



CERN-TH.4853/87
UWThPh-1987-31

RADIATIVE KAON DECAYS AND CP VIOLATION
IN CHIRAL PERTURBATION THEORY

Gerhard Ecker
Institut für Theoretische Physik
Universität Wien, A-1090 Wien

Antonio Pich
CERN - Geneva

and

Eduardo de Rafael
Department of Physics, Boston University
Boston, MA 02215

and

Centre de Physique Théorique*, Section II
CRNS Luminy, F-13288 Marseille Cedex 9

ABSTRACT

Chiral perturbation theory is a very useful framework for testing the Standard Model in processes where long-distance effects are expected to play an essential rôle. We analyze the rare K decays $K^0 \rightarrow \gamma \ell^+ \ell^-$, $K^+ \rightarrow \pi^+ \gamma \gamma$ and $K_L \rightarrow \pi^0 \ell^+ \ell^-$ in the effective chiral formulation of the Standard Model. These processes, like the decays $K^0 \rightarrow \gamma \gamma$, $K^+ \rightarrow \pi^+ \ell^+ \ell^-$, $K_S \rightarrow \pi^0 \ell^+ \ell^-$ and $K^0 \rightarrow \pi^0 \gamma \gamma$ discussed in previous work, have the property that the corresponding amplitudes vanish to lowest order in chiral perturbation theory. Precise predictions for decay rates and spectra are made in terms of a few coupling constants not restricted by softly broken chiral symmetry alone. Special consideration is given to various possible tests of CP non-invariance in these decays, in particular to effects due to intrinsic CP violating observables such as the charge asymmetries in $K^\pm \rightarrow \pi^\pm \gamma \gamma$ and $K^\pm \rightarrow \pi^\pm \ell^+ \ell^-$, the one-photon exchange contribution to $K_L \rightarrow \pi^0 e^+ e^-$ and the transverse polarization in $K_L \rightarrow \pi^0 \mu^+ \mu^-$. Detailed numerical results are shown.

*Laboratoire propre du Centre National de la Recherche Scientifique (permanent address).

1. INTRODUCTION

There is a revival of theoretical interest in the subject of rare kaon decays. This is mostly due to the prospect of significantly improved experiments, some of them already under way [1], but also to recent theoretical developments. Theorists are becoming aware of the fact that an effective chiral perturbation theory formulation of the Standard Model [2] is an ideal framework to describe K decays. This is because in K decays the only physical states which appear are pseudoscalar mesons, photons and leptons and because the characteristic momenta involved are small compared to the natural scale ($4\pi f_\pi \simeq 1.2$ GeV) of chiral symmetry breaking.

Chiral perturbation theory is a systematic expansion in momenta and pseudoscalar meson masses within a specific, albeit non-renormalizable, Lagrangian quantum field theory. It was developed in the 60's, first as an effective dynamical realization of Current Algebra [3], and later formulated as a non-linear sigma model field theory [4] whose configurations are maps from Minkowski space to the homogeneous space $SU(3)_L \times SU(3)_R / SU(3)_V$. With the advent of Quantum Chromodynamics (QCD) [5] and subsequent progress in the understanding of the realization of its chiral symmetry at low energies, it has become apparent that an effective Lagrangian of the non-linear sigma model type (with Wess-Zumino (WZ) terms [6] incorporated, as required by the global anomaly structure of QCD [7]) gives an explicit dynamical formulation of the strong interactions among the octet of pseudoscalar mesons when both gluonic and quark degrees of freedom are integrated out. Our ignorance of the details of the latter step is reflected by the appearance of coupling constants in the effective Lagrangian which are not fixed by symmetry requirements alone and which, although determined in principle by the confinement dynamics of QCD, are not yet calculable. Only in a few cases some of these constants have been determined semi-phenomenologically using QCD duality sum rules [8] which relate the long-distance hadronic realization of two-point functions to their calculable short-distance behaviour [9].

The explicit structure of the effective chiral Lagrangian for strong and electromagnetic interactions to fourth order in derivatives and masses is now completely known through the work of Gasser and Leutwyler [10]. Some progress in including weak perturbations was discussed in Ref. [11]. Recent applications of the effective chiral Lagrangian approach to rare K decays include $K_{L,S} \rightarrow \gamma\gamma$ [12], $K^+ \rightarrow \pi^+ \ell^+ \ell^-$ and $K_S \rightarrow \pi^0 \ell^+ \ell^-$ [11], $K_L \rightarrow \pi^0 \ell^+ \ell^-$ [13] and $K_{L,S} \rightarrow \pi^0 \gamma\gamma$ [14,15]. Some aspects of the phenomenology of CP violation have also been dis-

cussed within the effective chiral Lagrangian framework, sometimes combined with $1/N_C$ -expansion techniques [16].

The main purpose of this paper is to continue the investigation of K decays involving two either real or virtual photons. In Sect. 2, we review the treatment of electroweak perturbations to the strong effective chiral Lagrangian. The counterterms relevant for a one-loop calculation of the processes under consideration are presented. The simultaneous diagonalization of the covariant kinetic and mass terms quadratic in pseudoscalar fields is performed explicitly to $O(G_F)$. The use of this diagonal basis leads to a considerable simplification in the calculation of one-loop diagrams compared to the standard basis. In Sect. 3, we calculate the one-loop amplitude for the transition $K_1^0 \rightarrow \gamma^* \gamma$ with one photon off-shell. This amplitude is needed to obtain the spectrum and the rate of the decays $K_1^0 \rightarrow \gamma \ell^+ \ell^-$. A comparison with an earlier dispersion theoretic analysis of Sehgal [17] shows that a precision experiment for the muon channel will provide a test of the constraints of softly broken chiral symmetry. The decays $K_2^0 \rightarrow \gamma \ell^+ \ell^-$ proceed via the WZ term. The problems involved in a systematic calculation of CP violating effects in $K^0(\bar{K}^0) \rightarrow \gamma \gamma$ decays are emphasized. The decays $K^\pm \rightarrow \pi^\pm \gamma \gamma$ are considered in Sect. 4. The one-loop amplitude is finite, but there are in addition tree level contributions involving the WZ term on the one hand and dimension-four counterterms on the other hand. Because of the appearance of a previously undetermined counterterm coupling constant, chiral perturbation theory does not predict the absolute rate but only a lower bound. There is, however, a correlation between the total rate and the corresponding spectrum in the $\gamma \gamma$ -invariant mass. The charge asymmetry $\Gamma(K^+ \rightarrow \pi^+ \gamma \gamma) - \Gamma(K^- \rightarrow \pi^- \gamma \gamma)$ is derived and an estimate of its numerical value is made by relating the octet counterterm in the chiral Lagrangian to the electromagnetic penguin operator of Gilman and Wise [18] to leading order in $1/N_C$. The decay $K_L \rightarrow \pi^0 \ell^+ \ell^-$ is discussed in Sect. 5. There are both a CP violating one-photon exchange amplitude and a CP conserving contribution from two-photon exchange. From a calculation of the two-photon absorptive part we confirm the expected dominance [13] of the CP violating amplitude for $K_L \rightarrow \pi^0 e^+ e^-$ although our results are rather different from those of Ref. [13]. The one-photon exchange amplitude with intrinsic CP violation can be related to the charge asymmetry $\Gamma(K^+ \rightarrow \pi^+ e^+ e^-) - \Gamma(K^- \rightarrow \pi^- e^+ e^-)$. For $K_L \rightarrow \pi^0 \mu^+ \mu^-$, the CP conserving and the CP forbidden amplitudes are roughly comparable leading to a non-vanishing transverse muon polarization. Sect. 6 contains a summary of our results. The kinematics of $K \rightarrow \gamma \gamma$ and $K \rightarrow \pi \gamma \gamma$ decays is compiled in an appendix.

2. ELECTROWEAK PERTURBATIONS TO THE STRONG EFFECTIVE CHIRAL LAGRANGIAN

The effective chiral Lagrangian we shall be concerned with has the following structure

$$\mathcal{L}_{st} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{em} + \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 (\mathcal{L}_{\Delta S=1} + \mathcal{L}_{\Delta S=1}^{em}) \quad (2.1)$$

where \mathcal{L}_{st} describes the strong interactions among the octet of pseudoscalar mesons, $F_{\mu\nu}$ is the electromagnetic tensor, \mathcal{L}_{em} denotes the hadronic electromagnetic interaction Lagrangian, $\mathcal{L}_{\Delta S=1}$ the non-leptonic strangeness changing weak interactions and $\mathcal{L}_{\Delta S=1}^{em}$ the same weak perturbation in the presence of electromagnetic interactions. The weak perturbation is modulated by an overall coupling where G_F denotes the Fermi constant and $s_1 c_1 c_3$ the product of Kobayashi-Maskawa (KM) [19] matrix elements $V_{ud} V_{us}^*$ for three generations. The specific form of the various terms in (2.1) has already been discussed in Ref. [11], but we reproduce them here for the sake of completeness.

With

$$U(x) = \exp\left(\frac{i}{f_\pi} \sum_{i=1}^8 \lambda_i \varphi_i(x)\right) \quad (2.2)$$

the 3×3 special unitary matrix which incorporates the eight pseudoscalar mesons appearing as Goldstone coordinate fields in the matrix representation

$$\Phi \equiv \frac{1}{\sqrt{2}} \sum_{i=1}^8 \lambda_i \varphi_i = \begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix}, \quad (2.3)$$

we have

$$\mathcal{L}_{st} = \frac{f_\pi^2}{4} \text{tr}(\partial_\mu U \partial^\mu U^\dagger) + v \text{tr}(\mathcal{M}U + U^\dagger \mathcal{M}) + \text{higher order terms}, \quad (2.4)$$

where $f_\pi = 93.3$ MeV (to lowest order in chiral perturbation theory), \mathcal{M} denotes the diagonal quark mass matrix $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$ and

$$v = \frac{f_\pi^2 m_{\pi^+}^2}{2(m_u + m_d)} = \frac{f_\pi^2 M_{K^+}^2}{2(m_u + m_s)} = \frac{f_\pi^2 M_{K^0}^2}{2(m_d + m_s)}. \quad (2.5)$$

In (2.4) and more generally in what follows, higher order terms involve more derivatives, quark masses or the electromagnetic potential A_μ ($F_{\mu\nu} = \partial_\mu A_\nu -$

$\partial_\nu A_\mu$). For obvious reasons, the terms explicitly written in (2.4) will be referred to as terms $O(p^2)$. The complete list of terms $O(p^4)$ relevant for the effective Lagrangian \mathcal{L}_{st} to one-loop accuracy can be found in Ref. [10].

With the matrix representation of the electric charge operator

$$Q = \frac{1}{2}(\lambda_3 + \lambda_8/\sqrt{3}) = \hat{Q} - \frac{1}{3}\mathbf{1}, \quad \hat{Q} = \text{diag}(1, 0, 0), \quad (2.6)$$

the hadronic electromagnetic interaction to lowest order $O(p^2)$ has the form

$$\mathcal{L}_{em} = -e A_\mu \text{tr}(\hat{Q} V^\mu) + \frac{e^2 f_\pi^2}{2} A_\mu A^\mu (1 - |U_{11}|^2) \quad (2.7)$$

where V_μ denotes the vector current

$$V_\mu = \frac{i}{2} f_\pi^2 [U, \partial_\mu U^\dagger] = i(\Phi \overleftrightarrow{\partial}_\mu \Phi) - \frac{i}{6 f_\pi^2} [\Phi, [\Phi, \Phi \overleftrightarrow{\partial}_\mu \Phi]] + \dots \quad (2.8)$$

The explicit form of the weak perturbation is most conveniently expressed in terms of the 3×3 -matrix L_μ representing the octet of V-A Noether currents, i.e.,

$$L_\mu = i f_\pi^2 U \partial_\mu U^\dagger. \quad (2.9)$$

To lowest order, $\mathcal{L}_{\Delta S=1}$ is dominated by a term transforming as $8_L \otimes 1_R$ under chiral $SU(3)$ rotations (octet or $\Delta I = 1/2$ enhancement). It has the general structure

$$\mathcal{L}_{\Delta S=1} = g_8 (L_\mu L^\mu)_{23} + \text{h.c.} + \text{non-octet terms} + \text{higher order terms} \quad (2.10)$$

where g_8 is a dimensionless coupling constant. From $K \rightarrow \pi\pi$ decays, one finds

$$|g_8| \simeq 5.1. \quad (2.11)$$

In the following, we shall often use the short-hand notation

$$G_8 = \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 g_8. \quad (2.12)$$

Throughout this paper, we shall neglect the non-octet part of the $\Delta S = 1$ non-leptonic weak interactions.

With the matrix

$$\Delta = U[\hat{Q}, U^\dagger] \quad (2.13)$$

the electromagnetically induced $\Delta S = 1$ non-leptonic Lagrangian from the octet term in (2.10) has the following form to lowest order $O(p^2)$:

$$\mathcal{L}_{\Delta S=1}^{em} = eg_8 f_\pi^2 A_\mu \{L^\mu, \Delta\}_{23} + e^2 g_8 f_\pi^4 A_\mu A^\mu (\Delta^2)_{23} + h.c.. \quad (2.14)$$

It is a common feature of non-leptonic K decays with at most one pion in the final state that the corresponding amplitudes vanish to lowest order in chiral perturbation theory, i.e. to order $O(p^2)$. This is obviously the case for K^0 decays because the photon does not couple directly to neutral particles. At first sight, it is much less obvious for $K^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ [11] and for $K^\pm \rightarrow \pi^\pm \gamma \gamma$ to be discussed in Sect. 4. As emphasized in Refs. [11,14], the vanishing of amplitudes in lowest order is due to a mismatch between the minimum number of powers of external momenta required by gauge invariance and the powers of momenta that the lowest order effective chiral Lagrangian can provide. The same mechanism is operative at the one-loop level for $K_2^0 \rightarrow \pi^0 \gamma \gamma$ [14] and for $K^\pm \rightarrow \pi^\pm \gamma \gamma$ in eliminating the invariant amplitude B defined in (A.8) of the appendix.

The assertion contained in the first sentence of the preceding paragraph can be proven most directly by performing a simultaneous diagonalization of the covariant kinetic and mass terms quadratic in the pseudoscalar fields. To order G_F , this diagonalization is achieved through the following transformations of pseudoscalar fields

$$\begin{aligned} \pi^+ &\rightarrow \pi^+ - \frac{2M_K^2 f_\pi^2 G_8}{M_K^2 - m_\pi^2} K^+ \\ K^+ &\rightarrow K^+ + \frac{2m_\pi^2 f_\pi^2 G_8^*}{M_K^2 - m_\pi^2} \pi^+ \\ \pi^0 &\rightarrow \pi^0 + \frac{\sqrt{2} M_K^2 f_\pi^2}{M_K^2 - m_\pi^2} (G_8 K^0 + G_8^* \bar{K}^0) \\ K^0 &\rightarrow K^0 - \frac{\sqrt{2} m_\pi^2 f_\pi^2 G_8^*}{M_K^2 - m_\pi^2} \pi^0 + \sqrt{\frac{2}{3}} \frac{M_\eta^2 f_\pi^2 G_8^*}{M_\eta^2 - M_K^2} \eta \\ \eta &\rightarrow \eta - \sqrt{\frac{2}{3}} \frac{M_K^2 f_\pi^2}{M_\eta^2 - M_K^2} (G_8 K^0 + G_8^* \bar{K}^0) \end{aligned} \quad (2.15)$$

with the transformations for π^- , K^- , \bar{K}^0 implied by hermitian conjugation. In this basis, there are no off-diagonal propagators like $K - \pi$ nor vertices of the

type $K\pi\gamma$ because the covariant kinetic terms have been diagonalized. All weak vertices involve at least three pseudoscalar fields which proves our assertion. In addition to its intrinsic interest, the diagonal basis just introduced leads to a considerable simplification in calculating loop diagrams. A rather instructive example is provided by the one-loop calculation of $K_2^0 \rightarrow \pi^0\gamma\gamma$ which involves four Feynman diagrams in the diagonal basis [14], but some additional twenty diagrams in the standard basis [15].

In order to make quantitative predictions for the processes under consideration, we must therefore go to the one-loop level in chiral perturbation theory including the appropriate terms $O(p^4)$ in the effective Lagrangian. This has the important physical implication that one should expect in general a chiral suppression factor of order $M_K^2/16\pi^2 f_\pi^2 \simeq 0.18$ for such amplitudes, which may compensate the underlying non-leptonic $\Delta I = 1/2$ enhancement. Such a suppression is indeed observed in the measured rate for $K^\pm \rightarrow \pi^\pm e^+ e^-$ [11].

The complete list of $O(p^4)$ terms in \mathcal{L}_{em} can be found in Ref. [10]. Two of those counterterms are relevant for our purposes which in the notation of Gasser and Leutwyler read

$$\mathcal{L}_{em}^{(4)} = -ieL_9 F^{\mu\nu} \text{tr}(QD_\mu U D_\nu U^\dagger + QD_\mu U^\dagger D_\nu U) + e^2 L_{10} F^{\mu\nu} F_{\mu\nu} \text{tr}(UQU^\dagger Q) \quad (2.16)$$

with the covariant derivative

$$D_\mu U = \partial_\mu U - ieA_\mu[Q, U]. \quad (2.17)$$

When combined with the lowest order $\Delta S = 1$ Lagrangian (2.10) the counterterm Lagrangian (2.16) gives rise to physical contributions to the various K decays we are concerned with.

Another source of $O(p^4)$ contributions are the possible counterterms in $\mathcal{L}_{\Delta S=1}^{em}$. Those with one explicit factor $F_{\mu\nu}$, which survive all the symmetry constraints of the underlying Standard Model Lagrangian, are [11]

$$\begin{aligned} \mathcal{L}_{\Delta S=1,em}^{(4)} = & -\frac{ieg_8}{2f_\pi^2} F^{\mu\nu} \{w_1 \text{tr}(Q\lambda_{6-i7}\mathcal{L}_\mu\mathcal{L}_\nu) + w_2 \text{tr}(Q\mathcal{L}_\mu\lambda_{6-i7}\mathcal{L}_\nu) + \\ & + \bar{w}_3 \epsilon_{\mu\nu\rho\sigma} \text{tr}(Q\mathcal{L}^\rho) \text{tr}(\lambda_{6-i7}\mathcal{L}^\sigma)\} + h.c. \end{aligned} \quad (2.18)$$

where \mathcal{L}_μ denotes the V-A current L_μ in (2.9) with the ordinary derivative replaced by the covariant derivative (2.17). In addition to the constraints of chiral symmetry, a discrete symmetry of the $\Delta S = 1$ Lagrangian at the quark level called CPS [20] is essential in eliminating [11] the counterterms of opposite

parity to (2.18). The term proportional to \tilde{w}_3 does not contribute to processes $K \rightarrow \pi\gamma^*$ because there are only two independent momenta available. It does not contribute to $K^0 \rightarrow \gamma\gamma$ or $K \rightarrow \pi\gamma\gamma$, either, because the leading terms (linear in pseudoscalar fields) for both $\text{tr}(Q\mathcal{L}^\rho)$ and $\text{tr}(\lambda_{6-i7}\mathcal{L}^\sigma)$ do not contain the photon field. It will contribute, however, to processes like $K \rightarrow \pi\pi\gamma$ which will be discussed elsewhere.

The Lagrangian (2.18) contains in particular the leading effective chiral realization of the so-called electromagnetic penguin operator appearing at the fundamental quark level. The leading diagram contributing to the electromagnetic penguin operator [18]

$$Q_7 = \alpha \bar{s} \gamma^\mu (1 - \gamma_5) d \bar{\ell} \gamma_\mu \ell \quad (2.19)$$

is shown in Fig. 1. The corresponding local counterterm in the chiral realization must transform as an $SU(3)$ octet and is therefore given by the term in (2.18) proportional to w_1 (the w_2 term transforms as a mixture of 10 and $\bar{10}$). The Wilson coefficient $C_7(\mu^2)$ pertaining to Q_7 is complex because of the CP violating phase in the KM matrix. This in turn implies a complex coupling $g_8 w_1$ with important implications for the phenomenology of CP violation as we shall see.

To the terms in (2.18) we have to add possible $O(p^4)$ terms with two explicit factors $F_{\mu\nu}$. As discussed in Ref. [14], a single term

$$\mathcal{L}_{\Delta S=1,em^2}^{(4)} = \frac{e^2 f_\pi^2}{2} g_8 w_4 F^{\mu\nu} F_{\mu\nu} \text{tr}(\lambda_{6-i7} Q U Q U^\dagger) + h.c. \quad (2.20)$$

survives the constraints of chiral and CPS symmetry. This completes the structure of the effective chiral Lagrangian at the $O(p^4)$ level necessary for a consistent one-loop calculation.

In the diagonal basis of pseudoscalar fields implemented by the transformations (2.15), the weak cubic vertices are as usual given by the lowest order term in (2.10). A little more care is required for the weak quartic vertices. The quartic terms relevant for the one-loop calculation of $K_2^0 \rightarrow \pi^0 \gamma \gamma$ can be found in Ref. [14]. Here, we give the corresponding terms which are needed for the one-loop calculation of $K^\pm \rightarrow \pi^\pm \gamma \gamma$ transitions in the form

$$\tilde{\mathcal{L}}_{\Delta S=1}^{eff} = \frac{G_8}{3} (\mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3) + h.c. \quad (2.21)$$

\mathcal{L}_i contains the relevant quartic terms of the usual $\Delta S = 1$ Lagrangian (2.10)

replacing L_μ by \mathcal{L}_μ :

$$\begin{aligned} \mathcal{L}_1 = & \pi^- D^\mu K^+ (\pi^- D_\mu \pi^+ - 3\pi^+ D_\mu \pi^- + K^- D_\mu K^+ + K^+ D_\mu K^-) + \\ & + K^+ D^\mu \pi^- (\pi^+ D_\mu \pi^- + \pi^- D_\mu \pi^+ + K^+ D_\mu K^- - 3K^- D_\mu K^+) + \\ & + \text{terms with neutral fields.} \end{aligned} \quad (2.22)$$

The component \mathcal{L}_2 is generated by the transformations (2.15) applied to the terms with two derivatives in (2.4) again replacing ordinary derivatives by covariant ones:

$$\mathcal{L}_2 = -2K^+ \overleftrightarrow{D}^\mu \pi^- (K^+ \overleftrightarrow{D}_\mu K^- + \pi^+ \overleftrightarrow{D}_\mu \pi^-) + \text{terms with neutral fields.} \quad (2.23)$$

In the same way, \mathcal{L}_3 is obtained from the lowest order mass terms in (2.4):

$$\mathcal{L}_3 = -K^+ \pi^- (M_K^2 K^+ K^- + m_\pi^2 \pi^+ \pi^-) + \text{terms with neutral fields.} \quad (2.24)$$

Finally, to calculate the amplitudes for $K \rightarrow \pi\gamma\gamma$ transitions to $O(p^4)$ we also need the anomalous WZ terms linear in meson fields with the familiar form [21]

$$\mathcal{L}_{WZ} = \frac{\alpha}{8\pi f_\pi} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} (\pi^0 + \eta/\sqrt{3}). \quad (2.25)$$

This completes the description of the explicit form of the effective Lagrangian required to perform the calculation of the various K decays discussed in this paper to order p^4 .

3. $K_{L,S} \rightarrow \gamma\ell^+\ell^-$ DECAYS

A study of the decays $K_{L,S} \rightarrow \gamma + \text{Dalitz pair}$ was made by Sehgal some time ago [17]. He emphasized in particular the fact that these processes may act as probes of the dynamical structure that underlies the $K_{L,S} \rightarrow \gamma\gamma$ vertices. The purpose of this section is to spell out the particular structure implied by chiral perturbation theory and thus by QCD. Unless otherwise stated, we shall assume CP invariance in this section.

3a. The $K_1^0 \rightarrow \gamma\ell^+\ell^-$ Transition

The relevant Feynman diagram is shown in Fig. 2. The transition amplitude reads

$$A(K_1^0 \rightarrow \gamma\ell^+\ell^-) = \frac{e}{q_2^2 + i\epsilon} M^{\mu\nu}(q_1, q_2) \epsilon_\mu(q_1) \bar{u}(k) \gamma_\nu v(k') \quad (3.1)$$

where $M^{\mu\nu}(q_1, q_2)$ is the amplitude defined in Equ. (A.5) of the appendix. For the $K_1^0 \rightarrow \gamma \ell^+ \ell^-$ transition, only the invariant amplitude $b(0, q_2^2)$ contributes. To lowest non-trivial order $O(p^4)$, this amplitude is uniquely determined by a one-loop calculation of the $K_1^0 \rightarrow \gamma^* \gamma$ transition with one photon off-shell. Referring to Ref. [12] for the relevant diagrams, we only give the final result in the form

$$b(0, q_2^2) = \frac{2\sqrt{2}G_8\alpha f_\pi}{\pi}(1 - r_\pi^2)H(z) \quad (3.2)$$

where

$$H(z) = \int_0^1 dx \int_0^{1-x} dy \frac{xy}{r_\pi^2 - (1-z)xy - zy(1-y)}, \quad (3.3)$$

$$z = \frac{q_2^2}{M_K^2}, \quad r_\pi = \frac{m_\pi}{M_K}, \quad r_\ell = \frac{m_\ell}{M_K},$$

$$4r_\ell^2 \leq z \leq 1.$$

The differential decay rate in terms of the normalized invariant mass squared of the Dalitz pair can then be written most conveniently as

$$\frac{d\Gamma(K_1^0 \rightarrow \gamma \ell^+ \ell^-)}{dz} = \Gamma(K_1^0 \rightarrow \gamma\gamma) \frac{2}{z}(1-z)^3 \left| \frac{H(z)}{H(0)} \right|^2 \frac{\text{Im} \Pi(z)}{\pi} \quad (3.4)$$

where $\text{Im} \Pi(z)/\pi$ is the electromagnetic spectral function associated to the lepton pair

$$\frac{1}{\pi} \text{Im} \Pi(z) = \frac{\alpha}{3\pi} (1 + 2r_\ell^2/z) \sqrt{1 - 4r_\ell^2/z} \theta(z - 4r_\ell^2). \quad (3.5)$$

The specific dynamical behaviour predicted by chiral perturbation theory is contained in the structure function $H(z)$. The other factors in (3.4) are of kinematical origin. The value of $H(z)$ at $z = 0$ governs the amplitude for $K_1^0 \rightarrow \gamma\gamma$ in chiral perturbation theory [12]. The corresponding decay rate is

$$\Gamma(K_1^0 \rightarrow \gamma\gamma) = \frac{|G_8\alpha|^2 f_\pi^2}{4\pi^3} M_K^3 (1 - r_\pi^2)^2 |H(0)|^2. \quad (3.6)$$

The function $H(z)$ can be written in the form

$$H(z) = \frac{1}{2(1-z)^2} \left\{ zF\left(\frac{z}{r_\pi^2}\right) - F\left(\frac{1}{r_\pi^2}\right) - 2z \left[G\left(\frac{z}{r_\pi^2}\right) - G\left(\frac{1}{r_\pi^2}\right) \right] \right\} \quad (3.7)$$

with [14]

$$F(z) = \begin{cases} 1 - 4[\arcsin(\sqrt{z}/2)]^2/z & z \leq 4 \\ 1 - \frac{\pi^2}{z} + \left(\ln \frac{1 - \sqrt{1 - 4/z}}{1 + \sqrt{1 - 4/z}}\right)^2/z + \frac{2i\pi}{z} \ln \frac{1 - \sqrt{1 - 4/z}}{1 + \sqrt{1 - 4/z}} & z \geq 4 \end{cases} \quad (3.8)$$

and

$$G(z) = \begin{cases} \sqrt{4/z - 1} \arcsin(\sqrt{z}/2) & z \leq 4 \\ \frac{1}{2} \sqrt{1 - 4/z} \left[\ln \frac{1 + \sqrt{1 - 4/z}}{1 - \sqrt{1 - 4/z}} - i\pi \right] & z \geq 4 \end{cases} . \quad (3.9)$$

Numerically, using $|G_8| \simeq 9.1 \cdot 10^{-6} \text{ GeV}^{-2}$, one finds

$$|H(0)| = 0.67 \quad (3.10)$$

leading to a decay rate

$$\Gamma(K_1^0 \rightarrow \gamma\gamma) = 1.4 \cdot 10^{-20} \text{ GeV}, \quad (3.11)$$

which corresponds to a branching ratio

$$\frac{\Gamma(K_1^0 \rightarrow \gamma\gamma)}{\Gamma(K_S \rightarrow \text{all})} = 2.0 \cdot 10^{-6} \quad (3.12)$$

to be compared with a recent measurement by the NA31 collaboration at CERN [22]:

$$B(K_S \rightarrow \gamma\gamma) = (2.4 \pm 1.2) \cdot 10^{-6}. \quad (3.13)$$

The decay rates for the Dalitz pair modes normalized to $\Gamma(K_1^0 \rightarrow \gamma\gamma)$ are compared in Table 1 with the phase space predictions and with the results of the dispersion model of Sehgal [17]. In Fig. 3 we also show the normalized spectrum for the $\mu^+\mu^-$ case. In the case of e^+e^- there is practically no difference between the spectrum predicted by chiral dynamics and phase space [$H(z) = H(0)$] because the spectrum is very much peaked towards small z . This can also be seen from the rates in Table 1 which are practically identical in all three cases. There is, however, a non-negligible difference for the muon pair spectrum between chiral perturbation theory, phase space and the model of Sehgal which should be

distinguishable by high-precision experiments. The dispersion model of Ref. [17], which is inconsistent with softly broken chiral symmetry, has an even sharper spectrum than the one shown in Fig. 3 corresponding to a smaller total rate compared to the chiral prediction.

3b. The $K_2^0 \rightarrow \gamma\ell^+\ell^-$ Transition

The relevant Feynman diagram is shown in Fig. 4. The amplitude structure is the same as given in (3.1) except that now the invariant amplitude $c(0, q_2^2)$ defined in the appendix contributes to $K_2^0 \rightarrow \gamma^*\gamma$. The coupling of the two photons to π^0 and η is governed by the WZ term (2.25). To lowest order in chiral perturbation theory, $c(0, q_2^2)$ vanishes because the π^0 and η contributions cancel exactly because of the Gell-Mann-Okubo relation for the pseudoscalar meson masses squared. A leading log calculation [23] has found the $K_2^0 \rightarrow \pi^0, \eta$ amplitude to be rather sensitive to $SU(3)$ breaking. There are also contributions from $\eta - \eta'$ mixing [24] and from possible $O(p^4)$ counterterms with unknown couplings which have not been considered so far. However, it is obvious that the same problems appear in trying to evaluate the rate for $K_2^0 \rightarrow \gamma\ell^+\ell^-$. Therefore, if we normalize to the $K_2^0 \rightarrow \gamma\gamma$ decay rate we find

$$\frac{d\Gamma(K_2^0 \rightarrow \gamma\ell^+\ell^-)}{dz} = \Gamma(K_2^0 \rightarrow \gamma\gamma) \frac{2}{z} (1-z)^3 \frac{\text{Im} \Pi(z)}{\pi} \quad (3.14)$$

which is, of course, identical to the phase space spectrum. Possible form factor effects in q_2^2 are still higher order in chiral perturbation theory. To lowest non-trivial order, the normalized rates $\Gamma(K_2^0 \rightarrow \gamma\ell^+\ell^-)/\Gamma(K_2^0 \rightarrow \gamma\gamma)$ are therefore predicted to be identical to the phase space values for $\Gamma(K_1^0 \rightarrow \gamma\ell^+\ell^-)/\Gamma(K_1^0 \rightarrow \gamma\gamma)$ given in Table 1.

Possible CP violation effects in $K_{L,S} \rightarrow \gamma\ell^+\ell^-$ are most likely governed by the CP behaviour of the $K_{L,S} \rightarrow \gamma\gamma$ transition. There are several phenomenological analyses of CP violation in $K_{L,S} \rightarrow \gamma\gamma$ decays [25]. The authors of Ref. [25b] emphasized in particular the interest of measuring the asymmetry of intensities

$$\frac{\Gamma(K^0 \rightarrow \gamma\gamma) - \Gamma(\bar{K}^0 \rightarrow \gamma\gamma)}{\Gamma(K^0 \rightarrow \gamma\gamma) + \Gamma(\bar{K}^0 \rightarrow \gamma\gamma)} \quad (3.15)$$

as a function of the K -proper time. This is precisely the type of experiment which twenty years later may become feasible with the LEAR facility at CERN [26]. Unfortunately, it is difficult to make a precise prediction of this asymmetry in the Standard Model. In the effective chiral Lagrangian approach, the problems in calculating the $K_2^0 \rightarrow \gamma\gamma$ amplitude were discussed in the first paragraph

of this subsection. In particular, the coupling constants of the so far neglected $O(p^4)$ counterterms contributing to the $K_2^0 \rightarrow \gamma\gamma$ transition could be complex giving rise to an intrinsic CP violation different from the usual mixing effect. It is not at all obvious how to relate the imaginary parts of those coupling constants to the underlying quark flavour mixing pattern. Several predictions of the asymmetry (3.15) have nevertheless been attempted in the literature [25c].

4. $K^\pm \rightarrow \pi^\pm \gamma\gamma$ TRANSITIONS AND CP VIOLATION

The general structure of this process in terms of invariant amplitudes is given in the appendix. To lowest non-trivial order $O(p^4)$ in chiral perturbation theory only the invariant amplitudes A and C contribute. The dominant amplitude A is calculated from the loop diagrams in Fig. 5 and from the tree level counterterms discussed in Sect. 2 with the result

$$A(x_1, x_2) \equiv A(z) = \frac{G_8 \alpha}{2\pi z} [(r_\pi^2 - 1 - z)F(\frac{z}{r_\pi^2}) + (1 - z - r_\pi^2)F(z) + \hat{c}z] \quad (4.1)$$

$$z = \frac{(q_1 + q_2)^2}{M_K^2} = 2(x_1 + x_2 - \frac{1}{2}) + r_\pi^2, \quad r_\pi = \frac{m_\pi}{M_K}$$

with

$$\hat{c} = 32\pi^2 [4(L_9 + L_{10}) - \frac{1}{3}(w_1 + 2w_2 + 2w_4)]. \quad (4.2)$$

The function F was defined in (3.8) and its behaviour in the physical region for $K \rightarrow \pi\gamma\gamma$ decays can be seen in Fig. 6. It is evident that the pion loop contribution $(r_\pi^2 - 1 - z)F(z/r_\pi^2)$ dominates by far over the kaon loop amplitude $(1 - z - r_\pi^2)F(z)$. It is for this reason that in a process like $K_2^0 \rightarrow \pi^0\gamma\gamma$ where $\hat{c} = 0$ [14] the prediction of chiral perturbation theory comes very close to the result of a dispersion theoretic model using the pion loop contribution only [27]. The analytic form of the amplitude is, however, different in the two approaches because only chiral perturbation theory incorporates the restrictions dictated by softly broken chiral symmetry.

It may be instructive to discuss the loop contributions to the amplitude (4.1) in terms of the Feynman diagrams of Fig. 5. Of all these diagrams only 5a and 5b give a contribution to the absorptive part of A :

$$\frac{1}{\pi} \text{Im} A(z) = \frac{G_8 \alpha}{2\pi z} [(r_\pi^2 - 1 - z) \frac{\text{Im} F(z/r_\pi^2)}{\pi} + (1 - z - r_\pi^2) \frac{\text{Im} F(z)}{\pi}]. \quad (4.3)$$

Since for $z \rightarrow \infty$

$$\text{Im } A(z) = O\left(\frac{\log z}{z}\right), \quad (4.4)$$

the corresponding unsubtracted dispersive amplitude

$$\tilde{A}(z) = P \int_{4r_+^2}^{\infty} \frac{dz'}{z' - z} \frac{\text{Im } A(z')}{\pi} \quad (4.5)$$

converges. In full generality, the physical amplitude $A(z)$, to the order $O(p^4)$ we are working, will be

$$A(z) = \tilde{A}(z) + i \text{Im } A(z) + \frac{G_8 \alpha}{2\pi} \hat{c} \quad (4.6)$$

where \hat{c} is a possible constant arising from contact terms without discontinuities. It is now rather straightforward to convince oneself that \hat{c} does not receive any contributions from the loop diagrams of Fig. 5. The reason is that only diagrams 5a and 5b can produce the tensor structures $q_{2\mu} q_{1\nu}$ and $q_1 \cdot q_2 g_{\mu\nu}$ necessary for amplitude A [cf. Equ. (A.8)]. The diagram 5c gives a divergent constant of dimension mass^2 times $g_{\mu\nu}$ which cancels with similar non-gauge invariant pieces from the other diagrams. Diagrams 5d and 5e, on the other hand, produce a structure proportional to $p_\mu p'_\nu$ which is again cancelled by the tadpole diagrams not shown in Fig. 5, because the invariant amplitude $B(x_1, x_2)$ of (A.8) vanishes at the one-loop level. Thus, \hat{c} can only be due to the local counterterms of Sect. 2 as explicitly demonstrated in (4.2). Furthermore, since the dispersive amplitude $\tilde{A}(z)$ in (4.5) is convergent the constant \hat{c} must be renormalization scale invariant.

What do we know theoretically about the value of the constant \hat{c} ? The combination $L_9 + L_{10}$ is a renormalization scale invariant coupling constant which is rather well determined from the so-called structure term in $\pi \rightarrow e\nu\gamma$ [10]:

$$L_9 + L_{10} = (1.39 \pm 0.38) \cdot 10^{-3}. \quad (4.7)$$

If we could neglect in \hat{c} the genuinely weak contribution $w_1 + 2w_2 + 2w_4$, which is, of course, also renormalization scale invariant, we would obtain $\hat{c} \simeq 2$. Thus, we may expect $\hat{c} = O(1)$ as a reasonable order of magnitude guess.

There is, in fact, another combination of counterterm coupling constants contained in (4.2) which we have already determined previously [11], i.e.,

$$\tilde{w}_+^r \equiv w_+ + \frac{1}{3} \log \frac{M_K m_\pi}{M_\pi^2} = -\frac{16\pi^2}{3} [w_1 - w_2 + 3(w_2 - 4L_9)]. \quad (4.8)$$

From the measured rate $\Gamma(K^+ \rightarrow \pi^+ e^+ e^-)$ [28] we obtained two possible solutions [11] renormalized at M_η

$$\tilde{w}_+^r = \begin{cases} 0.66 \pm 0.08 \\ -1.07 \pm 0.08 \end{cases}, \quad (4.9)$$

which are again consistent with the estimate $\hat{c} = O(1)$. Unfortunately, however, the coupling constant w_4 is unknown and we can only discuss physical quantities in terms of the parameter \hat{c} .

We next describe the calculation of the amplitude $C(x_1, x_2)$ defined in (A.8). To $O(p^4)$, this amplitude is due to the Feynman diagram shown in Fig. 7 involving the anomalous couplings (2.25). There are in principle two different weak vertices in Fig. 7. Since on-shell the decay $K^+ \rightarrow \pi^+ \pi^0$ can only proceed via the 27-plet part of the non-leptonic weak interactions, the dominant contribution to the amplitude C actually comes from this 27-plet which we have always been neglecting. However, although the corresponding branching ratio

$$B(K^+ \rightarrow \pi^+ \pi^0 \rightarrow \pi^+ \gamma \gamma) = B(K^+ \rightarrow \pi^+ \pi^0) \cdot B(\pi^0 \rightarrow 2\gamma) = 0.21 \quad (4.10)$$

is huge, the contribution to the spectrum is very much concentrated at $z = r_\pi^2 \simeq 0.08$ and can therefore easily be cut away. For larger z where the loop amplitude A will be concentrated, as we shall soon discuss, the contribution of the 27-plet is as usually negligible.

There is nevertheless a small, but non-negligible contribution from the octet vertex in Fig. 7:

$$C(x_1, x_2) \equiv C(z) = \frac{G_8 \alpha}{\pi} \left[\frac{z - r_\pi^2}{z - r_\pi^2 + i r_\pi \Gamma_{\pi^0} / M_K} - \frac{z - \frac{2+r_\pi^2}{3}}{z - r_\eta^2} \right], \quad (4.11)$$

$$r_\eta = M_\eta / M_K.$$

The first contribution in (4.11), due to the pion pole, can serve as an instructive example of the relevance of chiral symmetry for the off-shell behaviour of weak vertices. Because of the derivative couplings in the chiral vertices there is a non-vanishing octet contribution when the π^0 is off-shell. This term is essentially constant for almost all z except at $z = r_\pi^2$. It would not be present in a dispersive approach where vertices are usually constructed without derivatives.

From (4.1) and (4.11) we find for the differential decay rate in terms of the normalized invariant mass² z of the photon pair

$$\frac{d\Gamma(K^+ \rightarrow \pi^+ \gamma \gamma)}{dz} = \frac{M_{K^+}^5}{2(8\pi)^3} \lambda^{1/2}(1, z, r_\pi^2) z^2 \{ |A(z)|^2 + |C(z)|^2 \}, \quad (4.12)$$

$$0 \leq z \leq (1 - r_\pi)^2$$

with

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ca). \quad (4.13)$$

Writing the integrated rate in the form

$$\Gamma(K^+ \rightarrow \pi^+ \gamma \gamma) = \Gamma_{loop} + \Gamma_{WZ} \quad (4.14)$$

we obtain

$$\begin{aligned} \Gamma_{loop} &= (2.80 + 0.87\hat{c} + 0.17\hat{c}^2) \cdot 10^{-23} \text{ GeV} \\ \Gamma_{WZ} &= 0.26 \cdot 10^{-23} \text{ GeV} \end{aligned} \quad (4.15)$$

showing the dominance of the loop over the anomalous contribution. The total rate as a function of \hat{c} and the absorptive part are shown in Fig. 8. From (4.14) and (4.15) one derives the lower bound

$$\Gamma(K^+ \rightarrow \pi^+ \gamma \gamma) \geq 2 \cdot 10^{-23} \text{ GeV}$$

or

$$B(K^+ \rightarrow \pi^+ \gamma \gamma) \geq 4 \cdot 10^{-7}, \quad (4.16)$$

well below the present experimental upper limit [29]

$$B(K^+ \rightarrow \pi^+ \gamma \gamma) \leq 8 \cdot 10^{-6}. \quad (4.17)$$

The predicted spectrum (4.12) is very characteristic. It is shown in Fig. 9 for three values of \hat{c} which cover a reasonable range for this parameter together with the phase space prediction. The peaking of the distribution at large z is due to the rapidly rising absorptive part of the $\pi\pi$ intermediate state (see also Fig. 6). Although, unlike $\Gamma(K_2^0 \rightarrow \pi^0 \gamma \gamma)$ [14,15], chiral perturbation theory cannot predict the rate $\Gamma(K^+ \rightarrow \pi^+ \gamma \gamma)$, it gives, up to a twofold ambiguity, a precise correlation between the rate and the spectrum.

So far, we have discussed the decay $K^+ \rightarrow \pi^+ \gamma \gamma$ only. The amplitude for $K^- \rightarrow \pi^- \gamma \gamma$ can be obtained from (4.1) and (4.11) by replacing G_8 and \hat{c} by their complex conjugates. The interference of $\text{Im } \hat{c}$ with the CP invariant absorptive amplitude (4.3) generates a charge asymmetry

$$\begin{aligned} &\Gamma(K^+ \rightarrow \pi^+ \gamma \gamma) - \Gamma(K^- \rightarrow \pi^- \gamma \gamma) = \\ &= \text{Im } \hat{c} \frac{|G_8 \alpha|^2 M_{K^+}^5}{(4\pi)^5} \int_{4r_\pi^2}^{(1-r_\pi)^2} dz \lambda^{1/2}(1, z, r_\pi^2) (r_\pi^2 - 1 - z) z \text{Im } F\left(\frac{z}{r_\pi^2}\right) = \end{aligned}$$

$$= 1.5 \text{Im} \hat{c} \cdot 10^{-23} \text{ GeV}. \quad (4.18)$$

In order to obtain an estimate of $\text{Im} \hat{c}$ we have to investigate the various counterterm coupling constants entering \hat{c} as given in (4.2). It is obvious that L_9 and L_{10} must be real because they derive from the strong + electromagnetic sector of the Standard Model. The genuinely weak $O(p^4)$ coupling constants w_1 , w_2 , w_4 will in general receive both long-distance and short-distance contributions. The long-distance contributions originate in amplitudes where the weak octet vertices governed by the constant g_8 are embedded in diagrams with additional strong and electromagnetic vertices. Clearly, such amplitudes will require counterterms with a phase determined by g_8 , the only weak coupling constant in these diagrams.

Short distance contributions, on the other hand, correspond to diagrams at the quark level where the weak interactions appear through operators different from the non-leptonic weak four-quark operators. The only such operator relevant for us is the Gilman-Wise operator (2.19) in the Lagrangian form (μ is a renormalization scale)

$$\mathcal{L}_{GW} = \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 C_7(\mu^2) \alpha \bar{s} \gamma^\mu (1 - \gamma_5) d \ell \gamma_\mu \ell \quad (4.19)$$

corresponding to the electromagnetic penguin diagram of Fig. 1 in zeroth order QCD with [18]

$$C_7(\mu^2) = \frac{2}{9\pi} \left[\log \frac{m_c^2}{\mu^2} + \tau \log \frac{m_t^2}{m_c^2} \right] \quad (4.20)$$

$$\tau = s_2^2 + s_2 c_2 s_3 e^{-i\delta} / c_1 c_3$$

in terms of the conventional KM parameters.

As already alluded to in Sect. 2, there must be a relation between the octet (4.19) at the fundamental quark level and the chiral counterterm in (2.18) proportional to w_1 which also transforms as an octet. This relation can be made precise in the large N_C approach because to leading order in $1/N_C$

$$\bar{s} \gamma^\mu (1 - \gamma_5) d \hat{=} i f_\pi^2 (U \partial^\mu U^\dagger)_{23}. \quad (4.21)$$

Using the field equations

$$\partial^\mu F_{\mu\nu} = e \ell \gamma_\nu \ell \quad (4.22)$$

and partial integration in the action

$$(U \partial^\mu U^\dagger)_{23} \partial^\nu F_{\nu\mu} \hat{=} - F_{\nu\mu} \partial^\nu (U \partial^\mu U^\dagger)_{23} = F_{\nu\mu} (U \partial^\nu U^\dagger U \partial^\mu U^\dagger)_{23} \quad (4.23)$$

the chiral realization of (4.19) to leading order in $1/N_c$ can be written as

$$\mathcal{L}_{GW}^{chiral} = \frac{3i\alpha C_7(\mu^2)}{2ef_\pi^2} \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 F^{\mu\nu} \text{tr}(Q\lambda_{6-i7}L_\mu L_\nu) \quad (4.24)$$

which has precisely the form of the w_1 term in (2.18) to first order in the electromagnetic field.

One may be tempted to equate the coefficients of (4.24) and of the octet term in (2.18). There is, however, no justification a priori for such an equality because the long-distance contributions to $C_7(\mu^2)$ and to w_1 may very well be completely different. On the other hand, it is reasonable to relate the imaginary parts of the two coefficients to each other because they are obviously short-distance effects¹. In this way we obtain the relation

$$\text{Im}(g_8 w_1) = -\frac{3}{4\pi} \text{Im} C_7 = \frac{c_2 s_2 s_3 \sin \delta}{3\pi^2 c_1 c_3} \log \frac{m_t}{m_c}. \quad (4.25)$$

In order to estimate the charge asymmetry (4.18) we need $\text{Im} \hat{c}$. Since of all the coupling constants in \hat{c} only w_1 gets a short-distance contribution, we have

$$\text{Im} \hat{c} = -\frac{32\pi^2}{3} \text{Im} w_1. \quad (4.26)$$

From the recent precision experiment of ϵ'/ϵ [30] we can estimate $\text{Im} g_8/\text{Re} g_8$. Although not really negligible, it suffices for an order of magnitude estimate of $\text{Im} w_1$ to neglect $\text{Im} g_8$ in (4.25). With $m_t/m_c = 60$ and with [31]

$$\frac{c_2 s_2 s_3 \sin \delta}{c_1 c_3} \simeq 1 \cdot 10^{-3} \quad (4.27)$$

one arrives at the final estimate

$$\begin{aligned} |\text{Im} w_1| &\simeq 3 \cdot 10^{-5} \\ |\text{Im} \hat{c}| &\simeq 3 \cdot 10^{-3}. \end{aligned} \quad (4.28)$$

In view of all the approximations made (in particular the neglect of QCD corrections) the result (4.28) should be considered as a rough order of magnitude estimate only. With (4.28) we can estimate the charge asymmetry (4.18) as

$$|\Gamma(K^+ \rightarrow \pi^+ \gamma\gamma) - \Gamma(K^- \rightarrow \pi^- \gamma\gamma)| \simeq 4 \cdot 10^{-26} \text{ GeV} \quad (4.29)$$

¹Note that $\text{Im} C_7(\mu^2)$ is independent of μ^2 .

and therefore

$$\frac{|\Gamma(K^+ \rightarrow \pi^+ \gamma \gamma) - \Gamma(K^- \rightarrow \pi^- \gamma \gamma)|}{\Gamma(K^+ \rightarrow \pi^+ \gamma \gamma) + \Gamma(K^- \rightarrow \pi^- \gamma \gamma)} \lesssim 1 \cdot 10^{-3} \quad (4.30)$$

because of the lower bound (4.16). A sensitivity of 10^{-10} in branching ratio is necessary to measure the asymmetry (4.29).

5. $K_L \rightarrow \pi^0 \ell^+ \ell^-$ AND CP VIOLATION

In the one-photon exchange approximation, the decay

$$K_1^0 \rightarrow \pi^0 \gamma^* \rightarrow \pi^0 \ell^+ \ell^- \quad (5.1)$$

was calculated in chiral perturbation theory [11], while

$$K_2^0 \rightarrow \pi^0 \gamma^* \rightarrow \pi^0 \ell^+ \ell^- \quad (5.2)$$

is forbidden in the limit of CP conservation. Experimentally, we have at present [29]

$$\begin{aligned} B(K_L \rightarrow \pi^0 e^+ e^-) &< 2.3 \cdot 10^{-6} \\ B(K_L \rightarrow \pi^0 \mu^+ \mu^-) &< 1.2 \cdot 10^{-6} \end{aligned} \quad (5.3)$$

but these upper limits on the branching ratios will be considerably improved in the near future [1]. It is then worthwhile to examine possible signals of CP violation in the $K_L \rightarrow \pi^0 \ell^+ \ell^-$ channels [13]. Here, the crucial question is the size of the CP violating one-photon exchange transition $K_L \rightarrow \pi^0 \gamma^* \rightarrow \pi^0 \ell^+ \ell^-$ as compared to the CP allowed transition $K_2^0 \rightarrow \pi^0 \gamma^* \gamma^* \rightarrow \pi^0 \ell^+ \ell^-$ via two photons. Before examining this question in some detail, we note that there is a "trivial" background

$$K_L \rightarrow \pi^0 \pi^0 \rightarrow \pi^0 e^+ e^- \quad (5.4)$$

with a comparatively big branching ratio

$$B(K_L \rightarrow \pi^0 e^+ e^-) |_{direct} = 3 \cdot 10^{-10}. \quad (5.5)$$

Since this background is strongly peaked in the invariant mass of the electron-positron pair around the pion mass, we shall disregard the amplitude for (5.4) in the following assuming an appropriate cut has been applied.

5a. The $K_L \rightarrow \pi^0 \gamma^* \rightarrow \pi^0 \ell^+ \ell^-$ Transition

The transitions $K^0 \rightarrow \pi^0 \gamma^* \rightarrow \pi^0 \ell^+ \ell^-$ have recently been studied [11] to lowest non-trivial order in chiral perturbation theory. The one-photon exchange amplitude has the form

$$A(K^0(p) \rightarrow \pi^0(p') \ell^+(k') \ell^-(k)) = -\frac{G_8 \alpha}{4\pi} d_{K^0}(z) \bar{u}(k) (\not{p} + \not{p}') v(k') \quad (5.6)$$

where

$$d_{K^0}(z) = \frac{1}{\sqrt{2}} (2\varphi(z) + w_S) \quad (5.7)$$

$$z = \frac{(k + k')^2}{M_K^2}.$$

The function $\varphi(z)$ is given by [11]

$$\begin{aligned} \varphi(z) &= \int_0^1 dx \left[\frac{1}{z} - x(1-x) \right] \log[1 - zx(1-x)] = \\ &= -\frac{1}{6} \left\{ 1 - \frac{z}{10} + O(z^2) \right\} \end{aligned} \quad (5.8)$$

and w_S is the renormalization scale invariant combination

$$w_S = -\frac{16\pi^2}{3} (w_1 - w_2) - \frac{2}{3} \log \frac{M_K}{M_\eta} \quad (5.9)$$

if w_1, w_2 are renormalized at M_η [11]. With CPT assumed, we have

$$A(\bar{K}^0 \rightarrow \pi^0 \ell^+ \ell^-) = \frac{G_8^* \alpha}{4\pi} d_{\bar{K}^0}(z) \bar{u}(k) (\not{p} + \not{p}') v(k') \quad (5.10)$$

where

$$d_{\bar{K}^0}(z) = \frac{1}{\sqrt{2}} [2\varphi(z) + w_S^*] = d_{K^0}(z)^* \quad (5.11)$$

since $\varphi(z)$ is a real function in the kinematic region of interest

$$4r_\ell^2 \leq z \leq (1 - r_\pi)^2 \quad (5.12)$$

$$r_\ell = \frac{m_\ell}{M_K}, \quad r_\pi = \frac{m_\pi}{M_K}.$$

From these equations and with

$$|K_{L,S}\rangle = |K_{2,1}\rangle + \rho |K_{1,2}\rangle \quad (5.13)$$

one obtains

$$A(K_L \rightarrow \pi^0 \ell^+ \ell^-) |_{1\gamma} = - \frac{\text{Re } G_8 \alpha}{4\pi} d_{K_L}(z) \bar{u}(k) (\not{p} + \not{p}') v(k') \quad (5.14)$$

$$d_{K_L}(z) = \epsilon d_{K_S}(z) + i \text{Im } w_S \quad (5.15)$$

$$d_{K_S}(z) \simeq d_{K_1}(z) = 2\varphi(z) + \text{Re } w_S \quad (5.16)$$

where

$$\epsilon = \rho + i \text{Im } G_8 / \text{Re } G_8 \quad (5.17)$$

is the standard CP violation parameter in $K \rightarrow \pi\pi$ decays and CP violation effects of higher order have been neglected in (5.15). We observe the usual appearance of two different sources of CP violation in (5.15): the first term proportional to ϵ is induced by CP violation in the $K^0 - \bar{K}^0$ mass matrix whereas the second term is due to an intrinsic or amplitude CP violation.

If there were no intrinsic CP violation ($\text{Im } w_S = 0$) we would get [11]

$$B(K_L \rightarrow \pi^0 \ell^+ \ell^-) \simeq 3 \cdot 10^{-3} B(K_S \rightarrow \pi^0 \ell^+ \ell^-) \quad (5.18)$$

or

$$B(K_L \rightarrow \pi^0 e^+ e^-) \simeq \begin{cases} 1.5 \cdot 10^{-12} \\ 1.5 \cdot 10^{-11} \end{cases} \quad (5.19)$$

for the two possible values² of w_S [11] with the $\mu^+ \mu^-$ branching ratios roughly a factor five smaller (see, however, Sect. 5d). In order to estimate the effect of amplitude CP violation we recall from Eqs. (4.2), (4.8) and (4.26)

$$\text{Im } w_S = \text{Im } w_+ = \text{Im } \hat{c} / 2. \quad (5.20)$$

Since ϵ has a phase $\simeq \pi/4$, the two terms in (5.15) will interfere and the rates will also depend on the sign of $\text{Im } w_S$. To get an idea of the relative importance of the two sources of CP violation we shall content ourselves to estimate the ratio of absolute values

$$R = \frac{|\text{Im } w_S|}{|\epsilon| (2\varphi(z) + \text{Re } w_S)}. \quad (5.21)$$

In (5.21) a certain average over the possible range (5.12) in z should be taken. However, the function $\varphi(z)$ varies very little over this range as evident from

²Octet dominance for both $K^+ \rightarrow \pi^+ \gamma^*$ and $K^0 \rightarrow \pi^0 \gamma^*$ implies the relation $w_S = w_+ + \frac{1}{3} \log \frac{M_\pi}{M_K}$ [11].

(5.8). Thus, setting $\langle 2\varphi(z) \rangle \simeq -1/3$ and using again the two possible solutions for $\text{Re } w_S$, we find

$$R \simeq \begin{cases} 1.7 \\ 0.5 \end{cases} \quad (5.22)$$

from the estimate (4.28) and (5.20). Thus, we may expect a sizable contribution from amplitude CP violation [13] especially in the case of the smaller branching ratio in (5.19). In view of the approximations that have gone into the estimate (4.28) for $\text{Im } \hat{c}$, it does not seem meaningful at this point to make more detailed predictions including the interference of mass matrix and amplitude CP violation.

Quite independently of these numerical estimates, it is important to notice that in view of (5.20) the same coupling constant responsible for the charge asymmetry (4.18) appears as a source of intrinsic CP violation (an ϵ' -like term) in the $K_L \rightarrow \pi^0 \gamma^* \rightarrow \pi^0 \ell^+ \ell^-$ amplitude. Moreover, the same coupling is also at the origin of a charge asymmetry in the $K^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ decays. With the amplitudes given in Ref. [11], one finds

$$\Gamma(K^+ \rightarrow \pi^+ \ell^+ \ell^-) - \Gamma(K^- \rightarrow \pi^- \ell^+ \ell^-) = \text{Im } w_+ \frac{|G_8 \alpha|^2 M_{K^+}^5}{3\pi(4\pi)^4} \cdot \int_{4r_\ell^2}^{(1-r_\pi)^2} dz \lambda^{3/2}(1, z, r_\pi^2) (1 - 4r_\ell^2/z)^{1/2} (1 + 2r_\ell^2/z) \text{Im } \varphi(z/r_\pi^2) \quad (5.23)$$

where the function φ is defined in (5.8) with

$$\text{Im } \varphi(z/r_\pi^2) = \frac{\pi}{6} (1 - 4r_\pi^2/z)^{3/2} \Theta(z - 4r_\pi^2). \quad (5.24)$$

Specializing to $\ell = e$ where the rate is larger, we get

$$\Gamma(K^+ \rightarrow \pi^+ \ell^+ \ell^-) - \Gamma(K^- \rightarrow \pi^- e^+ e^-) = 1.6 \text{Im } w_+ \cdot 10^{-25} \text{ GeV} \quad (5.25)$$

and therefore, in view of (5.20), an asymmetry more than two orders of magnitude smaller than for $K^\pm \rightarrow \pi^\pm \gamma \gamma$ as given in (4.18). With the estimate (4.28) for $\text{Im } \hat{c} = 2 \text{Im } w_+$, one obtains

$$\frac{|\Gamma(K^+ \rightarrow \pi^+ e^+ e^-) - \Gamma(K^- \rightarrow \pi^- e^+ e^-)|}{\Gamma(K^+ \rightarrow \pi^+ e^+ e^-) + \Gamma(K^- \rightarrow \pi^- e^+ e^-)} \simeq 8 \cdot 10^{-6} \quad (5.26)$$

which does not seem very promising even for the forthcoming high-precision experiments. Nevertheless, we want to emphasize again that chiral perturbation

theory yields a definite relation between the intrinsic CP violation signals in $K^\pm \rightarrow \pi^\pm \gamma \gamma$, $K^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ and $K_L \rightarrow \pi^0 \ell^+ \ell^-$.

So far, we have only considered the one-photon exchange amplitude for $K_L \rightarrow \pi^0 \ell^+ \ell^-$. In order to estimate the size of the two-photon exchange amplitude, we now turn to a discussion of the transition $K_2^0 \rightarrow \pi^0 \gamma \gamma$.

5b. The $K_2^0 \rightarrow \pi^0 \gamma \gamma$ Transition

With CP invariance assumed, the most general form of the amplitude for $K_2^0 \rightarrow \pi^0 \gamma \gamma$ with both photons on-shell depends on two invariant amplitudes A and B defined in the appendix. Because of the tensor structure of the amplitude $M^{\mu\nu}(p, q_1, q_2)$ in (A.8), only the amplitude A is non-vanishing to lowest non-trivial order $O(p^4)$ in chiral perturbation theory. From our previous work [14] we know that the order $O(p^4)$ contribution comes exclusively from one-loop Feynman graphs with the result

$$A(x_1, x_2) \equiv A(z) = \frac{G_8 \alpha}{\pi z} [(z - r_\pi^2)F(z/r_\pi^2) - (z - 1 - r_\pi^2)F(z)] \quad (5.27)$$

$$z = \frac{(q_1 + q_2)^2}{M_K^2} = 2(x_1 + x_2) - 1 + r_\pi^2$$

with the function F defined in (3.8).

To order $O(p^4)$, we have

$$B(x_1, x_2) = 0. \quad (5.28)$$

This has an interesting implication. Because of the tensor structure of $M^{\mu\nu}(p, q_1, q_2)$, the contribution from A in (5.27) to the absorptive part of the $K_2^0 \rightarrow \pi^0 \ell^+ \ell^-$ amplitude vanishes in the limit where the lepton mass goes to zero [13]. Actually, it can be shown that to lowest non-trivial order in chiral perturbation theory the complete two-photon exchange contribution to the $K_2^0 \rightarrow \pi^0 e^+ e^-$ amplitude including the dispersive part is suppressed by a factor $O(m_e/M_K)$ suggesting the dominance of the CP violating one-photon exchange amplitude discussed in the previous subsection.

However, contrary to a recent conjecture [13], the m_e -suppression factor is not a general property of the two-photon exchange contribution to $K_2^0 \rightarrow \pi^0 e^+ e^-$. In general, the amplitude $B(x_1, x_2)$ in (A.8) does not have to vanish and it contributes to $K_2^0 \rightarrow \pi^0 e^+ e^-$ even in the limit $m_e \rightarrow 0$.

We conclude this subsection by presenting an explicit $O(p^6)$ term in the effective chiral Lagrangian which contributes to the amplitude B and hence to

the two-photon exchange amplitude for $K_2^0 \rightarrow \pi^0 e^+ e^-$ even in the limit $m_e \rightarrow 0$. A possible $O(p^6)$ term satisfying all the symmetry constraints is

$$\frac{G_8}{f_\pi^2} \cdot \frac{\omega}{16\pi^2 f_\pi^2} e^2 F^{\mu\rho} F^\nu{}_\rho \text{tr}(Q\lambda_{6-i7}\{\mathcal{L}_\mu, \mathcal{L}_\nu\}UQU^\dagger) + \text{h.c.} \quad (5.29)$$

where ω denotes a new unknown coupling constant, presumably of the same order of magnitude as the couplings $w_1, w_2, \tilde{w}_3, w_4$ in (2.18) and (2.20). The two extra powers in derivatives are compensated by the chiral symmetry breaking scale factor $16\pi^2 f_\pi^2$ in the denominator. The effective Lagrangian (5.29) contributes to both amplitudes A and B with the result

$$A(x_1, x_2) = \frac{2G_8\alpha}{9\pi} \frac{M_K^2}{16\pi^2 f_\pi^2} (4\pi)^2 \omega (x_1 + x_2) \quad (5.30a)$$

$$B(x_1, x_2) = -\frac{4G_8\alpha}{9\pi} \frac{M_K^2}{16\pi^2 f_\pi^2} (4\pi)^2 \omega. \quad (5.30b)$$

This proves our claim that $B(x_1, x_2)$ arises at the order $O(p^6)$ corresponding to the two-loop level in chiral perturbation theory. The amplitude (5.30a) is a higher order correction to (5.27) and will be disregarded in what follows.

5c. The $K_2^0 \rightarrow \pi^0 \gamma\gamma \rightarrow \pi^0 e^+ e^-$ Amplitude

We are now in a position to make a realistic estimate of the two-photon exchange contribution to $K_2^0 \rightarrow \pi^0 e^+ e^-$. We can calculate unambiguously the absorptive part of $A(K_2^0 \rightarrow \pi^0 e^+ e^-)$ due to the two-photon discontinuity. We shall consider the result of this calculation as an educated guess of the actual size of the complete amplitude [13].

The contribution from the two-photon intermediate state to the absorptive part of $A(K_2^0 \rightarrow \pi^0 e^+ e^-)$ shown in Fig. 10 can be written as

$$A(K_2^0 \rightarrow \pi^0 e^+ e^-)|_{2\gamma} = \frac{1}{2} \int \frac{d^4 q_1}{(2\pi)^3} \delta_+(q_1^2) \frac{d^4 q_2}{(2\pi)^3} \delta_+(q_2^2) (2\pi)^4 \delta^{(4)}(p - p' - q_1 - q_2) \cdot \\ \cdot (-ie^2) M_{\mu\nu}(p, q_1, q_2) \bar{u}(k) \gamma^\mu \frac{\not{k} - \not{q}_1 + m_e}{(k - q_1)^2 - m_e^2 - i\epsilon} \gamma^\nu v(k') \quad (5.31)$$

where $M_{\mu\nu}(p, q_1, q_2)$ denotes the amplitude (A.8) for the transition $K_2^0 \rightarrow \pi^0 \gamma\gamma$ with both photons on-shell. We shall take for $A(x_1, x_2)$ the explicit calculation of lowest non-trivial order $O(p^4)$ in chiral perturbation theory given in (5.27) and for $B(x_1, x_2)$ the order $O(p^6)$ expression (5.30b). In the limit $m_e \rightarrow 0$,

only the term proportional to $q_1 \cdot q_2 p_\mu p_\nu$ in $M_{\mu\nu}(p, q_1, q_2)$ gives a non-vanishing contribution of the form

$$B p \cdot (k - k') \bar{u}(k) \not{p} v(k'). \quad (5.32)$$

Neglecting the other contributions from the amplitude B which are proportional to m_e , we find after integration over the two-photon phase space the following result

$$A(K_2^0 \rightarrow \pi^0 e^+ e^-) |_{2\gamma} = \frac{iG_8 \alpha^2}{4\pi} \bar{u}(k) [m_e E(z) + \not{p} K \frac{p \cdot (k - k')}{M_K^2}] v(k') \quad (5.33)$$

where

$$E(z) = \frac{1}{\beta z} \log\left(\frac{1-\beta}{1+\beta}\right) [(z - r_\pi^2) F(z/r_\pi^2) - (z - 1 - r_\pi^2) F(z)] \quad (5.34)$$

$$\beta = \sqrt{1 - 4r_e^2/z}$$

and K is a constant

$$K = \frac{4}{9} \frac{M_K^2}{16\pi^2 f_\pi^2} (4\pi)^2 \omega. \quad (5.35)$$

The Dalitz plot density associated to (5.33) is given by [13]

$$\frac{\partial^2 \Gamma}{\partial \epsilon_- \partial \epsilon_+} \sim m_e^2 (q^2 - 4m_e^2) |E(z)|^2 + (\epsilon_- - \epsilon_+)^2 (4\epsilon_- \epsilon_+ - q^2) |K|^2 - 4m_e^2 (\epsilon_- - \epsilon_+)^2 K \operatorname{Re} E(z) \quad (5.36)$$

$$q^2 = M_K^2 z = 2M_K(\epsilon_+ + \epsilon_-) - M_K^2 + m_\pi^2$$

with ϵ_- and ϵ_+ the electron and positron energies in the K rest frame:

$$\epsilon_- = \frac{p \cdot k}{M_K}, \quad \epsilon_+ = \frac{p \cdot k'}{M_K}. \quad (5.37)$$

To estimate the decay rate associated to the two-photon exchange amplitude (5.33) we neglect the interference term in (5.36) and compare the rates induced by the amplitudes A (proportional to m_e^2) and B (non-zero for $m_e \rightarrow 0$). The decay rate associated to $m_e E(z)$ is

$$\begin{aligned} & \Gamma(K_2^0 \rightarrow \pi^0 \gamma \gamma \rightarrow \pi^0 e^+ e^-) |_{A=} \\ &= \frac{2|G_8 \alpha^2|^2 M_K^3 m_e^2}{16^3 \pi^5} \cdot \int_{4r_e^2}^{(1-r_\pi)^2} dz \lambda^{1/2}(1, z, r_\pi^2) z (1 - 4r_e^2/z)^{3/2} |E(z)|^2 = \end{aligned}$$

$$= 1 \cdot 10^{-31} \text{ GeV} \quad (5.38)$$

which corresponds to a branching ratio

$$\frac{\Gamma(K_2^0 \rightarrow \pi^0 \gamma \gamma \rightarrow \pi^0 e^+ e^-) |_A}{\Gamma(K_L \rightarrow \text{all})} = 8 \cdot 10^{-15}. \quad (5.39)$$

The branching ratio (5.39) is considerably smaller than the estimate of Donoghue et al. [13] and at least two orders of magnitude smaller than the estimates (5.19) corresponding to the one-photon exchange amplitude for $\text{Im } w_S = 0$. Thus, the contribution of the amplitude $m_e E(z)$ in (5.33) is indeed negligible.

The decay rate associated to the K -term in (5.33) is

$$\begin{aligned} \Gamma(K_2^0 \rightarrow \pi^0 \gamma \gamma \rightarrow \pi^0 e^+ e^-) |_B &= \\ &= \frac{|G_8 \alpha^2|^2 |K|^2}{512 \pi^5 M_K} \int d\epsilon_- d\epsilon_+ (\epsilon_- - \epsilon_+)^2 [4\epsilon_- \epsilon_+ - 2M_K(\epsilon_+ + \epsilon_-) + M_K^2 - m_\pi^2] = \\ &= 5 \cdot (4\pi)^4 |\omega|^2 \cdot 10^{-32} \text{ GeV}. \end{aligned} \quad (5.40)$$

For reasonable values of the coupling constant ω [$(4\pi)^2 \omega = O(1)$] the rates (5.38) and (5.40) are comparable.

We conclude from this analysis that an order of magnitude estimate of the CP allowed $K_2^0 \rightarrow \pi^0 e^+ e^-$ transition via two-photon exchange gives a significantly smaller rate than the CP violating contribution (5.14) via one-photon exchange. The decay amplitude for $K_2^0 \rightarrow \pi^0 e^+ e^-$ is dominantly CP violating.

5d. Transverse Muon Polarization in $K_L \rightarrow \pi^0 \mu^+ \mu^-$ Decay

The situation for the $\mu^+ \mu^-$ mode is completely different. Although we may safely neglect the contribution of the amplitude B arising to order $O(p^6)$ in chiral perturbation theory, the CP allowed two-photon exchange amplitude to order $O(p^4)$ (proportional to m_μ) can now be comparable to the CP violating one-photon exchange.

A distinctive signal of the interference between the two amplitudes and thus of CP violation is the existence of a transverse polarization of the muons. As the last topic of this section, we proceed to a calculation of this polarization within the framework of the effective chiral realization of the Standard Model.

The transition amplitude for $K_L \rightarrow \pi^0 \mu^+ \mu^-$ can be written in the form

$$A(K_L \rightarrow \pi^0 \mu^+ \mu^-) = \frac{\text{Re } G_8 \alpha}{4\pi} \bar{u}(k)[im_\mu h - (\not{p} + \not{p}')g]v(k') \quad (5.41)$$

where h and g are the CP conserving and CP violating amplitudes, respectively. Comparing with (5.14), we find immediately the one-photon exchange amplitude

$$g(z) = \epsilon[2\varphi(z) + \text{Re } w_S] + i \text{Im } w_S. \quad (5.42)$$

As an estimate of the amplitude h induced by the two-photon exchange mechanism, we use the same two-photon absorptive contribution as in the previous subsection for $K_2^0 \rightarrow \pi^0 \gamma \gamma \rightarrow \pi^0 e^+ e^-$. To leading order $O(p^4)$,

$$h(z) = \alpha E(z) \quad (5.43)$$

where $E(z)$ is given in (5.34) with m_e replaced by m_μ .

Denoting with $N^\uparrow(z)$ and $N^\downarrow(z)$ the event densities for given z with the two possible muon spin orientations orthogonal to the decay plane, the z -dependence of the up-down asymmetry is computed as

$$\frac{N^\uparrow(z) - N^\downarrow(z)}{N^\uparrow(z) + N^\downarrow(z)} = \frac{-\frac{\pi}{2} r_\mu \lambda^{1/2}(1, z, r_\pi^2)(z - 4r_\mu^2)^{1/2} [\text{Re } h(z) \text{Re } g(z) + \text{Im } h(z) \text{Im } g(z)]}{r_\mu^2(z - 4r_\mu^2)|h(z)|^2 + \frac{2}{3} \lambda(1, z, r_\pi^2)(1 + 2r_\mu^2/z)|g(z)|^2} \quad (5.44)$$

The average transverse muon polarization $\langle \xi \rangle$ is given by

$$\langle \xi \rangle = \frac{-\frac{\pi}{2} r_\mu \int_{4r_\mu^2}^{(1-r_\pi)^2} dz \sqrt{z} \lambda(1, z, r_\pi^2)(1 - 4r_\mu^2/z) [\text{Re } h(z) \text{Re } g(z) + \text{Im } h(z) \text{Im } g(z)]}{\int_{4r_\mu^2}^{(1-r_\pi)^2} dz \lambda^{1/2}(1, z, r_\pi^2)(1 - 4r_\mu^2/z)^{1/2} [r_\mu^2(z - 4r_\mu^2)|h(z)|^2 + \frac{2}{3} \lambda(1, z, r_\pi^2)(1 + 2r_\mu^2/z)|g(z)|^2]} \quad (5.45)$$

and the integrated decay rate is calculated to be

$$\Gamma(K_L \rightarrow \pi^0 \mu^+ \mu^-) = \frac{|G_8 \alpha|^2 M_{K^0}^5}{2(4\pi)^5} \int_{4r_\mu^2}^{(1-r_\pi)^2} dz \lambda^{1/2}(1, z, r_\pi^2)(1 - 4r_\mu^2/z)^{1/2} \cdot [r_\mu^2(z - 4r_\mu^2)|h(z)|^2 + \frac{2}{3} \lambda(1, z, r_\pi^2)(1 + 2r_\mu^2/z)|g(z)|^2]. \quad (5.46)$$

In Fig. 11, we show the up-down asymmetry (5.44) as a function of z for the two possible values [11] of $\text{Re } w_S$

$$\begin{aligned} \text{Re } w_S^{(1)} &= 0.73 \pm 0.08 \\ \text{Re } w_S^{(2)} &= -1.00 \pm 0.08 \end{aligned} \quad (5.47)$$

and for three different values of $\text{Im } w_S$ which cover the expected range for this parameter. Positive (negative) values of $\text{Re } w_S$ lead to destructive (constructive)

interference between the loop and counterterm contributions in (5.16) and (5.42) and, consequently, to smaller (larger) rates, as shown explicitly in (5.19) for $K_L \rightarrow \pi^0 e^+ e^-$. For the same choice of values for w_S , the average transverse muon polarization $\langle \xi \rangle$, the integrated decay rate and the branching ratio are given in Table 2. From both Fig. 11 and Table 2 it is evident that the CP violating asymmetries are large. Because of the influence of the two-photon amplitude (5.43) the branching ratios are bigger than could be expected from the corresponding $e^+ e^-$ branching ratios (5.19) and phase space.

From Table 2 we infer that the decay $K_L \rightarrow \pi^0 \mu^+ \mu^-$ could be within reach of experiments currently under way [1]. Including the full two-photon exchange amplitude instead of the absorptive part (5.43) may be expected to yield still higher rates. A complete calculation of the two-photon amplitude h in (5.41) will be necessary to make really quantitative predictions, but even the estimates presented here strongly suggest the existence of a large CP violating signal in this decay mode.

6. CONCLUSIONS

Rare K decays have been the subject of many theoretical investigations [32]. More often than not, it is difficult in many treatments to distinguish between genuine aspects of the Standard Model and additional assumptions of variable credibility usually related to the problem of long-distance dynamics. Chiral perturbation theory, on the other hand, allows for a clear distinction. The low-energy amplitudes of the Standard Model are calculable in chiral perturbation theory except for some coupling constants which are not restricted by the symmetries of the underlying Lagrangian at the quark level. Those constants reflect our lack of understanding of the QCD confinement mechanism and must be determined experimentally for the time being. Further theoretical progress in QCD can only improve our knowledge of those constants, but it cannot modify the low energy structure of amplitudes.

K decays are ideally suited for the effective chiral Lagrangian approach because the characteristic momenta are small compared to the scale of chiral symmetry breaking and because the only particles involved are pseudoscalar mesons, photons and leptons. We have shown that all non-leptonic radiative K decays with at most one pion in the final state have vanishing amplitudes to lowest order $O(p^2)$ in chiral perturbation theory. The proof makes use of a

transformation to the diagonal basis of pseudoscalar fields which in addition entails a substantial simplification in the calculation of loop diagrams. At the order $O(p^4)$ corresponding to the one-loop level, we may distinguish three different cases. In the first class of decay amplitudes ($K_1^0 \rightarrow \gamma\gamma$, $K_2^0 \rightarrow \pi^0\gamma\gamma$, $K_1^0 \rightarrow \gamma\ell^+\ell^-$) chiral symmetry forbids all possible counterterms of order $O(p^4)$ and thus the loop amplitudes are necessarily finite. Both the rates and the spectra are unambiguously predicted in terms of the single octet coupling constant g_8 if we neglect the 27-plet piece of the non-leptonic weak interactions. In the second case ($K^\pm \rightarrow \pi^\pm\gamma\gamma$), the loop amplitude is still finite but chiral symmetry permits a renormalization scale invariant counterterm amplitude. Finally, if the loop amplitude is divergent, chiral symmetry must allow for a counterterm amplitude which needs to be renormalized ($K^\pm \rightarrow \pi^\pm\gamma^* \rightarrow \pi^\pm\ell^+\ell^-$, $K^0 \rightarrow \pi^0\gamma^* \rightarrow \pi^0\ell^+\ell^-$). In general, the coupling constants of the effective chiral Lagrangian receive both long-distance and short-distance contributions. Long-distance contributions are related to the non-renormalizability of the effective theory and they include effects of higher mass states which are not explicitly represented in the chiral Lagrangian. We have set up the complete list of electroweak counterterms to order $O(p^4)$ with at least one photon field, restricted to the dominant octet part of non-leptonic weak interactions.

The off-shell amplitude $K_1^0 \rightarrow \gamma^*\gamma^*$ shares with the on-shell transition the property of being uniquely given by the finite loop amplitude. The off-shell structure of chiral vertices predicted by QCD can be tested in $K_1^0 \rightarrow \gamma\mu^+\mu^-$ where both the rate and the spectrum can be distinguished from phase space or dispersion models with constant off-shell vertices. In $K_1^0 \rightarrow \gamma e^+e^-$, on the other hand, the spectrum is so much peaked towards small invariant masses of the lepton pair that QCD effects cannot be seen in the relevant ratio $d\Gamma(K_1^0 \rightarrow \gamma e^+e^-)/\Gamma(K_1^0 \rightarrow \gamma\gamma)$. To lowest non-trivial order in chiral perturbation theory, also $d\Gamma(K_2^0 \rightarrow \gamma\ell^+\ell^-)/\Gamma(K_2^0 \rightarrow \gamma\gamma)$ is given by phase space.

As in the case of $K_2^0 \rightarrow \pi^0\gamma\gamma$, the loop amplitude for $K^\pm \rightarrow \pi^\pm\gamma\gamma$ is finite but chiral symmetry allows nevertheless for a tree level counterterm contribution. Therefore, we can only derive a one-parameter relation between the rate and the spectrum and a lower bound for the branching ratio $B(K^\pm \rightarrow \pi^\pm\gamma\gamma) \geq 4 \cdot 10^{-7}$. We have also calculated the charge asymmetry $\Gamma(K^+ \rightarrow \pi^+\gamma\gamma) - \Gamma(K^- \rightarrow \pi^-\gamma\gamma)$ as a measure of intrinsic or amplitude CP violation in terms of the imaginary part of a certain counterterm coupling constant. Employing an approximate relation between the short-distance part of chiral counterterms and the semi-leptonic electromagnetic penguin operator, an estimate of the charge asymmetry was given suggesting a sensitivity of order 10^{-10} in the branching

ratio necessary for its experimental detection.

Quite independently of specific numerical estimates, it is important to emphasize that chiral perturbation theory yields a precise relation between different signals of CP violation. In particular, it predicts the charge asymmetry $\Gamma(K^+ \rightarrow \pi^+ e^+ e^-) - \Gamma(K^- \rightarrow \pi^- e^+ e^-)$ to be more than two orders of magnitude smaller than for $K^\pm \rightarrow \pi^\pm \gamma \gamma$. Moreover, these charge asymmetries can be related unambiguously to the size of the $K_L \rightarrow \pi^0 \ell^+ \ell^-$ amplitude with intrinsic CP violation. A similar numerical estimate as for the charge asymmetries suggests that the intrinsic CP violation in $K_L \rightarrow \pi^0 \ell^+ \ell^-$ is non-negligible and may even dominate over the usual ϵ -like CP violation from the $K^0 - \bar{K}^0$ mass matrix.

We have carefully estimated the CP conserving amplitude $K_L \rightarrow \pi^0 \gamma^* \gamma^* \rightarrow \pi^0 \ell^+ \ell^-$ by calculating the two-photon absorptive part. Although in detail our results differ considerably from those of Donoghue et al. [13], we confirm their conjecture that the CP violating one-photon exchange dominates by far in the $e^+ e^-$ mode. The situation is, however, rather different for the $\mu^+ \mu^-$ mode where the CP violating and CP conserving amplitudes are comparable. In addition to yielding a larger rate for $K_L \rightarrow \pi^0 \mu^+ \mu^-$ than expected from the $e^+ e^-$ rate and phase space, the interference between the two amplitudes produces a large transverse muon polarization as a spectacular signal of CP violation. With only the two-photon absorptive part in the CP conserving amplitude, we find a very pronounced dependence of the up-down asymmetry on the invariant mass of the lepton pair and an average transverse muon polarization in the range between 20 and 60%. A sensitivity of several 10^{-12} will be necessary to detect this decay mode, which could be within reach of ongoing experiments.

The effective chiral Lagrangian approach to rare K decays offers many possibilities for crucial tests of the Standard Model. Measuring the decay rates will shed light on the unsolved problem of whether the Standard Model can account for the $\Delta I = 1/2$ rule (octet enhancement). The chiral structure of weak vertices predicted by QCD with its softly broken chiral symmetry will be apparent in the spectra of the various decay channels. Finally, in the purely weak sector, chiral perturbation theory relates different observables sensitive to the elusive intrinsic CP violation of the Standard Model.

ACKNOWLEDGEMENTS

The work of one of us (A.P.) has been partly supported by C.A.I.C.Y.T. under "Plan movilizador de la Física de Altas Energías".

APPENDIX: KINEMATICS

In this appendix we review the general kinematic features of decays of the type

$$\begin{aligned} M(p) &\rightarrow \gamma(q_1) + \gamma(q_2) \\ M(p) &\rightarrow M'(p') + \gamma(q_1) + \gamma(q_2) \end{aligned} \quad (A.1a, b)$$

where M, M' denote (pseudo)scalar particles which in our case will always be on-shell:

$$p^2 = M^2, \quad p'^2 = M'^2. \quad (A.2)$$

The amplitude for $M \rightarrow \gamma\gamma$ transitions, with both photons off-shell, has the general form compatible with gauge invariance

$$\begin{aligned} M^{\mu\nu}(q_1, q_2) = & (g^{\mu\nu} - \frac{q_1^\mu q_1^\nu}{q_1^2} - \frac{q_2^\mu q_2^\nu}{q_2^2} + \frac{q_1 \cdot q_2}{q_1^2 q_2^2} q_1^\mu q_2^\nu) M^2 a(q_1^2, q_2^2) + \\ & + [q_2^\mu q_1^\nu - q_1 \cdot q_2 (\frac{q_1^\mu q_1^\nu}{q_1^2} + \frac{q_2^\mu q_2^\nu}{q_2^2} - \frac{q_1 \cdot q_2}{q_1^2 q_2^2} q_1^\mu q_2^\nu)] b(q_1^2, q_2^2) + \\ & + \epsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma} c(q_1^2, q_2^2) \end{aligned} \quad (A.3)$$

where a, b, c are invariant amplitudes free of kinematic singularities. Bose symmetry implies that all three amplitudes are symmetric functions of q_1^2, q_2^2 . The amplitudes a, b on the one hand, and c on the other hand, have opposite parity transformation properties. In $M \rightarrow \gamma\gamma$ transitions, the final eigenstates of P are also eigenstates of CP. Hence, with CP invariance assumed, the amplitudes a, b will contribute to $K_1^0 \rightarrow \gamma^* \gamma^*$ transitions only while c contributes to $K_2^0 \rightarrow \gamma^* \gamma^*$ only.

When one of the photons is on-shell, say $q_1^2 = 0$,

$$M^2 a(0, q_2^2) + q_1 \cdot q_2 b(0, q_2^2) = 0 \quad (A.4)$$

and $M^{\mu\nu}(q_1, q_2)$ reduces to two invariant amplitudes

$$\epsilon_\mu(q_1) M^{\mu\nu}(q_1, q_2) = \epsilon_\mu(q_1) \{ (-q_1 \cdot q_2 g^{\mu\nu} + q_2^\mu q_1^\nu) b(0, q_2^2) + \epsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma} c(0, q_2^2) \}. \quad (A.5)$$

With both photons on-shell, two invariant amplitudes remain.

The general amplitude structure for $M(p) \rightarrow M'(p') + \gamma(q_1) + \gamma(q_2)$ transitions, with all particles on-shell

$$p^2 = M^2, \quad p'^2 = M'^2, \quad q_1^2 = q_2^2 = 0 \quad (A.6)$$

is

$$\epsilon_\mu(q_1)\epsilon_\nu(q_2)M^{\mu\nu}(p, q_1, q_2) \quad (A.7)$$

where gauge invariance restricts $M^{\mu\nu}(p, q_1, q_2)$ to depend on four invariant amplitudes:

$$\begin{aligned} M^{\mu\nu}(p, q_1, q_2) = & A(x_1, x_2)[-q_1 \cdot q_2 g^{\mu\nu} + q_2^\mu q_1^\nu] + \\ & + B(x_1, x_2)[-M^2 x_1 x_2 g^{\mu\nu} - \frac{q_1 \cdot q_2}{M^2} p^\mu p^\nu + x_1 q_2^\mu p^\nu + x_2 p^\mu q_1^\nu] + \\ & + C(x_1, x_2)\epsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma} + \\ & + D(x_1, x_2)[\epsilon^{\mu\nu\rho\sigma}(x_2 q_{1\rho} + x_1 q_{2\rho})p_\sigma + (p^\mu \epsilon^{\nu\alpha\beta\gamma} + p^\nu \epsilon^{\mu\alpha\beta\gamma})\frac{p_\alpha q_{1\beta} q_{2\gamma}}{M^2}] \end{aligned} \quad (A.8)$$

where

$$x_i = \frac{p \cdot q_i}{M^2} \quad (i = 1, 2), \quad q_1 \cdot q_2 = M^2(x_1 + x_2 - \frac{1}{2}) + \frac{M'^2}{2}. \quad (A.9)$$

Bose symmetry yields the relations

$$\begin{aligned} A(x_1, x_2) &= A(x_2, x_1), & B(x_1, x_2) &= B(x_2, x_1) \\ C(x_1, x_2) &= C(x_2, x_1), & D(x_1, x_2) &= -D(x_2, x_1). \end{aligned} \quad (A.10)$$

The invariant amplitudes A, B have opposite parity transformation properties to C, D . In the limit where CP is conserved, the amplitudes A, B contribute only to $K_2^0 \rightarrow \pi^0 \gamma \gamma$ whereas $K_1^0 \rightarrow \pi^0 \gamma \gamma$ involves only the other two amplitudes C, D . Of course, such a distinction does not apply to the charged decay modes $K^\pm \rightarrow \pi^\pm \gamma \gamma$.

TABLE 1

Normalized rates $\frac{\Gamma(K_1^0 \rightarrow \gamma \ell^+ \ell^-)}{\Gamma(K_1^0 \rightarrow \gamma \gamma)}$

	$\ell = e$	$\ell = \mu$
chiral dynamics [Eqs. (3.4), (3.7)]	$1.60 \cdot 10^{-2}$	$3.75 \cdot 10^{-4}$
dispersion model [17]	$1.59 \cdot 10^{-2}$	$3.30 \cdot 10^{-4}$
phase space [$H(z) = H(0)$]	$1.59 \cdot 10^{-2}$	$4.09 \cdot 10^{-4}$

TABLE 2

Average transverse muon polarization $\langle \xi \rangle$, decay rate $\Gamma(K_L \rightarrow \pi^0 \mu^+ \mu^-)$ and branching ratio $B(K_L \rightarrow \pi^0 \mu^+ \mu^-)$

Re w_S	Im w_S	$\langle \xi \rangle$	$\Gamma(K_L \rightarrow \pi^0 \mu^+ \mu^-)/10^{-28}$ GeV	$B(K_L \rightarrow \pi^0 \mu^+ \mu^-)$
0.73	-10^{-3}	-0.05	0.68	$5.4 \cdot 10^{-12}$
	0	-0.24	0.69	$5.5 \cdot 10^{-12}$
	10^{-3}	-0.37	0.80	$6.3 \cdot 10^{-12}$
-1.00	-10^{-3}	0.52	1.29	$10.2 \cdot 10^{-12}$
	0	0.51	1.06	$8.4 \cdot 10^{-12}$
	10^{-3}	0.45	0.92	$7.2 \cdot 10^{-12}$

REFERENCES

- [1] Brookhaven Nat. Lab. experiments E777, E780, E787 and E791.
- [2] S.L. Glashow, Nucl. Phys. 22 (1961) 579;
S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264;
A. Salam, Proc. 8th Nobel Symposium, Aspenäsgrården, 1968, ed. N. Svartholm (Almqvist and Wiksell, Stockholm, 1968).
- [3] S.L. Adler and R.F. Dashen, Current Algebras and Applications to Particle Physics, Benjamin, New York, 1968.
- [4] K. Meetz, J. Math. Phys. 10 (1969) 589;
S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2239;
C. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2247;
B.W. Lee, Chiral Dynamics, Gordon and Breach, New York, 1972.
- [5] D.J. Gross and F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343;
H.D. Politzer, Phys. Rev. Lett. 30 (1973) 1346.
- [6] J. Wess and B. Zumino, Phys. Lett. 37B (1971) 95.
- [7] E. Witten, Nucl. Phys. B223 (1983) 422, 433.
- [8] A. Pich and E. de Rafael, Phys. Lett. B158 (1985) 477 and B189 (1987) 369;
B. Guberina, A. Pich and E. de Rafael, Phys. Lett. B163 (1985) 198;
A. Pich, B. Guberina and E. de Rafael, Nucl. Phys. B277 (1986) 197.
- [9] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (1979) 385, 448.
- [10] J. Gasser and H. Leutwyler, Ann. of Phys. 158 (1984) 142; Nucl. Phys. B250 (1985) 465, 517, 539.
- [11] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. B291 (1987) 692.
- [12] G. D'Ambrosio and D. Espriu, Phys. Lett. B175 (1986) 237;
J.L. Goity, Z. Phys. C34 (1987) 341.
- [13] J.F. Donoghue, B.R. Holstein and G. Valencia, Phys. Rev. D35 (1987) 2769.

- [14] G. Ecker, A. Pich and E. de Rafael, Phys. Lett. B189 (1987) 363.
- [15] L. Cappiello and G. D'Ambrosio, INFN-Napoli preprint (June 1987).
- [16] J. Bijnens and M.B. Wise, Phys. Lett. 137B (1984) 245;
J.F. Donoghue, E. Golowich, B.R. Holstein and J. Trampetić, Phys. Lett. B179 (1986) 361;
A.J. Buras and J.-M. Gérard, Phys. Lett. B192 (1987) 156.
- [17] L.M. Sehgal, Phys. Rev. D7 (1973) 3303.
- [18] F.J. Gilman and M.B. Wise, Phys. Rev. D21 (1980) 3150.
- [19] M. Kobayashi and T. Maskawa, Progr. Theor. Phys. 49 (1973) 652.
- [20] C. Bernard et al., Phys. Rev. D32 (1985) 2343.
- [21] S.L. Adler, Phys. Rev. 177 (1969) 2426;
J.S. Bell and R. Jackiw, Nuovo Cimento 60A (1969) 47.
- [22] H. Burkhardt et al. (CERN-NA31), preprint CERN/EP 87-146.
- [23] J.F. Donoghue, B.R. Holstein and Y.-C.R. Lin, Nucl. Phys. B277 (1986) 651.
- [24] J.F. Donoghue, B.R. Holstein and Y.-C.R. Lin, Phys. Rev. Lett. 55 (1985) 2766.
- [25] a) L.M. Sehgal and L. Wolfenstein, Phys. Rev. 162 (1967) 1362;
b) B.R. Martin and E. de Rafael, Nucl. Phys. B8 (1968) 131;
c) R. Decker, P. Pavlopoulos and G. Zoupanos, Z. Phys. C28 (1985) 117;
L.-L. Chau and H.-Y. Cheng, Phys. Rev. Lett. 54 (1985) 1768 and Univ. of California Davis preprint UCD-87-09;
J.O. Eeg and I. Picek, Univ. Oslo preprint 87-19.
- [26] E. Gabathuler and P. Pavlopoulos, Proc. Workshop on Physics at LEAR, Erice 1982, eds. U. Gastaldi and R. Klapisch, Plenum Press, New York (1982), p. 747.
- [27] L.M. Sehgal, Phys. Rev. D6 (1972) 367.
- [28] P. Bloch et al., Phys. Lett. 56B (1975) 201.

- [29] Review of Particle Properties, Phys. Lett. 170B (1986).
- [30] I. Mannelli (CERN-NA31), Int. Symposium on Lepton and Photon Interactions at High Energies, Hamburg, 1987, preprint CERN-EP/87-177.
- [31] J.F. Donoghue, T. Nakada, E.A. Paschos and D. Wyler, Phys. Lett. B195 (1987) 285.
- [32] In addition to papers already quoted, here is an incomplete list of earlier references on rare K decays:
 B.R. Martin, E. de Rafael and J. Smith, Phys. Rev. D2 (1970) 179;
 L.M. Sehgal, Nucl. Phys. B19 (1970) 445;
 F.J. Ynduráin, Progr. Theor. Phys. 46 (1971) 990;
 M. Moshe and P. Singer, Phys. Rev. D6 (1972) 1379; Phys. Lett. 51B (1974) 367;
 J. Smith and Z.E.S. Uy, Phys. Rev. D8 (1973) 3056;
 M.K. Gaillard and B.W. Lee, Phys. Rev. D10 (1974) 897;
 D.V. Nanopoulos and G.G. Ross, Phys. Lett. 56B (1975) 279;
 A.I. Vainshtein, V.I. Zakharov, L.B. Okun and M.A. Shifman, Yad. Fiz. 24 (1976) 820 [Sov. J. Nucl. Phys. 24 (1976) 427];
 R. Rockmore and A.N. Kamal, Phys. Rev. D17 (1978) 2503;
 J.O. Eeg and F. Ravndal, Lett. Nuovo Cimento 22 (1978) 397;
 Z.E.S. Uy, Phys. Rev. D19 (1979) 1623;
 J.O. Eeg, Lett. Nuovo Cimento 29 (1980) 197; Phys. Rev. D11 (1981) 2596;
 L.E. Ibañez, C. López and F.J. Ynduráin, Phys. Rev. D21 (1980) 1428;
 E. Ma and A. Pramudita, Phys. Rev. D24 (1981) 2476;
 L. Bergström, E. Masso and P. Singer, Phys. Lett. 131B (1983) 229;
 J. Ellis and J.S. Hagelin, Nucl. Phys. B217 (1983) 189;
 F.J. Gilman and J.S. Hagelin, Phys. Lett. 133B (1983) 443;
 Z.E.S. Uy, Phys. Rev. D29 (1984) 574;
 G. Eilam and M.D. Scadron, Phys. Rev. D31 (1985) 2263;
 L. Bergström and P. Singer, Phys. Rev. Lett. 55 (1985) 2633;
 A.A. Belkov, Yu.L. Kalinowskii, V.N. Pervushin and N.A. Sarikov, Yad. Fiz. 44 (1986) 690 [Sov. J. Nucl. Phys. 44 (1986) 448];
 J.O. Eeg, Nucl. Phys. B289 (1987) 673.

FIGURE CAPTIONS

- Fig. 1 Leading diagram contributing to the electromagnetic penguin operator.
- Fig. 2 Feynman diagram for the $K_1^0 \rightarrow \gamma \ell^+ \ell^-$ transition.
- Fig. 3 Normalized q_2^2 -distribution for $K_S \rightarrow \gamma \mu^+ \mu^-$ in the limit of CP conservation (full curve). Also shown for comparison is the phase space distribution (dashed curve). q_2^2 is the invariant mass squared of the lepton pair.
- Fig. 4 Tree level diagram for $K_2^0 \rightarrow \gamma \ell^+ \ell^-$ involving the WZ term of the chiral Lagrangian. The open box denotes the $\Delta S = 1$ weak vertex.
- Fig. 5 One loop diagrams for $K^+ \rightarrow \pi^+ \gamma \gamma$ in the diagonal basis described in Sect. 2. The weak (and weak + electromagnetic) vertices denoted by an open (full) box are given by the Lagrangian (2.21). There are in addition tadpole diagrams not shown in the figure.
- Fig. 6 Behaviour of the one-loop function $F(z)$ defined in (3.8) in the physical region for $K \rightarrow \pi \gamma \gamma$ decays. The real and imaginary parts of $F(z/r_\pi^2)$ are also shown.
- Fig. 7 Tree level diagram for $K^+ \rightarrow \pi^+ \gamma \gamma$ involving the anomalous WZ term.
- Fig. 8 Total rate for $K^+ \rightarrow \pi^+ \gamma \gamma$ (full curve) as a function of the renormalization scale invariant \hat{c} defined in (4.2). The absorptive contribution is indicated by the dashed line.
- Fig. 9 Normalized q^2 -distribution for $K^+ \rightarrow \pi^+ \gamma \gamma$ for $\hat{c} = -4$ (dotted curve), $\hat{c} = 0$ (full curve) and $\hat{c} = 4$ (dashed curve). Also shown is the pure phase space prediction (dash-dotted curve).
- Fig. 10 Contribution from the two-photon intermediate state to the absorptive part of $A(K_2^0 \rightarrow \pi^0 e^+ e^-)$.
- Fig. 11 Up-down asymmetry (5.44) as a function of z for the two possible values of $\text{Re } w_S$ given in (5.47) and for three different values of $\text{Im } w_S$ covering the expected range for this parameter. The different curves correspond to the following values of $(\text{Re } w_S, \text{Im } w_S)$:

$$\begin{array}{ll}
 (0.73, -10^{-3}) \sim [\text{---} \bullet \text{---}], & (0.73, 0) \sim [\text{---} \text{---}], \\
 (0.73, +10^{-3}) \sim [\text{---} \bullet \text{---}], & (-1.00, -10^{-3}) \sim [\text{---} \text{---}], \\
 (-1.00, 0) \sim [\text{---} \text{---}] & \text{and} \quad (-1.00, +10^{-3}) \sim [\bullet \bullet \bullet].
 \end{array}$$

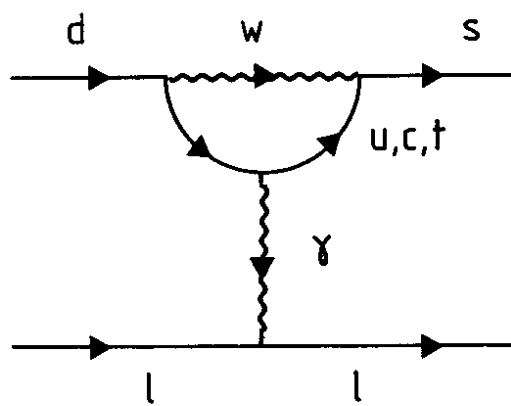


Fig. 1

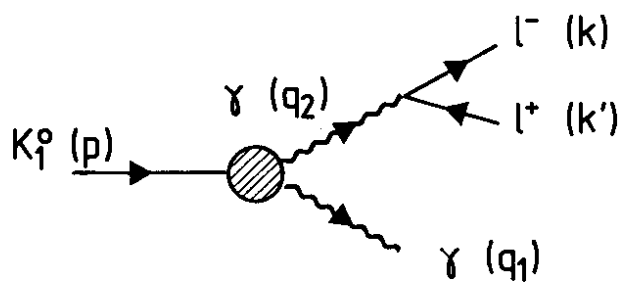


Fig. 2

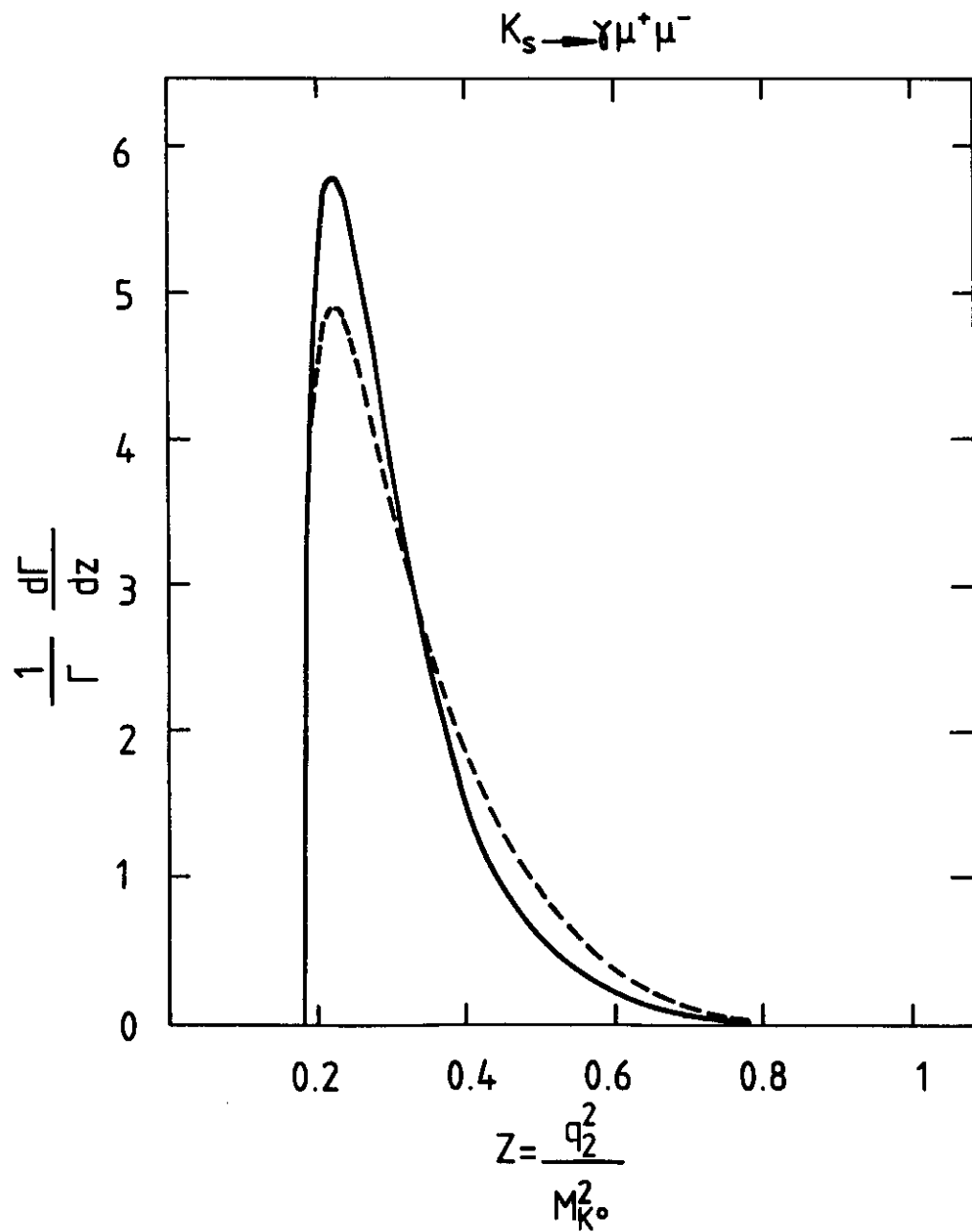


Fig. 3

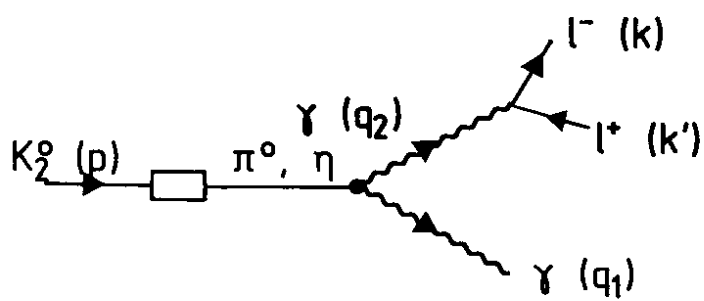


Fig. 4

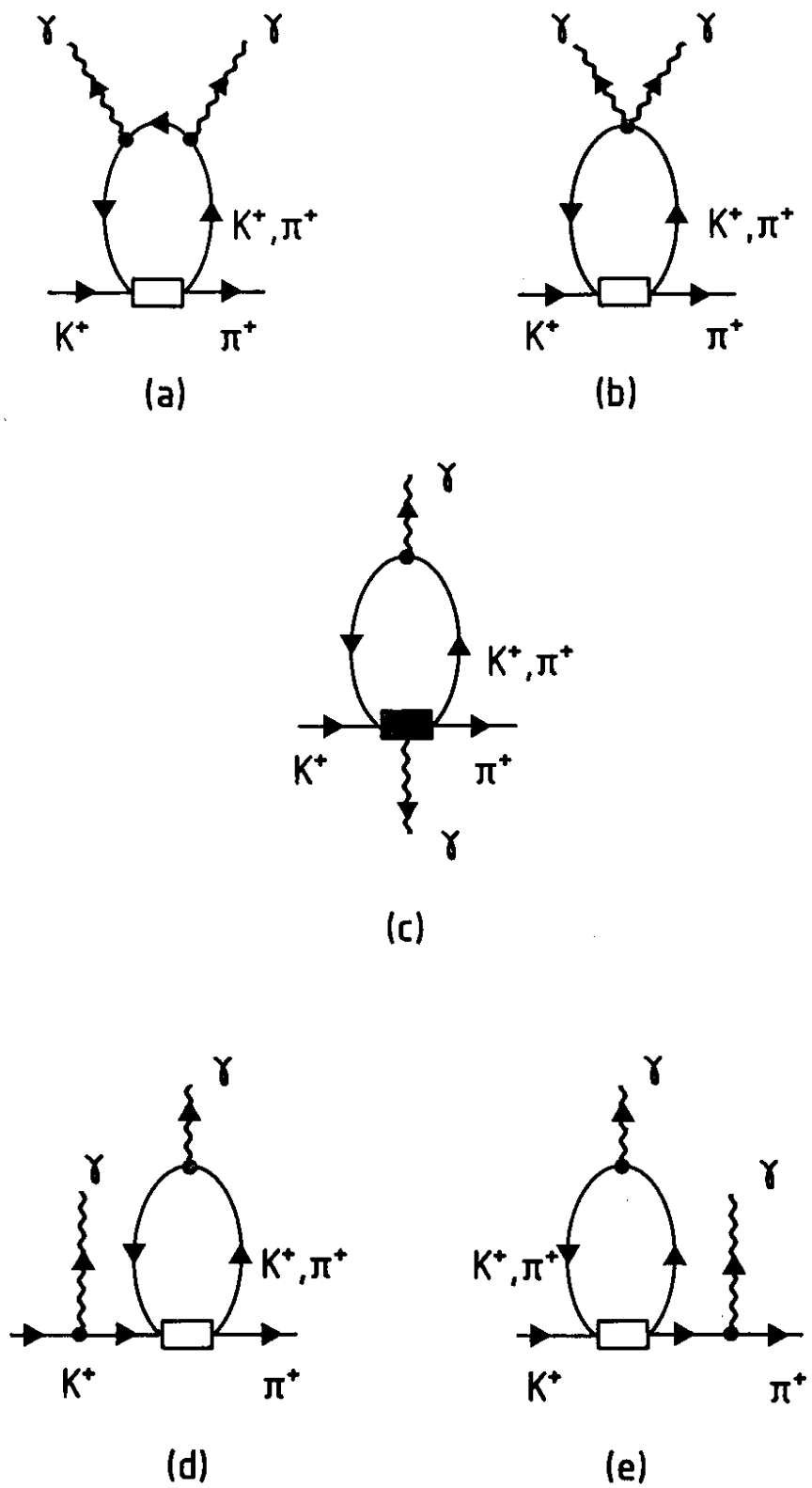


Fig. 5

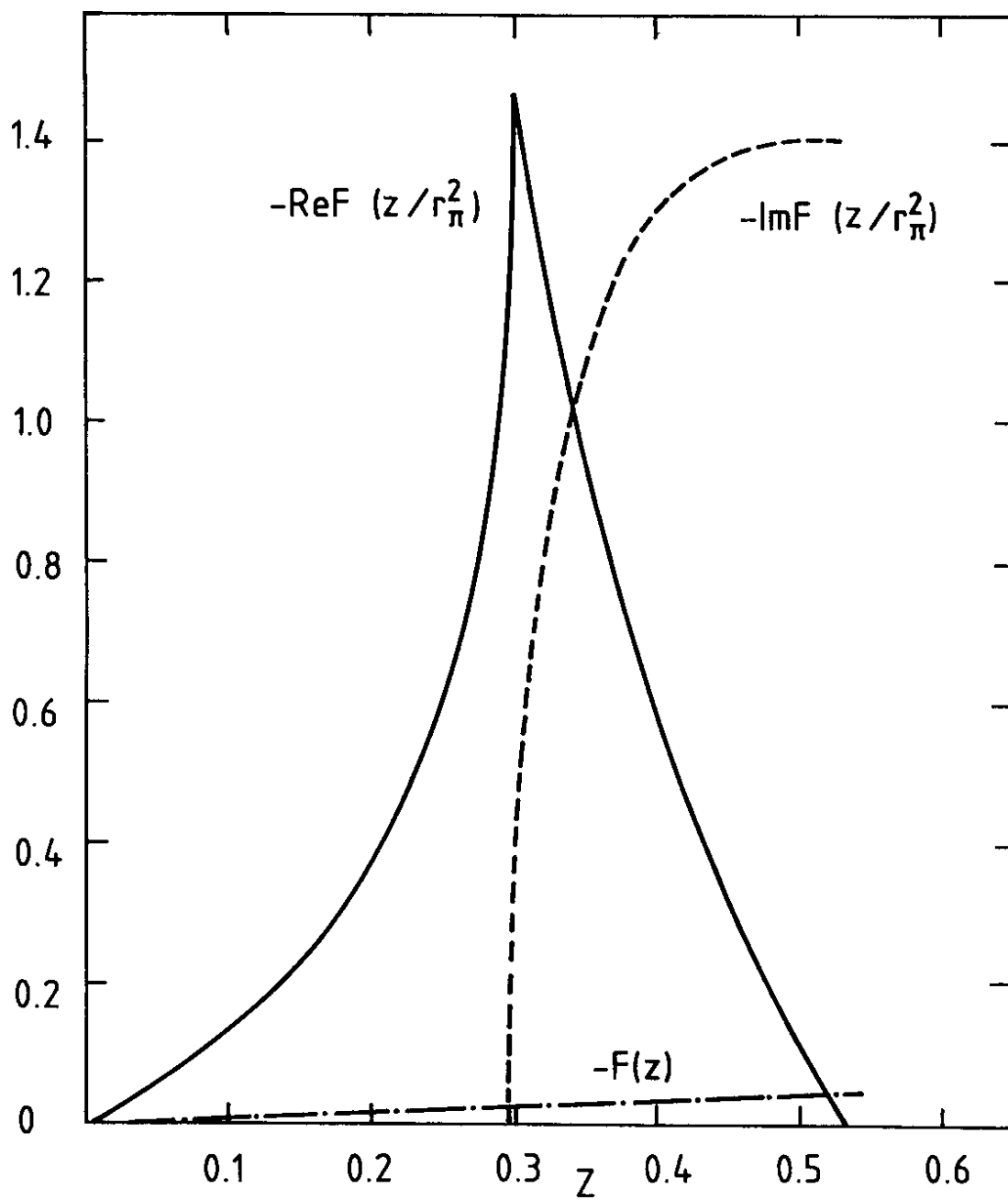


Fig. 6

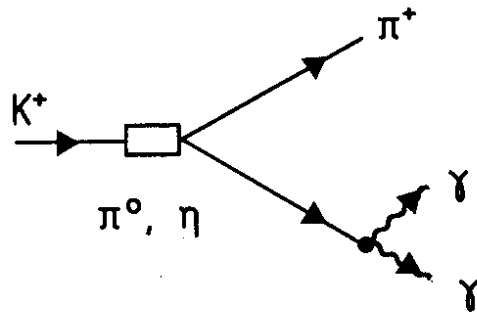


Fig. 7

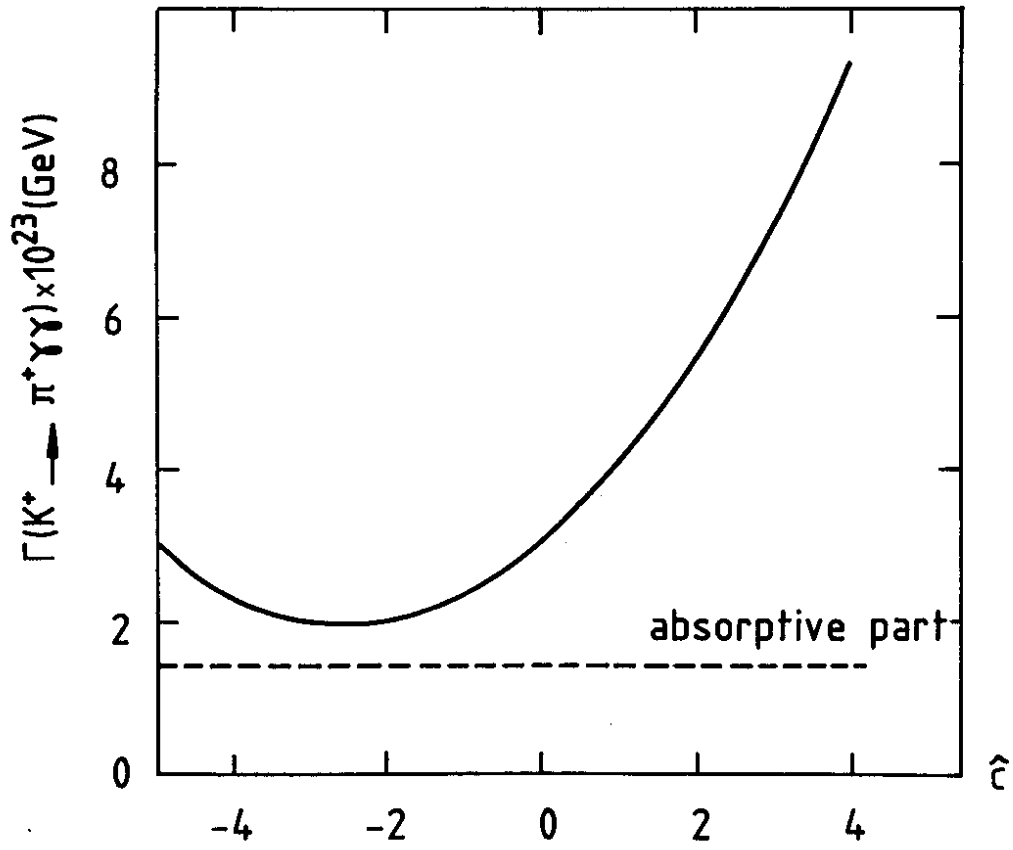


Fig. 8

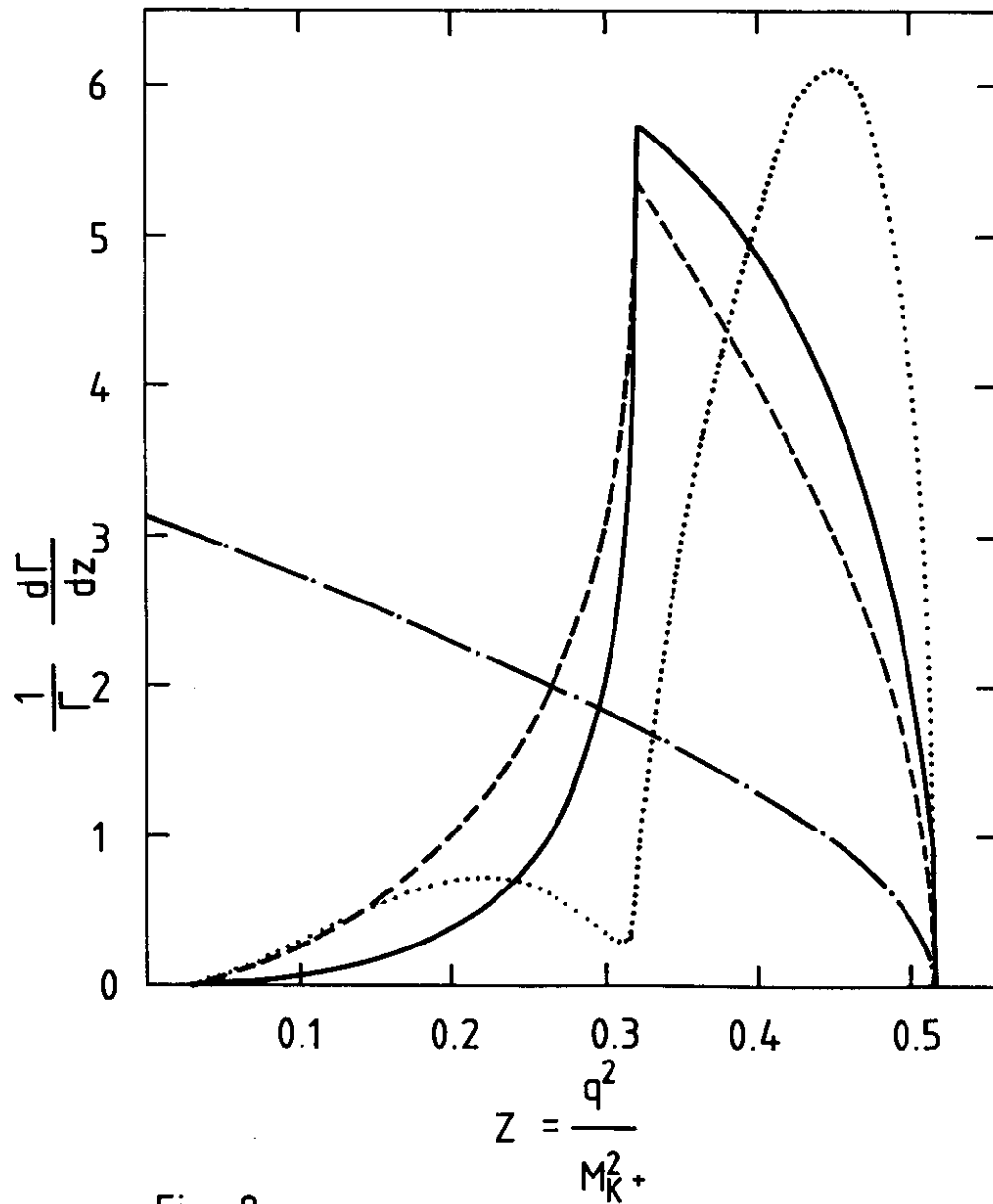


Fig. 9

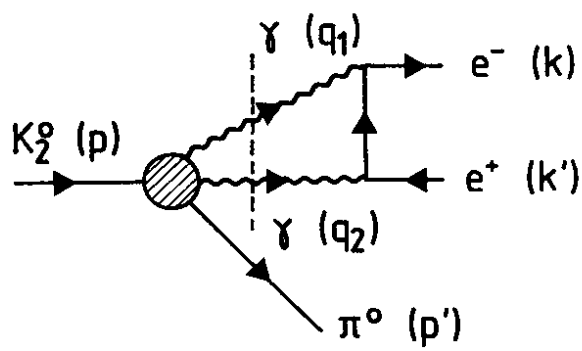


Fig. 10

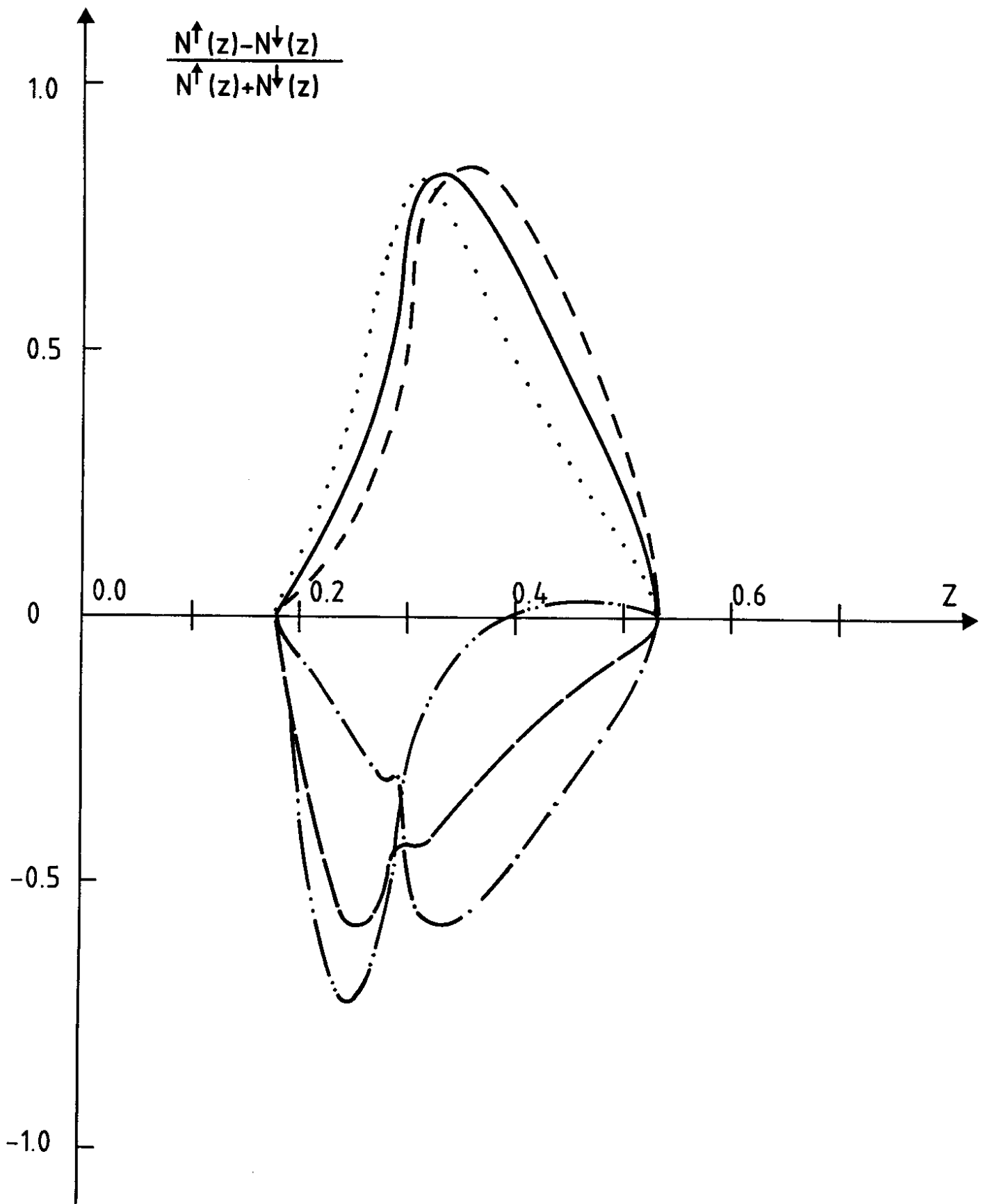


Fig. 11