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## TAU-SPIN CORRELATIONS AT THE Z PEAK: APLANARITIES OF THE DECAY PRODUCTS

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### ABSTRACT

The production of  $\tau$  pairs at the Z-peak leads to interesting dynamical information contained in their spin density matrix. Besides the longitudinal polarization of each  $\tau$ -lepton, there are two independent spin-spin correlations associated with the transverse (within the production plane) and normal (to the production plane) polarization components. The transverse-normal spin correlation is both a parity-odd and time reversal-odd observable. We analyze the correlated angular distribution of the decay products as polarization analyzers of the  $\tau^+\tau^-$  system. We find two azimuthal asymmetries of the decay products and the incoming beam, which are able to disentangle these spin correlation observables without reconstructing the  $\tau$ -direction.

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## 1. Introduction

The LEP experiments make it possible to study a variety of processes with high precision and in extremely clean conditions [1], [2], including the production of  $\tau^-\tau^+$  events. When the production mechanism is restricted to chirality conserving interactions, such as those associated with gauge theories, the non-vanishing observables at high energy are limited by helicity conservation. Besides the production cross section one finds, for unpolarized  $e^-e^+$  collisions, the forward-backward asymmetry [3], the longitudinal polarization [4], the longitudinal spin correlation (equal to unity), the transverse spin correlation and the T-odd transverse (within the production plane) - normal (to the production plane) spin correlation [5]. Around the Z-peak, all these observables get non-vanishing values at tree level from the Z-exchange amplitude, except the T-odd correlation which, in the standard model, only gets contributions from the absorptive parts generated at one-loop level as unitarity corrections [5]. In models with chirality-flipping couplings to fermions [6], [7], [8] one finds more possibilities of building non-vanishing observables.

The use of  $\tau$ -decays as polarization analyzers of the  $\tau$ -spin state has been considered in the preparation of LEP, following the principles discussed in the pioneering work of Tsai [9]. The energy spectrum of the decay product (either a charged lepton or a hadron) in the laboratory frame, i.e. the Z rest frame, carries information about the longitudinal polarization of the  $\tau$ . The sensitivity of a given decay channel to this longitudinal polarization depends on the spin of the decay product whose spectrum is measured. For the  $\rho$  and  $a_1$  channels, the sensitivity can be increased by measuring the helicity of these particles [10], [11]. First experimental results on the mean longitudinal  $\tau$ -polarization at LEP measuring the energy spectrum of the decay product have been presented recently [12]. Up to now, particular emphasis has been placed on the measurement of the longitudinal  $\tau$ -polarization, due to its sensitivity to the value of the electroweak mixing angle  $\sin^2 \theta_W$  ( $\equiv s_W^2$ ) of the standard model. However, it is important to carry out a more general test of the coupling structure of the theory beyond the mere determination of its parameters. In this respect, the measurement of the energy correlation function has been considered in the literature [13], [14].

The direct measurable quantities are the momenta of hadrons and charged leptons coming from the decay of the taus. In this paper we analyze the angular distributions of the decay products of both taus, to look for the manifestations of the  $\tau$ -spin correlations. We find two azimuthal asymmetries of the plane defined by the two final charged particles

with respect to the beam direction, able to measure independently the transverse spin correlation and the T-odd transverse-normal spin correlation. It is crucial to notice that the information is not lost when the  $\tau$ -direction is integrated out. The experimental determination of these asymmetries would be of great interest.

## 2. $\tau$ Production at the Z Peak

Let us consider the process

$$e^-(k_1) + e^+(k_2) \rightarrow \tau^-(p_1, s_1) + \tau^+(p_2, s_2) \quad (2.1)$$

around the Z-peak. The momenta of the particles are as indicated and  $s_1^\mu$  ( $s_2^\mu$ ) is the covariant spin vector of the  $\tau^-$  ( $\tau^+$ ). When the incoming helicity states are averaged, the transition probability is given by the general structure

$$\sum |M(s_1, s_2)|^2 = \frac{1}{4} |P(q^2)|^2 \{A + B_{1\mu} s_1^\mu + B_{2\mu} s_2^\mu + C_{\mu\nu} s_1^\mu s_2^\nu\}, \quad (2.2)$$

where  $q = k_1 + k_2$  and

$$P(q^2) = \frac{e^2}{16 s_W^2 c_W^2 q^2 - M_Z^2 + i q^2 \Gamma_Z/M_Z} q^2 \quad (2.3)$$

contains the Z-coupling normalization and the Z-propagator. The terms  $A, \dots, C_{\mu\nu}$  depend on the theory used to describe the transition. We include the Z-exchange amplitude, with effective (complex) vector and axial-vector couplings, and, when needed, its interference with the  $\gamma$ -exchange amplitude. In fact, this interference is of some importance only for the T-odd spin correlation [5]. In the standard model, its magnitude is comparable to that induced by the absorptive parts of the electroweak radiative corrections to the  $Z \rightarrow \tau^-\tau^+$  decay amplitude.

A straightforward calculation of the right-hand side of eq. (2.2) allows the identification of the spin-independent term,

$$A = C_0(1 + \cos^2 \theta) + 2C_1 \cos \theta. \quad (2.4)$$

Here  $\theta$  is the  $\tau^-$ -polar angle with respect to the  $e^-$ -direction, and the coefficients  $C_0$  and  $C_1$  contain the information on the Z-couplings:

$$C_0 = (|v_e|^2 + |a_e|^2)(|v_\tau|^2 + |a_\tau|^2), \quad (2.5a)$$

$$C_1 = 4 Re(v_e a_e^*) Re(v_\tau a_\tau^*). \quad (2.5b)$$

One recognizes in eqs. (2.4) and (2.5a, b) the well-known terms associated with the integrated cross section,  $\sigma = \frac{|E(g^*)|^2}{12\pi} C_0$ , and the forward-backward asymmetry,  $A_{FB} = \frac{3}{4} \frac{C_1}{C_0}$ .

In order to discuss the dependence on the  $\tau$ -spin vectors, we decompose  $s_1^\mu$  and  $s_2^\mu$  in their longitudinal, transverse (within the collision plane) and normal (to the collision plane) components,  $s_i^\mu = s_i^L s_i^\mu + s_i^T s_i^\mu + s_i^N s_i^\mu$  ( $i=1,2$ ). To leading order in the tau-lepton mass  $m_\tau$ , the terms linear in  $s_i^\mu$  ( $i=1,2$ ) are given by

$$B_{1\mu} s_1^\mu + B_{2\mu} s_2^\mu = W_\tau + W_e, \quad (2.6)$$

where

$$W_\tau = -D_0(s_1^L - s_2^L)(1 + \cos^2 \theta), \quad (2.7a)$$

$$W_e = -D_1(s_1^L - s_2^L)2 \cos \theta. \quad (2.7b)$$

Within our theoretical framework, other possible terms associated with transverse and/or normal polarizations of single taus are suppressed by lepton mass factors.  $W_\tau$  contains the longitudinal polarization of the taus for unpolarized Z's,  $P_\tau = -D_0/C_0$ , whereas  $W_e$  is related to the angular asymmetry of the longitudinal polarization of the taus induced by the polarized Z's,  $P_Z = -D_1/C_0$ . The information provided by  $W_e$  is the same as that of the left-right asymmetry for longitudinally polarized beams. The explicit expressions for the coefficients  $D_0$  and  $D_1$ , in terms of the Z-couplings, are

$$D_0 = 2(|v_e|^2 + |a_e|^2) \text{Re}(v_e a_e^*), \quad (2.8a)$$

$$D_1 = 2 \text{Re}(v_e a_e^*)(|v_\tau|^2 + |a_\tau|^2). \quad (2.8b)$$

Note that due to the factorization induced by single Z-exchange  $A_{FB} = \frac{3}{4} P_Z P_\tau$ .

The quadratic terms in the polarization of both taus contain interesting spin correlations, which introduce additional dependences on the vector and axial-vector Z-couplings besides those already present in the rate, the forward-backward asymmetry and the longitudinal polarization of the tau. We find

$$C_{\mu\nu} s_1^\mu s_2^\nu = C_0 h_0 + C_1 h_1 + C_2 h_2 + D_2 h_3, \quad (2.9)$$

where

$$h_0 = -s_1^L s_2^L (1 + \cos^2 \theta), \quad (2.10a)$$

$$h_1 = -s_1^L s_2^L 2 \cos \theta, \quad (2.10b)$$

$$h_2 = (s_1^N s_2^N - s_1^T s_2^T) \sin^2 \theta, \quad (2.10c)$$

$$h_3 = (s_1^N s_2^T + s_1^T s_2^N) \sin^2 \theta. \quad (2.10d)$$

The fact that both  $h_0$  and  $h_1$  have the same couplings,  $C_0$  and  $C_1$  respectively, as those of the spin independent term A (eq. (2.4)) leads to a longitudinal spin correlation equal to unity. This is nothing but a consequence of helicity conservation.

$C_2$  and  $D_2$  are the  $\tau$ -spin correlations associated with the transverse-transverse and transverse-normal components of both polarizations,

$$C_{TR} \equiv \frac{C_2}{C_0} = \frac{|\alpha_r|^2 - |\nu_r|^2}{|\alpha_r|^2 + |\nu_r|^2}, \quad (2.11a)$$

$$C_{TN} \equiv \frac{D_2}{C_0} = -\frac{2 \text{Im}(\nu_r \alpha_r^*)}{|\nu_r|^2 + |\alpha_r|^2} - \frac{2 s_W^2 c_W^2 \text{Re}(\nu_r \alpha_r) \Gamma_Z / M_Z}{(|\nu_r|^2 + |\alpha_r|^2)(|\nu_r|^2 + |\alpha_r|^2)}. \quad (2.11b)$$

The first is a parity-even observable which measures the interference between the left- and right-handed amplitudes in the Z-decay: it gives the difference in the probability for the decays from the charge conjugation state with  $C = +1$  or  $C = -1$ . The  $C_{TN}$  correlation is both a parity-odd and time reversal-odd observable which, as shown in eq. (2.11b), can be generated from absorptive parts in the electroweak amplitudes. The second term of eq. (2.11b) comes from the interference of  $\gamma$ -exchange with the imaginary part of the Z-exchange amplitude. In some extended models  $C_{TN}$  could get a contribution from CP-violating amplitudes. The purpose of this paper is to find a method to measure these spin correlations.

### 3. $\tau$ - Decays as Polarization Analyzers

To look for the signal of the spin correlations, we let both taus decay to a charged particle, which will be detected, plus missing neutrinos. This will translate the spin correlations into angular correlations of the decay products. The momenta of the two decay products  $X_1$  from  $\tau^-$  and  $X_2$  from  $\tau^+$  are indicated by  $q_1$  and  $q_2$  respectively. The angular distribution of  $X_1$  ( $X_2$ ) from an arbitrary polarized  $\tau^-$  ( $\tau^+$ ) can be written in the rest frame of the tau as

$$\frac{d\Gamma(s_i)}{\Gamma_{X_i}}(\tau \rightarrow X_i + \dots) = \frac{1}{4\pi} [1 - \alpha_i m_\tau \frac{(q_i \cdot s_i)}{(q_i \cdot p_i)}] d\Omega_i^*, \quad (3.1)$$

where  $\alpha_i$  is the polarization analyzer of the spin  $s_i^\mu$ ,  $i = 1, 2$ .

We have calculated the values of  $\alpha_i$  for the dominant decay channels of the  $\tau$ , assuming general vector and axial vector couplings for the  $\tau$  decay. In the leptonic decay mode, we find

$$\alpha_{1\pm} = \pm \xi/3, \quad (3.2)$$

where  $\xi$  is the Michel polarization parameter for  $\tau \rightarrow l + \nu_\tau + \nu_l$  [16]. For the hadronic decays, we introduce the covariant amplitudes

$$\mathcal{M} = \frac{G}{\sqrt{2}} \bar{\nu}_\tau \gamma_\mu (g_V - g_A \gamma_5) \tau J_H^\mu, \quad (3.3)$$

with  $G$  the Fermi constant and  $J_H^\mu$  the weak hadronic current, and define the chiral polarization parameter

$$f_H = \frac{2 \text{Re}(g_V g_A^*)}{|g_V|^2 + |g_A|^2}. \quad (3.4)$$

The polarization analyzer in the decay mode  $\tau \rightarrow \pi + \nu_\tau$  is then simply given by:

$$\alpha_{\pi^\pm} = \mp \xi_\pi. \quad (3.5)$$

The  $\rho$  channel is slightly more complicated. If the angular distribution of eq. (3.1) refers to the  $\rho$  direction,<sup>1</sup> it is straightforward to obtain the following result in the narrow width approximation,

$$\alpha_{\rho^\pm} = \mp \xi_\rho \frac{m_\tau^2 - 2m_\rho^2}{m_\tau^2 + 2m_\rho^2} \frac{m_\tau^2 + m_\rho^2}{m_\tau^2 - m_\rho^2} = \mp 0.67 \xi_\rho. \quad (3.6)$$

For the  $\tau \rightarrow a_1 + \nu_\tau$  channel, the value of the corresponding  $\alpha_{a_1^\pm}$  is also given by eq. (3.6) with the appropriate mass changes ( $m_{a_1}$  instead of  $m_\rho$ ); the result is however smaller and very sensitive to the value of the  $a_1$ -mass ( $4 \times 10^{-3}$  for  $m_{a_1} = 1260$  MeV and 0.13 for  $m_{a_1} = 1200$  MeV), so we will not consider this decay mode any further.

The angular distribution of the sequential process  $e^- e^+ \rightarrow \tau^- \tau^+ \rightarrow X_1 X_2 + \dots$  in the laboratory frame, where the taus move with velocity  $\beta = |\vec{p}|/E$ , may be written in the following way [9], [16]:

$$\begin{aligned} d\sigma(e^- e^+ \rightarrow X_1 X_2 + \dots) &= Br(\tau^- \rightarrow X_1 + \dots) Br(\tau^+ \rightarrow X_2 + \dots) \frac{|P(q^2)|^2}{64\pi^2 q^2 (4\pi)^2} \\ &\times \{A + B_{1\mu} n_1^\mu + B_{2\mu} n_2^\mu + C_{\mu\nu} n_1^\mu n_2^\nu\} \frac{d\Omega_1 d\Omega_2}{\gamma^4 (1 - \beta \cos \chi_1)^2 (1 + \beta \cos \chi_2)^2}. \end{aligned} \quad (3.7)$$

In the result (3.7), the Jacobian of the transformation for the angular variables from the rest system of the tau to the laboratory frame has been obtained in the limit of vanishing final masses. The dilation factor  $\gamma = E/m_\tau$  controls the range of the opening angle  $\chi_1$  of the decay product  $X_1$  with respect to  $\tau^-$ , and similarly for  $(\pi - \chi_2)$ . The bracket  $\{ \dots \}$  is obtained from eq. (2.2) under the recipe [16]:

$$s_i^\mu \rightarrow n_i^\mu = -\alpha_i \frac{m_\tau}{(p_i \cdot q_i)} \eta_i^{\mu\nu} q_{i\nu} \quad (i = 1, 2), \quad (3.8)$$

where  $\eta_i^{\mu\nu} = -g^{\mu\nu} + p_i^\mu p_i^\nu / m_\tau^2$ .

#### 4. Angular Correlations

To proceed further, we choose the following coordinate system in the laboratory frame, as indicated in fig. 1,

$$\begin{aligned} \hat{q}_1 &= (0, 0, 1) \\ \hat{q}_2 &= (\sin \theta_{12}, 0, \cos \theta_{12}) \\ \hat{k} &= (\sin \theta_1 \cos \phi_1, \sin \theta_1 \sin \phi_1, \cos \theta_1) \\ \hat{p} &= (\sin \chi_1 \cos \psi, \sin \chi_1 \sin \psi, \cos \chi_1). \end{aligned} \quad (4.1)$$

In terms of these angles, one can write the other directional variables  $\theta$  [angle between  $\hat{k}$  and  $\hat{p}$ ],  $\chi_2$  [angle between  $\hat{p}$  and  $\hat{q}_2$ ] and  $\theta_2$  [angle between  $\hat{k}$  and  $\hat{q}_2$ ]. Our choice of angular variables is adequate, as we shall see, to discuss angular correlations between the decay products. The aim is to analyze the resulting correlations when the tau direction  $\Omega(\chi_1, \psi)$  is integrated out in eq. (3.7). A different strategy to study energy correlations between the decay products was discussed by Nelson [14].

We can expand the resulting expression for eq. (3.7) to the dominant order in the dilation factor  $\gamma$ , taking into account that due to the boost one has  $\chi_1 \sim \gamma^{-1}$  and  $\theta_{12} \equiv \pi - \epsilon$  with  $\epsilon \sim \gamma^{-1}$ . To leading order in  $\chi_1$  and  $\epsilon$ , the bracket involved in the cross section (3.7) is given by

$$A + B_{1\mu} n_1^\mu + B_{2\mu} n_2^\mu + C_{\mu\nu} n_1^\mu n_2^\nu = G_0 (1 + \cos^2 \theta_1) + G_1 2 \cos \theta_1 + G_2 (\phi_1) \sin^2 \theta_1. \quad (4.2)$$

As seen, the angular distribution in  $\theta_1$  (the direction of one daughter product with respect to the electron beam) reproduces the form in  $\theta$  that the parent tau had with respect to the beam (see eqs. (2.4), (2.7) and (2.10)). Besides the angular dependences explicitly indicated,  $G_0$ ,  $G_1$  and  $G_2(\phi_1)$  are functions of the three angular variables  $(\chi_1, \psi, \epsilon)$ .  $G_0$

<sup>1</sup> Since the  $\rho$  decays before being detected, one may think that it would be cleaner to use the angular distribution of the final charged pion in the decay chain:  $\tau^+ \rightarrow \rho^\mp + \nu_\tau \rightarrow \pi^\mp + \pi^0 + \nu_\tau$ . Unfortunately, this procedure washes out the sensitivity to the  $\tau$ -polarization, as we find  $\alpha_{\rho^\pm} = \mp \xi_\rho f(m_\rho)$ , where  $f(m_\rho) \sim 6 \times 10^{-2}$ . As a consequence, we will use the  $\rho$ -direction as analyzer of the  $\tau$ -polarization, although experimentally the direction of the charged pion can be measured more accurately.

gets contributions from the  $C_0$  term of  $A$  and  $C_{\mu\nu}$ , and the  $D_0$  term of  $B_\mu$ ; its information on the parent process corresponds to the decay rate and the longitudinal polarization of the tau,  $P_\tau$ .  $G_1$  gets contributions from the  $C_1$  term of  $A$  and  $C_{\mu\nu}$  and the  $D_1$  term of  $B_\mu$ ; its information on the tau production process corresponds to the forward-backward asymmetry,  $A_{FB}$ , and the polarization of the  $Z$ ,  $P_Z$ , for unpolarized beams.  $G_2(\phi_1)$  is the function which contains the information on the spin correlations  $C_{T\bar{T}}$  and  $C_{TN}$ , discussed in eq. (2.11a,b).

When the azimuth  $\psi$  between the tau and the plane defined by both decay products is integrated out, the  $\theta_1$ -distribution has the same form as in eq. (4.2), with new functions

$$g_i \equiv \int_0^{2\pi} d\phi \frac{G_i}{(1 + \beta \cos x_2)^2} \quad (i = 0, 1, 2). \quad (4.3)$$

We find

$$\begin{aligned} g_i &= \frac{8\pi\gamma^4}{R^{3/2}} \left\{ C_i \left[ (1 + x + \gamma^2 \epsilon^2)[1 - \alpha_1 \alpha_2(1 - \frac{2}{1+x})] \right. \right. \\ &\quad \left. + \frac{2\alpha_1 \alpha_2}{R} (1 - \frac{2}{1+x})[(1 + x + \gamma^2 \epsilon^2)^2 + 2\gamma^2 \epsilon^2 x] \right] \\ &\quad + D_i \left[ (\alpha_1 - \alpha_2)(1 + x + \gamma^2 \epsilon^2) - \alpha_1 \frac{2(1 + x + \gamma^2 \epsilon^2)}{1+x} \right. \\ &\quad \left. + \alpha_2 \frac{2}{R} [(1 + x + \gamma^2 \epsilon^2)^2 + 2\gamma^2 \epsilon^2 x] \right] \Big\} \\ g_2(\phi_1) &= 8\pi\gamma^4 \alpha_1 \alpha_2 [C_2 \cos 2\phi_1 + D_2 \sin 2\phi_1] \frac{12\gamma^2 \epsilon^2 x(x - 1 - \gamma^2 \epsilon^2)}{(1+x)R^5/2}, \end{aligned} \quad (4.4a)$$

where  $x \equiv (\gamma x_1)^2$  and  $R \equiv x^2 + 2(1 - \gamma^2 \epsilon^2)x + (1 + \gamma^2 \epsilon^2)^2$ .

As seen in eq. (4.4b), the two spin correlations  $C_2$ ,  $D_2$  of interest can be separated out, by looking at the azimuthal distribution of the two decay products with respect to the electron beam. We can integrate over  $x$  without losing the information on  $C_2$ ,  $D_2$ ; so the reconstruction of the tau direction is unnecessary.

When  $d\sigma$  is integrated over the tau-direction  $X_1$ , the correlated angular distribution for the decay products  $X_1$  and  $X_2$  is obtained, to leading order in  $\epsilon \equiv \pi - \theta_{12}$ , as

$$d\sigma = K(q^2) F(\theta_1, \phi_1; \theta_{12}) d\Omega_1 d\Omega_2, \quad (4.5)$$

where

$$K(q^2) = B_T(\tau^- \rightarrow X_1 + \dots) B_T(\tau^+ \rightarrow X_2 + \dots) \frac{|P(q^2)|^2 \gamma^2}{(4\pi)^3 q^2}, \quad (4.6)$$

and

$$F(\theta_1, \phi_1; \theta_{12}) = F_0(\epsilon)(1 + \cos^2 \theta_1) + F_1(\epsilon)2 \cos \theta_1 + F_2(\epsilon, \phi_1) \sin^2 \theta_1. \quad (4.7)$$

The three functions  $F_i(\epsilon)$  ( $i = 0, 1, 2$ ) of eq. (4.7) are given by

$$F_i(\epsilon) \equiv \frac{1}{8\pi\gamma^4} \int dx \frac{g_i}{(1+x)^2} \quad (i = 0, 1, 2). \quad (4.8)$$

These functions have the following interesting dependences on the polarization analyzers  $\alpha_1$ ,  $\alpha_2$  of the channels  $X_1, X_2$ :

$$F_1(\epsilon) = C_i [Q_1(\epsilon) + \alpha_1 \alpha_2 Q_2(\epsilon)] + D_i (\alpha_1 - \alpha_2) Q_3(\epsilon) \quad (i = 0, 1), \quad (4.9a)$$

$$F_2(\epsilon, \phi_1) = \alpha_1 \alpha_2 [C_2 \cos 2\phi_1 + D_2 \sin 2\phi_1] Q_4(\epsilon). \quad (4.9b)$$

The four functions  $Q_i$  ( $i = 1, 2, 3, 4$ ) are represented in fig. 2. We reproduce that the characteristic width of the  $\epsilon$ -distribution is  $\sim \gamma^{-1}$ .

To single out the spin-correlation terms associated with  $C_2$  and  $D_2$ , we define the following aplanarities for fixed values of  $(\theta_1, \epsilon)$ :

$$A_c(\theta_1, \epsilon) = \frac{\int_{\cos 2\phi_1 > 0} d\sigma - \int_{\cos 2\phi_1 < 0} d\sigma}{\int_{\cos 2\phi_1 > 0} d\sigma + \int_{\cos 2\phi_1 < 0} d\sigma}, \quad (4.10a)$$

$$A_s(\theta_1, \epsilon) = \frac{\int_{\sin 2\phi_1 > 0} d\sigma - \int_{\sin 2\phi_1 < 0} d\sigma}{\int_{\sin 2\phi_1 > 0} d\sigma + \int_{\sin 2\phi_1 < 0} d\sigma}. \quad (4.10b)$$

In particular, for  $\theta_1 = \pi/2$  we present in fig. 3 the results of

$$A_c(\pi/2, \epsilon) = \frac{2 \alpha_1 \alpha_2 C_2 Q_4(\epsilon)}{\pi F_0(\epsilon)} \quad (4.11)$$

for the dominant decay modes of the tau. The coefficients  $C_2$ ,  $C_0$  and  $D_0$  have been fixed to the values expected in the standard theory, as well as the  $\tau$  decay parameters,  $\xi = \xi_\tau = \xi_p = 1$ . Similarly, these results can be translated into values of  $A_s(\pi/2, \epsilon)$  by the substitution  $C_2 \rightarrow D_2$ . The highest sensitivity of the method to determine the correlations  $C_{T\bar{T}}$  and  $C_{TN}$  appears for values of  $\gamma\epsilon \sim 2$ , although the width of the  $\epsilon$ -distribution of events is typically  $\gamma\epsilon \sim 1$ .

One can consider aplanarities integrated over  $\epsilon$ , keeping in each case the whole set of events within the plane defined by the azimuth  $\phi_1$ . Some sensitivity is lost in these  $\epsilon$ -integrated aplanarities, which are presented in Table 1 for all the channels we have analyzed.

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channel	$A_c(\pi/2)$	$A_s(\pi/2)$
$\pi^+ \pi^-$	$2.1 \times 10^{-1}$	$2.6 \times 10^{-3}$
$\pi^+ l^-$	$-7.0 \times 10^{-2}$	$-8.6 \times 10^{-4}$
$\pi^+ \rho^-$	$1.4 \times 10^{-1}$	$1.7 \times 10^{-3}$
$\rho^+ \rho^-$	$9.4 \times 10^{-2}$	$1.2 \times 10^{-3}$
$\rho^+ l^-$	$-4.7 \times 10^{-2}$	$-5.7 \times 10^{-4}$
$l^+ l^-$	$2.3 \times 10^{-2}$	$2.9 \times 10^{-4}$

Table 1: Integrated azimuthal asymmetries.

### 5. Discussion

The correlated angular distribution of the decay products of the tau-pairs produced at the  $Z$  peak in  $e^+ e^-$  collisions contains interesting information about the spin density matrix of both taus. A convenient choice of the angular variables for the decay product and beam three-momenta, without any reference to the tau direction, is shown in Fig. 1. We have found that the two aplanarities  $A_c$  and  $A_s$ , defined in eqs. (4.10) are able to measure the tau spin correlations  $C_{TT}$  and  $C_{TN}$ , associated with the transverse and normal polarizations of both taus.

The first observable is parity even and it gives the relative magnitude between the vector and the axial probabilities for the  $Z \rightarrow \tau^+ + \tau^-$  decay. The signal of  $A_c$  in the standard theory, either as a function of the acollinearity angle  $\epsilon$  between both decay products or in its  $\epsilon$ -integrated version, is given in Fig. 3 and Table 1, respectively. It appears to be big enough to be detected in forthcoming analyses of LEP experiments.

The spin correlation  $C_{TN}$ , as well as the corresponding observable aplanarity  $A_s$ , are of great interest because of their T-odd character. In the standard theory, the small non-vanishing value of  $A_s$  is due [5] to the generation of absorptive parts from the electroweak radiative corrections to the  $Z$ -decay amplitude and to the interference between the  $Z$ -exchange and  $\gamma$ -exchange amplitudes. This prediction for  $A_s$ , shown in Table 1 for the  $\epsilon$ -integrated version, should be considered as the standard reference to look for manifestations of new physics related to CP-violation. The observation of a value for  $A_s$  above the standard prediction would be of major interest.

## References

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## Figure Captions

- Fig. 1.** Coordinate system in the LAB frame used in the calculation of the angular distribution for  $e^+ + e^- \rightarrow \tau^+ + \tau^- \rightarrow X_1 + X_2 + \dots$
- Fig. 2.** Functions which appear in the angular distribution of  $e^+ + e^- \rightarrow \tau^+ + \tau^- \rightarrow X_1 + X_2 + \dots$  after integrating out the  $\tau$  solid angle (eq. (4.9a,b)). Dash-dotted line:  $Q_1$ ; continuous line:  $Q_2$ ; dotted line:  $Q_3$ ; dashed line:  $Q_4$ .
- Fig. 3.** Results of the azimuthal asymmetry  $A_c(\pi/2, \epsilon)$  for the dominant  $\tau$  decay modes.

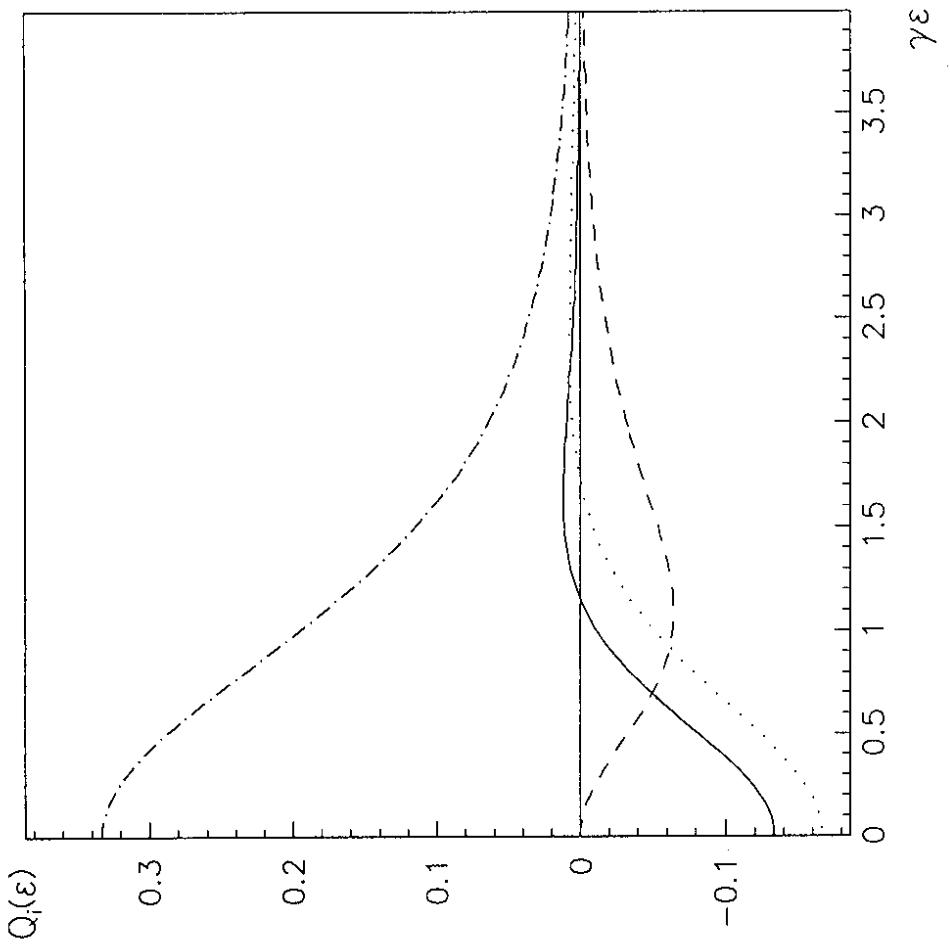


Fig. 2.

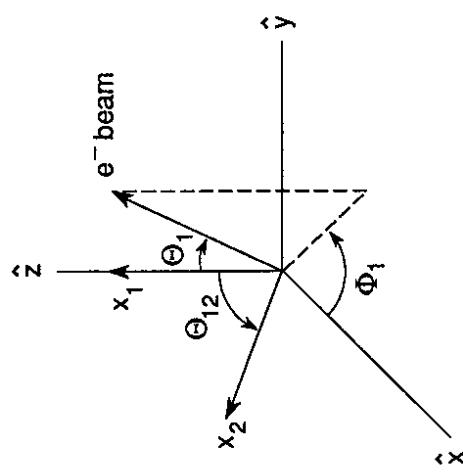


Fig. 1.

$A_c(\varepsilon)$

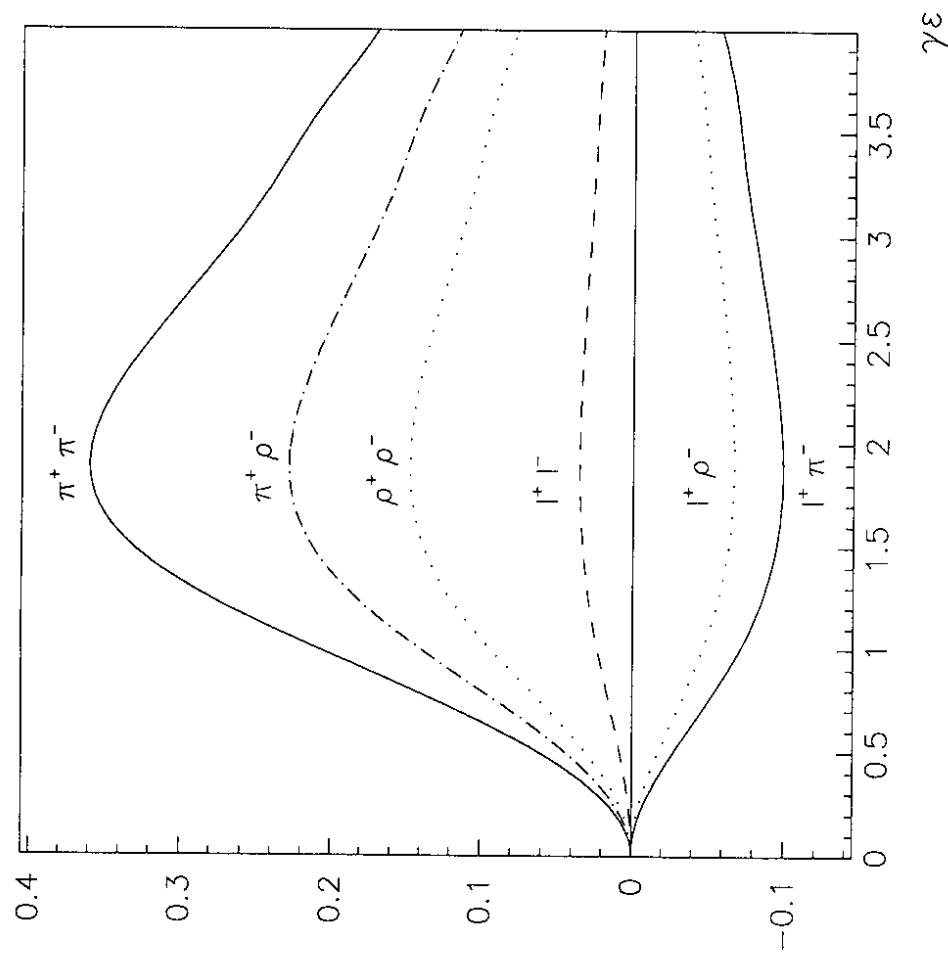


Fig. 3.