$K^{\circ} \rightarrow \pi^{\circ}\gamma\gamma$ DECAYS IN CHIRAL PERTURBATION THEORY

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Abstract

The decays $K_{L,S} \rightarrow \pi^{\circ}\gamma\gamma$ are calculated within the framework of chiral perturbation theory. The amplitude for $K_L \rightarrow \pi^{\circ}\gamma\gamma$ is found to be finite at the one-loop level yielding a branching ratio of $6.8 \cdot 10^{-7}$. The decay spectra of both decays are very characteristic and provide good tests of the effective chiral symmetry realization of the Standard Model.

In a recent paper [1] we have extended the methods of chiral perturbation theory to non-leptonic electroweak processes and applied them to $K \to \pi\ell^+\ell^-$ decays. The purpose of this letter is to present the results of a new application within this framework to $K_{L,S} \to \pi^0\gamma\gamma$ decays. At present, there is only an experimental upper limit for the $K_L \to \pi^0\gamma\gamma$ branching ratio [2]

$$\frac{\Gamma(K_L \to \pi^0\gamma\gamma)}{\Gamma(K_L \to \text{all})} < 2.4 \cdot 10^{-4}, \qquad (1)$$

but improved experiments on rare K-decays, which are under way, may well be able to measure this decay mode in the near future. Chiral perturbation theory* is ideally suited to describe such decays because the only hadrons involved are pseudoscalar mesons and, furthermore, the characteristic momenta are small compared to the natural scale of chiral symmetry breaking. As we shall see, the predictions for $K_{L,S} \to \pi^0\gamma\gamma$ decays are free from arbitrary parameters and provide good tests of the effective chiral symmetry realization of the Standard Model.

The decay $K_L \to \pi^0\gamma\gamma$ shares with $K_S \to \gamma\gamma$ [4] the remarkable property of being finite at the one-loop level in chiral perturbation theory. This can be understood by investigating the possible local counterterms of dimension four in the effective chiral Lagrangian which can induce these transitions. The counterterms have to be constructed with $U(x)$, the 3×3 special unitary matrix which incorporates non-linearly the octet of pseudoscalar fields, covariant derivatives $D_\mu U(x)$ and external gauge fields $F_L^{\mu\nu}(x)$, $F_R^{\mu\nu}(x)$ [1]. Neglecting the part of the non-leptonic $\Delta S = 1$ weak Hamiltonian transforming like a $27_L \times 1_R$ operator, and up to arbitrary coupling constants, there are three possible counterterms with the required transformation properties under chiral $SU(3)_L \times SU(3)_R$:

$$C_1 = \text{tr}(\lambda_{6-i7} F_L^{\mu\nu} U F_{R,\mu\nu} U^\dagger), \qquad (2a)$$

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*) See, e.g., Refs. [3] where earlier references can also be found.

$$C_2 = \text{tr}(\lambda_{6-i7} U F_R^{\mu\nu} U^\dagger F_{L,\mu\nu}), \qquad (2b)$$

$$C_3 = \text{tr}(\lambda_{6-i7} U F_R^{\mu\nu} F_{R,\mu\nu} U^\dagger). \qquad (2c)$$

Notice that there are no explicit mass terms of the required dimension, and counterterms with $D_\mu U$ cannot contribute because $D_\mu \phi = \partial_\mu \phi$ for neutral fields, and the corresponding expressions with $F^{\mu\nu}F_{\mu\nu}$ replaced by $\varepsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$ are forbidden by CPS-symmetry [5]. For external photons we have

$$F_L^{\mu\nu} = F_R^{\mu\nu} = u \, Q \, F^{\mu\nu} \qquad (3a)$$

$$[\lambda_{6-i7}, Q] = 0 \qquad (3b)$$

and therefore

$$C_1 = C_2 = - C_3. \qquad (4)$$

Expanding C_1 in powers of meson fields ϕ, one finds

$$C_1 \sim F^{\mu\nu} F_{\mu\nu}(K^+\pi^- + O(\phi^3)), \qquad (5)$$

with no linear or quadratic K^0, $K^0\pi^0$ terms in the brackets.

In principle, there could also be counterterms of strong + electro-magnetic origin which become relevant when combined with the $\Delta S = 1$ effective Lagrangian. With two external gauge fields present, the only counterterm available with dimension four is [3]

$$L_{10} \text{tr}(U F_R^{\mu\nu} U^\dagger F_{L,\mu\nu}). \qquad (6)$$

Similar to (5), there are again no neutral meson fields in the linear or quadratic terms of (6). From this analysis we then conclude that there are indeed counterterms of dimension four for $K^+ \to \pi^+\gamma\gamma$, but neither for $K_S \to \gamma\gamma$ nor for $K_L \to \pi^0\gamma\gamma$. Therefore, both $K_S \to \gamma\gamma$ and $K_L \to \pi^0\gamma\gamma$ must be

finite at the one-loop level. The finiteness of $K_S \to \gamma\gamma$ was confirmed by two recent calculations [4]. We want to emphasize, however, that the two counterterms discussed in Ref. [4a] are not relevant counterterms for the one-loop amplitude contrary to the authors' claim.

The Lagrangian for non-leptonic $\Delta S = 1$ electroweak transitions has been given in Ref. [1]. From the calculational point of view, we have found it a considerable simplification to diagonalize simultaneously the covariant kinetic and mass terms of order G_F (Fermi coupling constant) which are quadratic in the pseudoscalar fields. In this diagonal basis, there is neither a $K \to \pi$ coupling nor a $K \to \pi\gamma$ vertex. The effective couplings relevant to the $K^0 \to \pi^0\gamma\gamma$ transition are then given by the Lagrangian

$$L_{eff} = \frac{G_8}{3\sqrt{2}} (L_1 + L_2 + L_3) + h.c. \tag{7a}$$

$$G_8 = \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 g_8 , \qquad |g_8| \approx 5.1 \tag{7b}$$

$$L_1 = 3K^+ \partial_\mu K^0 K^- \overset{\leftrightarrow}{D^\mu}\pi^0 + 2K^+ D_\mu K^- K^0 \overset{\leftrightarrow}{\partial^\mu}\pi^0 + 2\pi^0 D_\mu K^+ K^- \overset{\leftrightarrow}{D^\mu}K^- +$$
$$+ K^0 D_\mu K^+ \pi^0 \overset{\leftrightarrow}{D^\mu}K^- + \pi^- \partial_\mu K^0 \pi^0 \overset{\leftrightarrow}{D^\mu}K^0 +$$
$$+ \pi^+ \partial_\mu K^0 \pi^0 \overset{\leftrightarrow}{D^\mu}\pi^0 + \pi^+ \partial_\mu \pi^0 \pi^0 \overset{\leftrightarrow}{D^\mu}K^0 \tag{7c}$$

$$L_2 = K^0 D_\mu K^- K^+ \overset{\leftrightarrow}{D^\mu}\pi^0 + K^+ D_\mu K^- K^0 \overset{\leftrightarrow}{\partial^\mu}\pi^0 + 2K^0 D_\mu \pi^- \pi^+ \overset{\leftrightarrow}{D^\mu}\pi^0 + 2\pi^0 \overset{\leftrightarrow}{D_\mu}\pi^- \pi^+ \overset{\leftrightarrow}{D^\mu}K^0 \tag{7d}$$

$$L_3 = 2M_K^2 K^0 \pi^+ K^- + m_\pi^2 K^0 \pi^0 \pi^+ \pi^- \tag{7e}$$

$$D_\mu \phi^\pm = \partial_\mu \phi^\pm \pm i e A_\mu \phi^\pm , \qquad D_\mu \phi^0 = \partial_\mu \phi^0 \qquad (\phi = K, \pi) . \tag{7f}$$

Here, L_1 contains the quartic terms of the usual $\Delta S = 1$ weak Lagrangian in the exponential parametrization of $U(x)$, while L_2 and L_3 arise from

the strong interaction Lagrangian (including the chiral symmetry breaking mass term) through the diagonalization procedure described above.

The Feynman diagrams for $K^0 \to \pi^0\gamma\gamma$ transitions at the one-loop level are shown in Fig. 1, where a small box denotes vertices generated by L_{eff} in Equ. (7). Note that in the diagonal basis the relation between the one-loop amplitudes for $K_L \to \pi^0\gamma\gamma$ and $K_S \to \pi^0\gamma\gamma$ is manifest. The diagrams of Fig. 1 are exactly the diagrams needed for the calculation of $K_S \to \gamma\gamma$ [4] except that there is an additional π^0 at the weak vertices. It is therefore no surprise that we do finally obtain the same loop functions for both amplitudes with slightly different vertex factors only. We have established the relation between the $K^0 \to \pi^0\gamma\gamma$ and $K^0 \to \gamma\gamma$ loop amplitudes diagram by diagram confirming the results of Refs. [4].

The final result for the $K_L \to \pi^0\gamma\gamma$ amplitude at the one-loop level is*

$$A(K_L(p) \to \pi^0(p') + \gamma(q_1) + \gamma(q_2)) = \frac{G_8 \alpha}{2\pi} \epsilon^\mu(q_1)\epsilon^\nu(q_2)(2q_{2\mu}q_{1\nu} - q^2 g_{\mu\nu}) \cdot$$
$$\cdot [(1 - m_\pi^2/q^2)F(q^2/m_\pi^2) - (1 - M_K^2/q^2 - m_\pi^2/q^2)F(q^2/M_K^2)] \tag{8}$$

with $q = q_1 + q_2$ and

$$F(z) = \begin{cases} 1 - 4[\arcsin \sqrt{z}/2]^2/z & z \leq 4 \\[2mm] 1 - \pi^2/z + (\ln \frac{1 - \sqrt{1-4/z}}{1 + \sqrt{1-4/z}})^2/z + \frac{2i\pi}{z} \ln \frac{1 - \sqrt{1-4/z}}{1 + \sqrt{1-4/z}} & z \geq 4 \end{cases}$$

(9a)
(9b)

$$F(z) \underset{z\to 0}{\sim} - z/12 .$$

Notice that the $\pi^+\pi^-$ and K^+K^- intermediate states give separately finite and gauge invariant contributions. Of the four possible invariant amplitudes which govern a general weak decay [6] $M_1 \to M_2\gamma\gamma$ with two (pseudo) scalar particles M_1, M_2 of the same charge, a single one appears at the one-loop level. Two of those invariant amplitudes are eliminated by our assumption of CP invariance which we shall drop later. The absence of the

*) We assume CP conservation at this point.

When restricted to external $\pi^0(\eta)\gamma\gamma$ fields, the WZW term has the simple form

$$L_{ABJ} = \frac{\alpha}{8\pi f_\pi} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}(\pi^0 + \eta/\sqrt{3}) . \quad (12)$$

The weak vertices are the usual ones in terms of the octet coupling constant G_8. Both the η'-contribution and the difference between f_π and f_η are higher order in chiral perturbation theory and are not included in (12). This is evident for $f_\pi - f_\eta$ and follows for the η'-contribution from the absence of a local counterterm for $K^0 \to \pi^0\gamma\gamma$ of dimension four. In any case, the η-contribution turns out to be very small and the $K_S \to \pi^0\gamma\gamma$ amplitude can be expected to be quite insensitive to higher order chiral corrections.

We want to emphasize that the situation for chiral perturbation theory is much more favourable for $K_S \to \pi^0\gamma\gamma$ than for $K_L \to \pi^0\gamma\gamma$ because of two reasons. The first observation is that $K_L \to \pi^0\gamma\gamma$ vanishes in lowest order because the π^0 and the η pole contributions exactly cancel. A leading log calculation [9] of the higher order corrections has found the $K_L \to \pi^0\gamma\gamma$ amplitude to be very sensitive to SU(3) breaking. A more detailed calculation involving also finite counterterms is necessary before any realistic prediction for the $K_L \to \pi^0\gamma\gamma$ decay rate can be given. In contrast to $K_L \to \pi^0\gamma\gamma$, the transition $K_S \to \pi^0\gamma\gamma$ is non-vanishing in lowest order chiral perturbation theory. Moreover, the invariant mass of the photon pair is obviously smaller for $K_S \to \pi^0\gamma\gamma$ because

$$q^2 \le (M_{K^0} - m_{\pi^0})^2 \approx 0.53 M_{K^0}^2 . \quad (13)$$

The diagram of Fig. 3 leads to the amplitude

$$A(K_S(p) \to \pi^0(p') + \gamma(q_1) + \gamma(q_2)) = \frac{G_8 \alpha}{\pi} \epsilon^\mu(q_1)\epsilon^\nu(q_2)\epsilon_{\mu\nu\rho\sigma}q_1^\rho q_2^\sigma \cdot$$

$$\cdot \left[\frac{2 - z - r_\pi^2}{z - r_\pi^2 + ir_{\pi^0}\Gamma_{\pi^0}/M_{K^0}} - \frac{f_\pi(2 - 3z + r_\pi^2)}{3f_\eta(z - r_\pi^2 + ir_\eta\Gamma_\eta/M_{K^0})}\right] , \quad (14)$$

second CP conserving amplitude is related to a rather general phenomenon of non-leptonic photonic weak decays. It is due to the mismatch between the minimum number of powers of momenta required by gauge invariance and those that chiral perturbation theory can provide at the one-loop level. The same phenomenon explains the absence of $K^+ \to \pi^+ \ell^+ \ell^-$ [1] and of $K^+ \to \pi^+\gamma\gamma$ [6] transitions to lowest order in chiral perturbation theory.

From (8) we obtain the total decay rate

$$\Gamma(K_L \to \pi^0\gamma\gamma) = \frac{|G_8 a|^2 M_{K^0}^5}{(4\pi)^5} \int_0^{(1-r_\pi)^2} dz \, \lambda^{1/2}(1,z,r_\pi^2) \cdot$$

$$\cdot |(z - r_\pi^2)F(z/r_\pi^2) + (1 + r_\pi^2 - z) F(z)|^2 , \quad (10)$$

$$z = q^2/M_{K^0}^2, \qquad r_\pi = m_{\pi^0}/M_{K^0},$$

where $\lambda(a,b,c) = a^2+b^2+c^2-2(ab+bc+ca)$. The predicted q^2-spectrum of this decay mode is very characteristic and is shown in Fig. 2 (full curve). For the sake of comparison, we have also plotted in the same figure the spectrum expected from phase space alone (dashed curve). The peaking of the predicted distribution at large z is due to the rapidly rising absorptive part of the $\pi^+\pi^-$ intermediate state. Altogether, the absorptive part contributes 69% to the total rate. Using $|g_8| = 5.1$ as determined from $K \to \pi\pi$ decays, we get from (10) for the $K_L \to \pi^0\gamma\gamma$ branching ratio

$$\frac{\Gamma(K_L \to \pi^0\gamma\gamma)}{\Gamma(K_L \to all)} = 6.8 \cdot 10^{-7} \quad (11)$$

which is significantly below the present experimental upper bound (1). Forthcoming high-statistics K-decay experiments are, however, expected to reach the required sensitivity to measure this branching ratio.

We now turn to a discussion of the decay $K_S \to \pi^0\gamma\gamma$ which proceeds via the diagram shown in Fig. 3. The $\pi^0(\eta)\gamma\gamma$ vertices are induced by the Adler [7a], Bell and Jackiw [7b] anomaly and are incorporated in chiral perturbation theory in the so-called Wess-Zumino-Witten (WZW) term [8].

$$r_\eta = M_\eta/M_{K0}, \qquad z = q^2/M^2_{K0},$$

keeping $f_\eta \neq f_\pi$ to display the sensitivity to chiral corrections. We observe that the amplitude (14) vanishes in the SU(3) limit and that there is again only one of two possible invariant amplitudes consistent with CP conservation. The absence of the second invariant amplitude is again due to the mismatch in powers of momenta required by gauge and chiral invariance, whereas the SU(3) limit can be understood most easily by employing a U-spin analysis. From Equ. (14) we obtain the decay rate

$$\Gamma(K_S \to \pi^0 \gamma\gamma) = \frac{|C_8\alpha|^2 M^5_{K0}}{(4\pi)^5} \int_0^{(1-r_\pi)^2} dz\, z^2\, \lambda^{1/2}(1,z,r^2_\pi) \cdot$$

$$\cdot \left| \frac{2 - z - r^2_\pi}{z - r^2_\pi + i r_\pi f_{\pi0}/M_{K0}} - \frac{f_\pi(2 - 3z + r^2_\pi)}{3f_\eta(z - r^2_\eta)} \right|^2 \qquad (15)$$

where the irrelevant imaginary part of the η-pole has been omitted. Of course, the total rate is completely dominated by the pion pole. Therefore, we restrict ourselves to the region

$$0.2 \leq z \leq (1 - r_\pi)^2 \qquad (16)$$

where the spectrum can be used to test the chiral structure of weak vertices. In Fig. 4 we show the normalized q^2-distribution for $K_S \to \pi^0 \gamma\gamma$ (full curve) with the cut $z \geq 0.2$. The pion pole contribution alone is also shown in the same figure (dotted curve). For the sake of comparison, the dashed curve shows the spectrum obtained with momentum independent weak vertices, i.e., with the expression in brackets in Equ. (14) replaced by

$$\frac{1}{z - r^2_\pi} - \frac{f_\pi}{f_\eta(z - r^2_\eta)} \qquad (17)$$

It is clear that even for large z the rate is very much dominated by the pion contribution. The smallness of the η-contribution is mainly due to the chiral vertex structure $(2 - 3z + r^2_\pi)/3$. The curves in Fig. 4 are drawn

using the one-loop corrected value [3b] $f_\eta/f_\pi = 1.3$, but, at least as far as the actual spectrum (15) is concerned, the difference to the symmetric value $f_\eta = f_\pi$ could not be seen in the figure. With sufficient statistics, the chiral structure of weak vertices could be experimentally tested. The relevant branching ratio is

$$\frac{\Gamma(K_S \to \pi^0 \gamma\gamma)|_{z \geq 0.2}}{\Gamma(K_S \to \text{all})} = 3.8\cdot10^{-8} . \qquad (18)$$

We shall finally discuss the question of CP violation in these decays. It is clear that the K^0_1 component in the K_L state can now induce a CP violating amplitude at the tree level

$$A(K_L \to \pi^0 \gamma\gamma)_{tree} = \varepsilon A(K^0_1 \to \pi^0 \gamma\gamma) , \qquad (19)$$

with $A(K^0_1 \to \pi^0 \gamma\gamma) \approx A(K_S \to \pi^0 \gamma\gamma)$ and with ε the usual CP violation parameter of $K \to \pi\pi$ decays [2] ($|\varepsilon| = 2.3\cdot10^{-3}$). In fact, the integrated rate for $K_L \to \pi^0 \gamma\gamma$ turns out to be dominated by (19). In order to test the chiral prediction for the one-loop amplitude (8) one must therefore perform a cut in q^2. From Fig. 2 it can be seen that a cut $z \geq 0.2$ makes practically no difference for the characteristic behaviour of the one-loop amplitude. On the other hand,

$$\frac{\Gamma(K_L \to \pi^0 \gamma\gamma)_{tree}}{\Gamma(K_L \to \pi^0 \gamma\gamma)_{loop}}\bigg|_{z \geq 0.2} \approx 2\cdot10^{-4} \qquad (20)$$

so that the amplitude (19) may safely be neglected for $z \geq 0.2$. To the accuracy quoted, the branching ratio for $K_L \to \pi^0 \gamma\gamma$ with a cut $z \geq 0.2$ is still given by Equ. (11).

In summary, we have shown that the decay $K_L \to \pi^0 \gamma\gamma$ is unambiguously calculable at the one-loop level in chiral perturbation theory because of the absence of possible counterterms. Eliminating the CP violating tree level amplitude with a small cut in q^2, we find a branching ratio of $6.8\cdot10^{-7}$ which should be experimentally accessible in the near future.

Furthermore, the spectrum in the invariant mass of the two photons is predicted to have a very characteristic behaviour. The decay $K_S \to \pi^0 \gamma\gamma$ is dominated by the pion pole contribution over the whole Dalitz plot because the η-contribution is here suppressed. The chiral structure of weak vertices can then be tested from a comparison with the predicted q^2-spectrum.

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Figure Captions

Fig. 1: One-loop diagrams for $K^o \to \pi^o\gamma\gamma$ in the diagonal basis described
        in the text. The weak vertices denoted by a box are given by
        the Lagrangian (7).

Fig. 2: Normalized q^2-distribution for $K_L \to \pi^o\gamma\gamma$ in the limit of CP
        conservation (full curve). Also shown for comparison is the
        phase space distribution (dashed curve). q^2 is the invariant
        mass squared of the photon pair.

Fig. 3: Tree level diagram for $K^o \to \pi^o\gamma\gamma$ involving the W2W term of the
        chiral Lagrangian.

Fig. 4: Normalized q^2-distribution for $K_S \to \pi^o\gamma\gamma$ in the region
        $0.2 \leq z \leq (1 - r_\pi)^2$ (full curve). The pion pole contribution
        alone is given by the dotted curve. The dashed curve shows
        the spectrum for momentum independent weak vertices for
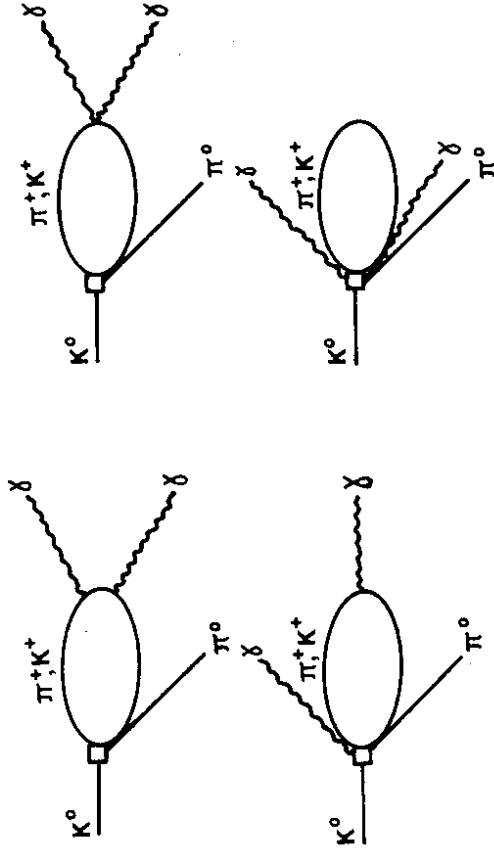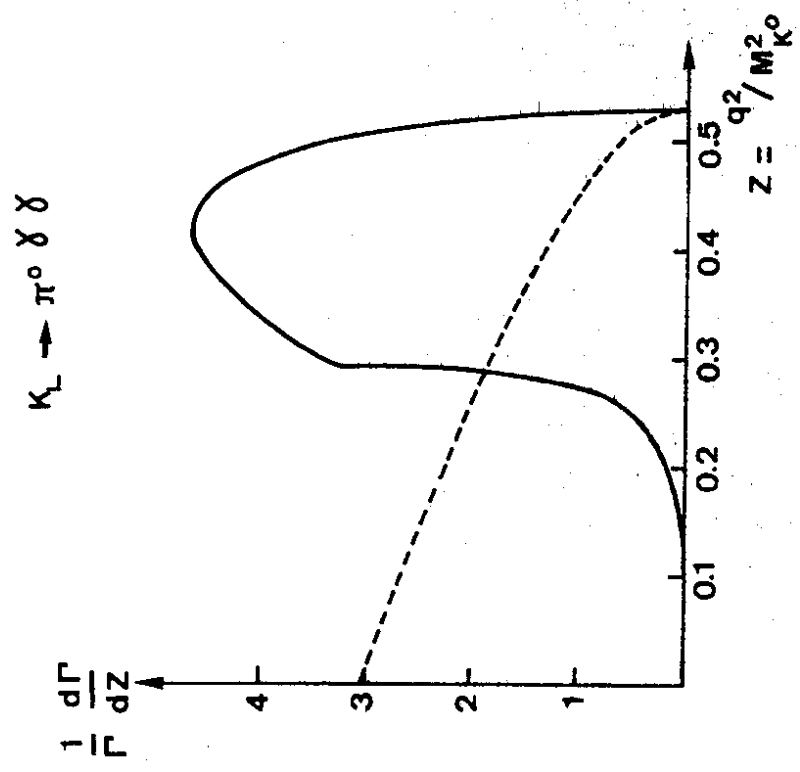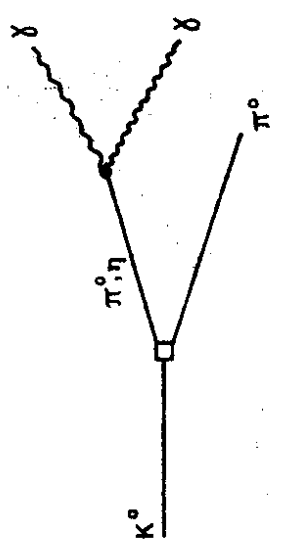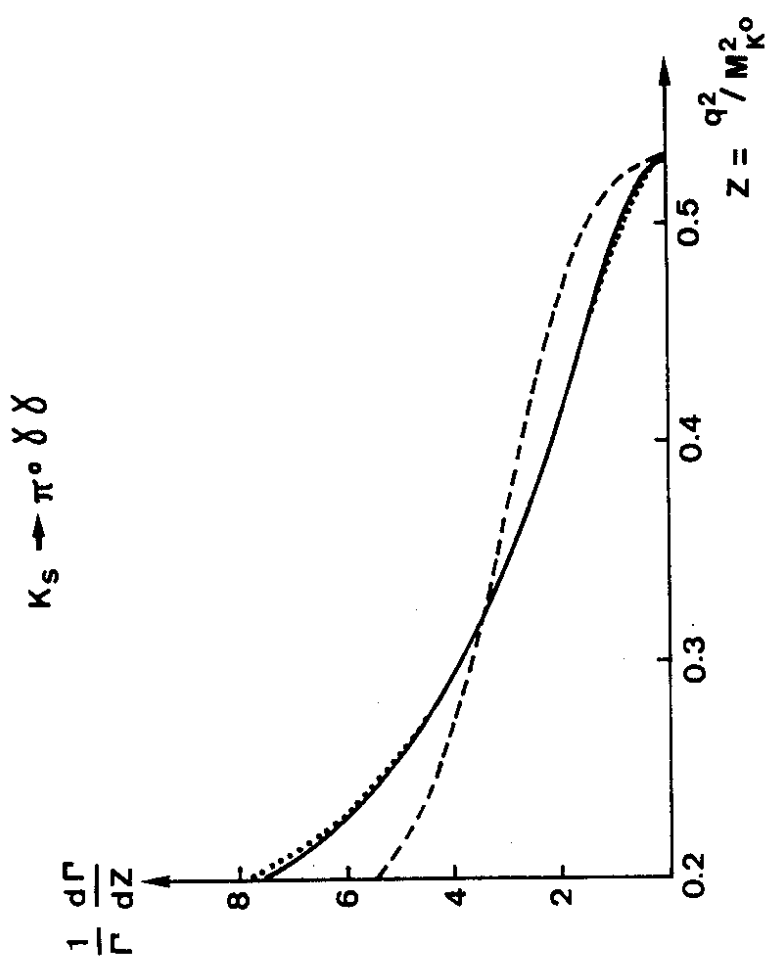   comparison (cf. Equ. (17)).

Fig. 1

$K_L \longrightarrow \pi^0 \gamma \gamma$

$\dfrac{1}{\Gamma}\dfrac{d\Gamma}{dZ}$

4

3

2

1

0.1 0.2 0.3 0.4 0.5

$z = q^2/M_{K^0}^2$

Fig.2

K^0

π^0, η

γ

γ

π^0

Fig.3

$K_S \longrightarrow \pi^\circ \gamma \gamma$

$\dfrac{1}{\Gamma}\dfrac{d\Gamma}{dZ}$

$Z = q^2/M^2_{K^\circ}$

Fig.4