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THE $\tau \to \nu_\tau \eta\pi$ PROCESS IN AND BEYOND QCD*)

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ABSTRACT

We estimate various sources of the $\tau \to \nu_\tau \eta\pi$ process by using QCD-duality sum rule information on the decay constants of standard and exotic mesons. We use the $\pi \to \ell\nu_\ell$ decay in order to constrain the effects of new interactions. We conclude from our analysis that in all cases, the $\tau \to \nu_\tau \eta\pi$ branching ratio is small. We also point out that the present data on the $\tau \to \nu_\tau K\pi$ process already provide an upper limit of 0.1% on the $\eta\pi$ mode while the $\tau \to \nu_\tau K\bar{K}$ data imply an upper limit of 0.5% if the $a_0(980)$-meson is a four-quark state.

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A sudden considerable interest has recently been focused[1] on the $\tau \to \nu_\tau \eta\pi$ decay, due to the reported measurement of a 5% branching ratio by the high resolution spectrometer (HRS) collaboration[2]. The $\eta\pi$-system in the final state has $I^G = 1^-$ and $J^P = 0^+$ or 1^- (for s- or p-waves, respectively) and its observation would therefore indicate a G-violation or a second-class current or new weak interactions. In this paper, we shall not attempt to explain this high rate obtained by the HRS, but, in turn, we will re-examine quantitatively various possible sources of the $\tau \to \eta_\tau \eta\pi$ process within and beyond the standard model. These include the $a_0(980) \equiv \delta$ meson in the $\bar{q}q$ and four-quark schemes, the $J^{PC} = 1^{-+}$ hybrid with odd G-parity and new non-standard weak interactions. We also point out that the data on $\tau \to \nu_\tau K\pi$ decay already provide a strong bound on the $\tau \to \nu_\tau \eta\pi$ process in the standard model while a measurement of the $\tau \to \nu_\tau K\bar{K}$ can give a test of the quark nature of the $a_0(980)$.

1. - NAMBU-GOLDSTONE REALIZATION OF CHIRAL SYMMETRY AND STANDARD MODEL

In the standard model, the $\tau \to \nu_\tau \eta\pi$ process is expected to be enhanced by the scalar resonance $a_0(980)$. Therefore the hadronic matrix element is governed by the a_0 decay amplitude normalized as:

$$\langle 0 | \bar{u}\gamma^\mu d | a_0 \rangle = \sqrt{2} f_{a_0} q^\mu \quad , \tag{1}$$

where within the Nambu-Goldstone realization of chiral symmetry, f_{a_0} is expected to vanish like the current quark mass difference $m_d - m_u$. There are several estimates of f_{a_0} from hadronic physics. A QCD sum rule analysis of the two-point correlator associated to the divergence of the vector current gives[3]:

$$f_{a_0} \simeq (1.3 \pm 0.2)\ \mathrm{MeV} \tag{2}$$

where the spectral function has been saturated by the $a_0(980)$ plus a QCD continuum and where one has used the combined running quark mass values obtained from a QCD sum rule analysis of the divergence of the axial current and the quark mass ratio obtained from low energy approaches. It is interesting that the result in Eq. (2) is reproduced within the errors by an effective Lagrangian treatment of the same two-point correlator[1e], where the divergence of the vector current in terms of the pseudoscalar meson fields is:

$$\partial_\mu V^\mu_{1+i2} = (-i)\left(\frac{m_d - m_u}{m_d + m_u}\right) m_\pi^2 \left(\sqrt{\tfrac{2}{3}}\,\eta\pi + K^+\bar{K}^0\right) \tag{3}$$

and the spectral function is parametrized in such a way that it reproduces the low energy chiral realization of QCD. An estimate of f_{a_0} using the hadronic kaon tadpole mass difference tends to give a slightly higher value of 1.8 ~ 2 MeV[4], which can be interpreted as an upper bound because of the assumption of single pole dominance used in this approach. In the same way, the $SU(3)_F$-partner of the a_0, the K_0^* (1350) ≡ κ is also expected to dominate the 0^{++} component of the $\tau \to \nu_\tau K\pi$ process. The decay constant of the K_0^* is also known from QCD sum rules to be[3]:

$$f_{K_0^*} \simeq (31 \pm 3)\ \mathrm{MeV} \quad , \tag{4}$$

within a narrow width approximation and for the standard value of the strange quark running mass. An effective Lagrangian estimate of $f_{K_0^*}$ including width corrections gives a larger value of about 45 MeV approaching the pole dominance result of 50 MeV[4c]. However, the most interesting observation is that different approaches predict:

$$\frac{f_{K_0^*}}{f_{a_0}} \simeq 24 \sim 30 \quad , \tag{5}$$

comparable with the strength of the current mass ratio $(m_s - m_u)/(m_d - m_u)$. Then Eq. (5) provides an unambiguous prediction of the branching ratio:

$$\frac{\mathcal{B}(\tau \to \nu_\tau K_0^*)}{\mathcal{B}(\tau \to \nu_\tau a_0)} \simeq tg^2\theta_c \left(\frac{f_{K_0^*}}{f_{a_0}}\right)^2 \left(\frac{1 - M_{K_0^*}^2/M_\tau^2}{1 - M_{a_0}^2/M_\tau^2}\right)^2 . \tag{6}$$

In order to exploit Eq. (6), we use the absolute bound:

$$\mathcal{B}(\tau \to \nu_\tau K_0^*) \lesssim \sin^2\theta_c \left(1 - \sum_{e,\mu} \mathcal{B}(\tau \to \nu_\tau \ell \bar{\nu}_\ell)\right) , \tag{7}$$

expressing that the $\tau \to \nu_\tau K_0^*$ process is smaller than the sum of all hadronic Cabibbo suppressed decays of the τ. Therefore, Eqs. (6) and (7) imply:

$$\mathcal{B}(\tau \to \nu_\tau \eta\pi) \lesssim 0.28\,\% \quad . \tag{8}$$

This bound can still be improved by stating that the $\tau \to \nu_\tau K\pi$ decay in the 0^{++} channel is smaller than the sum of all $K\pi$ branching ratios which are known to be about 1%. This gives:

$$\mathcal{B}(\tau \to \nu_\tau \eta\pi) < 0.1\,\% \quad . \tag{9}$$

From Eqs. (6) to (9), we want to stress that an accurate measurement of the $K\pi$ final states in the 0^{++} channel which might be easier than the $\eta\pi$ one should provide a strong constraint on the $\eta\pi$ branching ratio. From the value of f_{a_0} in Eq. (3), the latter ratio is expected to be ridiculously small in the standard model [see also Ref. 5)]

$$\mathcal{B}(\tau \to \nu_\tau \eta\pi) \simeq 1.5\; 10^{-5} \quad . \tag{10}$$

2. - EXOTIC MESONS OF QCD AND THE $\tau \to \nu_\tau\, \eta\pi$ PROCESS

The most relevant exotic contributions to the $\eta\pi$ process come from possible four-quark and hybrid ($\bar{q}qg$) mesons. The former might still be the a_0^- which has an unclear nature, while the latter might be the $\tilde{\rho}(1^{-+})$ hybrid.

A) Four-quark nature of the a_0^-

If the a_0 is a four-quark state it can be described by the interpolating operators[6),7)]:

$$O_4 = \frac{1}{M_H^2}\left\{\sum_{\Gamma \equiv 1,\gamma_5} \bar{u}\Gamma d\, \bar{s}\Gamma s + t \sum_{\Gamma} \bar{u}\Gamma\lambda_a d\, \bar{s}\Gamma\lambda^a s\right\}, \tag{11}$$

where t is the mixing parameter between the two operators estimated to be about[7)] $0.2 \sim 0.4$, in such a way that the model reproduces the $a_0 \to \eta\pi$ width. M_H is an effective normalization scale which controls the relative strength of the four-quark operator with respect to the familiar divergence of vector current described by quark bilinears. One could expect that M_H is typically of the order of 1 GeV. The a_0-decay constant normalized as:

$$\langle 0 | O_4 | a_0 \rangle = \sqrt{2}\, f_{a_0} M_{a_0}^4 / M_H^2 \tag{12a}$$

has been estimated to be[6)]:

$$f_{a_0} \simeq 2.7 \text{ MeV}. \tag{12b}$$

Then, one obtains the normalized branching ratio:

$$\frac{B(\tau \to \nu_\tau \eta\pi)}{B(\tau \to \nu_\tau \pi)} \simeq \left(\frac{g_{a_0}}{g_\pi}\right)^2 \left(\frac{M_{a_0}}{M_H}\right)^4 \left(\frac{1 - M_{a_0}^2/M_\tau^2}{1 - m_\pi^2/M_\tau^2}\right)^2 , \tag{13a}$$

which corresponds to

$$B(\tau \to \nu_\tau \eta\pi) \simeq 4\,10^{-4} \left(\frac{1\,GeV}{M_H}\right)^4 . \tag{13b}$$

The result in Eq. (13) is as small as the one in Eq. (10) which corresponds to a $\bar{q}q$ assignment of the a_0. In a four-quark scheme, we might also expect to have a strong coupling of the a_0 to $K\bar{K}$ if the operator O_2 [Eq. (11)] is present in the a_0-wave function. Using the estimate[7]:

$$g_{a_0 K^- K_0} \simeq (5 \sim 8)\ GeV , \tag{14a}$$

one can deduce:

$$B(\tau \to \nu_\tau K\bar{K}) \simeq (3 \sim 8)\,10^{-5} \left(\frac{1\,GeV}{M_H}\right)^4 , \tag{14b}$$

where we have used the following parametrization of the hadronic matrix element:

$$\langle 0 | \bar{u}\gamma^\mu d | K\bar{K} \rangle = k^\mu \sqrt{2}\, g_{a_0} \left(\frac{M_{a_0}}{M_H}\right)^2 \frac{g_{a_0 K^- K^0}}{k^2 - M_{a_0}^2 + i M_{a_0}\Gamma_{a_0}} . \tag{14c}$$

The result in Eq. (14) is consistent with the experimental upper limit of 0.4%. Instead, one can also combine the results in Eqs. (13) and (14) in order to obtain a constraint which is independent of M_H. One obtains:

$$\frac{B(\tau \to \nu_\tau \eta\pi)}{B(\tau \to \nu_\tau K\bar{K})} \simeq 0.5 \sim 1.3 , \tag{15a}$$

which implies, by using the experimental upper limit on $\tau \to \nu_\tau K\bar{K}$:

$$B(\tau \to \nu_\tau \eta\pi) < 0.5\,\% . \tag{15b}$$

B) Hybrids or hermaphrodite mesons

Another possible source of the $\eta\pi$ mode might be the $\tilde{\rho}(1^{-+})$ hybrid because of its negative G-parity and because it is the lowest mass hybrid. In fact, its mass is expected to range from 1.4 to 2.2 GeV from different QCD-inspired model approaches[8)], while the mass of the 0^{--} is of the order of 2-3 GeV. The hybrid $\tilde{\rho}$ interpolating field is:

$$\mathcal{O}^{\mu}_{\tilde{\rho}} = \frac{g}{M_H^2}\, \bar{u}\, \gamma_\alpha \lambda^a d\, G_a^{\mu\alpha} \, , \tag{16}$$

to which corresponds the decay amplitude:

$$\langle 0 | \mathcal{O}^{\mu}_{\tilde{\rho}} | \tilde{\rho} \rangle = \epsilon^{\mu} \sqrt{2}\, f_{\tilde{\rho}}\, M_{\tilde{\rho}}^3 / M_H^2 \, , \tag{17}$$

which has been estimated to be[8d)]:

$$f_{\tilde{\rho}} \simeq (17 \sim 21)\ \mathrm{MeV} \, . \tag{18}$$

The $\tilde{\rho}$ branching ratio into $\eta\pi$ has been estimated to be about 1%[8d)] by using an η-η' mixing angle of 17°. The quark part contribution of the η is chirally suppressed like other Goldstone modes of the $\tilde{\rho}$ and has been neglected. Therefore, the $\tilde{\rho}$ effect to the $\eta\pi$ decay is:

$$\frac{\mathcal{B}(\tau \to \nu_\tau \eta\pi)_{\tilde{\rho}}}{\mathcal{B}(\tau \to \nu_\tau \pi)} \simeq \left(\frac{f_{\tilde{\rho}}}{f_\pi}\right)^2 \left(\frac{M_{\tilde{\rho}}}{M_H}\right)^4 \left(1 - \frac{M_{\tilde{\rho}}^2}{M_\tau^2}\right)^2 \left(1 + \frac{2M_{\tilde{\rho}}^2}{M_\tau^2}\right) \mathcal{B}(\tilde{\rho} \to \eta\pi) \, , \tag{19a}$$

which for $M_{\tilde{\rho}} \geq 1.4$ GeV gives:

$$\mathcal{B}(\tau \to \nu_\tau \eta\pi)_{\tilde{\rho}} \lesssim 4.3\, 10^{-4} \left(\frac{1\,\mathrm{GeV}}{M_H}\right)^4 \, , \tag{19b}$$

which is still negligible for reasonable values of the scale M_H. The leading effects of the $\tilde{\rho}$ in τ-decay are given by its $\rho\pi$ decays which correspond to $Br(\tau \to \nu_\tau \rho\pi)$ less than 0.5% if M_H is equal or larger than 1 GeV. However, this contribution would be difficult to separate from the tail of $a_1(1^{++})$ meson which has a mass lighter than the $\tilde{\rho}$ meson.

3. - POSSIBLE EFFECTS OF TENSOR MESONS

One might also expect that the $\eta\pi$ final states are issued from the tensor a_2-meson, which might couple to a new spin-two weak boson. However:

a) this is unlikely due to the fact that the coupling of a tensor boson to a fermion pair is a derivative coupling $\bar{\tau}\overleftrightarrow{\partial}_\mu \gamma_\nu \nu_\tau$; therefore, this effect should be suppressed like $(M_\tau/\Lambda_H)^2$ compared to the one mediated by a W-exchange[9] (Λ_H being the weak breaking or some compositeness scale);

b) even, if there is another mechanism which enhances the $\eta\pi$ mode through an a_2-meson, then one cannot avoid an enhancement of the $\rho\pi$ mode which is the dominant decay of the a_2 *); however, there is no excess of $\rho\pi$ events in τ-decay as the data appear to be well described by the 1^{++} a_1-meson around the a_1-mass.

4. - NEW INTERACTIONS AND THE $\tau \to \nu_\tau \eta\pi$ DECAY

The most immediate new interaction obeying the "sacred" chirality constraint can indeed be induced by the contact term

$$\mathcal{L}_S = \frac{g^2}{M_S^2} \bar{\nu}_\tau (1+\gamma_5) \tau \; \bar{u}(1+g_A\gamma_5) d \, , \tag{20}$$

where M_S is a scalar mass.

a) If we have a universal coupling at the fermion vertices, one can derive the following constraint from the 2% precision measurement of the $\pi^- \to e^- \bar{\nu}_e$ decay

$$\frac{\sqrt{2}\, g_A g^2}{M_S^2 G_F} < 3.6\, 10^{-6} \left(\frac{m_u + m_d}{1\,\mathrm{MeV}}\right) , \tag{21}$$

which implies:

$$\frac{\mathcal{B}(\tau \to \nu_\tau \eta\pi)_{SCALAR}}{\mathcal{B}(\tau \to \nu_\tau \pi)_{STANDARD}} \lesssim \left\{ 3.6\, 10^{-3}\, \mathrm{GeV}^{-1} \frac{M_{a_0}^2}{M_\tau} \left(\frac{m_u + m_d}{m_d - m_u}\right) \frac{f_{a_0}}{f_\pi g_A} \cdot \frac{1 - M_{a_0}^2/M_\tau^2}{1 - m_\pi^2/M_\tau^2} \right\}^2 \simeq 4.7\, 10^{-9}/g_A^2 \, . \tag{22}$$

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*) We thank P. Singer for this remark.

Therefore, in a chiral theory where $g_A^2 \simeq 1$, the $\eta\pi$ mode of the τ is negligibly small.

b) In the case of a Higgs-type coupling at the leptonic vertex, i.e., $g_\ell \sim g m_\ell / M_S$, the constraint is weaker. The 0.1% accuracy of the $\pi^- \to \mu^- \bar{\nu}_\mu$ measurement then gives

$$\frac{\mathcal{B}(\tau \to \nu_\tau \eta \pi)_{SCALAR}}{\mathcal{B}(\tau \to \nu_\tau \pi)_{STANDARD}} < 3\,10^{-3} / g_A^2 \tag{23}$$

which is still very small in a chiral theory. The constraints in Eqs. (22) and (23) are as strong as the ones obtained in Ref. 1d) but the latter can be affected by the uncertain structure of the a_0. One should also note that the present accuracy on the $\tau \to \nu_\tau \pi$ branching ratio only provides a 3% upper limit on the $\tau \to \nu_\tau \eta\pi$ branching ratio in a chiral theory. This bound can be improved by an accurate measurement of this decay.

CONCLUSIONS

We have discussed various eventual sources of the $\tau \to \eta\pi\nu_\tau$ decay mode. We deduce that in all cases the $\tau \to \nu_\tau \eta\pi$ branching ratio is much smaller than the HRS data[2)].

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