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BOUNDS ON THE STRENGTH OF $\Delta I = 1/2$ WEAK AMPLITUDES

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ABSTRACT

Upper bounds to the strength of non-leptonic K-decay amplitudes with $\Delta I = 1/2$ are derived from first principles : perturbative QCD at very short distances and effective chiral perturbation theory at very long distances.

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The origin of the empirically observed enhancement of strangeness changing non-leptonic weak amplitudes with isospin transfer $\Delta I = 1/2$ is an old question in particle physics which, in spite of the great progress made in this field, has not yet been given a satisfactory explanation within the framework of the Standard Model [1]. A positive step towards a dynamical explanation of this so called $\Delta I = 1/2$-rule was the short-distance enhancement found in the Wilson coefficient of one of the four-quark operators which appears when, in the presence of gluon exchanges, the heavy W-field is removed from explicitly appearing in the $\Delta S = 1$ effective non-leptonic Hamiltonian [2], [3]. The enhancement thus found represents a factor of 2 to 3, too small to account for the bulk of the observed effect. Since then , it has been expected that the matrix elements of at least some of the four-quark operators in the effective Hamiltonian will eventually provide the required extra enhancement factor. In spite of many efforts, a mechanism of the $\Delta I = 1/2$ rule has not yet been exhibited.

There are at present three different approaches, within the framework of the Standard Model, to an understanding of non-leptonic weak amplitudes in general and the $\Delta I = 1/2$ rule in particular. In order of increasing technological complexity they are :

1. The 1/N - expansion approach [4], [5], [6].

2. The QCD - hadronic duality approach [7].

3. The lattice - QCD approach [8], [9], [10].

The three approaches are based on simple principles and therefore, in our opinion, they deserve attention.

As emphasized by the authors of ref. [5], the large N-expansion, even with the effect of the so called Penguin operators [3a] incorporated to lowest non-trivial order, predicts much too small a rate for $\Delta I = 1/2$ non-leptonic K-decays. There are however recent efforts

to reformulate this approach [6] with claims in the opposite direction, but only provided extreme input values of the strange quark mass[1] and Λ_{QCD} are used.

The QCD-hadronic duality approach to non-leptonic weak amplitudes has been used to evaluate the B-parameter of K^o-$\overline{K^o}$ mixing [7a] with the result[2] $|\hat{B}| = 0.33 \pm 0.09$. The same approach - with no free parameters - has been successful in reproducing the observed size of the $\Delta I = 3/2$ amplitude $K^+ \to \pi^+\pi^o$ [7b], but fails - by an order of magnitude - to reproduce the observed enhancement of the $K \to \pi\pi$ $\Delta I = 1/2$ amplitudes [7c].

The lattice QCD approach should be the ultimate test of our capability to understand the $\Delta I = 1/2$ rule within the Standard Model, provided of course that the technical difficulties inherent to the various methods used are well under control. Progress in this direction has been reported in refs. [9] and [10].

The purpose of this letter is to present a simplified version of the second approach above. We want to reduce the ingredients which go into the calculations reported in ref. [7c] to the strict minimum, and yet show that it is possible to derive a constraining upper bound to the strength of $K \to \pi\pi$ amplitudes with $\Delta I = 1/2$.

The basic tool in our approach is the two-point function ($Q^2 \equiv -q^2$)

$$\psi(Q^2) = i \int d^4x\, e^{iq\cdot x} \langle 0 | T\left(\mathcal{H}_{eff}(x), \mathcal{H}_{eff}^{\dagger}(0)\right) | 0 \rangle \,, \quad (1)$$

associated to that part of the effective non-leptonic Hamiltonian $\mathcal{H}_{eff}(x)$ which transforms like a $8_L \times 1_R$ operator and induces strangeness changing transitions with $\Delta I = 1/2$. In the chiral limit of massless u,d and s quarks, $\mathcal{H}_{eff}(x)$ reduces to a sum of four independent four-quark local operators. We shall choose the diagonal basis of

[1] For a recent determination of the light quark masses, where earlier references can be found, see ref.[11].

[2] $\hat{B}$ refers to the renormalization scale invariant quantity $\alpha_S(\mu^2)^{-2/9} B(\mu^2)$. For a comparison with other predictions see e.g. ref. [12]

multiplicatively renormalizable operators proposed in ref. [3f]; i.e., the same we used in ref.[7c]. The short-distance behaviour of $\Psi(Q^2)$ is governed by QCD-perturbation theory and fixes the number of subtractions required to define the two-point function in (1) : $\Psi(Q^2)$ obeys a dispersion relation in Q^2 up to an arbitrary polynomial in Q^2 of degree four. Five derivatives of $\Psi(Q^2)$ are thus required to get rid of this arbitrariness, with the result

$$F(Q^2) \equiv -\frac{\partial^5 \psi(Q^2)}{(\partial Q^2)^5} = 5! \int_0^\infty \frac{dt}{(t+Q^2)^6} \frac{1}{\pi} \mathrm{Im}\,\psi(t) . \qquad (2)$$

The function $F(Q^2)$, at sufficiently large Q^2-values , can then be calculated using perturbative QCD with leading non-perturbative $1/Q^2$ power corrections incorporated, if necessary, à la S.V.Z. [13]. The spectral function $\pi^{-1}\,\mathrm{Im}\Psi(t)$ in the r.h.s. of eq. (2) is by construction (see eq.(1)) a positive semidefinite quantity at all t-values. There are two-extreme regimes where we know something about this spectral function : at sufficiently high t-values, where (like for $F(Q^2)$ at sufficiently high Q^2) a perturbative-QCD evaluation is possible; and at very low t-values, where the hadronic matrix elements which define the spectral function i.e.,

$$\frac{1}{\pi}\mathrm{Im}\,\psi(t) = \sum_{\Gamma = \mathrm{Hadrons}} \int d\Gamma \, \langle 0|\partial \mathcal{L}_{\mathrm{eff}}(0)|\Gamma\rangle \langle \Gamma|\partial \mathcal{L}_{\mathrm{eff}}^{\dagger}(0)|0\rangle (2\pi)^3 \delta^{(4)}(q - \Sigma q_\Gamma), \qquad (3)$$

are governed by an effective chiral Lagrangean. To leading order in derivatives and light quark masses, the effect of $\Delta S=1$ non-leptonic interactions at low energies can be incorporated as a weak perturbation to the non-linear sigma model Lagrangean $\mathcal{L}_{\mathrm{st}}(x)$ which describes the effective chiral realization of QCD at low energies i.e.,

$$\mathcal{L}(x) = \mathcal{L}_{st}(x) + \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 \left\{ g \left(L_\mu(x) L^\mu(x) \right)_{23} \right.$$

$$\left. + h \frac{f_\pi^4 m_\pi^2}{m_u + m_d} \left(M U^\dagger + U M \right)_{23} + h.c. + \text{non-octet terms} \right\}. \quad (4)$$

Here U(x) is the usual 3x3 special unitary matrix which incorporates the Goldstone octet of pseudoscalar meson fields $\vec{\varphi}(x)$,

$$U(x) = \exp\left(\frac{i}{f_\pi} \vec{\lambda} . \vec{\varphi}(x) \right) \quad \text{and} \quad L_\mu(x) = i f_\pi^2 U(x) \partial_\mu U^\dagger(x),$$

where $\vec{\lambda}$ are the Gell-Mann matrices and $f_\pi \simeq 93.3$ MeV; M denotes the diagonal quark mass matrix : M=diag (m_u, m_d, m_s); G_F is the Fermi constant; $s_1c_1c_3$ is the product of Cabibbo-Kobayashi-Maskawa matrix elements V_{ud} V_{us} for three generations; and g and h are dimensionless coupling constants which are finite in the chiral symmetry limit, but cannot be fixed by chiral symmetry requirements alone. Phenomenologically, from the observed rate of $K \to \pi\pi$ decays one finds [3] that

$$|g|_{Exp.} \simeq 5.1 \,. \quad (5)$$

Our aim here is to derive an upper bound for $|g|$ from first principles alone.For this purpose we split the full domain of integration in the r.h.s. of eq.(2) into three regions : a very low energy region $0 \le t \le \Lambda_\chi^2$; an intermediate region $\Lambda_\chi^2 < t < Q^2$; and a very high energy region $Q^2 \le t \le \infty$. Here Λ_χ^2 denotes a chiral cut-off ($\Lambda_\chi^2 \ll Q^2$) which will be fixed sufficiently low to justify a chiral perturbation theory evaluation of the spectral function (3) in the region $0 \le t \le \Lambda_\chi^2$, using the chiral effective Lagrangean in (4). We shall later discuss what we consider to be a reasonable choice for Λ_χ^2. On the other hand,

[3] There is no contribution from the term proportional to h in eq. (4) for on-shell transitions like $K \to \pi\pi$.

the value of Q^2 must be fixed sufficiently high so as to justify the perturbative-QCD evaluation of $F(Q^2)$ and $\pi^{-1}\,\mathrm{Im}\Psi(t)$ for $t \geq Q^2$. Here, a criterion for the actual choice of Q^2 is the size of the corrections to the leading asymptotic freedom behaviour which will be required to be sufficiently small. Since $\pi^{-1}\,\mathrm{Im}\,\Psi(t) \geq 0$ for $0 \leq t \leq \infty$, the following inequality follows from eq.(2),

$$F(Q^2) - 5!\int_{Q^2}^{\infty} \frac{dt}{(t+Q^2)^6}\,\frac{1}{\pi}\,\mathrm{Im}\,\psi(t) \;\geq\; 5!\int_{0}^{\Lambda_\chi^2} \frac{dt}{(t+Q^2)^6}\,\frac{1}{\pi}\,\mathrm{Im}\,\psi(t) \qquad (6)$$

This inequality becomes an identity in the extreme case where the spectral function vanishes in the intermediate region ; i.e.,

$$\frac{1}{\pi}\,\mathrm{Im}\,\psi(t) = 0 \qquad \text{for} \qquad \Lambda_\chi^2 < t < Q^2 \,. \qquad (7)$$

The evaluation of the l.h.s. of (6) in QCD can be extracted from our work in ref. [7c]. Up to an overall $(G_F S_1 c_1 c_3/\sqrt{2})^2$-factor, we find

$$\hat{F}(Q^2) - 5!\int_{Q^2}^{\infty} \frac{dt}{(t+Q^2)^6}\,\frac{1}{\pi}\,\mathrm{Im}\,\psi(t) = \frac{1}{(16\pi^2)^3}\,\frac{12}{Q^2}\,W(Q^2)\,, \qquad (8a)$$

and

$$W(Q^2) = \sum_{ij} X_i X_j\, \alpha_s(Q^2)^{\gamma_i+\gamma_j} \left\{ a_{ij} + \frac{3}{2} b_{ij}\,\frac{\bar{m}_s^2(Q^2)}{Q^2} + \frac{8}{3}\,\frac{\langle \Omega_{ij} \rangle}{Q^4} + \ldots \right\} \qquad (8b)$$

where X_i , i=1,2,3,5 are the Wilson coefficients in the diagonal basis we already mentioned [3f], modulo their $\alpha_s(\mu^2)^{\gamma_i}$ dependence which cancels with factors $\alpha_s(\mu^2)^{-\gamma_i\,-\gamma_j}$ which appear in the evaluation of the two-point functions of the various four-quark anomalous operators. The actual values of the X_i coefficients and the

anomalous dimensions γ_i, as well as the numerical values of the matrices a, b and Ω in the terms in brackets in eq. (8b) can be found in the tables of ref. [7c]. Strictly speaking the Wilson coefficients X_i have been evaluated in the chiral limit only. The finite strange quark mass corrections in the r.h.s. of (8b) represent only the leading effect in the various two-point functions of four-quark operators. In principle, there are other effects like those due to the mixing of four-quark operators with mass-dependent operators of dimensions three and five which have not been taken into account. However, at the large Q^2-values we shall be concerned with, these finite quark mass effects are expected to be rather small and will be neglected.

For simplicity, we shall first derive an upper bound for the coupling constant g in the chiral limit. To leading order in chiral perturbation theory and in the chiral limit, where pseudoscalar masses vanish , the spectral function in (3) goes as t^2 and receives contributions from the πK and ηK intermediate states only. Other intermediate states will contribute corrections $O(t/16\pi^2 f_\pi^2)$ to the leading t^2-behaviour. Up to an overall $(G_F S_1 c_1 c_3/\sqrt{2})^2$ -factor one easily finds the result

$$\frac{1}{\pi}\,\mathrm{Im}\,\psi(t)\Big|_{\text{chiral limit}} = |g|^2\,\frac{1}{16\pi^2}\,f_\pi^4\,\frac{3}{2}\left(1+\frac{1}{9}\right)t^2\left\{1+O\left(\frac{t}{16\pi^2 f_\pi^2}\right)\right\}. \quad (9)$$

Inserting eq. (8) and (9) in the inequality (6) and neglecting terms of $O(\Lambda_\chi^2/Q^2)$ with respect to one we find

$$|g| \le \frac{1}{\sqrt{2}}\,\frac{3}{5}\left(\frac{Q^2}{16\pi^2 f_\pi^2}\right)\left(\frac{Q^2}{\Lambda_\chi^2}\right)^{3/2}\left(\sum_{ij} X_i X_j\,\alpha_s(Q^2)^{\gamma_i+\gamma_j}\left[a_{ij}+\frac{8}{3}\,\frac{\langle\Omega_{ij}\rangle}{Q^4}+\dots\right]\right)^{1/2}, \quad (10)$$

where, for consistency, finite quark mass corrections in (8b) have been neglected. Obviously, the bound increases as Q^2 increases and Λ_χ^2 decreases. Let us examine its numerical value for reasonable choices of Q^2 and Λ_χ^2. At $Q^2 \simeq 4\mathrm{GeV}^2$, the leading non-perturbative QCD corrections are only 3%. We then consider that this is a safe choice. The

natural choice for Λ_χ^2 would be $\Lambda_\chi^2 = 16\pi^2 f_\pi^2 \simeq 1.37\ \mathrm{GeV}^2$. However, due to the fact that in the chiral limit all the mass thresholds go to zero, corrections $O(t/16\pi^2 f_\pi^2)$ in the spectral function, which we neglect, when integrated from t = 0 to $t = \Lambda_\chi^2$ could become dangerously large. This forces the choice of the chiral limit Λ_χ^2 to be sensibly smaller than $16\pi^2 f_\pi^2$. We suggest $\Lambda_\chi^2 \simeq M_\rho^2 = 0.59\ \mathrm{GeV}^2$ as a safe choice. For these input values the upper-bound (10) is then $|g| \leq 4.3$ at $Q^2 = 4\mathrm{GeV}^2$ and $\Lambda_\chi^2 = 0.59\ \mathrm{GeV}^2$, already smaller then the empirical determination in eq. (5) .

We want to investigate how this bound could change when finite quark mass effects, and hence finite pseudoscalar masses, are taken into account. To lowest order in chiral perturbation theory the spectral function in (3) receives now contributions both from the g-coupling and the h-coupling in the effective Lagrangean of eq. (4). There is a K-pole contribution from h^2 terms and, in particular, interference terms from the two couplings which can be destructive. The simple spectral function in (9) is then replaced by a quadratic form in the coupling constants g and h (assumed to be real, since we are neglecting CP-violation effects throughout) with the result :

$$\frac{1}{\pi}\left|\mathrm{Im}\,\psi(t)\right|_{\text{chiral pert. th.}} = |h|^2 \frac{3}{2}\frac{f_\pi^6}{16\pi^2}(M_K^2 - m_\pi^2)^2\,\delta(t - M_K^2) + \frac{3}{2}\frac{f_\pi^4}{16\pi^2}\left\{\theta[t-(M_K+m_\pi)^2]\,\rho_{K\pi} + \frac{1}{9}\,\theta[t-(M_K+M_\eta)^2]\,\rho_{K\eta}\right\}, \tag{11a}$$

where $[\,\lambda(a,b,c) = a^2+b^2+c^2-2(ab+bc+ca)\,]$

$$\rho_{K\pi} \equiv \lambda^{1/2}\left(1, \frac{M_K^2}{t}, \frac{m_\pi^2}{t}\right)\left[g^2(t-M_K^2-m_\pi^2)^2 + 2gh\,M_K^2(t-M_K^2-m_\pi^2) + h^2 M_K^4\right], \tag{11b}$$

and the same expression for $\rho_{K\eta}$ with m_π replaced by M_η. Inserting this spectral function in the r.h.s. of the inequality (6) constrains the coupling constants g and h to lie inside an ellipse defined by the equation

$$\left(g + a(\Lambda_\chi^2, Q^2)h\right)^2 + b(\Lambda_\chi^2, Q^2)h^2 \leq \frac{1}{15}\left(\frac{Q^2}{16\pi^2 f_\pi^2}\right)^2 \left(\frac{Q^2}{\Lambda_\chi^2}\right)^3 \frac{W(Q^2)}{\alpha(\Lambda_\chi^2, Q^2)} \qquad (12a)$$

where $W(Q^2)$ is the QCD-function already defined in eq. (8b), and

$$a(\Lambda_\chi^2, Q^2) = \frac{M_K^2}{\Lambda_\chi^2}\frac{\beta}{\alpha} \quad , \quad b(\Lambda_\chi^2, Q^2) = \left(\frac{M_K^2}{\Lambda_\chi^2}\right)^2 \frac{1}{\alpha^2}(\gamma\alpha - \beta^2) \qquad (12b,c)$$

with α, β and γ functions of Λ_χ^2/Q^2 and pseudoscalar mass ratios to Λ_χ^2 defined by the following integral of the spectral function in eq. (11) :

$$\int_0^{\Lambda_\chi^2} \frac{dt}{(t+Q^2)^6} \frac{1}{\pi} \mathrm{Im}\, \psi(t)\Big|_{\text{chiral pert. th.}} = \frac{3}{32\pi^2}\left(\frac{f_\pi^2}{Q^2}\right)^2 \left(\frac{\Lambda_\chi^2}{Q^2}\right)^3 \frac{1}{Q^2}\left[\alpha g^2 + \frac{M_K^2}{\Lambda_\chi^2}\beta 2gh + \left(\frac{M_K^2}{\Lambda_\chi^2}\right)^2 \gamma h^2\right] \qquad (13)$$

In the chiral limit where pseudoscalar masses vanish $\alpha \Rightarrow 1/3\ (1+1/9)$; $\beta \Rightarrow 1/2\ (1+1/9)$ and $\gamma \Rightarrow 1 + 1/9 + 4/3\ \ 16\pi^2 f_\pi^2/\Lambda_\chi^2$. Then, the ellipse in (12a) degenerates into two parallel lines to the h-axis and the bound for |g| becomes the one quoted in eq. (10). In general, with finite pseudoscalar masses and finite coupling constant h, the extremum in g is reached by the projection of the ellipse on the g-axis which correponds to the value

$$|g_{Max}| = \frac{1}{\sqrt{15}} \frac{Q^2}{16\pi^2 f_\pi^2} \left(\frac{Q^2}{\Lambda_\chi^2}\right)^{3/2} \sqrt{\frac{W}{\alpha}\left(1 + \frac{a^2}{b}\right)} \qquad (14)$$

The ellipse defined by eq. (12) is shown in Fig. 1 for the input value $Q^2 \simeq 4\mathrm{GeV}^2$ and the choice $\Lambda_\chi^2 = 16\pi^2 f_\pi^2 = 1.37\ \mathrm{GeV}^2$. This higher value of Λ_χ^2 is now justified because finite mass threshold effects are taken into account. By contrast, the finite

strange quark mass corrections in the QCD-function $W(Q^2)$ of eq. (8b) are now dominant and, at $Q^2=4$ GeV^2, the total correction of b and Ω terms in (8b) with respect to the leading asymptotic freedom a-term represents an effect of 23 %. This explains the difference between the two ellipses shown in the figure. The dotted line ellipse is the one corresponding to asymptotic freedom only; the continuous line shows the same ellipse when the leading $1/Q^2$ effects of finite strange quark mass corrections and non-perturbative $1/Q^4$ power corrections are also incorporated. It can be seen from this continuous curve that the coupling constant g reaches its maximum when $|h| \simeq 1.4$ with the result

$$|g| \lesssim 4.0 \text{ at } Q^2 = 4 \text{ GeV}^2 \text{ and } \Lambda_\chi^2 = 16\pi^2 f_\pi^2 = 1.37 \text{ GeV}^2 \qquad (15)$$

still smaller than the empirical determination in eq.(5) !

What is the significance of this bound ? Admittedly we can find upper bounds for g compatible with the experimental value in (5) if only we lower Λ_χ^2 and/or raise Q^2; for example, at $Q^2=4$ GeV^2 but $\Lambda_\chi^2 = 1.10$ GeV^2 the maximum of |g| is reached at $|g_{max}|=7.2$ compatible with (5). However, we find very striking that for g to reach the upper bound value it requires an extremely peculiar behaviour of the spectral function which has to vanish in a large intermediate energy region $\Lambda_\chi^2 < t < Q^2$, where resonance production is copious. We are rather inclined to conclude, as we already did in our previous work [7c], that here we are missing something fundamental : perhaps in the short-distance formulation of the effective $\Delta I = 1/2$ Hamiltonian with integrated heavy quarks, perhaps in its chiral realization at long distances.

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FIGURE CAPTIONS

Fig. 1 The coupling constants g and h defined in eq. (4) are required to lie inside of the ellipse defined by the equality in eq. (12a). The dotted line ellipse corresponds to the asymptotic freedom expression for $W(Q^2)$ in eq. (8b) with only the a-terms; the continuous line ellipse when the correction terms b and Ω are also incorporated. These figures correspond to the choice $\Lambda_\chi^2 = 16\pi^2 f_\pi^2 = 1.37\ \mathrm{GeV}^2$ and $Q^2 = 4\ \mathrm{GeV}^2$.

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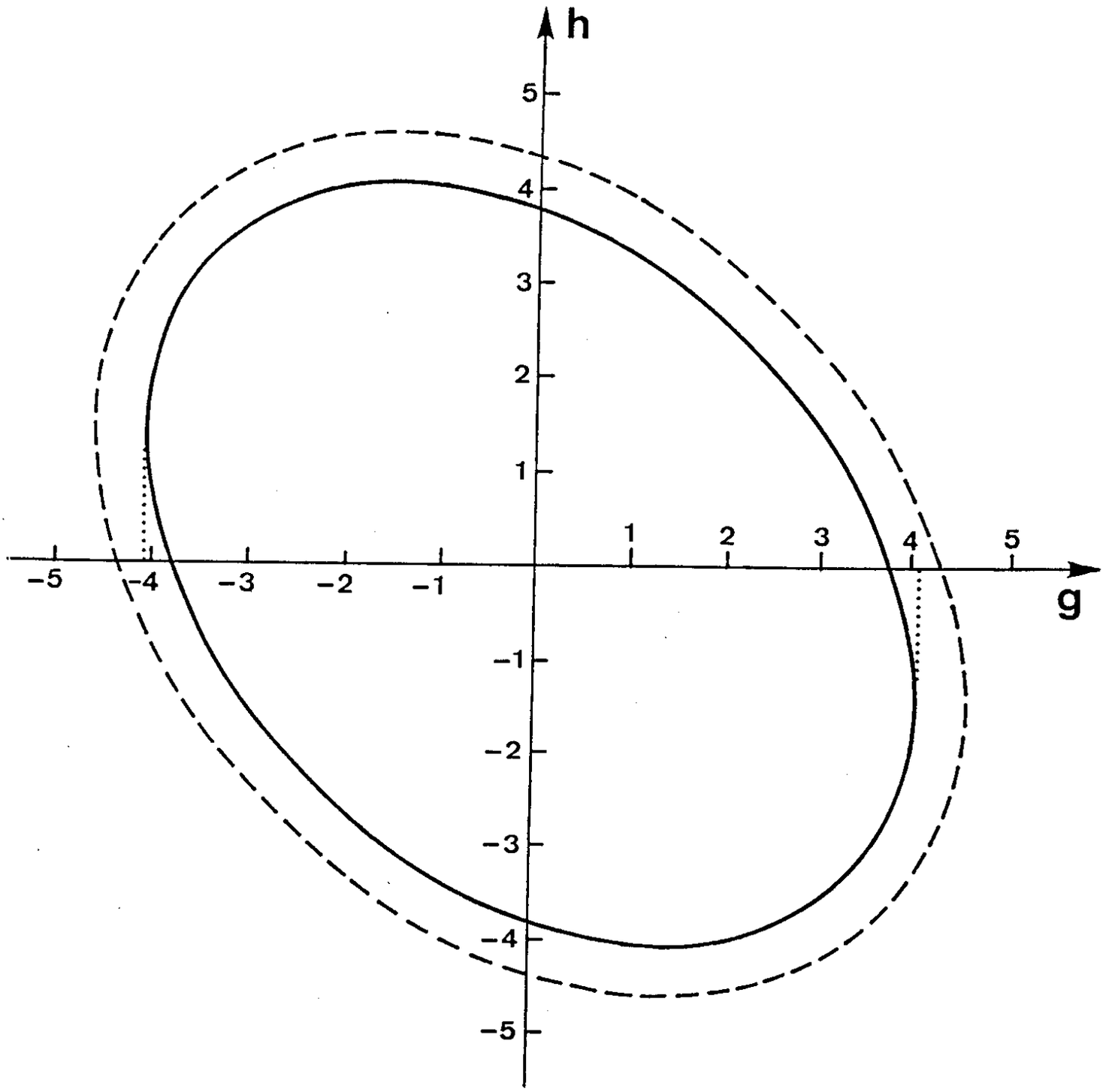

Fig. 1