$\Gamma(Z \rightarrow b\bar{b})$: A SIGNATURE OF HARD MASS TERMS FOR A HEAVY TOP

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A B S T R A C T

We calculate analytically the weak radiative corrections to the weak neutral current gauge boson-bottom fermion vertex, keeping the mass m_t of the internal fermion line for the relevant diagrams. We find, to order α, a hard mass-term dependence m_t^2/M_W^2 of the amplitude, for large m_t values. Its origin comes from the unphysical charged Higgs coupling to fermions in the renormalizable gauge or, equivalently, from the longitudinal charged gauge boson couplings. The diagonal Z° decay width to b-quarks decreases, due to these weak radiative corrections, by 0.6%-2.5% when the top mass m_t varies from 45 to 200 GeV.

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A striking feature of the spectrum of known fermions is that their masses increase by large factors in going from one generation to the next. The consistency of the standard electroweak theory, for three generations, with the present experimental information and, in particular with the observed [1] large $B_d^\circ - \bar{B}_d^\circ$ mixing seems to demand [2] a top quark as heavy as or heavier than the charged weak boson. Therefore, it is not only theoretically appealing but phenomenologically compelling to study the behaviour of weak amplitudes in the presence of virtual fermions such that $r \equiv (m_t/M_W)^2 > 1$. From this viewpoint, the manifestation of a heavy top quark, through its virtual effects, has been discussed from the determination [3] of the Z-boson mass combined with existing neutral current data. Present analyses [4] give the upper bound (for $M_H < 1$ TeV)

$$m_t < 200 \text{ GeV} \qquad\qquad (90\% \text{ c.l.})$$

If one assumes $M_H < 100$ GeV, this bound becomes $m_t < 180$ GeV. On the other side, a recent UA1 analysis [5] of the $p\bar{p} \to t\bar{t}X$ channel, using the most pessimistic assumptions on the cross-section uncertainties, suggests

$$m_t > 44 \text{ GeV} \qquad\qquad (95\% \text{ c.l.})$$

while a less conservative estimate of $p\bar{p} \to t\bar{t}X$ gives the more stringent bound $m_t > 56$ GeV. We shall consider that the top quark mass could be anywhere from 45 to 200 GeV.

It is known that in low-energy weak processes there are situations [6] in which the heavy fermions are not decoupled, so that flavour-changing neutral current amplitudes acquire at the one-loop level values which grow linearly with r, at large values of r. This somewhat counterintuitive behaviour constitutes an evasion of the decoupling theorem [7] for spontaneously broken gauge theories. This effect is clearly manifested in the processes $K \to \pi\nu\bar{\nu}$ [6,8] and $b \to s\ell\bar{\ell}$ [9], in which the flavour-changing neutral hadronic current amplitudes arise from Z-exchange. A similar phenomenon occurs not only from Z-exchange diagrams but also in box diagrams [6,10] responsible of $K-\bar{K}$ or $B-\bar{B}$ mixing. It is not difficult to imagine how this heavy fermion effect comes about. In the renormalizable gauge, the Yukawa coupling of unphysical scalars is proportional to the fermion mass. Therefore, powers of scalar couplings could compensate inverse powers of the fermion mass arising from heavy fermion propagators.

We address the question of the importance of the heavy fermion non-decoupling for the diagonal weak neutral current vertex, in a situation in which the hard mass term dependence effect cannot be compensated by arguments of small intergeneration mixing excuses. We put all fermion masses to zero, but one, such that r is allowed

to vary to arbitrarily large values. Phenomenologically this study selects the Z° → $b\bar{b}$ vertex as being the process of interest. As the presence of these hard terms is a consequence of the fermion mass generating Higgs mechanism of the standard theory, their detection would not only be a manifestation of the existence of a heavy top quark but also of the symmetry-breaking mechanism. The experimental measurement of the weak radiative corrections to the Z° → $b\bar{b}$ decay provides a window to these phenomena.

The diagrams which contain the interesting top quark mass effect arise from W-exchange in the one-loop correction to the $Zb\bar{b}$ vertex, and they are depicted in Fig. 1 in the renormalizable 't Hooft-Feynman gauge. There are also Z-exchange contributions, without the top quark running in the loop, and "uninteresting" QED radiative corrections. The one-loop renormalized neutral weak current vertex has been calculated [11] in the on-shell renormalization scheme, in the limit, when possible, of zero mass fermions. The QED correction [12] needs the addition of soft photon bremsstrahlung diagrams to exactly cancel the infrared singularities. The resulting QED contribution is thus well known, dependent of the particular experiment and typically of a size $(\alpha/\pi) \ln(M_Z^2/m_b^2) \simeq 1\%$. It will not be discussed anymore here, but its inclusion is necessary and straightforward.

We follow the terminology of Ref. [13] to call the remaining electroweak contribution as "weak" or "non-QED" correction. The Z-exchange contribution to the renormalized vertex does not depend on the top quark and thus the zero mass limit result of Böhm et al. [11] is appropriate for us. In addition to this Z-exchange contribution, we have the interesting W-exchange diagrams of Fig. 1, which are studied now.

We calculate the invariant T-matrix element for the W-exchange part of the $Zb\bar{b}$ vertex in the renormalizable 't Hooft-Feynman gauge, as explicit in the diagrams of Fig. 1. We obtain

$$T = - \frac{g}{\cos\theta_w} \bar{u}(p_1,\lambda_1) \Gamma^{\nu} v(p_2,\lambda_2) \varepsilon_{\nu}(q,\lambda) \tag{1}$$

where, in the limit of zero <u>external</u> quark mass, the induced current at the one-loop level, mediated by the W, is of the V-A type

$$\Gamma_W^{\nu} = \frac{\alpha}{2\pi} \gamma^{\nu} L \; I(s,r) \tag{2}$$

In Eq. (1), g is the SU(2) gauge coupling and θ_w the weak mixing angle. In Eq. (2), α is the fine structure constant, L the left-handed projector $L \equiv \frac{1}{2}(1-\gamma_5)$ and

$I(s,r)$ is a form factor to be evaluated at $s = (M_Z/M_W)^2$ and $r \equiv (m_t/M_W)^2$. The limit of the zero quark mass for external legs is well defined in each diagram by itself, except for the self-energy ones in which one has to sum diagrams (1c)+(1d) and (2c)+(2d) of Fig. 1.

The ultraviolet divergences are dealt with by using dimensional continuation to regulate them. Diagrams (1e) and (1f) are finite. The divergences in diagrams (2a) to (2d) are r-dependent, but they cancel among themselves, as it should be. All these finite contributions go like r for $r \to 0$ and, therefore, they do not contribute to the renormalized vertex at $r = 0$. We are left with the first four diagrams (1a), (1b) and (1c)+(1d), for which the pole term in the dimensionally regularized amplitude is r-independent. As a consequence, given a mass-independent renormalization scheme, we can write for the renormalized form factor of Eq. (2)

$$I_R(s,r) = I_R(s,r=0) + F(s,r)$$

$$F(s,r) \equiv I(s,r) - I(s,r=0) \tag{3}$$

where $F(s,r)$ is finite, as explained above, and $I_R(s,r = 0)$ corresponds to the W-exchange contribution to the renormalized vertex for vanishing fermion masses [11]. In Fig. 2 we present the results obtained by us for $F(s,r)$, as function of r, at the physical $s = s_0 = (M_Z/M_W)^2$. The constant imaginary part of F, for $r > s/4$, is an artifact of the subtraction made at $r = 0$, and it is cancelled for $I_R(s,r > s/4)$. Below $r = s/4$, the imaginary part of F depends on r. The discontinuity at $r = s/4$, the threshold for $t\bar{t}$ production, is also clearly manifested in the real part of the form factor $F(s,r)$.

We are mainly interested in the situation $r \gtrsim s/4$, for which real top quark production is kinematically forbidden. The real part of $F(s,r)$ has different behaviour, at large r, for the diagrams (1) and for the set of diagrams (2), as explicitly shown in Fig. 2. These results have been obtained from exact calculations at each value of r. Due to the interest in the large r behaviour of the amplitude, we give the asymptotic behaviour of each diagram contributing to $F(s,r)$ for $r \gg 1$, at fixed s. We obtain

$$F^{(1a)} \sim \frac{1}{6} \left(1 - \frac{3}{4 \sin^2 \theta_w}\right) \ln r$$

$$F^{(1b)} \sim \frac{3}{4} \cot^2 \theta_w \ln r$$

$$F^{(1c+1d)} \sim \frac{1}{12} \left(1 - \frac{3}{2 \sin^2 \theta_w}\right) \ln r \tag{4.1}$$

$$F^{(1e+1f)} \sim \frac{1}{2} \ln r$$

$$F^{(2a)} \sim \frac{r}{12} \left[\ln r + \Delta + \frac{3}{2 \sin^2 \theta_w}\right] + \frac{1}{6} \ln r \left(1 - \frac{3}{4 \sin^2 \theta_w}\right)$$

$$F^{(2b)} \sim -\frac{1}{8} \left(1 - \frac{1}{2 \sin^2 \theta_w}\right) \left\{ r \left[\ln r + \Delta\right] + \left(2 - \frac{s}{3}\right) \ln r \right\}$$

$$F^{(2c+2d)} \sim \frac{1}{24} \left(1 - \frac{3}{2 \sin^2 \theta_w}\right) \left\{ r \left[\ln r + \Delta\right] + 2 \ln r \right\} \tag{4.2}$$

where $\Delta \equiv 1/\varepsilon + \gamma_E - \ln(4\pi) - 3/2 + \ln(M_W^2/v^2)$.

We have kept in Eqs. (4) the leading terms with r, up to a constant. The s-dependence of these terms, if any, is particularly simple. The behaviour $r \ln r$, present in each of the diagrams (2), cancels in their sum together with the divergent Δ-terms. Diagrams (1), and their sum, have a soft quark mass dependence, as corresponds to a leading $\ln(r)$ behaviour. The hard mass term dependence of $F(s,r)$, for large r, comes entirely from diagram (2a), with the power dependence m_t^2/M_W^2.

The asymptotic (in r) expressions given above for all s are not enough to reproduce the correct values for the phenomenologically interesting values of r, corresponding to a top quark mass between 45 and 200 GeV. We have obtained such an expansion up to terms of order r^{-3}. Adding the contributions of the different diagrams, the large r-behaviour of $F(s,r)$, for the on-shell $s = s_0 \equiv M_Z^2/M_W^2$ value, is given by

$$F(s_0, r) = \frac{1}{8 \sin^2\theta_w} \left\{ r + 2.880 \, \ell_n r - 6.716 - i \, 1.484 \right.$$
$$+ (8.368 \, \ell_n r - 3.408)/r$$
$$+ (9.126 \, \ell_n r + 2.260)/r^2 \tag{5}$$
$$+ (4.043 \, \ell_n r + 7.410)/r^3 + \ldots \left. \right\}$$

For $r \gtrsim 2$, the differences between Eq. (5) and the exact result plotted in Fig. 2 are of the order of percent or less.

We see that, in the renormalizable gauge of the spontaneously broken gauge theory, the hard mass term originates from the diagram with the unphysical charged Higgs particle in the loop. It is rather natural to find this behaviour in this gauge, because the Yukawa coupling of this boson to fermions is proportional to the fermion mass:

$$m_b L - m_t R \tag{6}$$

However, in the unitary gauge there is nothing related to these would-be-Nambu-Goldstone modes. In this gauge, the behaviour of the longitudinal W_L boson degrees of freedom is playing an equivalent rôle [14].

Combining all our results, and including the renormalized vertex function for zero quark masses in the Z-exchange contribution [11], we calculate the rate ratio between the weak radiative correction and the tree-level value for the diagonal $Z \to b\bar{b}$ decay, which is given by

$$\frac{\delta \Gamma^{(weak)}}{\Gamma(Z^0 \to b\bar{b})} \simeq 2 \, \frac{v \, \delta^w v + a \, \delta^w a}{v^2 + a^2} \tag{7}$$

where

$$v = -\frac{1}{4} + \frac{1}{3} \sin^2\theta_w \qquad ; \qquad a = -\frac{1}{4} \tag{8}$$

are the tree-level vector and axial parts of the vertex Γ^μ defined in Eq. (1), while

$$\delta^{W} v \; \simeq \; \frac{\alpha}{4\pi} \; [\; 0.945 \; + \; Re(\; F(s_{0}, r)) \;]$$

$$\delta^{W} a \; \simeq \; \frac{\alpha}{4\pi} \; [\; 0.942 \; + \; Re(\; F(s_{0}, r))]$$

(9)

contain the real part of the pure weak corrections to v and a respectively. As the $O(\alpha)$ correction to the decay width comes from the interference between the tree-level vertex and its radiative corrections, the imaginary part of the radiatively corrected vertex does not play any rôle to this order. The small difference between $\delta^{W} v(r = 0)$ and $\delta^{W} a(r = 0)$ is due to the Z-exchange contribution.

The behaviour of Eq. (7) vs. the top quark mass is displayed in Fig. 3. We see that the hard mass dependence for a heavy top quark is clearly manifested, and the weak radiative corrections have a relative size which monotically increases (in absolute value) from 0.6% to 2.5% when the top mass m_{t} varies from 45 to 200 GeV.

Our calculation for the function $F(s, r)$, at large values of r, can be adapted with minor changes to estimate the flavour-changing decay rate [15] of the Z° to $b\bar{s}$. The contribution to this flavour-changing amplitude is again dominated by the top quark running in the loop. If we define a rate ratio between this decay and the diagonal decay rate to $b\bar{b}$, we obtain

$$R \equiv \frac{\Gamma(Z^{\circ} \to b\bar{s}) + \Gamma(Z^{\circ} \to \bar{b}s)}{\Gamma(Z^{\circ} \to b\bar{b})} \; \simeq \; 1.5 \; 10^{-5} \; (A \, \lambda^{2})^{2} \; | F(s_{0}, r) |^{2}$$

(10)

where $\lambda = 0.22$ and $A \sim 1$ are the parameters of the Wolfenstein presentation [16] of the Kobayashi-Maskawa matrix. For large values of r, R behaves now as the fourth power m_{t}^{4}/M_{W}^{4} with the top mass. When m_{t} varies from 45 to 200 GeV, the use of the real and imaginary parts of $F(s, r)$, as given in Fig. 2, leads to R-values which increase from $2.2 \; 10^{-8}$ to $5.8 \; 10^{-7}$.

To conclude, we have calculated analytically the weak radiative corrections to the renormalized $Z^{\circ}b\bar{b}$ vertex, studying the dependence of the W-exchange part of the amplitude with the top quark mass. We find a power behaviour for the mass dependence and for large mass one has, instead of decoupling, an enhancement due to the heavy quark effect. The weak radiative corrections tend to decrease the

diagonal Z° decay width to b-quarks: its relative size goes monotically from 0.6% to 2.5% when the top quark mass varies between 45 and 200 GeV. It remains to be seen whether such an important signature of the underlying Higgs mechanism for the fermion mass generation in the standard theory could be disentangled in forthcoming experiments.

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FIGURE CAPTIONS

Fig. 1: W-exchange contributions to the $Zb\bar{b}$ vertex in the 't Hooft-Feynman gauge. External particles and momenta are indicated in diagram 1a.

Fig. 2: Real (———) and imaginary (----) parts of the form factor $F(s,r)$ at $s = (M_Z/M_W)^2$ as function of $r \equiv (m_t/M_W)^2$. The separate contributions to $\text{Re}\{F(s,r)\}$ coming from type 1 (-•-•-•-) and type 2 (••••) diagrams are also displayed. The value $\sin^2\theta_W = 0.23$ for the Weinberg angle has been used.

Fig. 3: Rate ratio between the weak radiative correction and the tree-level value, for the diagonal $Z \to b\bar{b}$ decay, as function of the top mass. The values $\sin^2\theta_W = 0.23$ and $M_W = 82$ GeV have been used.

Fig. 1
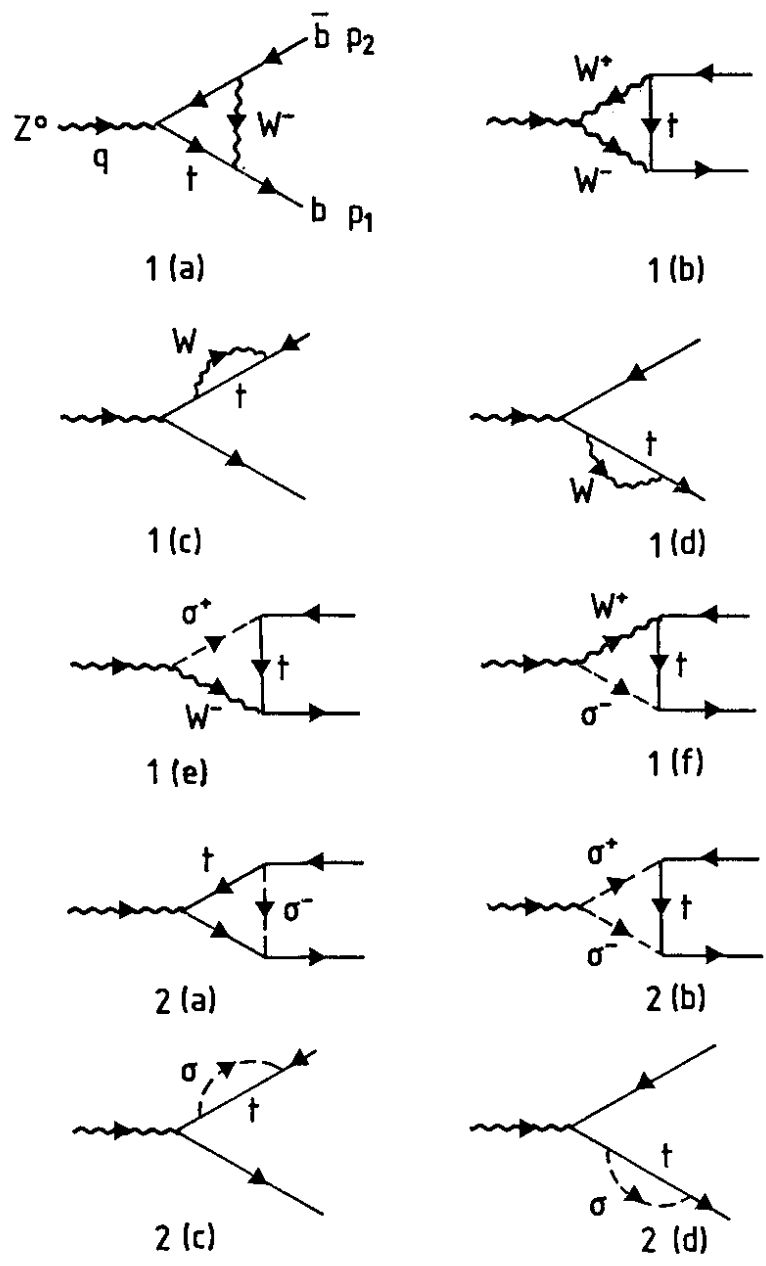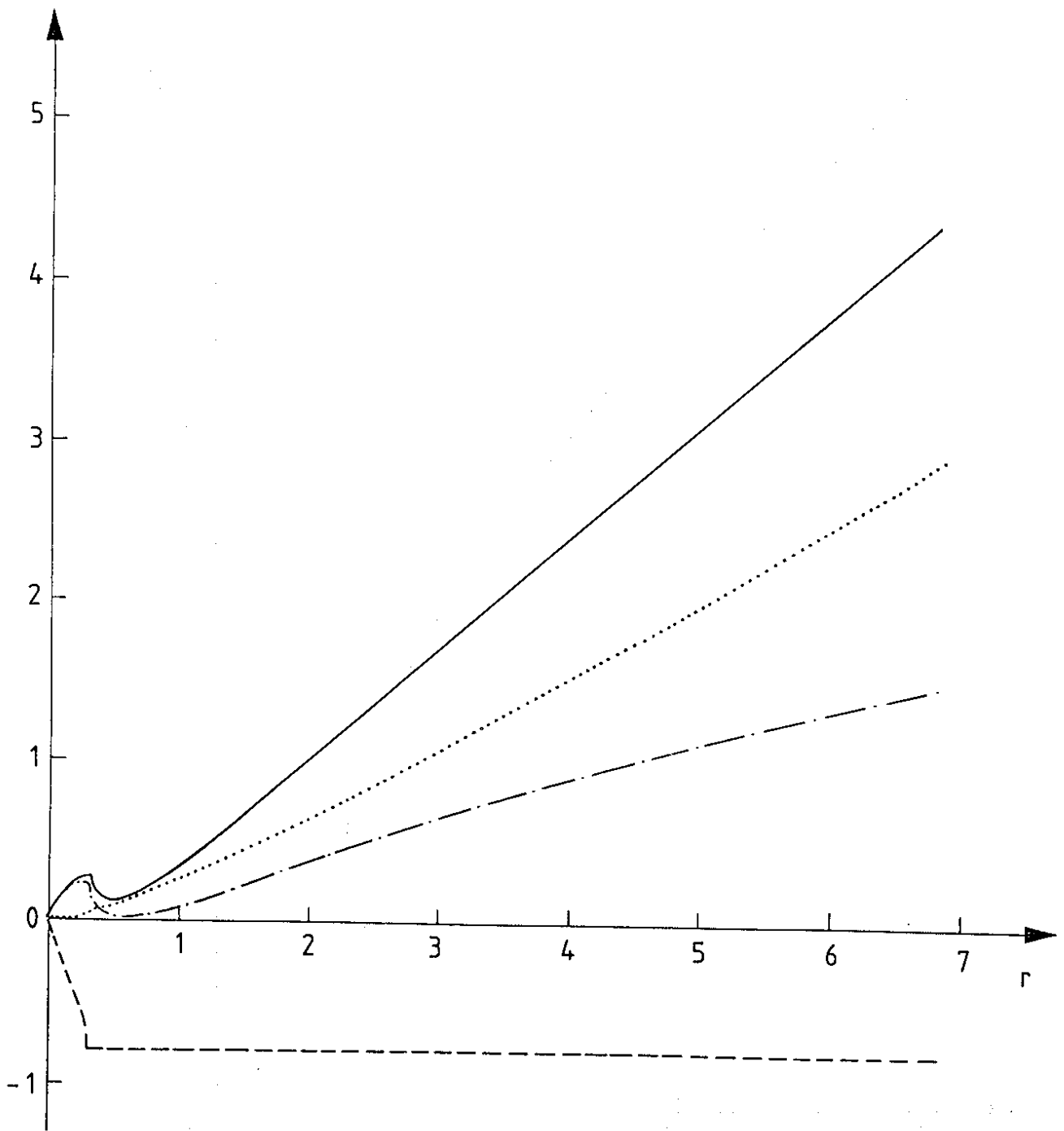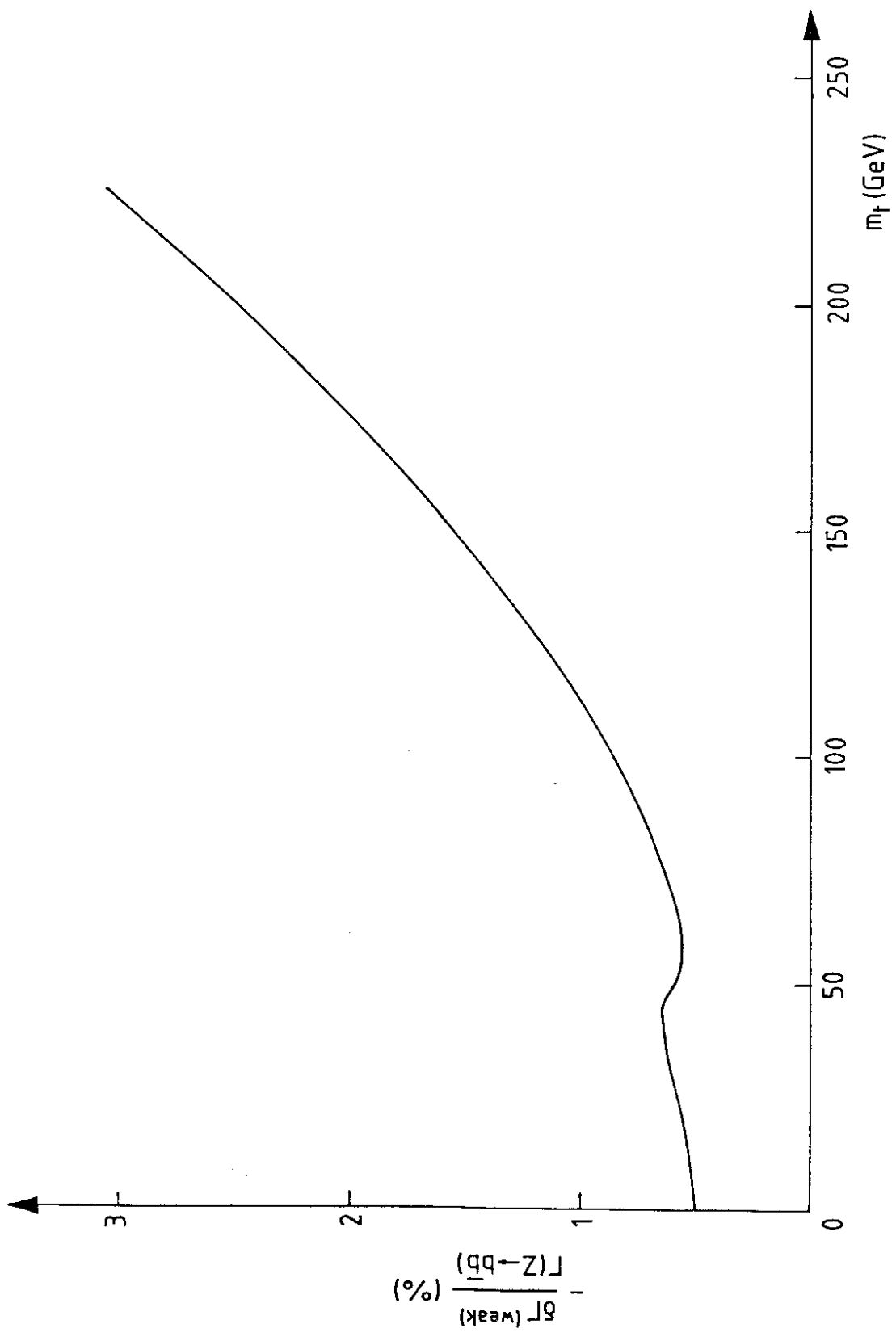
Fig. 2

Fig. 3

$-\dfrac{\delta\Gamma(\text{weak})}{\Gamma(Z\to b\bar{b})}\ (\%)$

m_t (GeV)