neutral Higgs interactions with the fermions, obtained from Eq. (52) can be written:

$$\begin{aligned} \mathcal{L}_Y(\text{neutral, lepton}) &= -\overline{l_L^0}\frac{1}{v}\,[m_l H^0 + N_l^0 R + iN_l^0 I]\, l_R^0 + \\ &- \overline{\nu_L^0}\frac{1}{v}\,[m_D H^0 + N_\nu^0 R + iN_\nu^0 I]\, \nu_R^0 + \text{h.c.}\ , \end{aligned} \tag{58}$$

with

$$N_l^0 = \frac{v_2}{\sqrt{2}}\Pi_1 - \frac{v_1}{\sqrt{2}}e^{i\theta}\Pi_2 \ , \tag{59}$$

$$N_\nu^0 = \frac{v_2}{\sqrt{2}}\Sigma_1 - \frac{v_1}{\sqrt{2}}e^{-i\theta}\Sigma_2 \ . \tag{60}$$

There is a new feature in the seesaw framework due to the fact that in the neutrino sector the light neutrino masses are not obtained from the diagonalization of m_D.

The 6×6 neutrino mass matrix $\mathcal{M}$ is diagonalized by the transformation:

$$V^T \mathcal{M}^* V = \mathscr{D} \ , \tag{61}$$

where $\mathscr{D} = \mathrm{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}, M_1, M_2, M_3)$, with $m_{\nu i}$ and M_i denoting the masses of the physical light and heavy Majorana neutrinos, respectively. It is convenient to write the matrices V and $\mathscr{D}$ in the following block form:

$$V = \begin{pmatrix} K & G \\ S & T \end{pmatrix} , \qquad \mathscr{D} = \begin{pmatrix} d & 0 \\ 0 & D \end{pmatrix} . \tag{62}$$

In the seesaw framework, with the scale of $M_R \gg v$ the matrix K coincides to an excellent approximation with the unitary matrix U that diagonalizes the effective mass matrix m_{eff} for the light neutrinos:

$$U^\dagger \ m_{eff} \ U^* = d \qquad \text{with} \qquad m_{eff} \equiv - \ m_D \frac{1}{M_R} m_D^T \ . \tag{63}$$

The matrix G verifies the exact relation [32]:

$$G = m_D T^* D^{-1} \ , \tag{64}$$

while S is given to an excellent approximation by [32]:

$$S^\dagger = -K^\dagger m_D M_R^{-1} \ , \tag{65}$$

It is clear from Eqs. (64) and (65) that G and S are of order m_D/M_R, therefore strongly suppressed. This in turn means that the 3×3 matrices K and T are unitary to an excellent approximation. The matrix T is also very approximately determined by:

$$T^\dagger M_R T^* = D \ . \tag{66}$$

The physical fermion fields l, ν and N are then related to the weak basis fields by:

$$l_L^0 = U_{lL} l_L \,, \quad l_R^0 = U_{lR} l_R \,, \quad \nu_L^0 = U \nu_L + G N_L \,, \quad \nu_R^0 = S^* \nu_L^c + T^* N_L^c \,. \quad (67)$$

In terms of physical fields the charged gauge current interactions become

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} (\overline{l_L} \gamma_\mu U_\nu \nu_L W^\mu + \overline{l_L} \gamma_\mu Q N_L W^\mu) + \text{h.c.} \,. \quad (68)$$

U_ν denotes the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix defined by the product $(U_{lL}^\dagger U)$. The second term of $\mathcal{L}_W$ in Eq. (68), with mixing given by $Q \equiv (U_{lL}^\dagger G)$ is suppressed by G and involves the heavy neutrinos N which are not relevant for low energy physics.

In general the couplings of Eq. (58) lead to arbitrary scalar FCNC at tree level. In order for these couplings to be completely controlled by the PMNS matrix we introduce the following Z_4 symmetry on the Lagrangian:

$$L_{L3}^0 \to \exp(i\alpha) \; L_{L3}^0 \,, \qquad \nu_{R3}^0 \to \exp(i2\alpha) \nu_{R3}^0 \,, \qquad \Phi_2 \to \exp(i\alpha) \Phi_2 \,, \quad (69)$$

with $\alpha = \pi/2$ and all other fields transforming trivially under Z_4. The most general matrices Π_i, Σ_i and M_R consistent with this Z_4 symmetry have the following structure:

$$\Pi_1 = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix} , \qquad \Pi_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix} , \quad (70)$$

$$\Sigma_1 = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{bmatrix} , \qquad \Sigma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix} , \qquad M_R = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{bmatrix} , \quad (71)$$

where $\times$ denotes an arbitrary entry while the zeros are imposed by the symmetry Z_4. Note that the choice of Z_4 is crucial in order to guarantee $M_{33} \neq 0$ and thus a non-vanishing $\det M_R$. The same choice was required in the previous subsection in order to allow for a non-vanishing determinant for the effective Majorana neutrino mass matrix. In this weak basis the following important relations are verified:

$$P_3 \Pi_2 = \Pi_2 \,, \qquad P_3 \Pi_1 = 0 \,, \qquad \text{with} \qquad P_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} , \quad (72)$$

as well as

$$P_3 \Sigma_2 = \Sigma_2 \,, \qquad P_3 \Sigma_1 = 0 \,. \quad (73)$$

3.3 Full Seesaw Higgs couplings

Let us now write the neutral scalar couplings of the charged leptons in the mass eigenstate basis:

$$\mathcal{L}_Y^l(\text{neutral}) = -\frac{H^0}{v}\,\bar{l}\,D_l\,l - \frac{R}{v}\,\bar{l}\,(N_l\gamma_R + N_l^\dagger\gamma_L))\,l + i\frac{I}{v}\,\bar{l}\,(N_l\gamma_R - N_l^\dagger\gamma_L))\,l \ , \quad (74)$$

where $\gamma_L = (1-\gamma_5)/2$, $\gamma_R = (1+\gamma_5)/2$ and $N_l \equiv U_{lL}^\dagger\ N_l^0\ U_{lR}$.

The fact that U given by Eq. (63) is block diagonal with no mixing for the third family leads to:

$$(N_l)_{ij} \equiv (U_{lL}^\dagger\ N_l^0\ U_{lR})_{ij} = \frac{v_2}{v_1}(D_l)_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)(U_\nu^\dagger)_{i3}(U_\nu)_{3j}(D_l)_{jj} \ . \quad (75)$$

U_ν is the PMNS matrix.

We obtain the neutrino couplings to neutral scalars from the last term of Eq. (58). It is useful to rewrite N_ν^0 in the form:

$$N_\nu^0 = \frac{v_2}{v_1}m_D - \frac{v_2}{\sqrt{2}}\left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)e^{-i\theta}\Sigma_2 \ . \quad (76)$$

Notice that the first term of N_ν^0 is proportional to m_D and therefore these couplings to the fields R and I have a structure similar to the H^0 couplings in Eq. (58). The couplings of the neutrino mass eigenstates ν_i (light), N_i (heavy) to the neutral scalars H^0, R and I are more involved than the couplings of the charged leptons, since they include light-light, light-heavy and heavy-heavy couplings. In the sequel, we shall consider each one of these terms, displaying their explicit form in the present model.

H^0 couplings

(i) light-light couplings.
These couplings can be written

$$\mathcal{L}_{\nu\nu}^{H^0} = \frac{A_{ij}}{v}\ \overline{\nu_{iL}}H^0\nu_{jL}^c + \text{h.c.} \ , \quad (77)$$

where

$$A = U^\dagger m_D S^* = d \ . \quad (78)$$

These couplings among light neutrinos are flavour diagonal and are proportional to the light neutrino masses. From the point of view of the effective low energy theory there are no scalar FCNC in the neutrino sector, since, as will be shown, the term of N_ν^0 in Σ_2 given by Eq. (76), corresponding to light-light couplings ($U^\dagger \Sigma_2 S^*$) will not generate nonzero off-diagonal entries. Its form is given explicitly, in the sequel, by Eq. (87).

(ii) light-heavy couplings.
We write these terms as

$$\mathcal{L}_{\nu N}^{H^0} = \frac{B_{ij}}{v}\ \overline{\nu_{Li}} H^0 N_{Lj}^c + \frac{E_{ij}}{v}\ \overline{N_{Li}} H^0 \nu_{Lj}^c + \text{h.c.}\ , \tag{79}$$

where

$$B = U^\dagger m_D T^*\ ,, \qquad E = G^\dagger m_D S^*\ . \tag{80}$$

From Eqs. (63) and (66) one can write

$$B = (i\sqrt{d}\, O^c \sqrt{D})\ , \tag{81}$$

where O^c is an orthogonal complex matrix. This expression readily follows from the Casas and Ibarra parametrization [33]. The fact that m_D as well as M_R are block diagonal implies that O^c is also block diagonal and can be parametrized as:

$$O^c = \begin{bmatrix} \cos Z & \pm\sin Z & 0 \\ -\sin Z & \pm\cos Z & 0 \\ 0 & 0 & 1 \end{bmatrix}\ , \tag{82}$$

with Z complex. These couplings, given by the matrix B, are not suppressed by the mixing matrices but the fact that the heavy neutrino fields, N, have masses of order M_R implies that they cannot be produced at low energies. Using Eqs. (64) and (65) it can be readily verified that:

$$E = -D^{-1}(i\sqrt{d}\, O^c \sqrt{D})^\dagger d\ , \tag{83}$$

These couplings, given by the matrix E, are suppressed by both matrices G and S therefore they are much smaller than those given by B, in addition, they also include a heavy neutrino.

(iii) heavy-heavy couplings.
One has for these couplings:

$$\mathcal{L}_{NN}^{H^0} = \frac{F_{ij}}{v}\ \overline{N_{Li}} H^0 N_{Lj}^c + \text{h.c.}\ , \tag{84}$$

where

$$F = G^\dagger m_D T^* \ , \tag{85}$$

it can be readily verified that:

$$F = D^{-1}(i\sqrt{d}\,O^c\sqrt{D})^\dagger(i\sqrt{d}O^c\sqrt{D}) \ . \tag{86}$$

These are couplings among heavy neutrinos and furthermore are suppressed by the mixing matrix G.

R and I couplings

Concerning the neutral couplings to R and I the first term of N_ν^0 given by Eq. (76) leads to currents with the same structure as those mediated by H^0. The second term of N_ν^0 leads to diagonal coupling matrices, due to the block structure of Σ_2 given by Eq. (71) and the fact that, as a result of the patterns given by Eq. (71) for the neutrino mass matrices, the matrices U, G, S and T are block diagonal with no mixing in the third row and column. The additional couplings to R and I are derived by replacing m_D by Σ_2 in A, B, E and F introduced by Eqs. (78), (80) and (85). From Eq. (78) and the definition of m_D given by Eq. (53) we obtain the following additional term for light-light couplings to R and I:

$$\frac{1}{\sqrt{2}} v_2 e^{-i\theta} U^\dagger \Sigma_2 S^* = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & d_3 \end{pmatrix} \ . \tag{87}$$

From Eqs. (80), (81) and (83) we obtain the following light-heavy coupling terms:

$$\frac{1}{\sqrt{2}} v_2 e^{-i\theta} U^\dagger \Sigma_2 T^* = i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sqrt{d_3}\sqrt{D_3} \end{pmatrix} \ , \tag{88}$$

$$\frac{1}{\sqrt{2}} v_2 e^{-i\theta} G^\dagger \Sigma_2 S^* = i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{d_3}{D_3}\sqrt{d_3}\sqrt{D_3} \end{pmatrix} \ . \tag{89}$$

Finally from Eqs. (85), (86) we obtain the following heavy-heavy coupling term:

$$\frac{1}{\sqrt{2}} v_2 e^{-i\theta} G^\dagger \Sigma_2 T^* = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & d_3 \end{pmatrix} \ . \tag{90}$$

Combining the two contributions from N_ν^0, the light-light couplings to R and I are diagonal. For the first two generations the coefficients are given by $\frac{v_2}{v_1}\frac{d_1}{v}$ and $\frac{v_2}{v_1}\frac{d_2}{v}$ respectively. For the third generation it is given by $-\frac{v_1}{v_2}\frac{d_3}{v}$.

The light-heavy couplings to R and I are block diagonal. Compared to the corresponding H^0 couplings the block (12) is multiplied by the ratio of vevs $\frac{v_2}{v_1}$ and the (33) coupling is multiplied by $-\frac{v_1}{v_2}$. Likewise for the heavy-heavy couplings to R and I.

From the point of view of low energy physics, the example given is a model of BGL type, with no Higgs mediated FCNC in the up sector (light neutrinos) and with the strength of the FCNC in the down sector controlled by the PMNS mixing matrix. All flavour changing neutral couplings with heavy neutrinos are parametrized by both light and heavy neutrino masses, and the product of matrices $i\sqrt{d}O^c\sqrt{D}$, with O^c of the form given by Eq. (82). Heavy neutrino decays may be the source of the baryon asymmetry of the universe through leptogenesis [34] with sphaleron processes [35], [36].

Next we write the Yukawa couplings to the charged Higgs, H^+ using Eqs. (87) and (90).

H^+ couplings

The charged Higgs interactions with the fermions, obtained from Eq. (52) are given by

$$\mathcal{L}_Y(\text{charged}) = \frac{\sqrt{2}H^+}{v}(\overline{\nu_L^0}N_l^0 l_R^0 + \overline{\nu_R^0}{N_\nu^0}^\dagger l_L^0) + \text{h.c.} \; . \tag{91}$$

In the fermion mass eigenstate basis these interactions become:

$$\begin{aligned}
\mathcal{L}_Y(\text{charged}) &= \frac{\sqrt{2}H^+}{v}[\overline{\nu_L}U_\nu^\dagger N_l l_R + \overline{N_L}Q^\dagger N_l l_R] + \\
&+ \frac{\sqrt{2}H^+}{v}\overline{\nu_L^c}\begin{pmatrix} \frac{v_2}{v_1}d_1 & 0 & 0 \\ 0 & \frac{v_2}{v_1}d_2 & 0 \\ 0 & 0 & -\frac{v_1}{v_2}d_3 \end{pmatrix} U_\nu^\dagger l_L + \\
&+ \frac{\sqrt{2}H^+}{v}\overline{N_L^c}\left(\frac{v_2}{v_1}F^\dagger - \left(\frac{v_2}{v_1}+\frac{v_1}{v_2}\right)\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & d_3 \end{pmatrix}\right) Q^\dagger l_L \; .
\end{aligned} \tag{92}$$

4 The scalar Potential

The Z_4 symmetry which we have imposed on the Lagrangian, forbids various gauge invariant terms in the scalar potential, such as $\phi_1^\dagger \phi_2$, $\phi_1^\dagger \phi_2 \phi_i^\dagger \phi_i$,, $\phi_1^\dagger \phi_2 \phi_1^\dagger \phi_2$. As a result, the Higgs potential has an exact ungauged accidental continuous symmetry, which is not a symmetry of the full Lagrangian. After spontaneous gauge symmetry breaking, the accidental symmetry leads to a pseudo-Goldstone boson. The simplest way of avoiding the pseudo-Goldstone boson is by breaking the Z_4 symmetry softly through the introduction of the term $m_{12}\phi_1^\dagger \phi_2 + h.c.$. This term avoids the pseudo-Goldstone boson which acquires a squared mass proportional to $|m_{12}|$. In order to discuss CP violation in this class of models, one has to consider separately the cases of explicit and spontaneous CP violation.

Explicit CP violation - If one does not impose CP invariance at the Lagrangian level, Yukawa couplings are complex. In spite of the special form of these couplings, due to the presence of the Z_4 symmetry, it can be readily checked that there is in general CP violation through the Kobayashi-Maskawa (KM) mechanism. The simplest way of verifying that this is the case, is by noting that $H_d \equiv M_d M_d^\dagger$ is a generic complex Hermitian matrix while H_u is a block diagonal matrix. One can easily compute $\mathrm{Tr}[H_u, H_d]^3$ and show that in general this weak-basis invariant does not vanish thus proving [37] that there is CP violation through the KM mechanism. In order to check whether there are in this model other sources of CP violation, one has to look at the scalar potential. It can be readily checked that the scalar potential, by itself, is CP invariant since the phase of m_{12} can be removed by rephasing the scalar doublets, thus rendering the potential real. The powerful Higgs-basis invariant CP-odd conditions derived in Ref. [38] would obviously provide the same answer, however this is a straightforward case. In this variant of the model, one has all CP violation arising from the KM mechanism. However, note that there are, for example, new contributions to $B_d - \bar{B}_d$ apart from the usual box diagrams of the Standard Model. These new contributions are mediated by tree level scalar interactions, which are proportional to $(V_{tb}V_{td}^*)^2$, therefore with the same phase as the SM box contribution.

Spontaneous CP violation - It can be readily checked that even in the presence of the soft breaking term $m_{12}\phi_1^\dagger \phi_2$, one cannot achieve spontaneous CP violation, without enlarging the scalar sector. On the other hand, one may obtain spontaneous CP violation by introducing scalar singlets. However in order for the phase arising from the vacuum to be able to generate a complex CKM matrix, one has to introduce vector-like quarks [39].

5 Conclusions

We have analysed how to extend to the leptonic sector, BGL models satisfying the minimal flavour violation (MFV) hypothesis. Both the cases of Dirac and Majorana neutrinos were considered. In the case of Dirac neutrinos the extension to the leptonic is straightforward with great similarity to the quark sector. We have shown that if type-I seesaw mechanism is adopted, the requirement of having a non-singular Majorana mass matrix for the righthanded neutrinos further restricts the choice of the discrete symmetry which allows for realistic BGL models in the leptonic sector. A striking result of our analysis is the fact that this restricted form of the symmetry is also required when considering the low energy effective theory with Majorana neutrinos. In particular, it was pointed out that BGL models satisfying the MFV paradigm can be extended in a natural and elegant way to the leptonic sector with Majorana neutrinos, through the introduction of a Z_4 symmetry, imposed on the full Lagrangian. Furthermore we derive the equations which guarantee calculability of Higgs FCNC in terms of masses, V_{CKM} and V_{PMNS} matrices showing that these equations are stable under renormalization. We have also analysed the scalar potential which acquires an exact ungauged accidental continuous symmetry arising from the absence of various terms forbidden by the Z_4 symmetry. We have pointed out that the simplest way of avoiding the resulting pseudo-Goldstone boson is through the addition of a quadratic term in the scalar potential, thus softly breaking the Z_4 symmetry. Finally, we emphasize that the relevance of BGL models stems in good part from the fact that the most general tree-level flavour violating neutral currents are naturally suppressed by small V_{CKM} elements like the combination $(V_{td}V_{ts}^*)$.

A full analysis of BGL models is beyond the scope of this paper and will be presented elsewhere [40]. It is clear that the extension of BGL models to the leptonic sector is essential in order to make possible the above analysis and furthermore, to allow for a consistent analysis of the renormalization group evolution.

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References

[1] S. Weinberg, Phys. Rev. Lett. **37** (1976) 657.

[2] S. L. Glashow and S. Weinberg, Phys. Rev. D **15** (1977) 1958.

[3] E. A. Paschos, Phys. Rev. D **15** (1977) 1966.

[4] A. Antaramian, L. J. Hall and A. Rasin, Phys. Rev. Lett. **69** (1992) 1871 [arXiv:hep-ph/9206205]; L. J. Hall and S. Weinberg, Phys. Rev. D **48** (1993) 979 [arXiv:hep-ph/9303241].

[5] A. S. Joshipura and S. D. Rindani, Phys. Lett. B **260** (1991) 149;

[6] N. Cabibbo, Phys. Rev. Lett. **10** (1963) 531.

[7] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652.

[8] A. J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini, Phys. Lett. B **500** (2001) 161 [arXiv:hep-ph/0007085].

[9] G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645** (2002) 155 [arXiv:hep-ph/0207036].

[10] V. Cirigliano, B. Grinstein, G. Isidori and M. B. Wise, Nucl. Phys. B **728** (2005) 121 [arXiv:hep-ph/0507001].

[11] S. Davidson and F. Palorini, Phys. Lett. B **642** (2006) 72 [arXiv:hep-ph/0607329].

[12] G. C. Branco, A. J. Buras, S. Jager, S. Uhlig and A. Weiler, JHEP **0709** (2007) 004 [arXiv:hep-ph/0609067].

[13] G. C. Branco, W. Grimus and L. Lavoura, Phys. Lett. B **380** (1996) 119 [arXiv:hep-ph/9601383].

[14] P. Tuzon and A. Pich, Acta Phys. Polon. Supp. **3** (2010) 215 [arXiv:1001.0293 [hep-ph]]. M. Jung, A. Pich and P. Tuzon, JHEP **1011** (2010) 003 [arXiv:1006.0470 [hep-ph]]. M. Jung, A. Pich and P. Tuzon, arXiv:1011.5154 [hep-ph].

[15] F. J. Botella, G. C. Branco and M. N. Rebelo, Phys. Lett. B **687** (2010) 194 [arXiv:0911.1753 [hep-ph]].

[16] F. J. Botella, M. Nebot and O. Vives, JHEP **0601** (2006) 106 [arXiv:hep-ph/0407349].

[17] B. Pontecorvo, Sov. Phys. JETP **7** (1958) 172 [Zh. Eksp. Teor. Fiz. **34** (1957) 247].

[18] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28** (1962) 870.

[19] B. Pontecorvo, Sov. Phys. JETP **26** (1968) 984 [Zh. Eksp. Teor. Fiz. **53** (1967) 1717].

[20] T. D. Lee, Phys. Rev. D **8** (1973) 1226.

[21] A. J. Buras, M. V. Carlucci, S. Gori and G. Isidori, JHEP **1010** (2010) 009 [arXiv:1005.5310 [hep-ph]].

[22] P. M. Ferreira, L. Lavoura and J. P. Silva, Phys. Lett. B **688** (2010) 341 [arXiv:1001.2561 [hep-ph]].

[23] B. Grzadkowski, M. Lindner and S. Theisen, Phys. Lett. B **198** (1987) 64.

[24] P. M. Ferreira and J. P. Silva, arXiv:1012.2874 [hep-ph].

[25] S. Weinberg, Phys. Rev. Lett. **43** (1979) 1566.

[26] W. Grimus and L. Lavoura, Eur. Phys. J. C **39** (2005) 219 [arXiv:hep-ph/0409231].

[27] P. Minkowski, Phys. Lett. B **67** (1977) 421.

[28] T. Yanagida, in *Proc. of the Workshop on Unified Theory and Baryon Number in the Universe*, KEK, March 1979.

[29] S. L. Glashow, in "Quarks and Leptons", Cargèse, ed. M. Lévy et al., Plenum, 1980 New York, p. 707.

[30] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, Stony Brook, Sept 1979.

[31] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44** (1980) 912.

[32] G. C. Branco, T. Morozumi, B. M. Nobre and M. N. Rebelo, Nucl. Phys. B **617** (2001) 475 [arXiv:hep-ph/0107164].

[33] J. A. Casas and A. Ibarra, Nucl. Phys. B **618** (2001) 171 [arXiv:hep-ph/0103065].

[34] M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45.

[35] F. R. Klinkhamer and N. S. Manton, Phys. Rev. D **30** (1984) 2212.

[36] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155** (1985) 36.

[37] J. Bernabeu, G. C. Branco and M. Gronau, Phys. Lett. B **169** (1986) 243.

[38] G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, Phys. Lett. B **614** (2005) 187 [arXiv:hep-ph/0502118].

[39] F. del Aguila and J. Cortes, Phys. Lett. B **156** (1985) 243; G. C. Branco and L. Lavoura, Nucl. Phys. B **278** (1986) 738; F. del Aguila, M. K. Chase and J. Cortes, Nucl. Phys. B **271** (1986) 61; Y. Nir and D. J. Silverman, Phys. Rev. D **42** (1990) 1477; D. Silverman, Phys. Rev. D **45** (1992) 1800; G. C. Branco, T. Morozumi, P. A. Parada

and M. N. Rebelo, Phys. Rev. D **48** (1993) 1167; V. D. Barger, M. S. Berger and R. J. N. Phillips, Phys. Rev. D **52** (1995) 1663 [arXiv:hep-ph/9503204]; M. Gronau and D. London, Phys. Rev. D **55** (1997) 2845 [arXiv:hep-ph/9608430]; F. del Aguila, J. A. Aguilar-Saavedra and G. C. Branco, Nucl. Phys. B **510** (1998) 39 [arXiv:hep-ph/9703410]; G. Barenboim, F. J. Botella, G. C. Branco and O. Vives, Phys. Lett. B **422** (1998) 277 [arXiv:hep-ph/9709369]; G. Barenboim, F. J. Botella and O. Vives, Phys. Rev. D **64** (2001) 015007 [arXiv:hep-ph/0012197]; G. Barenboim, F. J. Botella and O. Vives, Nucl. Phys. B **613** (2001) 285 [arXiv:hep-ph/0105306]; K. Higuchi and K. Yamamoto, arXiv:0911.1175 [hep-ph].

[40] To appear in a future publication