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THE ROLE OF RESONANCES IN CHIRAL PERTURBATION THEORY

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Abstract

The strong interactions of low-lying meson resonances (spin ≤ 1) with the octet of pseudoscalar mesons (π, K, η) are considered to lowest order in the derivative expansion of chiral $SU(3)$. The resonance contributions to the coupling constants of the $O(p^4)$ effective chiral lagrangian involving pseudoscalar fields only are determined. These low-energy coupling constants are found to be dominated by the resonance contributions. Although we do not treat the vector and axial-vector mesons as gauge bosons of local chiral symmetry, vector meson dominance emerges as a prominent result of our analysis. As a further application of chiral resonance couplings, we calculate the electromagnetic pion mass difference to lowest order in chiral perturbation theory with explicit resonance fields.

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1 Introduction

At low energies the strong, electromagnetic and weak interactions of pseudoscalar mesons can be described by an effective chiral lagrangian. This lagrangian depends on a number of low-energy coupling constants which cannot be determined from the symmetries of the fundamental theory only. They are in principle determined by the underlying QCD dynamics in terms of the renormalization group invariant scale Λ and the heavy quark masses $(m_c, m_b, \ldots)$.

The coupling constants of the effective chiral lagrangian will in general receive contributions from different sources, in particular from meson resonances, but also from other hadronic states or even direct short-distance contributions. The purpose of this paper is to demonstrate that the coupling constants of the effective chiral lagrangian for strong interactions at order p^4 [1,2] are essentially saturated by meson resonance exchange . This extends a previous analysis [1] of ρ exchange in $SU(2)_L \times SU(2)_R$ to general meson resonance exchange in the framework of chiral $SU(3)$. The non-leptonic weak interactions will not be considered here.

Independently of the general problem of including resonances in chiral perturbation theory (CHPT), it is of interest to find out which hadronic states are especially important for low-energy hadronic interactions in a consistent chiral framework. Many years of phenomenological analysis in both nuclear and particle physics have provided ample evidence for the special rôle of vector mesons in this respect. They have therefore been included in chiral lagrangians from the early days on [3], usually with the assumption that vector and axial-vector mesons are at least in some approximate sense the gauge bosons of local chiral symmetry. Comprehensive reviews of such attempts emphasizing especially the more recent ideas of "hidden" local chiral symmetry can be found in Ref. [4]. In spite of this attractive hypothesis it must be stressed that there is no proof for the existence of dynamical gauge bosons of local chiral symmetry in QCD.

From the point of view of chiral symmetry, there is nothing special about vector and axial-vector mesons compared to scalar, pseudoscalar or any other meson resonances. All meson resonance fields will be treated in this paper on the same level: they carry non-linear realizations of chiral $SU(3)$ which are uniquely determined by the known transformation properties under the vectorial subgroup $SU(3)_V$ (octets and singlets). In spite of this democratic treatment of all meson resonances with spin ≤ 1 the special rôle of vector mesons will emerge very clearly from our analysis.

In Sect. 2 we recall the basic features of CHPT [1,2,5] to calculate the generating functional of Green functions of quark currents in a systematic expansion in powers of external momenta and quark masses. At lowest order p^2, the effective action is provided by the non-linear sigma model coupled to external fields. Of special interest for the present investigation is the local action of order p^4. The corresponding coupling constants $L_1, \ldots, L_{10}$ were determined some time ago by comparison with experiment [2].

In Sect. 3 we introduce resonance fields of type V, A, S and P carrying non-linear realizations of chiral $SU(3)$. Their transformation properties under $SU(3)_V$ specify their interactions with the pseudoscalar mesons. For our purpose we only need the lowest order couplings in the chiral expansion which are linear in the resonance fields. A complete list of such couplings allowed by chiral symmetry, P and C invariance is given for octets and singlets of type V, A, S and P.

The values of the corresponding resonance parameters (masses and couplings) are determined as far as possible in Sect. 4. While the vector meson parameters can be directly taken from experiment, we use Weinberg's sum rules [6] in the resonance approximation to fix the mass and coupling of the axial-vector meson octet. As a check for the axial coupling, we calculate $\Gamma(A_1 \to \pi\gamma)$ to lowest order. While only octets couple to lowest order for V and A, both octets and singlets can in principle contribute for S and P. These couplings cannot be reliably estimated from decay processes alone.

The contributions of meson resonances to the p^4 effective chiral lagrangian are worked out[1] in Sect. 5. We find clear evidence for the importance of vector (and to a lesser extent axial-vector) meson contributions which account for the bulk of the low-energy coupling constants. The coupling parameters unaffected by spin-1 exchange are then shown to be dominated by pseudoscalar singlet (η') and very likely by scalar octet exchange. As a check for the scalar dominance assumption we calculate $\Gamma(a_0 \to \eta\pi)$ in good agreement with experiment. We collect the various contributions and find evidence for a complete resonance dominance of the coupling constants $L_1, \ldots, L_{10}$.

In Sect. 6 we calculate the electromagnetic pion mass difference in the chiral limit including explicit resonance fields. Unlike in the previous section where resonance exchange was restricted to tree diagrams, we must now consider loop diagrams involving both the photon and the resonance fields. Due to the Weinberg sum rules [6], the one-loop mass shift is finite and reduces to the old result of Das et al. [7] in resonance approximation. Phrasing the result in a different way, we find that in analogy to the coupling constants $L_1, \ldots, L_{10}$ also the single low-energy constant of $O(e^2p^0)$ is completely dominated by resonance (loop) contributions.

Our conclusions are summarized in Sect. 7. Appendix A contains a short discussion of antisymmetric tensor fields, which we use to describe massive spin-1 particles. Finally, resonance contributions to the effective chiral lagrangian for the case of $SU(2)_L \times SU(2)_R$ are considered in Appendix B.

2 Green Functions at Low Energies

The Green functions of the vector, axial-vector, scalar and pseudoscalar quark currents built out of the three flavours u, d and s are generated by the vacuum-to-vacuum transition amplitude

$$e^{iZ[v,a,s,p]} = \langle 0|Te^{i\int d^4x \mathcal{L}(x)}|0\rangle \tag{2.1}$$

associated with the lagrangian

$$\mathcal{L}(x) = \mathcal{L}^0_{QCD} + \bar{q}\gamma^\mu(v_\mu + \gamma_5 a_\mu)q - \bar{q}(s - i\gamma_5 p)q. \tag{2.2}$$

[1]We have been informed by J. Donoghue that a similar investigation is being performed by himself, C. Ramirez and G. Valencia.

$\mathcal{L}^0_{QCD}$ is the QCD lagrangian with the masses of the three light quarks set to zero. The external fields v_μ, a_μ, s and p are hermitian 3×3 matrices in flavour space. The quark mass matrix

$$\mathcal{M} = \text{diag}(m_u, m_d, m_s) \tag{2.3}$$

is contained in the field $s(x)$. In the following we disregard the $SU(3)$ singlet vector, axial-vector and pseudoscalar currents and put

$$\text{tr}\, a_\mu = \text{tr}\, v_\mu = \text{tr}\, p = 0.$$

The lagrangian (2.2) exhibits a local $SU(3)_L \times SU(3)_R$ symmetry

$$\begin{aligned} q &\rightarrow g_R \frac{1}{2}(1+\gamma_5)q + g_L \frac{1}{2}(1-\gamma_5)q \\ v_\mu \pm a_\mu &\rightarrow g_{R,L}(v_\mu \pm a_\mu)g^\dagger_{R,L} + i g_{R,L}\partial_\mu g^\dagger_{R,L} \\ s + ip &\rightarrow g_R(s+ip)g^\dagger_L \\ g_{R,L} &\in SU(3)_{R,L}. \end{aligned} \tag{2.4}$$

The generating functional Z admits an expansion in powers of the external momenta and of quark masses. Approximating Z by a given order in this expansion is called chiral perturbation theory (CHPT) [1,2,5]. As a consequence of chiral symmetry and its spontaneous breakdown, the generating functional Z coincides in the meson sector at *leading order* in CHPT with the classical action

$$Z = \int d^4x \mathcal{L}_2(U, v, a, s, p). \tag{2.5}$$

$\mathcal{L}_2$ is the non-linear σ model lagrangian coupled to the external fields v, a, s, p

$$\mathcal{L}_2 = \frac{f^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle \tag{2.6}$$

where

$$D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu), \qquad \chi = 2B_0(s+ip), \tag{2.7}$$

and $\langle A\rangle$ stands for the trace of the matrix A. U is a unitary 3×3 matrix

$$U^\dagger U = \mathbf{1}, \qquad \det U = 1,$$

which transforms as

$$U \rightarrow g_R U g^\dagger_L \tag{2.8}$$

under $SU(3)_L \times SU(3)_R$. U incorporates the fields of the eight pseudoscalar Goldstone bosons. The parameters f and B_0 are the only free constants at $O(p^2)$[2]: f is the pion decay constant in the chiral limit, $f_\pi = f(1 + O(m_{quark}))$, whereas B_0 is related to the condensate, $\langle 0|\bar{u}u|0\rangle = -f^2 B_0(1 + O(m_{quark}))$.

At order p^4 the generating functional consists of three terms [2]:

[2] f is denoted by F_0 in Refs. [1,2].

i) A contribution to account for the chiral anomaly.

ii) The one-loop functional originating from the lagrangian (2.6).

iii) An explicit local action of order p^4.

A functional which reproduces the anomaly was constructed by Wess and Zumino [8], whereas the one-loop functional may be found in [2]. In this article we are concerned with the local action of order p^4 which is generated by the lagrangian $\mathcal{L}_4$:

$$\begin{aligned}\mathcal{L}_4 \;=\;& L_1\langle D_\mu U^\dagger D^\mu U\rangle^2 + L_2\langle D_\mu U^\dagger D_\nu U\rangle\langle D^\mu U^\dagger D^\nu U\rangle \\ & + L_3\langle D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U\rangle + L_4\langle D_\mu U^\dagger D^\mu U\rangle\langle \chi^\dagger U + \chi U^\dagger\rangle \\ & + L_5\langle D_\mu U^\dagger D^\mu U(\chi^\dagger U + U^\dagger \chi)\rangle + L_6\langle \chi^\dagger U + \chi U^\dagger\rangle^2 + L_7\langle \chi^\dagger U - \chi U^\dagger\rangle^2 \qquad (2.9) \\ & + L_8\langle \chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger\rangle - iL_9\langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U\rangle \\ & + L_{10}\langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu}\rangle + H_1\langle F_{R\mu\nu}F_R^{\mu\nu} + F_{L\mu\nu}F_L^{\mu\nu}\rangle + H_2\langle \chi^\dagger \chi\rangle\end{aligned}$$

where

$$F_{R,L}^{\mu\nu} = \partial^\mu(v^\nu \pm a^\nu) - \partial^\nu(v^\mu \pm a^\mu) - i[v^\mu \pm a^\mu, v^\nu \pm a^\nu]. \qquad (2.10)$$

$L_1, \ldots, L_{10}$ are ten real low-energy coupling constants which, together with f and B_0, completely determine the low-energy behaviour of pseudoscalar meson interactions to $O(p^4)$ (H_1, H_2 are of no physical significance).

The new parameters $L_1, \ldots, L_{10}$ that arise at order p^4 are in general divergent (except L_3, L_7). They absorb the divergences of the one-loop functional referred to above. Consequently, they will depend on a renormalization scale μ which will, of course, drop out in all observable quantities. The renormalized parameters are denoted by $L_i^r(\mu)$ in the following.

It seems worthwhile to dwell upon the physical meaning of the coupling constants f, B_0, $L_1^r, \ldots, L_{10}^r$. In the language of CHPT, they parametrize the most general solution to the constraints imposed on the generating functional Z by chiral symmetry, P and C invariance and unitarity at order p^4. They are fixed by the dynamics of the underlying theory through the renormalization group invariant scale Λ and the heavy quark masses $m_c, m_b, \ldots$. With present techniques it is, however, not possible to calculate them directly from the QCD lagrangian (for several attempts see [9]). In the absence of such a calculational scheme they have been determined [2] by comparison with experimental low-energy information and by using large-N_C arguments . The result of that analysis is shown in the first column of Table 1, where we quote the values of $L_i^r(\mu)$ at the scale $\mu = M_\rho$. The entries are taken from Ref.[2], except for L_9 and L_{10}. The central value of L_9 is the same, whereas its error has been changed according to a recent accurate determination of the pion charge radius [10] (see also Bijnens and Cornet [11]). The value of L_{10} corresponds to a recent determination of the structure term associated with the decay $\pi \to e\nu\gamma$ [12]. The scale dependence of the running coupling constants is of some importance later in this article. We therefore list the central values of L_i^r at $\mu = 0.5$ GeV and $\mu = 1$ GeV in the second column of Table 1.

In Ref. [1] it was shown that the observed values of the corresponding coupling constants in the $SU(2)_L \times SU(2)_R$ case are quite well reproduced if one assumes that they are exclusively due to ρ exchange at a scale of order $\mu = 0.5$ GeV or $\mu = 1$ GeV (see App. B for details) . It is the purpose of this article to extend that analysis to the $SU(3)_L \times SU(3)_R$ case and to estimate the contributions of all low-lying resonances to the L_i^r and therefore to the effective chiral lagrangian at order p^4. We shall consider vector (V), axial-vector (A), scalar (S) and pseudoscalar (P) contributions and write the renormalized coupling constants $L_i^r(\mu)$ as sums

$$L_i^r(\mu) = \sum_{R=V,A,S,P} L_i^R + \hat{L}_i(\mu) \tag{2.11}$$

of resonance contributions L_i^R and a remainder $\hat{L}_i(\mu)$. The choice of the renormalization scale μ is arbitrary. However, it is rather obvious that we can only expect the resonances to dominate (if at all) the $L_i^r(\mu)$ when μ is not too far away from the resonance region. Therefore, we shall adopt $\mu = M_\rho$ as a reasonable choice in what follows.

In order to evaluate the resonance contributions L_i^R we have to include in the effective chiral lagrangian $\mathcal{L}_2$ [Eq. (2.6)] vector, axial-vector, scalar and pseudoscalar degrees of freedom in a chiral invariant manner. This is done in the following section.

3 Chiral Couplings of Resonances

From the point of view of chiral symmetry only, vector and axial-vector mesons do not have any special status compared to scalar, pseudoscalar or any other meson resonances. In particular, in a systematic low-energy expansion in powers of the momenta these massive particles do not play any special rôle – their presence only manifests itself indirectly in the values of the low-energy constants L_i^r. As we pointed out already in the Introduction, we shall therefore investigate the chiral couplings of vector and axial-vector mesons to Goldstone bosons along the lines outlined in Ref. [1] for the ρ meson couplings, i.e., not considering them as gauge bosons of any kind. With respect to transformations of the chiral group $G = SU(3)_L \times SU(3)_R$, all resonances are treated on the same footing. They carry non-linear realizations of G depending on their transformation properties under the diagonal subgroup $SU(3)_V$.

A non-linear realization of spontaneously broken chiral symmetry is defined [13] by specifying the action of G on the elements $u(\varphi)$ of the coset space $SU(3)_L \times SU(3)_R/SU(3)_V$:

$$u(\varphi) \xrightarrow{G} g_R u(\varphi) h(\varphi)^\dagger = h(\varphi) u(\varphi) g_L^\dagger \tag{3.1}$$

where φ^i ($1 \le i \le 8$) are the Goldstone fields parametrizing coset space and the equality in (3.1) is due to parity. Whenever an explicit form of $u(\varphi)$ is required we shall use the exponential parametrization

$$u(\varphi) = \exp\{-\frac{i}{\sqrt{2}}\,\Phi/f\}, \qquad \Phi = \frac{1}{\sqrt{2}}\sum_{i=1}^{8} \lambda_i \varphi^i. \tag{3.2}$$

The compensating $SU(3)_V$ transformation $h(\varphi)$ defined by (3.1) is the wanted ingredient for a non-linear realization of G. In practice, we shall only be interested in resonances transforming as octets or singlets under $SU(3)_V$. Denoting the multiplets generically by R (octet) and R_1 (singlet), the non-linear realization of G is given by

$$\begin{aligned} R &\xrightarrow{G} h(\varphi) R h(\varphi)^\dagger \\ R_1 &\xrightarrow{G} R_1 \end{aligned} \tag{3.3}$$

with the usual matrix notation for the octet

$$R = \frac{1}{\sqrt{2}} \sum_{i=1}^{8} \lambda_i R^i. \tag{3.4}$$

Since the non-linear realization of G on the octet field R in (3.3) is local we are led to define a covariant derivative

$$\nabla_\mu R = \partial_\mu R + [\Gamma_\mu, R] \tag{3.5}$$

with

$$\Gamma_\mu = \frac{1}{2}\{u^\dagger[\partial_\mu - i(v_\mu + a_\mu)]u + u[\partial_\mu - i(v_\mu - a_\mu)]u^\dagger\} \tag{3.6}$$

ensuring the proper transformation

$$\nabla_\mu R \xrightarrow{G} h(\varphi) \nabla_\mu R h(\varphi)^\dagger. \tag{3.7}$$

Without external fields, Γ_μ is the usual natural connection on coset space.

From (3.1) one infers the well-known linear representation

$$U(\varphi) \xrightarrow{G} g_R U(\varphi) g_L^\dagger \tag{3.8}$$

for the quantity $U(\varphi) = u(\varphi)^2$ [cf. (2.8)].

We shall now discuss the chiral couplings of meson resonances of the type $V(1^{--})$, $A(1^{++})$, $S(0^{++})$ and $P(0^{-+})$ to the pseudoscalar Goldstone fields. As far as the vector and axial-vector mesons are concerned, we shall describe the relevant degrees of freedom in terms of antisymmetric tensor fields [1] instead of the more familiar vector fields. This formulation is especially convenient when considering interactions with external gauge fields such as the electromagnetic field. Another advantage is that even in the presence of interactions the spin-1 character of the field is not modified. This is in contrast to the usual vector field formulation where couplings of the form

$$V_\mu \partial^\mu S \tag{3.9}$$

with a scalar field S may arise requiring a redefinition of the spin-1 vector field. A well-known example is provided by $a_1 - \pi$ mixing in the usual framework [4]. The description of massive spin-1 fields in terms of antisymmetric tensors is not very popular in phenomenological particle physics. We find it therefore useful to elaborate the method in some detail. In order not to interrupt the argument we relegate the discussion to App. A.

To determine the resonance exchange contributions to the effective chiral lagrangian we need the lowest order couplings in the chiral expansion which are linear in the resonance fields. With the coset element $u(\varphi)$ defined in (3.1) we obtain the following list of terms which can couple to those fields and which are at most of order p^2.

Octets

$$\begin{aligned} u_\mu &= iu^\dagger D_\mu U u^\dagger = u_\mu^\dagger \\ u_\mu u_\nu & \\ u_{\mu\nu} &= iu^\dagger D_\mu D_\nu U u^\dagger \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u \\ f_\pm^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u \end{aligned} \tag{3.10}$$

Singlets

$$\langle u_\mu u_\nu \rangle, \qquad \langle u_{\mu\nu} \rangle, \qquad \langle \chi_\pm \rangle. \tag{3.11}$$

The term $\nabla_\mu u_\nu = \partial_\mu u_\nu + [\Gamma_\mu, u_\nu]$ with Γ_μ defined in (3.6) is omitted in the list because of the relation

$$\nabla_\mu u_\nu = u_{\mu\nu} + \frac{i}{2}(u_\mu u_\nu + u_\nu u_\mu). \tag{3.12}$$

Invoking P and C invariance (cf. Table 2), it turns out that all the couplings linear in the fields V, A, S and P start at order p^2.

We merge all resonance couplings in a lagrangian

$$\mathcal{L}_{res} = \sum_{R=V,A,S,P} \{\mathcal{L}_{kin}(R) + \mathcal{L}_2(R)\} \tag{3.13}$$

with kinetic terms

$$\begin{aligned} \mathcal{L}_{kin}(R) &= -\frac{1}{2}\langle \nabla^\lambda R_{\lambda\mu} \nabla_\nu R^{\nu\mu} - \frac{M_R^2}{2} R_{\mu\nu} R^{\mu\nu} \rangle - \frac{1}{2}\partial^\lambda R_{1,\lambda\mu}\partial_\nu R_1^{\nu\mu} + \frac{M_{R_1}^2}{4} R_{1,\mu\nu} R_1^{\mu\nu}, \quad R = V, A \\ \mathcal{L}_{kin}(R) &= \frac{1}{2}\langle \nabla^\mu R \nabla_\mu R - M_R^2 R^2 \rangle + \frac{1}{2}\{\partial^\mu R_1 \partial_\mu R_1 - M_{R_1}^2 R_1^2\}, \quad R = S, P \end{aligned} \tag{3.14}$$

where M_R, M_{R_1} are the corresponding masses in the chiral limit. The interactions $\mathcal{L}_2(R)$

read

$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle \tag{3.15a}$$

$$\mathcal{L}_2[A(1^{++})] = \frac{F_A}{2\sqrt{2}} \langle A_{\mu\nu} f_-^{\mu\nu} \rangle \tag{3.15b}$$

$$\mathcal{L}_2[S(0^{++})] = c_d \langle S u_\mu u^\mu \rangle + c_m \langle S \chi_+ \rangle + \tilde{c}_d S_1 \langle u_\mu u^\mu \rangle + \tilde{c}_m S_1 \langle \chi_+ \rangle \tag{3.15c}$$

$$\mathcal{L}_2[P(0^{-+})] = i d_m \langle P \chi_- \rangle + i\tilde{d}_m P_1 \langle \chi_- \rangle. \tag{3.15d}$$

All coupling constants are real. In deriving the lagrangians (3.15) we have used the field equations [2] for D^2U and the relation

$$u^{[\mu\nu]} = iu^\dagger [D^\mu, D^\nu] U u^\dagger = -f_-^{\mu\nu}. \tag{3.16}$$

In the matrix notation (3.4)

$$V_{\mu\nu} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} & K^{*0} \\ K^{*-} & \overline{K^{*0}} & -\frac{2\omega_8}{\sqrt{6}} \end{pmatrix}_{\mu\nu} \tag{3.17}$$

and similarly for the other octets. We observe that for V and A only octets can couple whereas both octets and singlets appear for S and P (always to lowest order p^2). We also note that there is no coupling that would induce the transitions $V \to P\gamma$ at $O(p^2)$ in the chiral expansion. The leading couplings allowing these transitions are then $O(p^4)$. The consequences of this fact will be elaborated elsewhere [14].

In order to calculate the contribution of $\mathcal{L}_{res}$ to the effective chiral lagrangian we have to pin down the coupling constants and masses occurring in $\mathcal{L}_{res}$. This is done in the next section.

4 Resonance Parameters

4.1 Vector Mesons

The mass parameter M_V (octet mass in the chiral limit) cannot be directly determined from the observed mass spectrum. Using the empirical fact that vector meson masses may well be described by the quark counting rule [15], we conclude that M_V must be rather close to M_ρ. Thus, we shall take $M_V = M_\rho$ for the numerical discussion in Sect. 5. Note that the error committed through this choice is of order p^6 in the effective lagrangian. Since there is no coupling of the singlet vector meson at $O(p^2)$, singlet exchange will not contribute at $O(p^4)$ in the effective lagrangian and the value of M_{V_1} is of no concern.

The octet couplings F_V and G_V can in principle be determined from the decay rates for $\rho^0 \to e^+e^-$ and $\rho \to 2\pi$, respectively. From the observed rate [16] $\Gamma(\rho^0 \to e^+e^-) = (6.9 \pm 0.3)$ keV we obtain [3]

$$|F_V| = 154\,\text{MeV}, \tag{4.1}$$

while $\Gamma(\rho \to 2\pi) = (153 \pm 2)$ MeV [16] implies

$$|G_V| = 69\,\text{MeV}. \tag{4.2}$$

Since chiral corrections are in general difficult to estimate and since we are more interested in the general features of resonance contributions than in detailed fits, we shall refrain from assigning errors to our coupling constants.

For the decay $\rho \to 2\pi$ in particular, chiral corrections are expected to be important since the pions are far from being soft. In this case we can actually obtain a rather reliable estimate of the chiral corrections by noting that the vector form factor is quite well reproduced by the vector meson dominance formula

$$1 + F_V^\rho(t) = \frac{M_V^2}{M_V^2 - t}. \tag{4.3}$$

$F_V^\rho(t)$ is the ρ contribution to the vector form factor and is found to be

$$F_V^\rho(t) = \frac{F_V G_V}{f^2} \frac{t}{M_V^2 - t} \tag{4.4}$$

from (3.15a). Comparison between (4.3) and (4.4) requires $F_V G_V > 0$ and

$$|G_V| = 57\,\text{MeV}. \tag{4.5}$$

Including also the contributions from chiral loops [2] reduces (4.5) to

$$|G_V| = 53\,\text{MeV}. \tag{4.6}$$

Since L_9^r is determined precisely from the pion charge radius, the value (4.6) for G_V amounts to the assumption that $L_9^r(M_\rho)$ is completely given by ρ exchange. For the analysis of Sect. 5 we shall use (4.6). Comparison with (4.2) gives an idea of the magnitude of chiral corrections.

4.2 Axial-Vector Mesons

Instead of determining F_A and the octet mass M_A in the chiral limit from experiment we appeal to Weinberg's sum rules [6]. The first sum rule is known to converge even in the presence of quark masses while the second one converges only in the chiral limit [17]. Since we are interested precisely in the chiral limit values for F_A and M_A we can safely make use of both sum rules. The relevant vector and axial-vector currents follow in a straightforward

[3] Unless stated otherwise, we use $f = f_\pi = 93.3$ MeV. This is a consistent procedure at the order in which we consider the low-energy expansion in the present article.

manner from the lagrangians (2.6) and (3.13). Saturating the corresponding spectral functions with the one-particle contributions yields the two sum rules in the familiar form

$$F_V^2 = F_A^2 + f^2 \tag{4.7a}$$

$$M_V^2 F_V^2 = M_A^2 F_A^2. \tag{4.7b}$$

Thus, the Weinberg sum rules (4.7) allow for a determination of F_A, M_A in terms of the already known parameters F_V, f_π and M_V:

$$\begin{aligned} F_A &= \sqrt{F_V^2 - f_\pi^2} = 123\,\mathrm{MeV} \\ M_A &= M_V/\sqrt{1 - f_\pi^2/F_V^2} = 968\,\mathrm{MeV}. \end{aligned} \tag{4.8}$$

The mass M_A, which we recall is the axial-vector octet mass in the chiral limit, compares reasonably well with two recent determinations of the a_1 mass[4] from τ decay [18,19]:

$$M_{a_1} = \begin{cases} (1056 \pm 20 \pm 15)\mathrm{MeV} & [18] \\ (1046 \pm 11)\mathrm{MeV} & [19] \end{cases} \tag{4.9}$$

We can also check the Weinberg prediction for F_A by calculating the decay $a_1 \to \pi\gamma$ to lowest order in CHPT. From (3.15b) we obtain

$$\Gamma(a_1 \to \pi\gamma) = \frac{\alpha F_A^2 M_{a_1}}{24 f_\pi^2}(1 - \frac{M_\pi^2}{M_{a_1}^2})^3. \tag{4.10}$$

Comparison with the experimental value [21]

$$\Gamma(a_1 \to \pi\gamma) = (640 \pm 246)\mathrm{keV} \tag{4.11}$$

and using $M_{a_1} = 1050$ MeV yields

$$F_A = (135 \pm 30)\mathrm{MeV} \tag{4.12}$$

in remarkable agreement with (4.8).

4.3 Scalar Mesons

The most promising way to determine c_d and c_m in lowest order CHPT seems to be the decay $a_0 \to \eta\pi$ where both final mesons are reasonably soft. The relevant term in (3.15c) is given by

$$\mathcal{L}_2(a_0\eta\pi) = \frac{2\sqrt{2}}{\sqrt{3}\,f^2}(c_d \vec{a}_0 \partial_\mu \vec{\pi} \partial^\mu \eta - c_m \overset{0}{M}{}_\pi^2 \vec{a}_0 \vec{\pi} \eta) \tag{4.13}$$

[4]Note, however, that the errors in (4.9) do not include the considerable uncertainties involved in parametrizing a large-width resonance like the a_1. In fact, other τ decay experiments [20] have extracted substantially bigger values of M_{a_1} from similar raw data.

where $\overset{0}{M}{}^2_\pi = B_0(m_u + m_d)$ is the first term in the quark mass expansion of M_π^2, $M_\pi^2 = \overset{0}{M}{}^2_\pi(1+O(m_{quark}))$. However, from the observed rate [16] $\Gamma(a_0 \to \eta\pi) \simeq \Gamma_{tot}(a_0) = (54\pm7)$ MeV we can only determine a linear combination of c_d and c_m. Thus, we leave their values undetermined for the time being. In analogy to the vector mesons we shall assume the octet mass M_S in the chiral limit to be given by $M_{a_0} = 983$ MeV.

The scalar singlet parameters $\tilde{c}_d$, $\tilde{c}_m$ and M_{S_1} are practically impossible to determine at present because the assignments of the 0^{++} states with $I = 0$ are still controversial. However, we can invoke large-N_C arguments [22] to relate the scalar singlet to the scalar octet parameters. For $N_C = \infty$, octet and singlet mesons become degenerate and thus

$$M_{S_1} = M_S. \tag{4.14}$$

Moreover, since the amplitude for a meson to decay into two other mesons is [22] $O(N_C^{-1/2})$ and since $f = O(N_C^{1/2})$ we conclude that c_d, c_m, $\tilde{c}_d$, $\tilde{c}_m$ are all $O(N_C^{1/2})$ [cf. Eq. (4.13)]. Anticipating the results of Sect. 5 where scalar octet and singlet exchange will contribute also to coupling constants $(2L_1 - L_2, L_4, L_6)$ which are $O(1)$ for large N_C [2] and taking into account (4.14), we find that the scalar couplings must obey the nonet relations

$$\tilde{c}_d = \frac{\varepsilon}{\sqrt{3}} c_d, \qquad \tilde{c}_m = \frac{\varepsilon}{\sqrt{3}} c_m, \qquad \varepsilon = \pm 1 \tag{4.15}$$

for $N_C = \infty$. We shall use the large-N_C estimates (4.14) and (4.15) for the numerical discussion in Sect. 5.

4.4 Pseudoscalar Resonances

Although we do not expect the pseudoscalar nonet (including, e.g., the $\pi(1300)$) to give rise to important contributions to the low-energy effective lagrangian we have included the octet P in $\mathcal{L}_{res}$ for completeness. We shall argue in the next section that we can safely disregard those contributions.

The more interesting case is the pseudoscalar singlet η_1 which becomes a Goldstone boson in the limit $N_C \to \infty$. The lagrangian (3.15d) gives rise to $\eta_1 - \pi^0$ and $\eta_1 - \eta_8$ mixing via[5]

$$\mathcal{L}_2(\text{mixing}) = \frac{4\tilde{d}_m B_0}{f}\{\eta_1\pi^0(m_u - m_d) - \frac{2\eta_1\eta_8}{\sqrt{3}}(m_s - \hat{m})\}, \qquad \hat{m} = \frac{1}{2}(m_u + m_d). \tag{4.16}$$

In the $SU(2)$ limit $m_u = m_d$ the mass terms relevant for $\eta - \eta'$ mixing are of the form

$$\mathcal{L}_{mass} = -\frac{1}{2}(\eta_8\eta_1)\begin{pmatrix} \bar{M}^2_{\eta_8} & \delta m^2 \\ \delta m^2 & \bar{M}^2_{\eta_1} \end{pmatrix}\begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix} \tag{4.17}$$

$$\delta m^2 = \frac{8\tilde{d}_m B_0}{\sqrt{3}\,f}(m_s - \hat{m}) = \frac{8\tilde{d}_m}{\sqrt{3}\,f}(\overset{0}{M}{}^2_K - \overset{0}{M}{}^2_\pi) \tag{4.18}$$

[5]We shall use from now on η_1 instead of P_1 to denote the pseudoscalar singlet field.

where $\overset{0}{M}{}_\pi^2$ and $\overset{0}{M}{}_K^2$ are the pseudoscalar octet masses to leading (linear) order in the quark mass expansion and $\bar{M}_{\eta_8}(\bar{M}_{\eta_1})$ is the octet (singlet) mass. $\bar{M}_{\eta_1}$ is related to M_{η_1} by

$$\bar{M}_{\eta_1}^2 = M_{\eta_1}^2 + d(2\overset{0}{M}{}_K^2 + \overset{0}{M}{}_\pi^2) + O(m_{quark}^2) \tag{4.19}$$

with $d = 1/3$ in the large-N_C limit [23,2].

We may now obtain an estimate for the coupling $\tilde{d}_m$ as follows. From (4.17) we have

$$\begin{aligned} \bar{M}_{\eta_8}^2 + \bar{M}_{\eta_1}^2 &= M_\eta^2 + M_{\eta'}^2 \\ \delta m^2 &= (\bar{M}_{\eta_8}^2 \bar{M}_{\eta_1}^2 - M_\eta^2 M_{\eta'}^2)^{1/2} \end{aligned} \tag{4.20}$$

with $M_\eta = 548.8$ MeV, $M_{\eta'} = 957.6$ MeV. In Sect. 5 we shall determine L_4 and L_6 from scalar exchange using L_5 and L_8 as input. The information on these couplings suffices to evaluate $\bar{M}_{\eta_8}$ from the quark mass expansion of the η mass squared given in [2]. We skip all details and just quote the result $\bar{M}_{\eta_8} = 639$ MeV. Using finally the values $\overset{0}{M}{}_\pi = 135$ MeV, $\overset{0}{M}{}_K = 487$ MeV and $f = 87.2$ MeV quoted in [2], we find from (4.18) and (4.20)

$$\begin{aligned} M_{\eta_1} &= 804\,\mathrm{MeV} \\ |\tilde{d}_m| &= 20\,\mathrm{MeV}. \end{aligned} \tag{4.21}$$

where we have used $d = 1/3$ and neglected terms of $O(m_{quark}^2)$ in (4.19). The estimate (4.21) for $\tilde{d}_m$ is in nice agreement with the large-N_C prediction $|\tilde{d}_m| = f/\sqrt{24} = 18$ MeV which follows by comparing the contribution (5.13) to L_7 and the corresponding expression in the large-N_C limit (see Ref. [2]).

5 Resonance Contributions to the Low-Energy Effective Chiral Lagrangian

The determination of the resonance contributions to the effective lagrangian is straightforward given the chiral couplings of Sects. 3 and 4. Since all those couplings are $O(p^2)$, resonance exchange will automatically produce contributions of $O(p^4)$ from the two vertices. This implies that only the non-derivative (momentum independent) parts of the resonance propagators are relevant for the L_i^R. Moreover, the resonance masses appearing in L_i^R will be the chiral limit values independent of the quark masses.

We shall be rather explicit for the vector meson contributions and only state the results for the remaining cases.

5.1 Vector Mesons

The lagrangian (3.15a) can be written as

$$\mathcal{L}_2(V) = \langle V_{\mu\nu} J^{\mu\nu} \rangle = \sum_{i=1}^{8} \frac{V_{\mu\nu}^i}{\sqrt{2}} \langle \lambda_i J^{\mu\nu} \rangle$$

with

$$J^{\mu\nu} = \frac{F_V}{2\sqrt{2}} f_+^{\mu\nu} + \frac{iG_V}{2\sqrt{2}} [u^\mu, u^\nu]. \tag{5.1}$$

Expanding around the classical solution for $V_{\mu\nu}$, we obtain the effective action S^V induced by V exchange

$$S^V = \frac{1}{2} \int d^4x \, \langle V_{\mu\nu} J^{\mu\nu} \rangle \tag{5.2}$$

where $V_{\mu\nu}$ satisfies the equation of motion

$$\nabla^\alpha \nabla_\rho V^{\rho\beta} - \nabla^\beta \nabla_\rho V^{\rho\alpha} + M_V^2 V^{\alpha\beta} = -2J^{\alpha\beta}. \tag{5.3}$$

Solving (5.3) by iteration, the contribution at order p^4 is found to be

$$\begin{aligned}
S^V &= \int d^4x \mathcal{L}_4^V(x) + O(p^6) \\
\mathcal{L}_4^V &= -M_V^{-2} \langle J^{\mu\nu} J_{\mu\nu} \rangle \\
&= \frac{G_V^2}{4M_V^2} \langle D_\mu U^\dagger D_\nu U D^\mu U^\dagger D^\nu U - D_\mu U^\dagger D_\nu U D^\nu U^\dagger D^\mu U \rangle \\
&\quad - \frac{iF_V G_V}{2M_V^2} \langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U \rangle \\
&\quad - \frac{F_V^2}{4M_V^2} \langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu} \rangle - \frac{F_V^2}{8M_V^2} \langle F_{R\mu\nu} F_R^{\mu\nu} + F_{L\mu\nu} F_L^{\mu\nu} \rangle.
\end{aligned} \tag{5.4}$$

In order to transform the first term in (5.4) into the basis employed for the lagrangian (2.9) we make use of the SU(3) relation [2]

$$\begin{aligned}
&\langle D_\mu U^\dagger D_\nu U D^\mu U^\dagger D^\nu U - D_\mu U^\dagger D_\nu U D^\nu U^\dagger D^\mu U \rangle = \\
&= -3 \langle D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U \rangle + \frac{1}{2} \langle D_\mu U^\dagger D^\mu U \rangle^2 + \langle D_\mu U^\dagger D_\nu U \rangle \langle D^\mu U^\dagger D^\nu U \rangle.
\end{aligned} \tag{5.5}$$

Inserting (5.5) into (5.4) and comparing with (2.9), we can directly read off the non-vanishing coupling constants L_i^V (including H_1^V for completeness):

$$\begin{aligned}
L_1^V &= \frac{G_V^2}{8M_V^2}, & L_2^V &= 2L_1^V, & L_3^V &= -6L_1^V, \\
L_9^V &= \frac{F_V G_V}{2M_V^2}, & L_{10}^V &= -\frac{F_V^2}{4M_V^2}, & H_1^V &= -\frac{F_V^2}{8M_V^2}.
\end{aligned} \tag{5.6}$$

5.2 Axial-Vector Mesons

Proceeding in exactly the same way as for the vector mesons with

$$\mathcal{L}_2(A) = \langle A_{\mu\nu} \hat{J}^{\mu\nu} \rangle, \qquad \hat{J}^{\mu\nu} = \frac{F_A}{2\sqrt{2}} f_-^{\mu\nu} \tag{5.7}$$

we easily obtain the axial-vector meson induced lagrangian of order p^4

$$\mathcal{L}_4^A = \frac{F_A^2}{4M_A^2}\langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu}\rangle - \frac{F_A^2}{8M_A^2}\langle F_{R\mu\nu}F_R^{\mu\nu} + F_{L\mu\nu}F_L^{\mu\nu}\rangle \tag{5.8}$$

and thus

$$L_{10}^A = \frac{F_A^2}{4M_A^2}, \qquad H_1^A = -\frac{F_A^2}{8M_A^2}. \tag{5.9}$$

Before proceeding with the scalar and pseudoscalar meson resonances it is very instructive to make a first comparison of the V and A contributions with the phenomenologically determined $L_i^r(\mu)$. As already discussed in Sect. 4, we prefer to determine G_V directly from L_9^r rather than from $\Gamma(\rho \to 2\pi)$. F_V, F_A, M_A are taken from (4.1) and the Weinberg sum rules (4.8) together with $M_V = M_\rho = 770$ MeV.

The results shown in Table 1 are a clear indication for the chiral version of vector (and to a lesser extent axial-vector) meson dominance. Whenever V and A contribute, they strongly dominate the low-energy coupling constants $L_i^r(M_\rho)$ leaving very little room for additional contributions. We emphasize once again that we did not have to make any assumptions about a possible gauge structure of the V and A interactions.

5.3 Scalar Mesons

In contrast to the spin-1 case both octet and singlet resonances contribute in this case. We denote the octet (singlet) mass in the chiral limit by M_S (M_{S_1}) and arrive at the following scalar contributions to the low-energy coupling constants:

Octet:

$$\begin{aligned} &L_1^S = -\frac{c_d^2}{6M_S^2}, \quad &&L_3^S = -3L_1^S, \quad &&L_4^S = -\frac{c_d c_m}{3M_S^2}, \quad &&L_5^S = -3L_4^S, \\ &L_6^S = -\frac{c_m^2}{6M_S^2}, \quad &&L_8^S = -3L_6^S, \quad &&H_2^S = \frac{c_m^2}{M_S^2}. \end{aligned} \tag{5.10}$$

Singlet:

$$L_1^{S_1} = \frac{\tilde{c}_d^2}{2M_{S_1}^2}, \qquad L_4^{S_1} = \frac{\tilde{c}_d \tilde{c}_m}{M_{S_1}^2}, \qquad L_6^{S_1} = \frac{\tilde{c}_m^2}{2M_{S_1}^2}. \tag{5.11}$$

5.4 Pseudoscalar Meson Resonances

Again both octet and singlet can in principle contribute although we expect the singlet η_1 contribution to be much more important in this case.

Octet:

$$L_7^P = \frac{d_m^2}{6M_P^2}, \qquad L_8^P = -3L_7^P, \qquad H_2^P = 6L_7^P. \tag{5.12}$$

Singlet:

$$L_7^{\eta_1} = -\frac{\tilde{d}_m^2}{2M_{\eta_1}^2}. \tag{5.13}$$

Referring to Table 1, we first concentrate on those coupling constants (L_5, L_7, L_8) which are definitely non-zero but do not get V or A contributions. Starting with L_7, we observe that η_1 exchange gives the right sign [2] while the contribution coming from the pseudoscalar octet resonances has the wrong sign. Therefore, neither the octet P nor a possible heavy singlet P_1 (the nonet partner of P) are expected to be of much relevance for the low-energy chiral lagrangian and we disregard those contributions in the sequel just as we neglect other resonances in the 1 – 2 GeV region. For L_7, this procedure is in addition supported by an argument based on large N_C [2]: the contribution (5.13) is of order N_C^2 in the large-N_C limit, while the other resonance contributions are of order N_C and thus suppressed. Using the values for M_{η_1} and $\tilde{d}_m$ quoted in (4.21) we find

$$L_7^{\eta_1} = -0.3 \cdot 10^{-3}. \tag{5.14}$$

Neglecting the octet P, the two remaining coupling constants of relevance, L_5 and L_8, only receive contributions from the scalar octet given in (5.10). We note that L_8^S necessarily has the correct positive sign. In Sect. 4 we discussed the problem of determining the scalar couplings c_d, c_m from scalar meson decays where only the decay $a_0 \to \eta\pi$ seems to be amenable to a trustworthy calculation in lowest order CHPT. We shall therefore turn the argument around and assume that L_5^S and L_8^S completely account for the phenomenological values $L_{5,8}^r(M_\rho)$ given in Table 1 to predict the rate $\Gamma(a_0 \to \eta\pi)$. In this way one computes with $M_S = M_{a_0} = 983$ MeV

$$\begin{aligned} |c_d| &= 3.2 \cdot 10^{-2}\,\mathrm{GeV} \\ |c_m| &= 4.2 \cdot 10^{-2}\,\mathrm{GeV} \\ c_d c_m &> 0. \end{aligned} \tag{5.15}$$

From (4.13) and using $\overset{0}{M}{}_\pi^2 \simeq M_\pi^2$ we can then calculate

$$\Gamma(a_0 \to \eta\pi)|_{theory} = 59\,\mathrm{MeV} \tag{5.16}$$

to be compared with $\Gamma(a_0 \to \eta\pi) \simeq \Gamma_{tot}(a_0) = (54 \pm 7)$ MeV from experiment. Even though this exercise cannot be considered as a definite proof for scalar dominance of L_5, L_8, the prediction (5.16) is at least a very convincing demonstration of its consistency. From (5.15) all other octet scalar contributions in (5.10) can be calculated and we collect the complete results for V, A, S, S_1 and η_1 exchange in Table 3 using the large-N_C estimates (4.14) and (4.15) for S_1 exchange.

The assumption of scalar dominance for L_5, L_8 has not only produced the successful prediction (5.16) for $\Gamma(a_0 \to \eta\pi)$, but it is also fully consistent with all the other low-energy information embodied in the L_i^r. The emerging picture of complete resonance saturation of all the low-energy constants $L_1, \ldots, L_{10}$ can be expressed in the concise form

$$\hat{L}_i(M_\rho) \simeq 0 \qquad (1 \leq i \leq 10) \tag{5.17}$$

in the notation of (2.11). In other words, there is no indication for the presence of any other contribution in addition to the meson resonances.

6 Electromagnetic Interactions and the Pion Mass Difference

In this section we show that the low-energy coupling constant which occurs at leading order in the effective lagrangian for electromagnetic interactions can also be estimated with resonance contributions.

We first disregard resonance contributions due to V, A, S and P exchange. For the evaluation of the mass shifts of the meson octet it is sufficient to evaluate the pole position of the relevant two-point functions of the meson fields defined in (3.2). Our calculation furthermore concerns the chiral limit $m_u = m_d = m_s = 0$, and we may therefore completely dispose of the external fields. The lowest order effective lagrangian including electromagnetism is then obtained from (2.6) by putting $v_\mu^3 + \frac{1}{\sqrt{3}} v_\mu^8$ proportional to the photon field A_μ, disregarding the remaining external fields and by adding the relevant kinetic term:

$$\mathcal{L} = -\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + \frac{f^2}{4} \langle (\partial_\mu U - ie[Q,U]A_\mu)(\partial^\mu U^\dagger - ie[Q,U^\dagger]A^\mu) \rangle \tag{6.1}$$

where $Q = \mathrm{diag}(2/3, -1/3, -1/3)$ is the charge matrix. The tree graphs associated with this lagrangian determine the leading term in the low-energy expansion. One-photon loops evaluated with (6.1) contribute corrections of order e^2. In addition to those loops, one has to add contributions from chiral invariant local terms of order e^2. The effective loop lagrangian at order e^2 is of the current $\times$ current type and transforms under $SU(3)_L \times SU(3)_R$ as (8,8), (8 $\times$ 8,1) and (1,8 $\times$ 8). The corresponding local counterterms are easily found by introducing spurions Q_R, Q_L which transform under $SU(3)_L \times SU(3)_R$ as

$$Q_I \to g_I Q_I g_I^\dagger, \qquad I = R, L. \tag{6.2}$$

At the end, one identifies Q_I with the charge matrix Q. One finds that the effective lagrangian contains a piece of order p^0

$$e^2 C \langle QUQU^\dagger \rangle, \tag{6.3}$$

where C is a low-energy constant, independent of the quark masses and not fixed by chiral symmetry alone. We shall not consider counterterms of order e^2p^2 in the following.

Now we show that the low-energy constant C determines the electromagnetic masses $\hat{M}_{\pi^\pm}$, $\hat{M}_{K^\pm}$ of the pions and kaons[6] in the chiral limit. In the parametrization (3.2) the term (6.3) has the form

$$e^2 C \langle QUQU^\dagger \rangle = -\frac{2e^2C}{f^2}(\pi^+\pi^- + K^+K^-) + O(\Phi^4). \tag{6.4}$$

[6]The index $\hat{}$ on meson masses denotes the chiral limit values.

It contributes an equal amount to the square of the masses of $\pi^{\pm}$, $K^{\pm}$ in accordance with Dashen's theorem [24]. It does not contribute to the masses of π^0, K^0, $\bar{K}^0$ or η, nor does it generate π^0-η mixing. Incidentally, it also does not contribute to $\eta \to 3\pi$ in accordance with Sutherland's theorem [25]. At the same order in the low-energy expansion there is also the one-photon loop contribution to the meson mass (Fig. 1a and 1b. The vertex in Fig. 1a denotes in the present case the pure photon coupling which follows from the lagrangian (6.1). Resonances are considered below). These contributions vanish, however, in the chiral limit in the dimensional regularization scheme, and we are thus left with

$$\hat{M}^2_{\pi^\pm} = \frac{2e^2 C}{f^2} \tag{6.5a}$$

$$\hat{M}^2_{K^\pm} = \hat{M}^2_{\pi^\pm} \tag{6.5b}$$
$$\hat{M}^2_{\pi^0} = \hat{M}^2_{K^0} = \hat{M}^2_{\bar{K}^0} = \hat{M}^2_{\eta} = 0.$$

This shows that C indeed fixes the electromagnetic mass of pions and kaons in the chiral limit.

The determination of C via resonance exchange cannot be carried over directly from Sects. 3 – 5: in the absence of photon loops, resonance exchange will not contribute to the counterterm (6.3). Nevertheless, it is possible to evaluate $\hat{M}^2_{\pi^\pm}$ via resonance contributions [26]. In fact, a surprisingly satisfactory estimate for this quantity will emerge. In analogy to (2.11) we write

$$C = \sum_R C^R + \hat{C} \tag{6.6}$$

where $\sum_R C^R$ will be calculated below and $\hat{C}$ stands for non-resonance contributions.

We add to (6.1) the lagrangian $\mathcal{L}_{res}$ [Eq.(3.13)] and introduce the electromagnetic interactions as described above. Then we evaluate $\hat{M}^2_{\pi^\pm}$ at the one-loop level. There are altogether four diagrams shown in Fig. 1. The vertex in Fig. 1a now stands for the sum of the two diagrams exhibited in Fig. 1e.

The sum of the diagrams 1a and 1b vanishes in the dimensional regularization scheme, and the two diagrams 1c, 1d yield the following contributions to the mass shifts:

$$\begin{aligned} \Delta M^2_{\pi^\pm}|_c &= -\frac{3\alpha F_V^2 M_V^2}{2\pi f^2}[(4\pi)^2\lambda + \frac{1}{3} + \frac{1}{2}\ln\frac{M_V^2}{\mu^2}] \\ \Delta M^2_{\pi^\pm}|_d &= \frac{3\alpha F_A^2 M_A^2}{2\pi f^2}[(4\pi)^2\lambda + \frac{1}{3} + \frac{1}{2}\ln\frac{M_A^2}{\mu^2}] \end{aligned} \tag{6.7}$$

where

$$\lambda = \frac{\mu^{d-4}}{(4\pi)^2}[\frac{1}{d-4} - \frac{1}{2}(\ln 4\pi + \Gamma'(1) + 1)]. \tag{6.8}$$

These contributions are divergent. The divergences are cancelled by renormalizing the coupling constant $\hat{C}$

$$\hat{C} = \hat{C}^r(\mu) + 3\lambda(F_V^2 M_V^2 - F_A^2 M_A^2) \tag{6.9}$$

and the electromagnetic contribution to the pion (mass)^2 becomes

$$\hat{M}^2_{\pi^\pm} = \frac{2e^2\hat{C}^r(\mu)}{f^2} + X_A(\mu) - X_V(\mu)$$

$$X_I(\mu) = \frac{3\alpha}{4\pi f^2} F_I^2 M_I^2 (\frac{2}{3} + \ln\frac{M_I^2}{\mu^2}), \qquad I = A, V. \tag{6.10}$$

The remaining mass shifts in the meson octet are found from the relations (6.5b).

Using the second Weinberg sum rule (4.7b), the divergences in the mass shift $\Delta M^2_{\pi^\pm} = \Delta M^2_{\pi^\pm}|_c + \Delta M^2_{\pi^\pm}|_d$ cancel[7]. One finds (identifying $\hat{C}^r(\mu)$ with $\hat{C}$ according to (6.9))

$$\hat{M}^2_{\pi^\pm} = \frac{2e^2\hat{C}}{f^2} + \frac{3\alpha}{4\pi f^2} F_V^2 M_V^2 \ln\frac{F_V^2}{F_V^2 - f^2}, \tag{6.11}$$

which, for $\hat{C} = 0$, reduces to the result of Das et al. [7] in resonance approximation. With the values of f, F_V and M_V given in Sect. 4 one obtains

$$\hat{M}^2_{\pi^\pm} = 1.29 \cdot 10^3\,\mathrm{MeV}^2, \tag{6.12}$$

very close to the observed mass difference $M^2_{\pi^\pm} - M^2_{\pi^0} = 1.26 \cdot 10^3$ MeV^2. Thus, we may conclude

$$\hat{C} \simeq 0, \tag{6.13}$$

a result analogous to what we already found in Eq.(5.17).

7 Summary and Conclusions

We have presented in this article a systematic treatment of all low-lying meson resonances of the type $V(1^{--})$, $A(1^{++})$, $S(0^{++})$ and $P(0^{-+})$ in the framework of CHPT. Incorporating P and C invariance, all possible chiral couplings to the pseudoscalar mesons linear in the resonance fields were constructed to lowest order in the chiral expansion. These couplings start at order p^2 and meson resonance exchange thus contributes to the coupling constants $L_1, \ldots, L_{10}$ of the $O(p^4)$ effective chiral lagrangian [2]. Determining the resonance couplings as far as possible directly from experiment and with a few additional plausible approximations we have been able to show that the renormalized coupling constants $L_i^r(\mu)$ are completely dominated by meson resonance exchange as long as the scale parameter μ is in the range between 0.5 and 1 GeV.

More explicitly, our findings can be summarized as follows.

i) Exchange of vector and axial-vector mesons, which we describe in terms of antisymmetric tensor fields, contributes to the constants L_1, L_2, L_3, L_9 and L_{10} (in Fig. 2 we visualize the contributions from resonance exchange to the quantities

[7] If we would not use dimensional regularization, there would in general also be a quadratic divergence proportional to $F_V^2 - F_A^2 - f^2$ which vanishes due to the first Weinberg sum rule (4.7a).

from which $L_1, \ldots, L_{10}$ were determined phenomenologically [2]). Since there are no chiral couplings to $O(p^2)$ for $SU(3)$ singlet vector or axial-vector mesons, only the V and A octets can in fact contribute to the $O(p^4)$ effective chiral lagrangian. Due to chiral corrections, the vector coupling constant G_V can be determined from $\Gamma(\rho \to 2\pi)$ only with rather big uncertainties. Although the qualitative conclusion is the same, we instead choose L_9 as input to fix G_V. The other parameters necessary for the evaluation of the V and A contributions to L_1, L_2, L_3 and L_{10} are taken from experiment and from the Weinberg sum rules [6]. The results shown in Table 1 clearly establish a chiral version of vector (and axial-vector) meson dominance: whenever they can contribute at all, V and A exchange seem to completely dominate the relevant coupling constants. Note that vector meson dominance as defined here is not an assumption but a result of our analysis.

ii) The four coupling constants L_4,L_5,L_6 and L_8 behave differently in the large-N_C limit: L_5,L_8 are $O(N_C)$, L_4 and L_6 are $O(1)$. Except for the negligible pseudoscalar octet resonances in the case of L_8, only scalar octet exchange contributes to L_5 and L_8. Since the experimental information is limited in the scalar sector, we assume L_5 and L_8 to be due exclusively to scalar octet exchange and investigate the implications of this assumption. On the one hand, we can then predict $\Gamma(a_0 \to \eta\pi)$ in good agreement with experiment. On the other hand, the scalar octet contributions to the other L_i are fixed. The scalar singlet exchange can be expressed in terms of the octet parameters using large-N_C arguments. For $N_C = \infty$, octet and singlet scalar exchange cancel in L_1,L_4 and L_6.

iii) Dismissing the pseudoscalar nonet (including, e.g., the $\pi(1300)$) as not really low-lying resonances, the only meson resonance contribution to L_7 is due to η' exchange. The magnitude of the η' contribution can be calculated (using L_4, L_5, L_6 and L_8 as input) from the quark mass expansion of the η mass squared. The result for L_7 is in close agreement with its experimental value. η' exchange does not contribute to any other L_i.

iv) The combined resonance contributions are compared with the phenomenologically determined renormalized coupling constants L_i^r in Table 3. The meson resonances appear to saturate the L_i^r almost entirely. Within the uncertainties of the approach, there is no need for invoking any additional contributions. Although we have made the comparison for $\mu = M_\rho$, it is obvious from the scale dependence of the $L_i^r(\mu)$ shown in Table 1 that a similar conclusion would apply for any value of μ in the low-lying resonance region between 0.5 and 1 GeV.

v) The effective chiral lagrangian with explicit resonance fields has a much larger range of applicability than discussed so far. In particular, we have used this lagrangian to calculate the electromagnetic mass differences of the eight pseudoscalar Goldstone bosons in the chiral limit at the one-loop level. The divergent piece in the mass shifts is proportional to $F_A^2 M_A^2 - F_V^2 M_V^2$ in the dimensional regularization scheme and thus vanishes if we make use of the second Weinberg sum rule (4.7b). The resonance contribution coincides with the expression obtained by Das et al. [7] using current algebra. In analogy to the resonance saturation of the constants L_i, this result can also be expressed in a different way: the single low-energy constant of $O(e^2p^0)$ is again completely dominated by resonance (one-loop) contributions.

To the accuracy one can reasonably ask for, the Green functions of quark currents can be calculated to $O(p^4)$ in two equivalent ways. Either one incorporates the local $O(p^4)$ action with phenomenologically determined coupling constants $L_1, \ldots, L_{10}$ in the generating functional or one uses the effective chiral lagrangian only to $O(p^2)$, but including explicit meson resonance fields with chiral couplings determined in this paper. In the latter case, the scale parameter appearing in the one-loop functional (generated by the lagrangian of order p^2) must be chosen in the resonance region, say $\mu = M_\rho$. It remains to be seen whether this remarkable equivalence extends beyond the one-loop level in CHPT.

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Appendix A: Spin-1 Particles in Terms of Antisymmetric Tensor Fields

We consider a lagrangian quadratic in the antisymmetric tensor field $W_{\mu\nu} = -W_{\nu\mu}$

$$\mathcal{L} = a\partial^\mu W_{\mu\nu}\partial_\rho W^{\rho\nu} + b\partial^\rho W_{\mu\nu}\partial_\rho W^{\mu\nu} + cW_{\mu\nu}W^{\mu\nu} \tag{A.1}$$

with a, b, c arbitrary constants. The field $W^{\mu\nu}$ contains six degrees of freedom. To describe massive spin-1 particles we ought to reduce them to three. This can be done with an appropriate choice of the constants a, b. Indeed, consider the equations of motion

$$a(\partial^\mu\partial_\sigma W^{\sigma\nu} - \partial^\nu\partial_\sigma W^{\sigma\mu}) + 2b\partial^\sigma\partial_\sigma W^{\mu\nu} - 2cW^{\mu\nu} = 0. \tag{A.2}$$

In components,

$$\begin{gathered}(a+2b)\ddot{W}^{0i} + a\partial_\ell \dot{W}^{\ell i} - a\partial^i\partial_\ell W^{\ell 0} - 2(b\Delta + c)W^{0i} = 0 \\ 2b\ddot{W}^{ik} + a[\partial^i(\dot{W}^{0k} + \partial_\ell W^{\ell k}) - \partial^k(\dot{W}^{0i} + \partial_\ell W^{\ell i})] - 2(b\Delta + c)W^{ik} = 0\end{gathered} \tag{A.3}$$

where dots denote time derivatives. For $a+2b=0$, the three fields W^{0i} do not propagate, whereas the three fields W^{ik} are frozen for the choice $b = 0$. The propagator of $W^{\mu\nu}$, defined to be the inverse of the differential operator in (A.1), contains poles at $k^2 = -c/b$ and $k^2 = -2c/(a+2b)$ which disappear for $b = 0$ or $a + 2b = 0$, respectively. In the following we choose $a = -1/2$, $b = 0$, $c = M^2/4$ and obtain

$$\mathcal{L} = -\frac{1}{2}\partial^\mu W_{\mu\nu}\partial_\rho W^{\rho\nu} + \frac{M^2}{4}W_{\mu\nu}W^{\mu\nu} \tag{A.4}$$

from where

$$\partial^\mu\partial_\sigma W^{\sigma\nu} - \partial^\nu\partial_\sigma W^{\sigma\mu} + M^2 W^{\mu\nu} = 0. \tag{A.5}$$

The lagrangian (A.4) describes free spin-1 particles of mass M.

In terms of the canonical momenta

$$\Pi^i = \frac{\partial\mathcal{L}}{\partial\dot{W}_{0i}} = -\partial_\sigma W^{\sigma i} \tag{A.6}$$

the equations of motion (A.3) read in the present case

$$\begin{gathered}\dot{\Pi}^i - \partial^i\partial_\ell W^{0\ell} - M^2W^{0i} = 0 \\ \partial^i\Pi^k - \partial^k\Pi^i - M^2W^{ik} = 0.\end{gathered} \tag{A.7}$$

It is easy to see that the initial values of Π^i, W^{0i} at $t = 0$, together with the equations of motion (A.7), suffice to fix all six components of $W^{\mu\nu} = -W^{\nu\mu}$ at $t \neq 0$.

With the definition

$$W_\mu = M^{-1}\partial^\nu W_{\nu\mu} \tag{A.8}$$

one obtains from (A.5) the familiar Proca equation

$$\partial_\rho(\partial^\rho W^\mu - \partial^\mu W^\rho) + M^2 W^\mu = 0. \tag{A.9}$$

From the lagrangian (A.4) one derives the free propagator

$$\langle 0|T\{W_{\mu\nu}(x), W_{\rho\sigma}(y)\}|0\rangle =$$

$$= iM^{-2}\int \frac{d^4k e^{-ik(x-y)}}{(2\pi)^4(M^2-k^2-i\varepsilon)}[g_{\mu\rho}g_{\nu\sigma}(M^2-k^2)+g_{\mu\rho}k_\nu k_\sigma - g_{\mu\sigma}k_\nu k_\rho - (\mu\leftrightarrow\nu)]. \tag{A.10}$$

The propagator (A.10) corresponds to the normalization

$$\langle 0|W_{\mu\nu}|W,p\rangle = iM^{-1}\{p_\mu\varepsilon_\nu(p) - p_\nu\varepsilon_\mu(p)\} \tag{A.11}$$

or

$$\langle 0|W_\mu|W,p\rangle = \varepsilon_\mu(p) \tag{A.12}$$

with the usual polarization vector $\varepsilon_\mu(p)$.

Appendix B: The Case of $SU(2)_L \times SU(2)_R$

If we only consider Green functions involving u or d quarks and, furthermore, ignore the isoscalar currents $\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d$, $\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d$, the generating functional at order p^4 reduces in the limit

$$p^2 \ll M_K^2; \qquad m_u, m_d \ll m_s$$

to the low-energy expansion for $SU(2)_L \times SU(2)_R$ which was analyzed in detail in Ref. [1]. In particular, the seven low-energy constants $\ell_1, \ldots, \ell_7$ and the three high-energy constants h_1, h_2 and h_3 which specify the general effective lagrangian of $SU(2)_L \times SU(2)_R$ at order p^4 can be expressed in terms of the parameters $L_1, \ldots, L_{10}$, H_1 and H_2 [2]:

$$\begin{aligned}
\ell_1^r &= 4L_1^r + 2L_3 - \frac{1}{24}\,\nu_K, \\
\ell_2^r &= 4L_2^r - \frac{1}{12}\,\nu_K, \\
\ell_3^r &= -8L_4^r - 4L_5^r + 16L_6^r + 8L_8^r - \frac{1}{18}\,\nu_\eta, \\
\ell_4^r &= 8L_4^r + 4L_5^r - \frac{1}{2}\,\nu_K, \\
\ell_5^r &= L_{10}^r + \frac{1}{12}\,\nu_K, \\
\ell_6^r &= -2L_9^r + \frac{1}{6}\,\nu_K, \\
\ell_7 &= \frac{f^2}{8B_0 m_s}(1 + \frac{10}{3}\,\bar{\mu}_\eta) + 4(L_4^r - L_6^r - 9L_7 - 3L_8^r + \frac{1}{8}\,\nu_K), \\
h_1^r &= 8L_4^r + 4L_5^r - 4L_8^r + 2H_2^r - \frac{1}{2}\,\nu_K, \\
h_2^r &= -\frac{1}{4}\,L_{10}^r - \frac{1}{2}\,H_1^r - \frac{1}{24}\,\nu_K, \\
h_3^r &= 4L_8^r + 2H_2^r - \frac{1}{2}\,\nu_K - \frac{1}{3}\,\nu_\eta + \frac{1}{96\pi^2},
\end{aligned} \tag{B.1}$$

where

$$\begin{aligned}
\nu_P &= \frac{1}{32\pi^2}(\ln\frac{\bar{M}_P^2}{\mu^2} + 1), \qquad P = K, \eta, \\
\bar{\mu}_\eta &= \frac{1}{32\pi^2}\,\frac{\bar{M}_\eta^2}{f^2}\,\ln\frac{\bar{M}_\eta^2}{\mu^2}
\end{aligned}$$

$$\bar{M}_K^2 = B_0 m_s, \qquad \bar{M}_\eta^2 = \frac{4}{3}\,\bar{M}_K^2. \tag{B.2}$$

The contributions ν_K, ν_η and $\bar{\mu}_\eta$ in (B.1) are due to eta and kaon loops, whereas the first term in ℓ_7 comes from $\pi^0 - \eta$ mixing at tree level.

For the following it is useful to introduce the scale independent constants $\bar{\ell}_i$,

$$\ell_i^r(\mu) = \frac{\gamma_i}{32\pi^2}(\bar{\ell}_i + \ln\frac{M^2}{\mu^2}), \qquad i = 1,\ldots,6,$$

$$M^2 = B_0(m_u + m_d), \tag{B.3}$$

$$\gamma_1 = \frac{1}{3}, \quad \gamma_2 = \frac{2}{3}, \quad \gamma_3 = -\frac{1}{2}, \quad \gamma_4 = 2, \quad \gamma_5 = -\frac{1}{6}, \quad \gamma_6 = -\frac{1}{3}.$$

Numerical values for $\ell_i^r(M_\rho)$ and $\bar{\ell}_i$ are listed in Table 4, together with ℓ_i^r at $\mu = 0.5$ GeV and 1 GeV. The entries in the table were obtained from the relations (B.1) and the numerical values for L_i^r as listed in Table 1. Some of the constants $\bar{\ell}_i$ (and consequently ℓ_i^r) have slightly changed their values compared to the ones given in [1], see the corresponding discussion for L_i^r in Sect. 2. With the exception of $\bar{\ell}_5$ and $\bar{\ell}_6$, the error bars in Table 4 are taken from [1]. In order to quote an error for ℓ_7, we would have to consider isospin breaking effects in more detail. Since this is outside the scope of this article, we just give the central value for ℓ_7 obtained from (B.1).

In a recent analysis [27] of $\pi\pi$ scattering, experimental information on the elastic $\pi\pi$ amplitude up to $\sqrt{s} \simeq 700$ MeV was used to pin down $\bar{\ell}_1$, $\bar{\ell}_2$ with the result $\bar{\ell}_1 = -6.6$, $\bar{\ell}_2 = 6.2$. This value of $\bar{\ell}_1$ deviates by slightly more than one standard deviation from $\bar{\ell}_1 = -2.3 \pm 3.7$ which was obtained from the experimental value of the D-wave scattering lengths quoted in Ref.[28], whereas $\bar{\ell}_2$ is very close to the value $\bar{\ell}_2 = 6.0 \pm 1.3$ (extracted from the same D-wave scattering lengths). Another phenomenological determination of ℓ_1 and ℓ_2 was made some time ago by Pham and Truong [29] using forward dispersion relations in $\pi\pi$ scattering. They already noticed, in qualitative agreement with our results, that ℓ_2 is ρ dominated while $\ell_1 + \ell_2$ gets its main contribution from the large I=0 S-wave. However, since they have neglected chiral loops a quantitative comparison with our values is not meaningful.

Consider now the decomposition

$$\ell_i^r(\mu) = \sum_{P=V,A,S,P} \ell_i^P + \sum_{P=K,\eta} \ell_i^P + \hat{\ell}_i(\mu), \qquad i = 1,\ldots,6,$$

$$\ell_7 = \sum_{P=V,A,S,P} \ell_7^P + \sum_{P=K,\eta} \ell_7^P + \hat{\ell}_7 \tag{B.4}$$

which is analogous to Eq. (2.11) and where we have explicitly included the contributions from η, K exchange (and loops) which come in addition to the resonance contributions V, A, S and P. In the present case the variation of ℓ_i^r with the scale μ is considerably larger than in the case of $SU(3)_L \times SU(3)_R$, in particular for ℓ_2, ℓ_3 and ℓ_4. The meaning of resonance saturation may therefore seem questionable for these constants. We note, however, that in physical quantities only the scale independent couplings $\bar{\ell}_i$ occur. According to (B.3) and (B.4), $\hat{\ell}_i(\mu) = 0$ leads to

$$\bar{\ell}_i = -\ln\frac{M^2}{\mu^2} + \frac{32\pi^2}{\gamma_i}\sum_P \ell_i^P = -\ln\frac{M^2}{\mu^2} + \sum_P \bar{\ell}_i^P, \qquad i = 1,\ldots,6. \tag{B.5}$$

The corresponding predictions for $\bar{\ell}_i$ thus vary by less than $1\frac{1}{2}$ unit in the range $\mu = 0.5$ GeV to $\mu = 1$ GeV.

In any case, the results of Sect. 5 and Eq. (B.1) allow to immediately evaluate resonance contributions in the $SU(2)_L \times SU(2)_R$ case. The results are shown in Table 5, both for the running constants $\ell_i^r(M_\rho)$ and for the scale independent quantities $\bar{\ell}_i$. In Table 4 we have summed up these individual contributions.

It follows from the results shown in Table 5 that one obtains a good estimate for $\bar{\ell}_1, \ldots, \bar{\ell}_5$ if we assume that the running coupling constants at a scale of the order of M_ρ are given by ρ and a_1 contributions alone [for $\bar{\ell}_6$ this is input, see Sect. 5]. The prediction for $\bar{\ell}_5$ from ρ exchange alone is $\bar{\ell}_5 = 22.4$, to be compared with the value $\bar{\ell}_5 = 13.4 \pm 0.5$ from $\pi \to e\nu\gamma$ [12]. It is amusing to see that axial-vector exchange brings $\bar{\ell}_5$ down to 14.8, close to its experimental value [compare the corresponding case of L_{10}^r in Table 1]. ℓ_7, which is scale independent and which describes isospin breaking effects, does not receive contributions from vector or axial-vector meson exchange. It is dominated by π^0-η, π^0-η' mixing and by scalar exchange. The experimental value of ℓ_7 is $\ell_7 = 7.1 \cdot 10^{-3}$, whereas resonance exchange predicts $\ell_7 = 4.5 \cdot 10^{-3}$ (see first and third row in Table 4). This apparent failure of resonance saturation occurs because L_7 contributes to ℓ_7 with the weight -36 (see (B.1)): a failure of saturation in L_7 is grossly enhanced in ℓ_7. This discrepancy in the prediction for ℓ_7 is thus of no significance.

Table 1

V and A contributions to the coupling constants $L_i^r(M_\rho)$ in units of 10^{-3}. The entries in the first column are from [2] except L_9^r and L_{10}^r (see text).To show the scale dependence, the mean values of L_i^r are also given in brackets for $\mu = 0.5$ GeV and $\mu = 1$ GeV.

	$L_i^r(M_\rho)$	[0.5 GeV, 1 GeV]	V	A	$V+A$
L_1^r	0.7 ± 0.3	[0.9, 0.5]	0.6	0	0.6
L_2^r	1.3 ± 0.7	[1.8, 1.0]	1.2	0	1.2
L_3^r	-4.4 ± 2.5	[-4.4, -4.4]	-3.6	0	-3.6
L_4^r	-0.3 ± 0.5	[0.1, -0.5]	0	0	0
L_5^r	1.4 ± 0.5	[2.4, 0.8]	0	0	0
L_6^r	-0.2 ± 0.3	[0.0, -0.3]	0	0	0
L_7^r	-0.4 ± 0.15	[-0.4, -0.4]	0	0	0
L_8^r	0.9 ± 0.3	[1.2, 0.7]	0	0	0
L_9^r	6.9 ± 0.2	[7.6, 6.5]	6.9*	0	6.9
L_{10}^r	-5.2 ± 0.3	[-5.9, -4.8]	-10.0	4.0	-6.0

*) input

Table 2

P and C transformation properties for octet fields $V_{\mu\nu}(1^{--})$, $A_{\mu\nu}(1^{++})$, $S(0^{++})$ and $P(0^{-+})$ and for the quantities defined in (3.10). Except for the matrix transposition under C, the singlet fields transform in the same way as the octets. Space-time arguments are suppressed. $\varepsilon(0) = 1$, $\varepsilon(1) = \varepsilon(2) = \varepsilon(3) = -1$.

	P	C
$V_{\mu\nu}$	$\varepsilon(\mu)\varepsilon(\nu)V_{\mu\nu}$	$-V_{\mu\nu}^T$
$A_{\mu\nu}$	$-\varepsilon(\mu)\varepsilon(\nu)A_{\mu\nu}$	$A_{\mu\nu}^T$
S	S	S^T
P	$-P$	P^T
u_μ	$-\varepsilon(\mu)u_\mu$	u_μ^T
$u_{\mu\nu}$	$-\varepsilon(\mu)\varepsilon(\nu)u_{\mu\nu}^\dagger$	$u_{\mu\nu}^T$
$\chi_\pm$	$\pm\chi_\pm$	$\chi_\pm^T$
$f_\pm^{\mu\nu}$	$\pm\varepsilon(\mu)\varepsilon(\nu)f_\pm^{\mu\nu}$	$\mp f_\pm^{\mu\nu\,T}$

Table 3

V, A, S, S_1 and η_1 contributions to the coupling constants L_i^r in units of 10^{-3}.

	$L_i^r(M_\rho)$	V	A	S	S_1	η_1	total
L_1^r	0.7 ± 0.3	0.6	0	−0.2	0.2**	0	0.6
L_2^r	1.3 ± 0.7	1.2	0	0	0	0	1.2
L_3^r	−4.4 ± 2.5	−3.6	0	0.6	0	0	−3.0
L_4^r	−0.3 ± 0.5	0	0	−0.5	0.5**	0	0.0
L_5^r	1.4 ± 0.5	0	0	1.4*	0	0	1.4
L_6^r	−0.2 ± 0.3	0	0	−0.3	0.3**	0	0.0
L_7^r	−0.4 ± 0.15	0	0	0	0	−0.3	−0.3
L_8^r	0.9 ± 0.3	0	0	0.9*	0	0	0.9
L_9^r	6.9 ± 0.2	6.9*	0	0	0	0	6.9
L_{10}^r	−5.2 ± 0.3	−10.0	4.0	0	0	0	−6.0

*) input
**) large-N_C estimate

Table 4

Values of low-energy constants $\ell_1, \ldots, \ell_7$ and total resonance contributions for $SU(2)_L \times SU(2)_R$. We did not work out an error for ℓ_7. The individual resonance contributions are listed in Table 5. The barred quantities are defined in (B.3) and (B.5).

	$10^3 \cdot \ell_i^r(M_\rho)$	[0.5 GeV, 1 GeV]	$10^3 \cdot \sum_P \ell_i^P$	$\bar{\ell}_i$	$\sum_P \bar{\ell}_i^P - \ln \frac{M^2}{M_\rho^2}$
ℓ_1	-6.1 ± 3.9	[-5.2,-6.7]	-3.6	-2.3 ± 3.7	0.04
ℓ_2	5.3 ± 2.7	[7.1,4.2]	4.7	6.0 ± 1.3	5.7
ℓ_3	0.9 ± 3.8	[-0.5,1.7]	1.4	2.9 ± 2.4	2.6
ℓ_4	3.4 ± 5.7	[8.8,0.1]	5.5	4.3 ± 0.9	4.4
ℓ_5	-5.2 ± 0.3	[-5.7,-5.0]	-6.0	13.4 ± 0.5	14.8
ℓ_6	-13.7 ± 0.3	[-14.7,-13.2]	-13.7	16.5 ± 0.3	16.5
ℓ_7	7.1		4.5		

Table 5

Resonance contributions ℓ_i^P and $\bar{\ell}_i^P$ evaluated from Table 3 and Eq.(B.1).

ℓ_i	η, K	V	A	S	S_1	η_1
$10^3 \cdot \ell_1^P$	~ 0	–4.7	0	0.4	0.7	0
$\bar{\ell}_1^P$	~ 0	–4.5	0	0.4	0.7	0
$10^3 \cdot \ell_2^P$	~ 0	4.7	0	0	0	0
$\bar{\ell}_2^P$	~ 0	2.2	0	0	0	0
$10^3 \cdot \ell_3^P$	–0.1	0	0	0.5	1.0	0
$\bar{\ell}_3^P$	~ 0	0	0	–0.3	-0.6	0
$10^3 \cdot \ell_4^P$	–0.1	0	0	1.9	3.7	0
$\bar{\ell}_4^P$	~ 0	0	0	0.3	0.6	0
$10^3 \cdot \ell_5^P$	~ 0	–10.0	4.0	0	0	0
$\bar{\ell}_5^P$	~ 0	18.9	–7.6	0	0	0
$10^3 \cdot \ell_6^P$	~ 0	–13.8	0	0	0	0
$\bar{\ell}_6^P$	~ 0	13.0	0	0	0	0
$10^3 \cdot \ell_7$	3.7	0	0	–11.2	0.7	11.3

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Figure Captions

Fig. 1: One-loop diagrams for the electromagnetic pion mass difference. The vertex in Fig. 1a is given by the off-shell pion form factor shown in Fig. 1e.

Fig. 2: Resonance exchange contributions to the processes from which the L_i are determined [2]. $\overline{= = = =}$ denotes the axial current $\bar{q}\gamma_\mu\gamma_5\lambda^{\pi,K}q$, – – – – stands for resonance exchange and the crosses denote (tadpole) quark mass insertions. L_1, L_2, L_3: $\pi\pi$ D-wave scattering, Zweig rule; L_4: f_π, L_5, Zweig rule; L_5: f_K/f_π; L_6: $M^2_{\pi^0}$, Zweig rule; L_7: Gell-Mann-Okubo, L_5, L_8; L_8: M_{K^+}-M_{K^0}, $(m_d - m_u)/(m_s - \hat{m})$, L_5; L_9: $\langle r^2\rangle^\pi_{em}$; L_{10}: $\pi \to e\nu\gamma$.

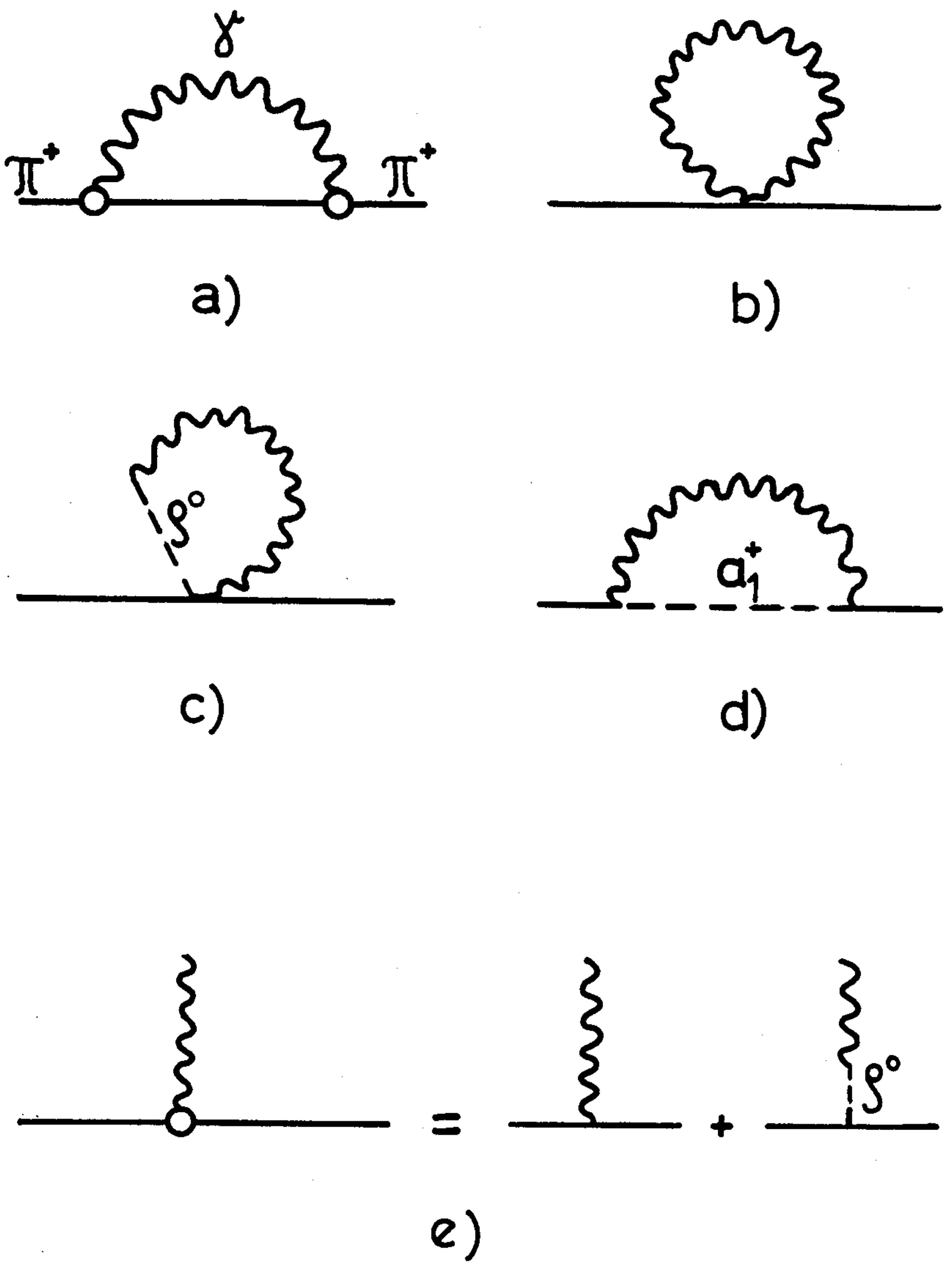

Fig.1

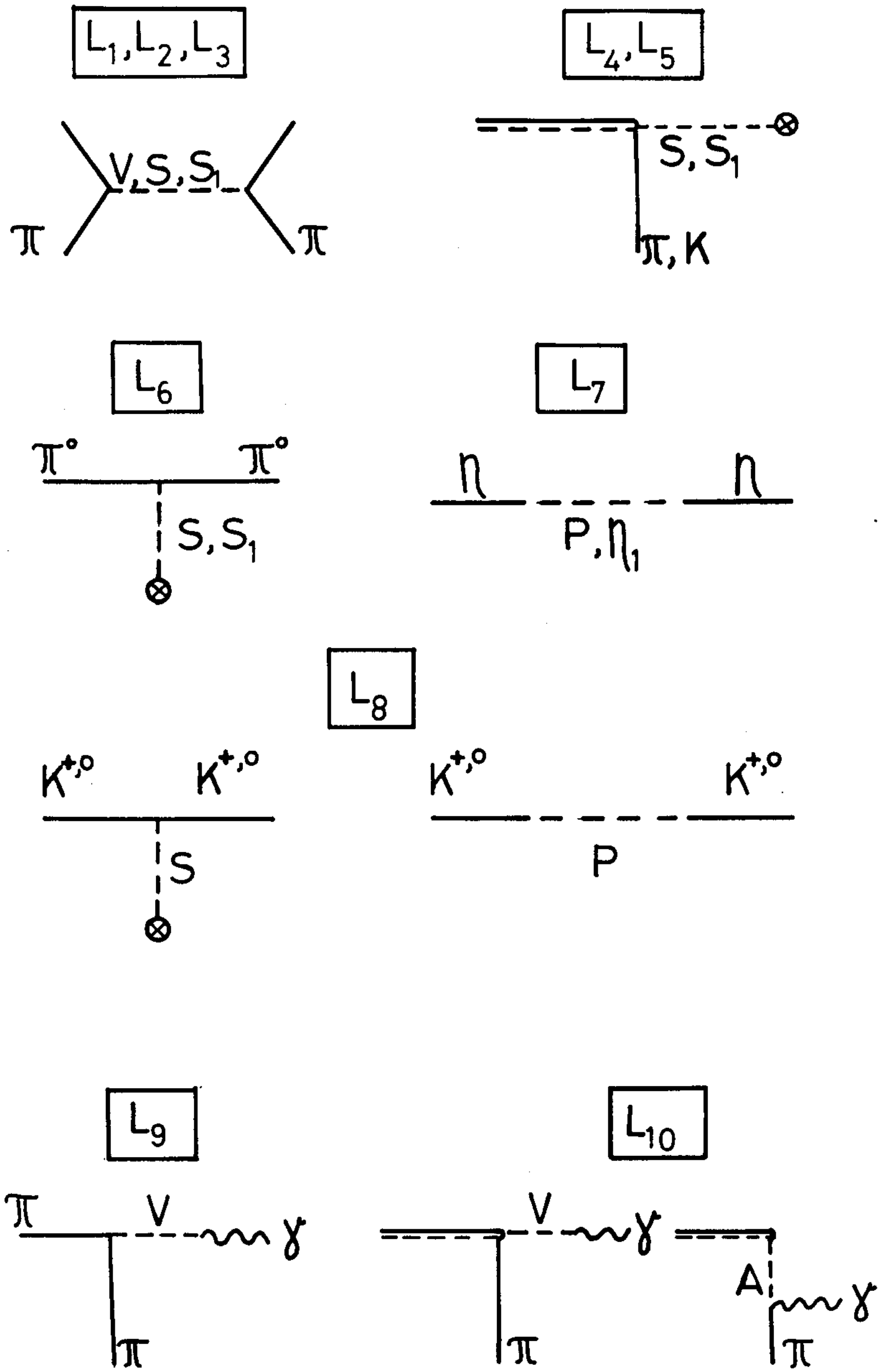
L1,L2,L3
V,S,S1
π
π
L4,L5
S,S1
π,K
L6
π°
π°
S,S1
L7
η
η
P,η1
L8
K+,o
K+,o
S
K+,o
K+,o
P
L9
π
V
γ
π
L10
V
γ
A
γ
π
π

Fig.2