UdeM-GPP-TH-06-143

Looking for $\Delta I = 5/2$ amplitude components in $B \to \pi\pi$ and $B \to \rho\rho$ experiments

F. J. Botella,[1] David London,[2] and João P. Silva[3,4]

[1]*Departament de Física Teòrica and IFIC, Universitat de València-CSIC, E-46100, Burjassot, Spain*

[2]*Physique des Particules, Université de Montréal, C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7*

[3]*Instituto Superior de Engenharia de Lisboa, Rua Conselheiro Emídio Navarro, 1900 Lisboa, Portugal*

[4]*Centro de Física Teórica de Partículas, Instituto Superior Técnico, P-1049-001 Lisboa, Portugal*

(Dated: February 2, 2008)

Abstract

We discuss how experiments measuring $B \to \pi\pi$ and $B \to \rho\rho$ may be used to search for a $\Delta I = 5/2$ amplitude component. This component could be the explanation for a recent (albeit very tentative) hint from $B(\bar{B}) \to \rho\rho$ decays that the isospin triangles do not close.

PACS numbers: 13.25.Hw, 11.30.Er, 12,60.-i, 14.40.Nd

Within the standard model (SM), CP violation is due to a complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. This phase information can be elegantly encoded in the unitarity triangle [1, 2], in which the interior CP-violating angles are called α, β and γ. Independent measurements of the sides and angles of the unitarity triangle allow tests of the SM explanation of CP violation.

The canonical decay mode for measuring α is $B^0(t) \to \pi^+\pi^-$. However, due to the fact that this decay receives both tree and penguin contributions, α cannot be extracted cleanly – there is penguin "pollution." On the other hand, if one uses isospin to combine measurements of $B^+ \to \pi^+\pi^0$, $B^0(t) \to \pi^+\pi^-$ and $B^0(t) \to \pi^0\pi^0$, as well as the CP-conjugate decays, then the penguin pollution can be removed, and α obtained cleanly [3].

The isospin analysis goes as follows. Due to Bose statistics and the fact that the final-state pions come from the decay of a spinless state, they must be in a symmetric isospin configuration. As a result, the final states are

$$\begin{aligned} \langle \pi^0\pi^0 | &= \sqrt{\frac{2}{3}} \langle 2,0| - \sqrt{\frac{1}{3}} \langle 0,0| ~, \\ \langle \pi^+\pi^- | &= \sqrt{\frac{1}{3}} \langle 2,0| + \sqrt{\frac{2}{3}} \langle 0,0| ~, \\ \langle \pi^+\pi^0 | &= \langle 2,1| ~. \end{aligned} \tag{1}$$

In the SM, short-distance diagrams contribute only to the $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions. Thus, the physical decay amplitudes are

$$\begin{aligned} A_{+-} \equiv \langle \pi^+\pi^- | T | B^0 \rangle &= -\sqrt{\frac{1}{3}} A_{1/2} + \sqrt{\frac{1}{6}} A_{3/2} ~, \\ A_{00} \equiv \langle \pi^0\pi^0 | T | B^0 \rangle &= \sqrt{\frac{1}{6}} A_{1/2} + \sqrt{\frac{1}{3}} A_{3/2} ~, \\ A_{+0} \equiv \langle \pi^+\pi^0 | T | B^+ \rangle &= \frac{\sqrt{3}}{2} A_{3/2} ~, \end{aligned} \tag{2}$$

where A_k $(k = 1/2, 3/2)$ are the relevant reduced matrix elements. The parametrization for the CP-conjugate modes is similar, with the isospin amplitudes replaced by $\bar{A}_k$. Because there are two transitions, but three decays, the B decay amplitudes obey a triangle relation:

$$\sqrt{2} A_{+0} = A_{+-} + \sqrt{2} A_{00} ~. \tag{3}$$

The measurement of the three decays allows one to extract $A_{3/2}$, while the CP-conjugate decays give $\bar{A}_{3/2}$. However, the penguin amplitude contributes only to $A_{1/2}$, so that the relative phase between $A_{3/2}$ and $(q/p)\bar{A}_{3/2}$ is 2α, where q/p describes B–$\bar{B}$ mixing. Thus, the penguin pollution has been removed.

Now, a generic $B \to \pi\pi$ transition contains $\Delta I = 1/2$, $\Delta I = 3/2$, and $\Delta I = 5/2$ terms, which contribute to the physical decay amplitudes as

$$\begin{aligned} A_{+-} &= -\sqrt{\frac{1}{3}}A_{1/2} + \sqrt{\frac{1}{6}}A_{3/2} - \sqrt{\frac{1}{6}}A_{5/2} \ , \\ A_{00} &= \sqrt{\frac{1}{6}}A_{1/2} + \sqrt{\frac{1}{3}}A_{3/2} - \sqrt{\frac{1}{3}}A_{5/2} \ , \\ A_{+0} &= \frac{\sqrt{3}}{2}A_{3/2} + \sqrt{\frac{1}{3}}A_{5/2} \ . \end{aligned} \tag{4}$$

The key point is that, in the presence of a nonzero $A_{5/2}$, the three $B \to \pi\pi$ amplitudes by themselves no longer obey a triangle relation. That relation is modified as follows:

$$\sqrt{2}A_{+0}\,(1-z) = A_{+-} + \sqrt{2}A_{00} \ , \tag{5}$$

with

$$y \equiv \frac{A_{5/2}}{A_{3/2}} = \frac{z}{1+\frac{2}{3}\,(1-z)} \ . \tag{6}$$

Although isospin symmetry was mentioned above, Eqs. (4) already take into account any possible isospin-breaking effects in the decay amplitudes, since the three isospin amplitudes are enough to encode all the information contained in the three experimental amplitudes.

Note also that, although $B \to \pi\pi$ decays were described above, the isospin analysis also holds for each final-state polarization of $B \to \rho\rho$ decays. In addition, it holds for the decay of any neutral isospin-1/2 meson. In particular, it applies if the initial meson is K or D.

As noted above, the SM contributes only to the $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions at short distance. The $\Delta I = 5/2$ transitions arise from rescattering effects, such as the combination of $A_{1/2}$ with a $\Delta I = 2$ electromagnetic rescattering of the two pions in the final state. This is naively estimated to be of order $|A_{5/2}| \sim \alpha|A_{1/2}|$, where $\alpha \sim 1/127$ is the electromagnetic coupling constant. There are also strong-interaction isospin-violating effects ($m_u \neq m_d$).

A $\Delta I = 5/2$ contribution was first identified in $K \to \pi\pi$ decays. In this case, $|A_{1/2}| \sim 20|A_{3/2}|$ (known as the $\Delta I = 1/2$ rule), meaning that $|A_{5/2}| \sim 0.1|A_{3/2}|$, thus influencing the decay $K^+ \to \pi^+\pi^0$ [4]. A detailed comparison between theory and experiment is rather involved; a recent analysis within chiral perturbation theory may be found in Ref. [5].

In contrast, in the B system it is expected that $|A_{1/2}| \sim |A_{3/2}|$ and $A_{5/2}$ is normally discarded (as above, in the isospin analysis). (Recent analyses including electromagnetic and strong isospin violation can be found in Ref. [6].) Our main purpose is to encourage experiments to scrutinize this assumption very closely, highlighting the fact that current data could be interpreted as showing some hints of $A_{5/2} \neq 0$. This is an important issue since, if $A_{5/2} \neq 0$, the isospin triangles do not close, and the extraction of α will be affected.

If the SM is valid and the arguments leading to $A_{5/2} = 0$ are correct, then four predictions can be made:

1. as noted above, the triangle in Eq. (3) and its conjugate version close.

2. all measurements of α will yield the same result. For example, the CP phase β has already been measured very precisely in $B^0(t) \to J/\psi K_S$: $\sin 2\beta = 0.726 \pm 0.037$ [7], which determines β up to a four-fold ambiguity. The phase γ can in principle be cleanly determined through CP violation in decays such as $B \to DK$ [8], or from a fit to a variety of other measurements (the latest analysis gives $\gamma = 58.2^{+6.7^\circ}_{-5.4}$ [9]). The phase α is then given by $\alpha_{UT} \equiv \pi - \beta - \gamma$. If $A_{5/2} = 0$, then $\alpha_{\rm fit} = \alpha_{UT}$, where $\alpha_{\rm fit}$ is determined from $B \to \pi\pi$ or $B \to \rho\rho$ decays.

3. the direct CP asymmetry in $B^+ \to \pi^+\pi^0$ (C_{+0}) vanishes.

4. because there is one more observable than independent parameters in $B \to \pi\pi$, the interference CP asymmetry parameter in $B^0 \to \pi^0\pi^0$ (S_{00}), may be written as a function of the other observables: $F(S_{00}, C_{00}, B_{00}, S_{+-}, C_{+-}, B_{+-}, C_{+0}, B_{+0}) = 0$. Here B, C, and S represent the CP-averaged branching ratio, the direct CP violation and the interference CP violation, respectively.

Of the four predictions, only the first and fourth are smoking-gun signals of $A_{5/2} \neq 0$; the others can be violated in the presence of physics beyond the SM with $A_{5/2} = 0$. The situation is summarized in Table I.

The most obvious test for a nonzero $A_{5/2}$ is the non-closure of the isospin triangle. In the following, we examine the present data on $B(\bar{B}) \to \pi\pi$ and $B(\bar{B}) \to \rho\rho$ decays with this in mind. In analyzing the $\rho\rho$ data we assume that these particles are completely longitudinally polarized. This is known experimentally to be an excellent approximation [10].

TABLE I: Strategies to utilize the experimental observables to distinguish three cases: neglecting isospin-violations in the SM (IC-SM); considering isospin-conserving new physics (NP); and considering $\Delta I = 5/2$ components.

	IC-SM	NP	$\Delta I = 5/2$
triangle	closes	closes	does not close
$\alpha_{\rm fit} - \alpha_{UT}$	$= 0$	$\neq 0$	$\neq 0$
C_{+0}	$= 0$	$\neq 0$	$\neq 0$
$F(S_{00}, \ldots)$	$= 0$	$= 0$	$\neq 0$

Note that, since $A_{5/2}$ is expected to be small, it can only be seen in those triangles which are relatively flat. This is the case for the $B(\bar{B}) \to \rho\rho$ triangles, since the branching ratios for $B^0 \to \rho^0\rho^0$ and $\bar{B}^0 \to \rho^0\rho^0$ are much less than those of the other decay channels. It is also, by chance, the case for the $B \to \pi\pi$ triangle, but not for that of $\bar{B} \to \pi\pi$.

TABLE II: Branching ratios B_f, direct CP asymmetries C_f, and interference CP asymmetries S_f (if applicable) for the three $B \to \pi\pi(\rho\rho)$ decay modes. Data comes from Refs. [11, 12, 13, 14, 15, 16]; averages (shown) are taken from Ref. [17].

	$B_f[10^{-6}]$	C_f	S_f
$B^+ \to \pi^+\pi^0$	5.5 ± 0.6	-0.01 ± 0.06	
$B^0 \to \pi^+\pi^-$	5.0 ± 0.4	-0.37 ± 0.10	-0.50 ± 0.12
$B^0 \to \pi^0\pi^0$	1.45 ± 0.29	-0.28 ± 0.40	
$B^+ \to \rho^+\rho^0$	26.4 ± 6.4	0.09 ± 0.16	
$B^0 \to \rho^+\rho^-$	26.2 ± 3.7	-0.03 ± 0.17	-0.21 ± 0.22
$B^0 \to \rho^0\rho^0$	≤ 1.1	$(-1, 1)$	

The current $B \to \pi\pi$ and $B \to \rho\rho$ experimental measurements are shown in Table II. This data can be turned into measurements of the $B \to f$ (A_f) and $\bar{B} \to f$ $(\bar{A}_f)$ decay amplitudes through:

$$
\begin{aligned}
|A_f|^2 &\propto B_f(1 + C_f) , \\
|\bar{A}_f|^2 &\propto B_f(1 - C_f) . \qquad (7)
\end{aligned}
$$

The proportionality constants involve two ingredients. First, there is the phase-space factor $K(m_B, m_f)$ which is essentially the same for all amplitudes in each channel. The second factor is the lifetime of the decaying B. Thus, B_+ and B_- must be multiplied by $x = \tau(B^0)/\tau(B^+)$, $1/x = 1.076 \pm 0.008$, due to the difference between the charged and neutral B lifetimes [2]. We present the norms $|A_f|$ and $|\bar{A}_f|$ in Table III in arbitrary units (i.e. we include the factor x but not $K(m_B, m_f)$).

TABLE III: The isospin amplitudes in $B(\bar{B}) \to \pi\pi$ and $B(\bar{B}) \to \rho\rho$ (in arbitrary units).

	$\sqrt{2}\|A_{+0}\|$	$\|A_{+-}\|$	$\sqrt{2}\|A_{00}\|$
$B \to \pi\pi$:	3.2 ± 0.3	1.8 ± 0.2	1.4 ± 0.6
$B \to \rho\rho$:	7.3 ± 1.5	5.0 ± 0.8	$< 1.5\sqrt{1 + C_{00}}$
	$\sqrt{2}\|\bar{A}_{+0}\|$	$\|\bar{A}_{+-}\|$	$\sqrt{2}\|\bar{A}_{00}\|$
$\bar{B} \to \pi\pi$:	3.2 ± 0.3	2.6 ± 0.2	1.9 ± 0.5
$\bar{B} \to \rho\rho$:	6.7 ± 1.4	5.2 ± 0.8	$< 1.5\sqrt{1 - C_{00}}$

We note in passing that, in addition, for the decays of the neutral B mesons in which S_f is measured, we also have access to the relative phase in

$$\lambda_f \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f} = \frac{\pm\sqrt{1 - C_f^2 - S_f^2} + iS_f}{1 - C_f} , \tag{8}$$

where q/p arises from B–$\bar{B}$ mixing. However, we will not use this information.

In order to see if the isospin triangles close, we proceed as follows. In the absence of $A_{5/2}$, the triangle relation of Eq. (3) holds. We therefore have

$$|\sqrt{2}A_{+0}| = |A_{+-} + \sqrt{2}A_{00}| \leq |A_{+-}| + |\sqrt{2}A_{00}| . \tag{9}$$

Thus, if $|\sqrt{2}A_{+0}|$ is larger than $|A_{+-}| + |\sqrt{2}A_{00}|$, the triangle cannot close. The logic is siimilar for the CP-conjugate triangle.

For the $\pi\pi$ final state we see from the data that the central values do close both the $B \to \pi\pi$ and $\bar{B} \to \pi\pi$ unitarity triangles (but just barely for $B \to \pi\pi$): $|\sqrt{2}A_{+0}| = 3.2$, $|A_{+-}| + |\sqrt{2}A_{00}| = 3.2$; $|\sqrt{2}\bar{A}_{+0}| = 3.2$, $|\bar{A}_{+-}| + |\sqrt{2}\bar{A}_{00}| = 4.5$.

However, the same is not true for $B \to \rho\rho$. Here, the data show that the $B(\bar{B}) \to \rho\rho$ isospin triangles *do not* close (we present a detailed analysis below). This is quite tantalizing:

is it simply a statistical flucturation, or is it a signal of a $\Delta I = 5/2$ component at a level larger than naive expectations?

Consider $B \to \rho\rho$. The length $\sqrt{2}|A_{00}|$ depends on the value of C_{00}, but for the purposes of illustration, suppose that $C_{00} = 0$. Then the central values give $|\sqrt{2}A_{+0}| = 7.3$, $|A_{+-}| + |\sqrt{2}A_{00}| < 6.5$, and the triangle does not close. This situation can be rectified by the inclusion of a $\Delta I = 5/2$ piece. For various values of C_{00}, the data require that

$$|y| = \left|\frac{A_{5/2}}{A_{3/2}}\right| \geq \begin{pmatrix} 0.01 \pm 0.19 \ ; & C_{00} = 1 \\ 0.04 \pm 0.19 \ ; & C_{00} = 0.5 \\ 0.07 \pm 0.19 \ ; & C_{00} = 0 \\ 0.11 \pm 0.19 \ ; & C_{00} = -0.5 \\ 0.21 \pm 0.19 \ ; & C_{00} = -1 \end{pmatrix} \tag{10}$$

For all values of C_{00}, a nonzero $A_{5/2}$ is required by the central values of the present data. However, a study of the errors shows that, at present, the effect is not yet statistically significant – it is at most at the level of 1σ ($C_{00} = -1$).

Turning to $\bar{B} \to \rho\rho$, the present data give

$$|\overline{y}| = \left|\frac{\overline{A}_{5/2}}{\overline{A}_{3/2}}\right| \geq \begin{pmatrix} 0.16 \pm 0.21 \ ; & C_{00} = 1 \\ 0.06 \pm 0.21 \ ; & C_{00} = 0.5 \\ 0.01 \pm 0.20 \ ; & C_{00} = 0 \\ \text{No Bound} \ ; & C_{00} = -0.5 \\ \text{No Bound} \ ; & C_{00} = -1 \end{pmatrix} \tag{11}$$

In this case, a nonzero value of $A_{5/2}$ is required only for certain values of C_{00} (and the effect is not yet statistically significant).

This summarizes the present hint for a $\Delta I = 5/2$ piece in $B \to \rho\rho$ and $\bar{B} \to \rho\rho$ decays, separately. However, the signals go in opposite directions in each decay: the size of $A_{5/2}$ in $B \to \rho\rho$ decays increases as C_{00} goes from $+1$ to -1, while $\bar{A}_{5/2}$ in $\bar{B} \to \rho\rho$ decays increases as C_{00} goes from -1 to $+1$. As a result, we may combine information from both sets of data, using

$$|\sqrt{2}A_{+0}| + |\sqrt{2}\bar{A}_{+0}| \leq |A_{+-}| + |\bar{A}_{+-}| + |\sqrt{2}A_{00}| + |\sqrt{2}\bar{A}_{00}| \ . \tag{12}$$

The presence of a $\Delta I = 5/2$ piece is implied if this inequality is not satisfied. The current data imply that

$$y \vee \overline{y} \geq \begin{pmatrix} 0.08 \pm 0.13 : & C_{00} = 1 \\ 0.04 \pm 0.12 : & C_{00} = 0.5 \\ 0.04 \pm 0.12 : & C_{00} = 0 \\ 0.04 \pm 0.12 : & C_{00} = -0.5 \\ 0.08 \pm 0.13 : & C_{00} = -1 \end{pmatrix} \quad (13)$$

As above, the present data suggest a nonzero $A_{5/2}$ piece for all values of C_{00}, but the effect is not yet statistically significant.

In summary, we have shown that if the usual $B(\bar{B}) \to \pi\pi$ or $B(\bar{B}) \to \rho\rho$ isospin triangles do not close, this may be due to a SM $\Delta I = 5/2$ piece ($A_{5/2}$) at a level much larger than expected. This is a crucial question since a $A_{5/2}$ piece can also mimic new-physics contributions to other observables, such as C_{+0} or $\alpha_{\rm fit} - \alpha_{UT}$ (see Table I). We have pointed out some strategies to disentangle $A_{5/2}$ from legitimate new physics.

At present, data on $B(\bar{B}) \to \rho\rho$ decays give a hint – not yet statistically significant – that the isospin triangles do not close. The purpose of this letter is to stress the need for experimental scrutiny of such a signal (and to continue to look for one in $B(\bar{B}) \to \pi\pi$). [A probe with $F(S_{00}, \ldots)$ is also possible (Table I), particularly for $B \to \rho\rho$, and advisable once the data become more precise.] If this signal remains, it may be a sign of a SM $\Delta I = 5/2$ piece.

We would like to thank the anonymous referee of [18] for making a comment which lead us to think about this problem. The work of D. L. is supported by NSERC of Canada. F. J. B. is partially supported by the spanish M. E. C. and FEDER under FPA2005-01678. J. P. S. is supported in part by the Portuguese *Fundação para a Ciência e a Tecnologia* (FCT) under the contract CFTP-Plurianual (777), and through the project POCTI/37449/FNU/2001, approved by the Portuguese FCT and POCTI, and co-funded by FEDER.

[1] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[2] S. Eidelman *et al.* [Particle Data Group Collaboration], Phys. Lett. B **592**, 1 (2004).

[3] M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990).

[4] See, for example, J. F. Donoghue, E. Golowich, and B. R. Holstein, *Dynamics of the Standard Model* (Cambridge University Press, Cambridge, 1992); G. C. Branco, L. Lavoura and J. P.

Silva, *CP Violation* (Oxford University Press, Oxford, 1999), pp. 97-99; S. Gardner and G. Valencia, Phys. Rev. D **62**, 094024 (2000).

[5] V. Cirigliano, G. Ecker, H. Neufeld, and A. Pich, Eur. Phys. J. C**33**, 369 (2004), and references therein.

[6] M. Gronau and J. Zupan, Phys. Rev. D **71**, 074017 (2005); S. Gardner, Phys. Rev. D **72**, 034015 (2005); E. Baracchini and G. Isidori, Phys. Lett. B **633** (2006) 309.

[7] K. Abe *et al.* [BELLE Collaboration], hep-ex/0408111; B. Aubert *et al.* [BABAR Collaboration], hep-ex/0408127.

[8] M. Gronau and D. Wyler, Phys. Lett. B **265**, 172 (1991); D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett. **78**, 3257 (1997). See also M. Gronau and D. London., Phys. Lett. B **253**, 483 (1991); I. Dunietz, Phys. Lett. B **270**, 75 (1991); N. Sinha and R. Sinha, Phys. Rev. Lett. **80**, 3706 (1998).

[9] The CKMfitter group, http://www.slac.stanford.edu /xorg/ckmfitter/ckm_results_winter2005.html.

[10] See Refs. [14, 15, 16] below.

[11] $B \to \pi\pi$ BR's: A. Bornheim *et al.* [CLEO Collaboration], Phys. Rev. D **68**, 052002 (2003); B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0408081 ($\pi^+\pi^0$, $\pi^0\pi^0$), Phys. Rev. Lett. **89**, 281802 (2002) ($\pi^+\pi^-$); Y. Chao *et al.* [Belle Collaboration], Phys. Rev. D **69**, 111102 (2004) ($\pi^+\pi^0$, $\pi^+\pi^-$), K. Abe *et al.* [Belle Collaboration], arXiv:hep-ex/0408101 ($\pi^0\pi^0$).

[12] A_{CP} for $B^+ \to \pi^+\pi^0$ and $B^0 \to \pi^0\pi^0$: B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0408081; Y. Chao *et al.* [Belle Collaboration], Phys. Rev. Lett. **93**, 191802 (2004) ($\pi^+\pi^0$), K. Abe *et al.* [Belle Collaboration], arXiv:hep-ex/0408101 ($\pi^0\pi^0$).

[13] A_{CP} and S_{CP} for $B^0 \to \pi^+\pi^-$: B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0408089; K. Abe *et al.* [Belle Collaboration], arXiv:hep-ex/0502035.

[14] $B^0 \to \rho^+\rho^-$: B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **93**, 231801 (2004); K. Abe *et al.*, arXiv:hep-ex/0507039.

[15] $B^+ \to \rho^+\rho^0$: B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **91**, 171802 (2003); J. Zhang *et al.* [BELLE Collaboration], Phys. Rev. Lett. **91**, 221801 (2003).

[16] $B^0 \to \rho^0\rho^0$: R. Godang *et al.* [CLEO Collaboration], Phys. Rev. Lett. **88**, 021802 (2002); B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **94**, 131801 (2005).

[17] The Heavy Flavor Averaging Group, http://www.slac.stanford.edu/xorg/hfag/

[18] S. Baek, F. J. Botella, D. London and J. P. Silva, Phys. Rev. D **72**, 036004 (2005). The notation in this article is related to ours by $A_0 = A_{1/2}/\sqrt{3}$ and $A_2 = -A_{3/2}/\sqrt{6}$.