FTUV/90-24
IFIC/90-20

THE τ-LEPTON AND ITS ASSOCIATED NEUTRINO

A. Pich

Departament de Física Teórica and Institut de Física
Corpuscular (C.S.I.C.), Universitat de Valencia,
46100 Burjassot, Spain.

ABSTRACT

The present knowledge on the τ lepton and the prospects for future improvements are discussed. It is shown how a better understanding of the τ properties could be used for testing fundamental aspects of the electroweak and strong interactions.

To be published in Modern Physics Letters A (Brief Reviews).

1. INTRODUCTION

Since its discovery[1] in 1975 at the SPEAR $e^+ e^-$ storage ring, the tau lepton has been a subject of extensive study. All experimental results obtained so far , seem to confirm the standard model scenario in which the τ is a sequential lepton, with its own quantum number and associated neutrino. However, there are still many unanswered questions and important properties of the τ remain to be tested.

Our knowledge of the τ lepton could drastically be improved in the near future, if the recently proposed[2] τ-charm factory storage ring is finally built. A low energy ($\sqrt{s} \sim$ 3-5 GeV) $e^+ e^-$ collider with a design luminosity of 10^{33} cm^{-2} sec^{-1} , would produce about 10^7 $\tau^+ \tau^-$ events per year, allowing an extensive programme of high-precision measurements to confront the Standard Model. In addition, large τ data samples may be also accumulated in future medium energy high luminosity $e^+ e^-$ colliders, such as B-meson factories ($\sqrt{s} \sim$ 8-14 GeV), or at the Z^0 - peak, in the LEP high luminosity option.

In the following, I will try to give a brief overview of what is presently known about the τ lepton, and to show how a better understanding of the τ properties could be used for testing fundamental aspects of the electroweak and strong interactions. A more detailed discussion can be found in refs. (3) and (4).

2. TAU PAIR PRODUCTION

In the standard model, tau pair production in $e^+ e^-$ annihilation proceeds through both the electromagnetic and neutral weak currents,

$$e^+ e^- \longrightarrow \gamma,\ Z^0 \longrightarrow \tau^+ \tau^- . \qquad (2.1)$$

At low energies ($s \ll M_{Z^0}{}^2$) the Z^0 contribution is very small, therefore the production cross section is only sensitive to the coupling of the τ to the photon. From the energy dependence of the production cross section near threshold, the spin of the τ has been determined to be 1/2 and its mass has been measured to be[5] $m_\tau = \left(1784.1 {}^{+2.7}_{-3.6}\right)$ MeV. At energies well above the threshold, the deviations from the QED prediction test the point-like nature of the τ. From PEP and PETRA data, is posible to set an upper limit of 10^{-3} fm on the τ charge radius. The same data can also be used to extract a value for the tau anomalous magnetic moment[6], $a_\tau = (5.2 \pm 4.8)\, 10^{-2}$; however, the experimental error in too big to check the standard model prediction[7] $(a_\tau)_{th} = (1.1741 \pm 0.0005)\, 10^{-3}$.

At high energies, where the Z^0-contribution is important, the study of the production cross-section allows to extract information on the lepton electroweak parameters. For unpolarized e^+ and e^- beams, the differential production cross section can be written, at lowest order, as

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \left[A\,(1 + \cos^2\vartheta) + B \cos\vartheta \right] . \qquad (2.2)$$

Here,

$$A = 1 + 2\, v_e v_\tau\ \mathrm{Re}(\chi) + (v_e^2 + a_e^2)\,(v_\tau^2 + a_\tau^2)\,|\chi|^2$$
$$B = 4\, a_e a_\tau\ \mathrm{Re}(\chi) + 8\, v_e v_\tau\ a_e a_\tau\,|\chi|^2 , \qquad (2.3)$$

where

$$v_e = v_\mu = v_\tau = -1 + 4 \sin^2\vartheta_W$$
$$a_e = a_\mu = a_\tau = -1 \qquad (2.4)$$

are the weak charges, and the parameter χ contains the Z propagator

$$\chi = \frac{1}{16 \sin^2\vartheta_W \cos^2\vartheta_W}\ \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z} . \qquad (2.5)$$

Only the term proportional to $(1+\cos^2\vartheta)$ will contribute to the total cross section. Therefore, A represents the normalization of the tau production cross section with respect to to the QED point cross section. Since $\sin^2\vartheta_W$ is close to 1/4, the vector coupling v_l is very small; thus the contribution of the γZ^o interference term to A is considerably suppressed.

The Z^o - exchange amplitude introduces a linear dependence on $\cos\vartheta$ in the cross section, which leads to a forward-backward asymmetry,

$$A^{FB}_{\tau\tau} = \frac{N_F - N_B}{N_F + N_B} = \frac{3}{8}\,\frac{B}{A}\,. \tag{2.6}$$

Here, N_F and N_B are the number of negative taus emerging in the forward and backward hemispheres, respectively, with respect to the electron direction. At $s \ll M_Z^2$, this asymmetry basically measures the product of the axial-vector couplings of the electron and the tau to the Z^o,

$$A^{FB}_{\tau\tau}\,(s \ll M_Z^2) \simeq \frac{3}{2}\, a_e a_\tau\, \mathrm{Re}(\chi). \tag{2.7}$$

The propagator contained in χ determines then the sign of the asymmetry to be negative.

Table 1

l	$v_e\, v_l$	$a_e\, a_l$	
e	-0.09 ± 0.11	0.82 ± 0.20	
μ	0.14 ± 0.06	1.07 ± 0.06	(1.06 ± 0.05)
τ	0.07 ± 0.12	0.85 ± 0.09	(0.93 ± 0.07)
Standard Model ($\sin^2\vartheta_w = 0.231$)	0.006	1	

A global analysis[6] of the $e^-e^+ \to l^- l^+$ ($l = e, \mu, \tau$) differential cross sections, incorporating data from all the experiments at PEP and PETRA, gives the results shown in table 1 for the separate weak charges. Assuming the validity of e-μ-τ universality, supported by the table, it is possible to determine universal v and a charges[6], $v^2 = 0.06 \pm 0.06$, $a^2 = 0.99 \pm 0.05$, in good agreement with the standard model predictions. The inclusion of the more recent TRISTAN data[8] gives the averages shown in the table within brackets.

At LEP/SLC energies, the dependence of the forward-backward asymmetry on the weak charges is quite different. For $s = M_Z^2$, the real part of the Z^o-propagator vanishes (i.e., $\mathrm{Re}(\chi) = 0$) and the photon exchange terms can be neglected in comparison with the Z^o-exchange contributions ($\Gamma_Z^2/M_Z^2 \ll 1$). Eq. (2.6) becomes then,

$$A^{FB}_{\tau\tau}\,(s = M_Z^2) = \frac{3}{4}\, P(e)\, P(\tau)\,, \tag{2.8}$$

where $P(l) = -2 v_l a_l/(v_l^2 + a_l^2)$ is the average longitudinal polarization of the lepton l, which depends on the ratio of the axial and vector couplings only. $P(l)$ is a sensitive function of $\sin^2\vartheta_W$.

Spin polarization of the produced taus is reflected in the distorted distribution of the decay products. Thus $P(\tau)$, can also be determined from a measurement of the spectrum of the final charged particle in the decay channels $\tau^- \to \nu_\tau\pi^-, \nu_\tau\rho^-, \nu_\tau e^-\bar{\nu}_e, \nu_\tau\mu^-\bar{\nu}_\mu$. A recent study of the decay sequence[9] $Z^o \to \tau^+\tau^- \to \mu^+ e^-$ neutrinos suggest that with 10^7 Z^o events a sensitivity of $\delta P(\tau) = 0.016$ could be achieved. The study of correlations between the decay products of both taus, has also been suggested recently[10] to search for T-odd effects in the $\tau^+\tau^-$ production vertex.

The measurement of the leptonic Z^o-widths provides information on $(v_l^2 + a_l^2)$. The present data agrees with the standard model. Assuming lepton universality, one gets[11] $\sin^2\vartheta_W = 0.231 \pm 0.003$.

3. LEPTONIC TAU DECAYS

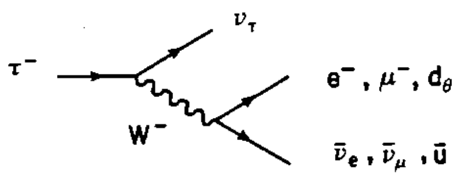

Fig.1. Feynman diagram for the decay of the τ lepton.

Within the Standard Model the τ lepton decays via the W-emission diagram shown in figure 1. Since the W coupling to the charged current is of universal strength, there are five equal contributions (if final masses and gluonic corrections are neglected) to the τ-decay width. Two of them correspond to the decay modes $\nu_\tau e^- \bar{\nu}_e$ and $\nu_\tau \mu^- \bar{\nu}_\mu$, while the other three are associated with the three possible colours of the quark-antiquark pair in the final $\nu_\tau d_\theta \bar{u}$ mode ($d_\theta = \cos\theta_c d + \sin\theta_c s$). Hence, the branching fractions for the different channels are expected to be approximately,

$$Br\ (\tau^- \to \nu_\tau l^- \bar{\nu}_l) \sim \frac{1}{5} = 20\%\ , \quad (l = e, \mu)$$

$$R_H \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} \simeq N_c = 3, \tag{3.1}$$

which should be compared with the formal experimental averages

$$Br\ (\tau^- \to \nu_\tau e^- \bar{\nu}_e) = (17.5 \pm 0.4)\%$$

$$Br\ (\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu) = (17.8 \pm 0.4)\%$$

$$R_H = (3.54 \pm 0.08)\% \ . \tag{3.2}$$

The agreement if fairly good. Notice that the measured tau hadronic width provides strong evidence for the colour degree of freedom. The difference between the measured value of R_H and the lowest order prediction $R_H = N_c$, allows to infer a value[12] for the QCD-scale $\Lambda_{\overline{MS}}$.

The tau-decay partial widths for the leptonic modes $\tau^- \to \nu_\tau\ l^- \bar{\nu}_l\ (l = e, \mu)$ are easily computed, with the result (neutrinos are assumed to be massless)

$$\Gamma(\tau^- \to \nu_\tau l^- \bar{\nu}_l) = \frac{G_F^2 m_\tau^5}{192\pi^3} f(m_l^2\ / m_\tau^2) r = \frac{f(m_l^2\ / m_\tau^2)}{1.60\ 10^{-12} \text{sec.}}\ , \tag{3.3}$$

where $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$. The factor $r \simeq 0.996$ takes into account[13] radiative corrections non included in the Fermi coupling constant G_F, and the non-local structure of the W-propagator. Eq. (3.3) gives a relation between the tau lifetime and the electronic branching ratio. Using the world average measured τ lifetime[11], $\tau_\tau = (3.04 \pm 0.09)\ 10^{-13}$ sec , one gets the prediction $Br(\tau^- \to \nu_\tau\ e^-\ \bar{\nu}_e)_{th} = (19.0 \pm 0.6)\%$, which is about two standard deviations higher than the measured branching fraction. Given the present limit[14] of $m_{\nu_\tau} < 35$ MeV (95% C.L.), this small discrepancy cannot be due to a non-zero value of the tau neutrino mass. The agreement is slightly better in the muonic channel; taking into account the phase space mass correction, $f(m_\mu^2/m_\tau^2) = 0.9728$, one predicts $Br(\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu)_{th} = (18.5 + 0.6)\%$. Note, however, that in both cases the experimental branching fractions are below the values extracted theoretically from the measured lifetime.

Precise measurements of the τ lifetime and its leptonic decay branching fractions can be used to test e-μ-τ universality. Allowing the value of the weak coupling g to depend on the lepton flavour considered, i.e. g_e, g_μ, g_τ, one has

$$\frac{Br(\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu)}{Br(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = \left(\frac{g_\mu}{g_e}\right)^2 f(m_\mu^2 / m_\tau^2)\ . \tag{3.4}$$

At present the measured branching fractions imply $|g_\mu/g_e| = 1.023 \pm 0.017$, which should be compared with the more accurate value5) $|g_\mu/g_e| = 1.006 \pm 0.006$ obtained from pion decay.

A test on the tau lepton coupling can be obtained from the expression

$$Br(\tau^- \to \nu_\tau e^- \bar{\nu}_e)\,\frac{\tau_\mu}{\tau_\tau} = \left(\frac{g_\tau}{g_\mu}\right)^2 \left(\frac{m_\tau}{m_\mu}\right)^5 \frac{r}{r_\mu}\,, \qquad (3.5)$$

which relates the electronic branching fraction and the ratio of the muon and tau lifetimes. Here r_μ is the corresponding r correction in muon decay. Present data allows to extract the estimate $|g_\tau/g_\mu| = 0.96 \pm 0.02$.

Future experiments are expected to improve significantly the accuracy of the tau inputs needed in the above formulae. The electronic branching ratio could be mesured6) at the 0.4% level in a Tau-Charm Factory, while high luminosity $e^+ e^-$ colliders running at higher center of mass energies could certainly provide a better value of the τ- lifetime.

An independent test of lepton universality has been obtained at the CERN proton-antiproton collider by comparing the ratios of the σ.B partial production cross-sections for the various $W^- \to l^- \bar{\nu}_l$ decay modes. The results of this analysis are7) $|g_\mu/g_e| = 1.00 \pm 0.07 \pm 0.04$ and $|g_\tau/g_e| = 1.01 \pm 0.10 \pm 0.06$.

The V-A structure of the charged current can be tested by studying the distribution of the final charged lepton in the leptonic decay modes of the tau, $\tau^- \to \nu_\tau l^- \bar{\nu}_l$ (l=e,μ). Assuming that the τ-ν_τ-W vertex is a linear combination of vector and axial currents, $v\, V^\mu + a\, A^\mu$, and using the standard V-A form for the l^--ν_l-W (l = e, μ) vertex, the so called Michel parameter is predicted to be

$$\rho = \frac{3}{4}\;\frac{(v-a)^2}{(v-a)^2 + (v+a)^2}\,. \qquad (3.6)$$

Note that one expects $\rho \le 3/4$, with the maximum allowed value $\rho = 3/4$ corresponding to the standard V-A structure. Pure vector or pure axial couplings would imply $\rho = 3/8$, while a vertex of the V+A type would result in $\rho = 0$.

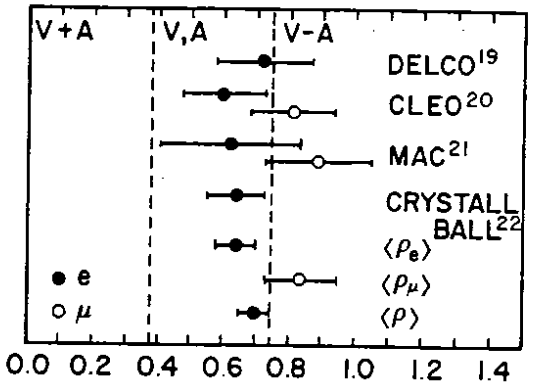

Fig. 2. Michel Parameter

Fig.2 summarizes the available data on the Michel parameter, both for the electron (ρ_e) and muon (ρ_μ) modes. The experimental measurements are in agreement with the V-A hypothesis, although ρ_e is systematically smaller than ρ_μ. Averaging the different experiments one has $\langle\rho_e\rangle = 0.64 \pm 0.06$ and $\langle\rho_\mu\rangle = 0.84 \pm 0.11$. The combined result $\langle\rho\rangle = 0.70 \pm 0.05$ excludes a vertex of pure V+A, V or A type. However, using this value in eq.(3.6), one can only get the upper limit (95% C.L.) $(v+a)/(v-a) < 0.50$ on a possible mixture of right-handed current structure; a very poor limit indeed. For comparison, the Michel parameter measured in μ-decay is5) $\rho = 0.7518 \pm 0.0026$, more than one order of magnitude better.

The exact form of the τ-ν_τ-W vertex, without assumptions, is certainly not determined by existing experimental data (the most general, local, derivative-free lepton-number conserving four-fermion interaction hamiltonian contains ten complex coupling constants). General vector-axial vector interactions always lead to $\rho \le 3/4$, while $\rho > 3/4$ is only possible if scalar and tensor interactions are present simultaneusly. If $\rho \neq 3/4$ could be established experimentally, transitions with equal chiralities of ν_τ and $\bar{\nu}_l$ should be necessarily present.

With 10^5 $\tau^- \rightarrow \nu_\tau l^- \bar{\nu}_l$ events, which could be probably collected in one year run at the Tau–Charm Factory, a precision of $\Delta\rho = 0.0033$ could be achieved[9,10]. This is comparable with what has been already obtained for μ–decay. With polarized τ's, two more decay parameters, ξ and δ, can be determined. This measurement is possible due to the fact that the spins of the two τ's produced in $e^+ e^-$ annihilation are strongly correlated. It has been estimated[9,10] that, in one year run at the Tau–Charm Factory, a precision of the order of 2% for ξ_e, ξ_μ, δ_e, δ_μ and the helicity of the tau neutrino, could be achieved. A similar precision could be obtained[9,10] in a B Factory. The measurement of the low energy parameter η will be difficult, but may be it would be possible for the $\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu$ mode, where the mass suppression is weaker. The next step would be to measure the polarization of the charged lepton emitted in the τ–decay. This could be possible, in principle, for the decay $\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu$ by stopping the muons and detecting their decay products. However, it is not clear whether this kind of experiment would be feasible in the near future.

4. LEPTON NUMBER VIOLATION

In the minimal standard model with massless neutrinos, there is a separately conserved additive lepton number for each generation. All present data is consistent with this conservation law. However, there is no strong theoretical reason forbidding a mixing among the different leptons, in the same way as happens in the quark sector. Many models in fact predict lepton–flavour or even lepton–number violation at some level. Experimental searches for these processes can provide information on the scale at which the new physics begins to play a significant role.

K, π and μ decays, together with μ–e conversion, neutrinoless double beta decays and neutrino oscillation studies, have put already stringent limits on lepton–flavour and lepton–number violation. However, given the present lack of understanding of the origin of fermion generations, one can imagine different patterns of violation of this conservation law for different mass scales. Moreover, the larger mass of the tau lepton opens the possibility of new types of decay which are kinematically forbidden for the muon. The present upper limits on the branching ratios of lepton–flavour and lepton–number violating decays of the tau are in the range of 10^{-3} to 10^{-5}, which is far away of the impressive bounds[5] obtained in μ–decay [$Br(\mu^- \rightarrow e^- \gamma) < 5\ 10^{-11}$, $Br(\mu^- \rightarrow e^- e^+ e^-) < 10^{-12}$, $Br(\mu^- \rightarrow e^- \gamma\gamma) < 7\ 10^{-11}$]. With future tau decay samples of 10^7 per year, an improvement of two orders of magnitude would be possible.

5. THE TAU NEUTRINO

All observed τ–decays are supossed to be accompanied by neutrino emission, in order to fulfill energy–momentum conservation requirements. The data shows that the ν_τ spin is 1/2 and, as seen in section 3, it is consistent with the ν_τ being a conventional sequential neutrino. Since taus are not produced by ν_e or ν_μ beams, we know that ν_τ is different from the electronic and muonic neutrinos, and an upper limit can be set on the couplings of the tau to ν_e and ν_μ[23],

$$g(\nu_e - \tau) < 0.073 \quad , \quad g(\nu_\mu - \tau) < 0.002 \qquad (90\%\ \text{C.L.}). \tag{5.1}$$

These limits can be interpreted in terms of $\nu_\mu/\nu_e \rightarrow \nu_\tau$ oscillations, to exclude a region in the neutrino mass difference and neutrino mixing angle space. LEP and SLC have confirmed recently[11] the existence of three (and only three) different light neutrinos, with standard couplings to the Z^0. However, no direct observation of ν_τ, that is, interactions resulting from neutrinos produced in tau decay, has been done so far.

To detect ν_τ–interactions, it is first necessary to know how to make a ν_τ–beam. The expected source of tau neutrinos in beam dump experiments is the decay of D_s mesons produced by interactions in the dump, i.e., $p+N \rightarrow D_s + \ldots$, followed by the decays $D_s \rightarrow \tau^- \bar{\nu}_\tau$ and $\tau^- \rightarrow \nu_\tau + \ldots$. Several experiments have searched for $\nu_\tau + N \rightarrow \tau^- + \ldots$.. interactions with negative results, and therefore only an upper limit on the production of ν_τ' s has been obtained. As shown in reference (24), the direct detection of the $\tau-$ neutrino should be possible at the future high energy colliders LHC and SSC, thanks to the large charm-production cross-section in these machines.

The possibility of a nonzero neutrino mass is obviously a very important question in particle physics. There is no fundamental principle requiring a null mass for the neutrino. On the contrary, many extensions of the Standard Model predict non-vanishing neutrino masses, which could have, in addition, important implications in cosmology and astrophysics. The present upper limit on the ν_τ - mass[4],

$$m_{\nu_\tau} < 35 \text{ MeV} \quad (95\,\%\ \text{C.L.}) \ , \tag{5.2}$$

has been set by studying the endpoint of the hadronic mass spectrum of the decay $\tau^- \rightarrow 2\pi^+ 3\pi^- \nu_\tau$. For comparison, the best limits on the muon and electron neutrinos are[5] $m_{\nu_\mu} < 250$ KeV (90% C.L.) and $m_{\nu_e} < 18$ eV (95% C.L.). Note however that, although the tau neutrino mass limit is about two and six orders of magnitude worse than the ν_μ and ν_e mass limits, respectively, in many models a mass hierarchy among different generations is expected, with the neutrino mass being proportional to some power of the mass of its charged lepton partner. Assuming for instance the fashionable relation $m_{\nu_\tau}/m_{\nu_e} \sim (m_\tau/m_e)^2$, the bound (5.2) would be equivalent to a limit of 3 eV in the mass of the electron neutrino. A relatively crude measurement of m_{ν_τ} may imply then strong constraints on neutrino mass model building.

The possibility of reducing the m_{ν_τ} upper-bound in a future Tau-Charm Factory has been considered recently. It has been shown that the study of the decay $\tau^- \rightarrow \nu_\tau\ l^-\ \bar{\nu}_l$ could result in a limit of the order of 20-30 MeV[25], thus

providing little improvement of the current limit. The prospects are much better for the hadronic modes $\tau^- \rightarrow 2\pi^+ 3\pi^- \nu_\tau$[26] and $\tau^- \rightarrow K^- K^+ \pi^- \nu_\tau$[27], were a mass limit of about 5 and 10 MeV, respectively, could be achieved. One obvious condition for obtaining this result is that the error on the tau mass should be much smaller than the present value of 3.2 MeV. A measurement of m_τ with the required precission could be, however, easily done in a Tau-Charm factory.

6. HADRONIC DECAYS

The τ is the only known lepton massive enough to decay into hadrons; therefore, its semileptonic decays are an ideal tool for studying strong interaction effects in very clean conditions. The semileptonic decay modes of the tau, $\tau^- \rightarrow \nu_\tau H^-$, probe the matrix element of the left-handed charged current between the vacuum and the final hadronic state H^-. For the decay modes with lowest multiplicity, $\tau^- \rightarrow \nu_\tau \pi^-$ and $\tau^- \rightarrow \nu_\tau K^-$, the relevant matrix elements are already known from the measured decays $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$ and $K^- \rightarrow \mu^- \bar{\nu}_\mu$. The corresponding τ-decay widths can then be predicted rather accurately. One gets,

$$\frac{\Gamma(\tau^- \rightarrow \nu_\tau \pi^-)}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \nu_e)} = \frac{24\,\pi^2}{m_\tau^2} f_\pi^2 \cos^2\vartheta_c \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 \hat{r} = 0.601$$

$$\frac{\Gamma(\tau^- \rightarrow \nu_\tau K^-)}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \nu_e)} = \frac{24\,\pi^2}{m_\tau^2} f_K^2 \sin^2\vartheta_c \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 \hat{r} = 0.0399 \ , \tag{6.1}$$

where $\hat{r} \sim 0.99$ takes into account the estimated electroweak radiative corrections[13]. These numbers are in good agreement with the experimental ratios, which are measured to be (0.617 ± 0.037) and (0.038 ± 0.011) respectively[5].

One can alternatively use the measured ratio between the $\tau \rightarrow \nu_\tau K^-$ and $\tau^- \rightarrow \nu_\tau \pi^-$ decay widths to obtain a value for the ratio of hadronic decay constants, $\tan^2\vartheta_c\, (f_K/f_\pi)^2 = (7.1 \pm 2.1)\ 10^{-2}$. This number is, however, an order of magnitude worse than the result $(7.56 \pm 0.02)10^{-2}$ obtained from $\Gamma(K^- \rightarrow \mu^- \bar{\nu}_\mu) / \Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)$.

In the Cabibbo allowed modes with $J^P = 1^-$, the matrix element of the vector charged current can also be obtained, through an isospin rotation, from the isovector part of the $e^+ e^-$ annihilation cross-section into hadrons, which measures the hadronic matrix element of the $I=1$ component of the electromagnetic current, The tau-decay width for these modes is then expressed as an integral over the corresponding $e^+ e^-$ cross-section. Taking into account the estimated electroweak radiative corrections[13], the available $e^+ e^- \rightarrow 2\pi$, 4π data imply [28]

$$\frac{\Gamma(\tau^- \rightarrow \nu_\tau \pi^- \pi^0)}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = 1.26 \qquad \left[\text{exp.: } 1.27 \pm 0.07\right]$$

$$\frac{\Gamma(\tau^- \rightarrow \nu_\tau 2\pi^- \pi^+ \pi^0)}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = 0.281 \qquad \left[\text{exp.: } 0.25 \pm 0.09\right]$$

$$\frac{\Gamma(\tau^- \rightarrow \nu_\tau \pi^- 3\pi^0)}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = 0.056 \qquad \left[\text{exp.: } 0.17 \pm 0.15\right] , \tag{6.2}$$

where the masured experimental ratios[5] are given inside brackets for comparison. The predictions agree quite well with the data.

The exclusive tau-decays into final hadronic states with $J^P = 1^+$ or Cabibbo suppressed modes with $J^P = 1^-$ cannot be predicted with the same degree of confidence. We can only make model-dependent estimates with an accuracy which depends on our ability to handle the strong interactions at low energies. However, that just indicates that the decay of the tau lepton is providing us new experimental hadronic information . Due to their semileptonic character, the hadronic tau-decay data can be then a unique and extremely useful tool to learn about the couplings of the low-lying mesons to the weak currents. A summary of QCD tests, which can be done using τ - decay data , is given in reference (4).

Table 2

Decay Mode	Branching Fraction (%)		
$(\tau^- \rightarrow \nu_\tau x^-)$	One Prong	Three Prongs	Five Prongs
$e^- \bar{\nu}_e$	17.5 ± 0.4	—	—
$\mu^- \bar{\nu}_\mu$	17.8 ± 0.4	—	—
π^-	10.8 ± 0.6	—	—
$\pi^- \pi^0$	22.3 ± 1.1	—	—
$(3\pi)^-$	7.5 ± 0.9	6.8 ± 0.6	—
$(4\pi)^-$	(3.0 ± 2.7)~1.0	4.4 ± 1.6	—
$3\pi^- 2\pi^+$	—	—	0.056 ± 0.016
$3\pi^- 2\pi^+ \pi^0$	—	—	0.051 ± 0.022
K^-	0.66 ± 0.19	—	—
$(K\pi)^-$	1.3 ± 0.3	0.3 ± 0.1	—
$(K\pi\pi)^- (\pi^0)$		$0.22^{+0.16}_{-0.13}$	—
$(K\bar{K}\pi)^-$		$0.22^{+0.17}_{-0.11}$	—
Total	77.9 ± 1.7	11.9 ± 1.7	0.107 ± 0.027
Inclusive Measurement	86.6 ± 0.3	13.3 ± 0.3	0.11 ± 0.03

The topological branching fractions for the inclusive decay of the tau lepton into one, three and five charged particles have been measured in many experiments. The formal average values are[5] $BR_1 = (86.6 \pm 0.3)$ %, $BR_3 = (13.3 \pm 0.3)$% and $BR_5 = (0.11 \pm 0.03)$%. These numbers are compared in table 2 with the branching ratios of the measured exclusive modes, classified according to the charged multiplicity of the final state[5]. For one-charged-particle final states, the sum of the exclusive branching fractions is significantly smaller than the inclusive measurement. Given the large uncertainty of the experimental value for the $\pi^- 3\pi^0$ mode, the CVC theoretical estimate[28] of 1.0% (obtained from eq. (6.2c) and the measured electronic branching ratio) has been used in the sum. With very mild theoretical assumptions, such as isospin conservation, and other data, it is possible to set an upper limit of 2.2% on the branching fraction due to unmeasured modes[28,29]. Thus, the sum of all one–charged–particle exclusive modes is bounded to be less than (80.1 ± 1.7)%, implying that more than 6% of the measured inclusive one–prong branching ratio is missing in the exclusive sum.

The reasons for this discrepancy are not understood at present. Several possibilities have been suggested in the literature (a detailed discussion is given in reference (3)), but no convincing explanation has been found. With present data, it is difficult to study the problem because the discrepancy is just at the limit of the statistical and systematic errors of a single experiment, and some caution has to be applied when combining results from different experiments. Larger data samples, collected at future high luminosity $e^+ e^-$ colliders, could greatly help in understanding the source of the problem. In this respect, a good identification of neutral particles would be important.

7. SUMMARY

Two basic properties make the tau particle an ideal laboratory for testing the Standard Model: the τ is a lepton, which means clean physics, and, moreover, it is heavy enough to produce a large variety of different decay modes.

The leptonic τ–decays, together with the $\tau^+\tau^-$ production cross section, probe the structure of the weak currents and the universality of their couplings to the gauge bosons. On the other hand, the semileptonic character of the hadronic τ–decays provide a unique and extremely useful tool to learn about the couplings of the low–lying mesons to the vector and axial–vector currents[4] . The invariant mass distribution of the final hadrons allows to test different aspects of strong interaction phenomena (resonance structures, Weinberg sum rules, pion mass difference, vacuum condensates, ...), and perturbative QCD predictions can be compared with the measured hadronic tau–decay width, to infer a value for the QCD–scale $\Lambda_{\overline{MS}}$.

Lepton number violation studies can greatly benefit from the large number of kinematically allowed decay modes which are forbidden by this conservation law. The tau neutrino, finally, deserves a careful investigation, in view of the important consequences which would have a non–vanishing neutrino mass.

At present, all experimental results on the tau lepton are consistent with the Standard Model. The accuracy of the tau data is, however, rather poor, in comparison with the high precision experiments done with the lighter leptons, and large room for new physics is still allowed. Present experimental discrepancies, like the missing fraction of one–prong modes or the conflict between lifetime and leptonic branching ratios measurements, remain open to any speculation.

ACKNOWLEDGEMENTS

This work has been partly supported by C.I.C.Y.T., Spain, undergrant No. AEN 90 - 0040.

REFERENCES

1. M.L. Perl et al. (MARK I), Phys. Rev. Lett. 35 (1975) 1489.

2. Proc. of the Tau-Charm Factory Workshop (SLAC 1989), ed. L. V. Beers, SLAC-Report-343 (1989).

3. A. Pich, Proc. of the XVIIth International Meeting on Fundamental Physics (Lekeitio 1989), ed. M. Aguilar-Benitez, World Scientific Pub. (Singapore 1990).

4. A. Pich, Proc. Tau-Charm Factory Workshop (ref. 2), p. 416.

5. M. Aguilar-Benitez et al., "Review of Particle Properties", Phys. Lett. B204 (1988) 1.

6. R. Marshall, Rep. Progr. Phys. 52 (1989) 1329.

7. S. Narison, J. Phys. G4 (1978) 1849.

8. K. Fujii, contribution to the EPS-89 Conference, Madrid 1989.

9. C. A. Nelson, Phys. Rev. Lett. 62 (1989) 1347; Phys. Rev. D40 (1989) 123.

10. F. Hoogeveen and L. Stodolsky, Phys. Lett. B212 (1988) 505;
    W. Bernreuther and O. Nachtmann, Phys. Rev. Lett. 63 (1989) 2787;
    J. Bernabéu, A. Pich and N. Rius, FTUV/90-26.

11. E. Fernández, contribution to the 1990 Conference on Weak interactions and Neutrinos, Geneva (1990).

12. A. Pich and S. Narison, Phys. Lett. B211 (1988) 183.
    E. Braaten, Phys. Rev. Lett. 60 (1988) 1606; Phys. Rev. D39 (1989) 1458.

13. W.J. Marciano and A. Sirlin, Phys. Rev. Lett. 61 (1988) 1815

14. H. Albrecht et. al (ARGUS), Phys. Lett. B202 (1988) 149

15. W.J. Marciano BNL-41498 (1988)

16. J. J. Gomez-Cadenas, Proc. Tau-Charm Factory Workshop (ref. 2), p. 48.

17. C. Albajar et al. (UA1), Z. Phys. C44 (1989) 15.

18. W. Fetscher, Proc. Tau-Charm Factory Workshop (ref. 2), p. 444.

19. W. Bacino et al. (DELCO), Phys. Rev. Lett. 42 (1979) 749

20. S. Behrends et al. (CLEO), Phys. Rev. D32 (1985) 2468

21. W.T. Ford et al (MAC), Phys. Rev. D36 (1987) 1971

22. H. Janssen et al. (CRYSTAL BALL), DESY 89-054 (1989)

23. N. Ushida et al. (Fermilab E531), Phys. Rev. Lett. 57 (1986) 2897

24. A. de Rújula and R. Rückl, Proc. of the ECFA-CERN Workshop on LHC in the LEP Tunnel (Lausanne-Geneva, 1984) CERN 84-10, Vol. II, p. 571.

25. J.J. Gomez-Cadenas and C.M. González-Garcia, Phys. Rev. D39 (1989) 1370.
    R.R. Mendel et al., Z. Phys. C32 (1986) 517.

26. J.J. Gomez-Cadenas et al., Phys. Rev. D41 (1990) 2179.

27. J.J. Gomez-Cadenas, C. M. González-Garcia and A. Pich, FTUV/90-09.

28. F.J. Gilman and S.H. Rhie, Phys. Rev. D31 (1985) 1066

29. F.J. Gilman, Phys. Rev. D35 (1987) 3541
    A. Pich, Phys. Lett. B196 (1987) 561.

NOTE ADDED IN PROOF:

After submission of this paper for publication, the ARGUS Collaboration has reported a new measurement of the Michel parameter, in the two leptonic decay modes of the Tau. The measured ARGUS values are[30)]

$$\langle \rho_e \rangle = 0.747 \pm 0.045 \pm 0.028 \quad ; \quad \langle \rho_\mu \rangle = 0.734 \pm 0.055 \pm 0.027,$$

in good agreement with a standard V-A couplig.

NEW REFERENCE:

30) H.Albrecht et al. (ARGUS), DESY 90-059