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FOUR-QUARK OPERATORS AND NON-LEPTONIC WEAK TRANSITIONS

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ABSTRACT

Matrix elements of four-quark operators are studied with two approaches. First inclusively; we consider two-point functions of four-quark operators. We compute their corresponding spectral functions at very short distances using perturbative QCD with inclusion of $O(\alpha_s)$ corrections; and at very long distances, using the chiral effective realization of QCD in terms of Goldstone particles. A qualitative picture emerges which requires, for consistency between the two extreme behaviours, a large coupling constant for $\Delta I=1/2$ transitions.

The other, more direct approach, consists of calculating the effective action of four-quark operators by integrating out the quark fields in a gluonic background and in the presence of a source term which triggers spontaneous chiral symmetry breaking. The procedure follows the method elaborated recently in ref. [1]. This way we compute the coupling constants of the lowest order effective chiral Lagrangian with $\Delta S=1$ ($\Delta I=1/2$ and $3/2$) and $\Delta S=2$ (the so called $\hat{B}$-factor.) The calculations include the well-known $O(N_c^2)$ contributions as well as the subleading $O(N_c)$ with inclusion of the $O(\alpha_s N_c)$ terms. The picture which emerges at this approximation is already very encouraging. Comparison with other approaches is also made.

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1.— Introduction

The Standard Model [2] predicts strangeness changing transitions with $\Delta S = 1$ via W-exchange between two weak charged currents as shown in Fig. 1. At long distances, and to a first approximation where gluonic exchanges between the two weak vertices are neglected, this is described by the effective Hamiltonian

$$H_{eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^{*} \bar{s}(x)\gamma^{\mu}(1-\gamma_5)u(x)\bar{u}(x)\gamma_{\mu}(1-\gamma_5)d(x) + h.c. \tag{1.1}$$

where G_F is the Fermi coupling ($1.166 \times 10^{-5} GeV^{-2}$) and V_{ud}, V_{us} are matrix elements of the Cabibbo [3a], Kobayashi - Maskawa [3b] flavour mixing matrix (CKM). The overall four-quark operator in eq. (1.1) is conventionally called the Q_2-operator :

$$Q_2(x) \equiv 4(\bar{s}_L\gamma^{\mu}u_L)(\bar{u}_L\gamma_{\mu}d_L), \tag{1.2a}$$

where e.g.,

$$(\bar{s}_L\gamma^{\mu}u_L) \equiv \sum_{\alpha} \bar{s}^{(\alpha)}(x)\gamma^{\mu}\left(\frac{1-\gamma_5}{2}\right)u^{(\alpha)}(x) \tag{1.2b}$$

with α the quark colour index. Because of its flavour content, the operator Q_2 induces $\Delta S = 1$ transitions with an isospin change $\Delta I = \frac{1}{2}$ as well as $\Delta I = \frac{3}{2}$.

There is experimental evidence, both from K-decays and hyperon decays [4], that the rates of non-leptonic $\Delta S = 1$ transitions with $\Delta I = \frac{1}{2}$ are particularly enhanced; a phenomenological fact which is referred to as "the $\Delta I = \frac{1}{2}$ rule". The explanation of this selection rule, presumably of dynamical origin, has been a continuous challenge to theorists for the last three decades !

To illustrate the problem let us first make an educated guess of the strength of the transitions expected for $K \to \pi\pi$ decays. For this purpose it would seem appropriate to consider the operator Q_2 as a factorized product of the two currents

$$(\bar{s}_L\gamma^{\mu}u_L) \quad \text{and} \quad (\bar{u}_L\gamma_{\mu}d_L)$$

and use an effective realization of these currents in terms of pseudoscalar fields. To lowest order in the number of derivatives, we know what the answer is from current algebra (for a review, see ref. [5]) :

$$(\bar{s}_L\gamma^{\mu}u_L) \to -\frac{1}{\sqrt{2}}f_{\pi}\partial^{\mu}K^{+} - \frac{i}{2\sqrt{2}}[(\pi^0\overleftrightarrow{\partial}_{\mu}K^{+}) + \sqrt{3}(\eta\overleftrightarrow{\partial}_{\mu}K^{+}) + \sqrt{2}(\pi^{+}\overleftrightarrow{\partial}_{\mu}K^{0})] + \ldots \tag{1.3a}$$

and

$$(\bar{u}_L\gamma_{\mu}d_L) \to -\frac{1}{\sqrt{2}}f_{\pi}\partial_{\mu}\pi^{-} - \frac{i}{2}[\sqrt{2}(\pi^{-}\overleftrightarrow{\partial}_{\mu}\pi^{0}) + (K^{0}\overleftrightarrow{\partial}_{\mu}K^{-})] + \ldots, \tag{1.3b}$$

where f_{π}, determined from $\pi \to \mu\nu$ decay, is $f_{\pi} = 93.3 MeV$ and the dots denote terms higher than quadratic in the number of pseudoscalar fields. The first term on the r.h.s. of

(1.3) is the axial current realization as given by $PCAC$ in the $SU(3)$ limit; the second term is the vector current realization. With this ansatz inserted in eq. (1.1) one can then proceed to the evaluation of the $K \to \pi\pi$ physical amplitudes in a straightforward manner. For later purposes, it is convenient to give the results in terms of isospin-irreducible amplitudes A_I, $I = 0, 2$. Bose statistics requires the 2π-final state to be in an even isospin state. In full generality, we normalize the A_I-amplitudes in such a way that

$$A(K^0 \to \pi^{+}\pi^{-}) = iA_0 e^{i\delta_0} + i\frac{A_2}{\sqrt{2}}e^{i\delta_2} \tag{1.4a}$$

$$A(K^0 \to \pi^{0}\pi^{0}) = iA_0 e^{i\delta_0} - i\sqrt{2}A_2 e^{i\delta_2} \tag{1.4b}$$

$$A(K^{+} \to \pi^{+}\pi^{0}) = \frac{3}{2}iA_2 e^{i\delta_2}, \tag{1.4c}$$

where δ_I are the $J = 0$ $\pi\pi$ phase shifts for $I = 0, 2$ at the CM energy of the K-mass. The factors $e^{i\delta_I}$ are induced by the $\pi\pi$ final state interactions. Electromagnetic corrections have however been neglected. The results of the factorization hypothesis are then

$$A_0 = -\frac{G_F}{\sqrt{2}}V_{ud}V_{us}^{*}\frac{2\sqrt{2}}{3}f_{\pi}(M_K^2 - m_{\pi}^2), \tag{1.5a}$$

$$A_2 = -\frac{G_F}{\sqrt{2}}V_{ud}V_{us}^{*}\,\frac{2}{3}\,f_{\pi}(M_K^2 - m_{\pi}^2), \tag{1.5b}$$

Experimentally, from the ratio of $\Gamma(K_S \to \pi^{+}\pi^{-})$ to $\Gamma(K_S \to \pi^{0}\pi^{0})$ and with neglect of radiative corrections one finds

$$Re\frac{A_0}{A_2}\Big|_{Exp} \simeq 22.2 \tag{1.6}$$

i.e., a factor of <u>sixteen</u> larger than the factorization estimate. In fact, from the comparison between the experimental rate for $K^{+} \to \pi^{+}\pi^{0}$ and the one predicted from (1.4c) and (1.5b), one is led to the conclusion that the factorization assumption <u>overestimates</u> the $\Delta I = \frac{3}{2}$ amplitude by a factor of <u>two</u>. The same factorization assumption <u>underestimates</u> the $\Delta I = \frac{1}{2}$ amplitude by a factor of <u>eight</u>.

One may be tempted to conclude that the factorization assumption we have used to make a first guess estimate must indeed be a very naive picture. It turns out however, as we shall later discuss, that this is precisely the result predicted by the leading behaviour of the $1/N_c$-expansion, where N_c - the number of colours in QCD - is taken to be large with $\alpha_S N_c$ fixed [6].

How does QCD overcome this catastrophic estimate ? The answer to this question is the main purpose of this paper. We study the problem with two approaches. First, following our previous work ([7] to [11]), we consider two-point functions of four-quark operators and compute their spectral functions at very short distances (section 3) and at very long distances (section 5). The short-distance behaviour is calculated using perturbative QCD with inclusion of $0(\alpha_s)$-corrections [11]. For the sake of clarity we also include in section

2 a brief summary of the short-distance reduction from the Standard Model Lagrangian to effective four-quark Hamiltonians for non-leptonic weak interactions. This section contains explicit expressions for the Wilson coefficients of the weak Hamiltonian which will be needed everywhere later.

The long-distance behaviour of the spectral functions associated to four-quark correlators is calculated within the framework of effective chiral Lagrangians [10]. This framework is briefly reviewed in section 4. It provides a very powerful way of parametrizing non-leptonic amplitudes in terms of a few fundamental coupling constants : $g_{\underline{8}}^{(1/2)}$, $g_{\underline{27}}^{(1/2)}$, $g_{\underline{27}}^{(3/2)}$ and the so called $\widehat{B}$-parameter. From the comparison of the two extreme behaviours (short-distance and long-distance) of the spectral functions we discuss in section 5 how a qualitative understanding of the $\Delta I = 1/2$ enhancement emerges.

The other more direct approach is to calculate the effective action of four-quark operators within an approximated version of QCD [1], which consists of integrating out the quark fields in the presence of a gluonic background as well as a source term which triggers spontaneous chiral symmetry breaking. Gluon fields are allowed to form condensates in the physical vacuum and this provides a parametrization of non-perturbative effects. Sections 6 and 7 describe the functional formalism to implement calculations. Sections 8 and 9 give the main steps which allow for a determination of the constants $g_{\underline{8}}^{(1/2)}$, $g_{\underline{27}}^{(3/2)}$, $g_{\underline{27}}^{(1/2)}$ and the $\widehat{B}$-parameter to order $0(N_c^2)$ and to $0(N_c)$ with inclusion of the $0(\alpha_s N_c)$ contributions. Section 10 gives our final results for these coupling constants and discusses their numerical evaluation. Finally, section 11 gives a comparative description with other approaches and summarizes our conclusions. To the reader who is not interested in technical details of "how we do it" we advise her or him to skip altogether sections 3, 7, 8 and 9.

2.— Short-Distance Reduction to Effective Hamiltonians

In QCD, the composite operator Q_2 in (1.2) is not multiplicatively renormalizable. As first pointed out by Gaillard and Lee [12] and by Altarelli and Maiani [13], the effect of gluon exchanges as indicated in Fig. 2 generates a new $\Delta S = 1$ operator

$$Q_1 \equiv 4(\bar{s}_L \gamma^\mu d_L)(\bar{u}_L \gamma_\mu u_L) \tag{2.1}$$

which mixes with the standard Q_2-operator

$$Q_2 \equiv 4(\bar{s}_L \gamma^\mu u_L)(\bar{u}_L \gamma_\mu d_L) \tag{2.2}$$

under renormalization. It was later noticed by Vainshtein, Zakharov and Shifman [14] that the effect of the so-called penguin diagrams shown in Fig. 3 brings further $\Delta S = 1$ operators ($q_R \equiv \frac{1}{2}(1+\gamma_5)q(x)$) :

$$Q_3 = 4(\bar{s}_L \gamma^\mu d_L) \sum_{q=u,d,s} (\bar{q}_L \gamma_\mu q_L) \tag{2.3}$$

$$Q_4 = 4 \sum_{q=u,d,s} (\bar{s}_L \gamma^\mu q_L)(\bar{q}_L \gamma_\mu d_L) \tag{2.4}$$

$$Q_5 = 4(\bar{s}_L \gamma^\mu d_L) \sum_{q=u,d,s} (\bar{q}_R \gamma_\mu q_R) \tag{2.5}$$

$$Q_6 = -8 \sum_{q=u,d,s} (\bar{s}_L q_R)(\bar{q}_R d_L) \tag{2.6}$$

which also mix with Q_2 and among themselves under renormalization. The operators

$$Q_- \equiv Q_2 - Q_1 \tag{2.2}$$

and $Q_i, i = 3,4,5,6$, induce pure $\Delta I = \frac{1}{2}$ transitions, but only four of these operators are independent since

$$Q_- + Q_3 - Q_4 = 0 \quad . \tag{2.7}$$

One can already see at this level a crucial feature of QCD concerning $\Delta S = 1$ non-leptonic transitions in the Standard Model. It is the fact that the full basis of possible four-quark operators with $I = \frac{1}{2}$ only appears when at least the order-α_S gluonic interactions are taken into account.

The reduction from the Lagrangian of the Standard Model to an effective electroweak Hamiltonian where only the light quark fields u, d, s appear has been well discussed elsewhere [15]. The procedure consists of using the asymptotic freedom property of QCD [16] to successively integrate out the fields with heavy masses down to scales $\mu^2 < m_c^2$. The appropriate technique is the operator product expansion [17] and the use of renormalization group equations [18] to compute the various Wilson coefficient functions of the four-quark operators $Q_i, i = 1, \cdots, 6$. In practice, this has only been fully carried out at the one-loop level (the leading logarithmic approximation). There exist in fact two-loop calculations, but only in the Q_2, Q_1 sector (i.e., in the absence of penguins) [19].

Most of the calculations integrate first the W-field and then successively the quark fields from the top to the bottom to the charm to a scale μ^2 appropriate to low energy physics. This implicitly assumes the order $M_W^2 > m_t^2 > m_b^2 > m_c^2 > \mu^2$. In view of the experimental fact that $m_t > M_W$ there have been recent revisions of the calculation of the Wilson coefficients of the effective four-quark $\Delta S = 1$ and $\Delta S = 2$ Hamiltonians which correct for this fact. The effects are important as far as CP-violation predictions (ϵ and ϵ' parameters) are cocerned, but they are practically negligible for the CP-conserving parts of the A_I-amplitudes which is what we are concentrating on in this first paper. By the same token we shall also neglect here the effect of the so-called electromagnetic penguins [20] which bring yet further four-quark operators. The effect of these extra operators, as well as the question of CP-violation effects in K-decays in general, will be discussed in forthcoming publications.

The mixing of the Q_i-operators under renormalization defines the anomalous dimension matrix $\gamma_{ij}(\alpha_s)$ as follows

$$\mu^2 \frac{d}{d\mu^2} < Q_i > = -\frac{1}{2}\gamma_{ij}(\alpha_s) < Q_j > \ . \tag{2.8}$$

At the one-loop level, and with N_c the number of colours and n_f the number of massless quark flavours, γ_{ij} is given by the 6×6 matrix [15c] :

$$\gamma_{ij} = \frac{\alpha_s}{\pi}\gamma_{ij}^{(1)} + 0(\frac{\alpha_s}{\pi})^2 \tag{2.9a}$$

with

$$\gamma_{ij}^{(1)} = \frac{N_c}{2} \begin{pmatrix} -\frac{3}{N_c^2} & \frac{3}{N_c} & 0 & 0 & 0 & 0 \\ \frac{3}{N_c} & -\frac{3}{N_c^2} & -\frac{1}{3N_c^2} & \frac{1}{3N_c} & -\frac{1}{3N_c^2} & \frac{1}{3N_c} \\ 0 & 0 & -\frac{11}{3N_c^2} & \frac{11}{3N_c} & -\frac{2}{3N_c^2} & \frac{2}{3N_c} \\ 0 & 0 & \frac{3}{N_c} - \frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} - \frac{3}{N_c^2} & -\frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} \\ 0 & 0 & 0 & 0 & \frac{3}{N_c^2} & -\frac{3}{N_c} \\ 0 & 0 & -\frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} & -\frac{n_f}{3N_c^2} & -3 + \frac{n_f}{3N_c} + \frac{3}{N_c^2} \end{pmatrix} . \tag{2.9b}$$

The pattern of this matrix in the large $N_c \to \infty$ limit is very revealing [21]. In the strict large N_c limit all entries go to zero except γ_{66} ; i.e., in this limit there is no mixing of Q_2 with the other operators, and only Q_6 gets renormalized in a multiplicative way. In order to take into account the mixing of Q_2 with the penguin operators while running from the scales m_t^2 and M_W^2 down to $\mu^2 < m_c^2$ it is clearly necessary to go beyond the strict large N_c limit approximation.

Keeping the full form of the lowest order anomalous dimension matrix, the structure of the effective low-energy Hamiltonian which results is the following [15c] :

$$H_{eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \Big\{ \frac{1}{2}C_+(\mu^2)(Q_2+Q_1) + \frac{1}{2}C_-(\mu^2)(Q_2-Q_1)$$

$$+ C_3(\mu^2)Q_3 + C_4(\mu^2)(Q_3+Q_-) + C_5(\mu^2)Q_5 + C_6(\mu^2)Q_6 \Big\} + h.c. \tag{2.10}$$

where the Wilson coefficients $C_\pm$ and C_i, $i = 3,4,5,6$ are known functions of the heavy masses and the renormalization scale μ^2, calculated at the leading logarithmic approximation. (In writing eq. (2.10) we have used the relation in eq. (2.7).)

In the strict large N_c-limit, $\alpha_S \sim \frac{1}{N_c}$ and therefore $C_i \to 0$, $i = 3,4,5,6$, while $C_+ = C_- \to 1$. In this limit we recover the Hamiltonian of eq. (1.1) with the operator Q_2 factorized as a product of two currents. As already discussed in the introduction the phenomenological implications of this limit are catastrophic.

To a good approximation, the result from the leading logarithm approximation to the calculation of the coefficients $C_\pm$ is the following

$$C_\pm(\mu^2) = \left(\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)}\right)^{a_\pm}, \tag{2.11}$$

where $\alpha_s(\mu^2)$ denotes the QCD running coupling constant at the one-loop level

$$\frac{\alpha_s(\mu^2)}{\pi} = \frac{1}{-\beta^{(1)}\frac{1}{2}ln\frac{\mu^2}{\Lambda_{QCD}^2}}, \qquad \beta^{(1)} = -\frac{11N_c - 2n_f}{6} \tag{2.12a, b}$$

and

$$a_\pm = \frac{\pm 3/2(1 \mp 1/N_c)}{-\beta^{(1)}}. \tag{2.12c}$$

As first found in refs. [12] and [13], there is a short-distance enhancement of the C_--coefficient of the Q_--operator, i.e., the operator which can only induce $\Delta I = 1/2$ transitions, and a slight suppression of the C_+-coefficient of the Q_+-operator responsible for $\Delta I = 3/2$ transitions. Historically, this has been the first step towards a dynamical understanding of the $\Delta I = 1/2$ rule.

The short-distance enhancement just discussed cannot be the full story however, and this for a very simple reason. The $C_\pm$-coefficients depend on an arbitrary renormalization scale μ^2, while the $K \to \pi\pi$ amplitudes, say, or any other physical observable, do not. In other words, the matrix elements of the $Q_\pm$-operators must produce a μ^2-dependence as well, which cancels the one from the $C_\pm$-coefficients. This observation implies that the evaluation of hadronic matrix elements of the four-quark operators has to be made within a framework which can keep track of gluonic interactions as well.

Let us come back to the effective Hamiltonian in eq. (2.10). Explicit expressions for the $C_\pm(\mu^2)$ and $C_i(\mu^2)$ coefficients calculated at the leading logarithmic approximation can be found in the literature. (See e.g., ref. [21]). Usually, the mass effects in the evolution from m_t and M_W down to $\mu < m_c$ have been taken into account in the form of successive step functions. In the latest evaluations, the top quark is simultaneously integrated out with the W-boson with neglect of the (presumably) short evolution between the t and W mass scales. The other five quarks are treated as massless in an effective theory with five flavours. At the threshold where $\mu^2 = m_b^2$ the b-quark field is removed and the remaining four quarks are treated as massless in an effective theory with four flavours. At the new threshold where $\mu^2 = m_c^2$ the c-quark field is removed with the u, d, s quarks massless. Here the short-distance evolution stops. As recently emphasized in ref. [22] it would be nice to improve this procedure to include more directly the effect of heavy quark masses in conjunction with a next-to-leading logarithmic approximation of the full anomalous dimension matrix.

For our purposes in this paper we need explicit expressions for the real parts of the $C_\pm$, C_4 and C_6 coefficients. We can dispense from C_3 and C_5 because, as we shall see in section 8, the effective actions of the C_3Q_3 and C_5Q_5 terms are higher order in α_s. For C_+ and C_- we shall simply take the expression in (2.11) with $\alpha_s(\mu^2)$ given in (2.12) and $n_f = 3$. For $ReC_4(\mu^2)$ and $ReC_6(\mu^2)$ we shall use the approximate expressions which Bardeen, Buras and Gerard [21] have obtained within the framework of the $1/N_c$-expansion using a running anomalous dimension. As they have shown, the effect of a smooth mass interpolation, instead of the step approximation described above, results in sizeable effects on the Wilson coefficients of the penguin operators. Their result is

$$ReC_i(\mu^2) = -\frac{1}{4}\int_{\mu^2}^{M_W^2} \frac{dQ^2}{Q^2}[C_+(Q^2) + C_-(Q^2)]\gamma_{2i}(Q^2)_{uc}\left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right)^{a_i}, i = 4,6 \qquad (2.13)$$

where

$$a_4 = 0 \quad ; \quad a_6 = -\frac{9}{11}\,. \qquad (2.14a,b)$$

In the step-like approximation, the anomalous dimension matrix element $\gamma_{2i}(Q^2)_{uc}$ would simply be

$$\gamma_{24}(Q^2)_{uc} = \gamma_{26}(Q^2)_{uc} = \begin{cases} 0 & \text{for} \quad Q^2 > m_c^2 \\ \frac{1}{6}\frac{\alpha_s(Q^2)}{\pi} & \text{for} \quad Q^2 \le m_c^2 \end{cases} \qquad (2.15)$$

The improvement proposed in ref. [21], which we shall adopt here, is to replace this by the integral

$$\gamma_{24}(Q^2)_{uc} = \gamma_{26}(Q^2)_{uc} = \frac{\alpha_s(Q^2)}{6\pi}\int_0^1 dx 6x^2(1-x)^2$$
$$\left\{\frac{1}{\frac{m_u^2}{Q^2} + x(1-x)} - \frac{1}{\frac{m_c^2}{Q^2} + x(1-x)}\right\}\,. \qquad (2.16)$$

There is an additional contribution to $ReC_i(\mu^2)$ from $\gamma_{24}(Q^2)_{ct}$ and $\gamma_{26}(Q^2)_{ct}$. However, it appears modulated by the flavour mixing factor : $V_{td}V_{ts}^*/V_{ud}V_{us}^*$ and therefore it only

adds a negligible amount. This completes the discussion of the short-distance reduction to a low-energy four-quark Hamiltonian for $\Delta S = 1$ transitions in the Standard Model.

The Standard Model also predicts strangeness changing transitions with $\Delta S = 2$ via two W-exchanges, the so-called box diagrams of Fig. 4. At long distances this results in an effective Hamiltonian [23] which is also proportional to a local four-quark operator

$$Q_{\Delta S=2} \equiv (\bar{s}_L\gamma^\mu d_L)(\bar{s}_L\gamma_\mu d_L)\,, \qquad (2.17)$$

modulated by products of CKM-matrix elements,

$$\lambda_q = V_{qd}^* V_{qs} \quad q = u, c, t \qquad (2.18)$$

times functions $F_{1,2,3}$ of the heavy masses $m_t^2, M_W^2, m_b^2, m_c^2$ of the fields which have been integrated out :

$$H_{eff}^{\Delta S=2}(x) = \frac{G_F^2 M_W^2}{4\pi^2}\left[\lambda_c^2 F_1\left(\frac{m_i^2}{M_W^2}\right) + \lambda_t^2 F_2\left(\frac{m_i^2}{M_W^2}\right) + 2\lambda_c\lambda_t F_3\left(\frac{m_i^2}{M_W^2}\right)\right]\times$$
$$\alpha_s(\mu^2)^{-a} \times (\bar{s}_L(x)\gamma^\mu d_L(x))(\bar{s}_L(x)\gamma_\mu d_L(x)) \qquad (2.19)$$

The operator $Q_{\Delta S=2}$ is multiplicatively renormalizable and has an anomalous dimension $\gamma(\alpha_s)$ defined by the equation

$$\mu^2\frac{d}{d\mu^2}\langle Q_{\Delta S=2}\rangle = -\frac{1}{2}\gamma(\alpha_s)\langle Q_{\Delta S=2}\rangle\,. \qquad (2.20)$$

At the one-loop level

$$\gamma(\alpha_s) = \frac{\alpha_s}{\pi}\gamma^{(1)} + 0\left(\frac{\alpha_s}{\pi}\right)^2 \quad , \quad \gamma^{(1)} = \frac{3}{2}\left(1 - \frac{1}{N_c}\right); \qquad (2.21)$$

hence the μ^2-dependence via $\alpha_s(\mu^2)$ in eq. (2.19) with the exponent a given by the ratio

$$a = \frac{\gamma^{(1)}}{-\beta^{(1)}} = 2/9\,. \qquad (2.22)$$

The explicit forms of the functions $F_{1,2,3}$ can be found in the literature [23], [22].

In what follows we shall be particularly concerned with the matrix element

$$\langle\bar{K}^o|Q_{\Delta S=2}(0)|K^o\rangle \equiv \frac{4}{3}f_K^2 M_K^2 B \qquad (2.23)$$

i.e., the so-called B-parameter which governs $K^o - \bar{K}^o$ mixing at short-distances. Theoretical knowledge of this parameter is urgently required for a useful confrontation of the observed CP-violation in the $K^o - \bar{K}^o$ system with the Standard Model predictions. The definition in eq. (2.23) is such that in the so-called vacuum saturation approximation [24] (VSA)

$$B\big|_{VSA} = 1. \tag{2.24}$$

As we shall discuss in section 4, in the leading large N_c-limit the operator $Q_{\Delta S=2}$ factorizes into a product of current-operators. The bosonic realization of the operator $(\bar{s}_L \gamma^\mu d_L)$ to leading order in the large N_c-limit is

$$\bar{s}_L \gamma_\mu d_L \rightarrow -\frac{1}{\sqrt{2}} f_K \partial_\mu K^o + \ldots \tag{2.25}$$

from which it follows that [35]

$$B\big|_{N_c \to \infty} = 3/4 \tag{2.26}$$

The B-factor, as defined by eq. (2.23), is μ^2-scale dependent. This μ^2-dependence should exactly cancel the $\alpha_s(\mu^2)^{-a}$ factor in the effective $\Delta S = 2$ Hamiltonian in (2.19). Both the VSA result (2.24) and the strict large N_c limit result (2.26) fail to reproduce this cancellation.

3.— Two-Point Functions of Four-Quark Operators at Short Distances

The physics described in the previous section has been obtained within the framework of perturbative QCD. Before we engage ourselves in the question of hadronic matrix elements of four-quark operators, an issue which will necessarily take us beyond the purely perturbative sector of QCD, it seems worthwhile to explore more properties of four-quark operators, but still within the framework of perturbative QCD, which may shed some light on the question of non-leptonic weak transitions.

In previous work [7-11] we have suggested looking at the properties of inclusive non-leptonic weak transitions. Technically, this means the study of two-point functions of the type

$$\psi_{ij}(q^2) \equiv i \int d^4x e^{iq.x} < 0|T(Q_i(x)Q_j^\dagger(0)|0 > \tag{3.1}$$

with Q_i one of the $\Delta S = 1$ operators listed in eqs. (2.1) to (2.6), or the $Q_{\Delta S=2}$-operator of eq. (2.17). The spectral functions associated to these two-point functions ($t \equiv q^2$)

$$\frac{1}{\pi} Im\psi_{ij}(t) = \sum_{\Gamma=Hadrons} \int d\Gamma < 0|Q_i(0)|\Gamma >< \Gamma|Q_j^\dagger(0)|0 > (2\pi)^3 \delta^{(4)}(q - \sum p_\Gamma), \tag{3.2}$$

when modulated with the appropriate Wilson coefficients, are quantities with definite physical information.

The asymptotic behaviour for t-large of the spectral functions (3.2) can be obtained from perturbative QCD. To lowest order, the leading behaviour is given by the absorptive part of three-loop quark diagrams of the type shown in Fig. 5. In the $\Delta S = 1$ sector, working with the set of independent operators Q_1, Q_2, Q_3, Q_5 and Q_6, one finds [9]

$$\frac{1}{\pi} Im\psi_{ij}(t) = \theta(t) \frac{t^4}{(16\pi^2)^3} A_{ij}, \tag{3.3}$$

where the matrix $A_{ij}(i, j = 1, 2, 3, 5, 6)$ is

$$A = \frac{4}{5}\frac{N_c^2}{9} \begin{pmatrix} 1 & \frac{1}{N_c} & 1 & 0 & 0 \\ \frac{1}{N_c} & 1 & \frac{1}{N_c} & 0 & 0 \\ 1 & \frac{1}{N_c} & n_f + \frac{2}{N_c} & 0 & 0 \\ 0 & 0 & 0 & n_f & \frac{n_f}{N_c} \\ 0 & 0 & 0 & \frac{n_f}{N_c} & n_f \end{pmatrix} \tag{3.4}$$

The spectral function associated to the $\Delta S = 2$ operator $Q_{\Delta S=2}$ is [7],

$$16 \frac{1}{\pi} Im\psi_{\Delta S=2}(t) = \theta(t) \frac{t^4}{(16\pi^2)^3} \frac{4}{5} \frac{N_c^2}{9} 2(1 + \frac{1}{N_c}). \tag{3.5}$$

[Note the factor 16 on the l.h.s. to compensate for the different normalization of the operator $Q_{\Delta S=2}$ in eq. (2.17) and the Q_i-operators in eqs. (2.1) to (2.6).] In eqs. (3.4) and (3.5) we have explicitly written the different colour (N_c) and flavour (n_f) factors to allow for an easier understanding of the results. The N_c^2 terms originate from the diagrams of the type shown in Fig. 5.a, while the diagrams of the type 5.b give rise to contributions with a single power of N_c. The zero entries in the matrix A are due to the different helicity structure of the Q_5 and Q_6 operators. At the lowest-order level (no gluons), all four quark operators have the same behaviour. The only difference between penguin and non-penguin operators comes from the sum over the three light flavours, appearing in the Q_i-definition, $i = 3,5,6$, which gives rise to the n_f factors in eq. (3.4). The particular flavour composition of each operator just selects the allowed topology (diagrams like 5.a, 5.b or both). For instance, the factor $2/N_c$ in A_{33} appears because the diagram 5.b is present for $q = d$ and $q = s$, but not for $q = u$. Similarly, there is a combinatorial factor of 2 in eq. (3.5), because the operator $Q_{\Delta S=2}$ contains two currents with the same flavour structure.

The spectral functions (3.2) describe in an inclusive way how the weak operators couple the vacuum to physical states of a given invariant mass. If the $|\Delta I| = 1/2$ enhancement is an intrinsic property of the $\Delta S = 1$ Hamiltonian, it should also show up at the inclusive level; i.e., the strength of the $|\Delta I| = 1/2$ spectral functions should be much bigger than the $|\Delta I| = 3/2$ one. This is not, however, what results from eq. (3.4), suggesting that very likely the gluonic corrections to the $\Delta I = 1/2$ spectral functions are important.

At the leading logarithmic approximation, the $0(\alpha_s)$ corrections in the $\Delta S = 1$ transitions are governed by the one-loop anomalous dimension matrix $\gamma_{ij}^{(i)}$ in eq. (2.9b). For instance, in the simpler case of $Q_{\Delta S=2}$ where no operator mixing is present, one has

$$C_{\Delta S=2}(\mu^2)\frac{1}{\pi}Im\psi_{\Delta S=2}(t) \sim \alpha_s(\mu^2)^{2\gamma^{(1)}/\beta^{(1)}}$$

$$\left\{1 + \frac{\alpha_s(\mu^2)}{\pi}\left(\gamma^{(1)} ln(t/\mu^2) + \text{cte}\right) + O(\frac{\alpha_s}{\pi})^2\right\}. \tag{3.6}$$

We observe that all the leading $[\alpha_s(\mu^2) ln(t/\mu^2)]^n$ contributions can be summed by doing the rescaling $\mu^2 = t$. Doing that in the slightly more complicated $\Delta S = 1$ sector results in the short-distance enhancement discussed in section 2, but now in an inclusive hadronic observable. It has been shown in refs. [9] and [10] that this enhancement is however not enough to explain the observed $K \to \pi\pi$ decay rates.

3a. Order α_s-corrections in the "no penguin" approximation.

The calculation of the non-logarithmic α_s-corrections to the $\Delta S = 1$ $\psi_{ij}(q^2)$ correlators is quite involved, due to the fact that we have to deal with several operators $(Q_1, Q_2, Q_3, Q_5, Q_6)$ which mix under renormalization. Fig. 6 shows the kind of diagrams contributing to this order. The exact results of this four-loop calculation are given in Appendix B. A preliminary version of this work can be found in ref . [11]. Here, we prefer to discuss a simplified version based on approximations which will allow us to write simple analytic expressions which contain however the bulk of the exact calculation. In what follows we use the standard dimensional regularization with anticommuting γ_5 and the $\overline{MS}$-renormalization scheme.

The mixing of Q_2 with $Q_j (j = 3,4,5,6)$ is generated by the penguin diagram of Fig. 3. If this diagram were absent, $\gamma_{2j}^{(1)} = 0 (j = 3,4,5,6)$, and we would only need to consider the operators Q_1 and Q_2. In this case, the associated 2×2 anomalous dimension matrix has the eigenvalues $\gamma_+^{(1)} = 1$ and $\gamma_-^{(1)} = -2$, which correspond to the operators $Q_+ = Q_2 + Q_1$ and $Q_- = Q_2 - Q_1$. Therefore, in the approximation where the penguin diagrams are neglected, $Q_\pm$ are multiplicatively renormalizable operators. Within this approximation, the diagram 6.c should not be considered when evaluating the corresponding two-point functions. At this level of approximation, one gets

$$\frac{1}{\pi}Im\psi_{++}(t)\big|_{\text{no penguin}} = \frac{32}{15}\frac{t^4}{(16\pi^2)^3}\left[1 + \frac{\alpha_s(\mu^2)}{\pi}[ln(t/\mu^2) - \frac{49}{20}] + O(\frac{\alpha_s}{\pi})^2\right] \tag{3.7a}$$

and

$$\frac{1}{\pi}Im\psi_{--}(t)\big|_{\text{no penguin}} = \frac{16}{15}\frac{t^4}{(16\pi^2)^3}\left[1 + \frac{\alpha_s(\mu^2)}{\pi}[-2ln(t/\mu^2) + \frac{47}{5}] + O(\frac{\alpha_s}{\pi})^2\right]. \tag{3.7b}$$

While the non-logarithmic α_s-correction to the ψ_{++} spectral function turns out to be moderate and negative, the one appearing in the $\Delta I = 1/2$ correlator ψ_{--} is very big and positive. Taking for instance $\mu^2 = t = 1$ GeV^2 and $\Lambda_{QCD} = 100$ MeV (200 MeV), this non-logarithmic α_s-correction represents a 91% (130%) increase of the spectral function $\frac{1}{\pi}Im\psi_{--}(t)$, with respect to the leading logarithmic result. The corresponding correction to $\frac{1}{\pi}Im\psi_{++}(t)$ amounts only to a 24% (34%) suppression. In fact, at moderate values of t of a few GeV^2, the perturbative corrections to the ψ_{--}- spectral function turn out to be much larger than the leading non-perturbative power corrections calculated in ref.[9].

Some insight into the origin of this big α_s-correction can be gained by doing the same computation in the large N_c limit, where only diagrams 6.a and 6.b (together with 5.a at lowest order) contribute. The anomalous dimensions vanish in this case and therefore no logarithmic corrections appear in the spectral functions. The short-distance enhancement of the Wilson coefficients is obviously also lost in the large N_c-limit. In this limit one gets

$$\frac{1}{\pi}Im\psi_{++}(t)\big|_{1/N_c} = \frac{1}{\pi}Im\psi_{--}(t)\big|_{1/N_c} = \frac{8}{5}\frac{t^4}{(16\pi^2)^3}\left[1 + \frac{3}{4}\frac{N_c\alpha_s(t)}{\pi} + 0(\frac{N_c\alpha_s}{\pi})^2\right] \tag{3.8}$$

which is clearly a very bad approximation to the results given in eq. (3.7). At leading order in $1/N_c$, not only the α_s-correction is moderate, but in addition it is the same for the two spectral functions. This is consistent with the estimate of the $K \to \pi\pi$ amplitudes, within the factorization hypothesis, done in eq. (1.5). The big enhancement of $\frac{1}{\pi}Im\psi_{--}(t)$ we have obtained in eq. (3.7b) and the moderate suppression of $\frac{1}{\pi}Im\psi_{++}(t)$ in eq. (3.7a) are produced by the same non-factorizable gluonic corrections, which produce the short-distance enhancement of the C_- (suppression of the C_+) Wilson coefficients.

The α_s-correction to the $\Delta S = 2$ spectral function (3.6) is the same as the one appearing in eq. (3.7a) for $\frac{1}{\pi}Im\psi_{++}(t)$; the only difference being that for $\psi_{\Delta S=2}$ this is an exact result, since no penguin contribution is allowed in this case. In fact, the moderate α_s-correction given in eq. (3.7a) is shared by all the correlators associated to operators transforming as $(27_L, 1_R)$ under the chiral flavour group $SU(3)_L \times SU(3)_R$ (QCD is flavour blind). Note that only the octet part of ψ_{++} can get a contribution from the neglected penguin diagram in fig. 6.c.

Although eqs. (3.7) are the result of a complete $0(\alpha_s)$ calculation of the spectral functions (in the no-penguin approximation described above), it is still necessary to include the corresponding next-to-leading logarithm contribution to the Wilson coefficients, in order to have a μ^2-independent result at this order. Using the two-loop expressions for the anomalous dimensions $\gamma_\pm$ and the β-function,

$$\gamma_\pm(\alpha_s) = \frac{\alpha_s}{\pi}\gamma_\pm^{(1)} + (\frac{\alpha_s}{\pi})^2\gamma_\pm^{(2)} + \dots$$

$$\beta(\alpha_s) = \frac{\alpha_s}{\pi}\beta^{(1)} + (\frac{\alpha_s}{\pi})^2\beta^{(2)} + \dots, \tag{3.9}$$

the solution of the renormalization group equations obeyed by $C_\pm(\mu^2)$ is easily found to be

$$C_\pm(\mu^2) = C_\pm(M_W^2)\left(\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)}\right)^{-\gamma_\pm^{(1)}/\beta^{(1)}}\left\{1 + \frac{\alpha_s(\mu^2) - \alpha_s(M_W^2)}{\pi}K_\pm + 0(\frac{\alpha_s}{\pi})^2\right\} \tag{3.10}$$

where

$$K_\pm = \frac{\beta^{(2)}}{\beta^{(1)}}\left(\frac{\gamma_\pm^{(2)}}{\beta^{(2)}} - \frac{\gamma_\pm^{(1)}}{\beta^{(1)}}\right) \tag{3.11}$$

and

$$C_\pm(M_W^2) = 1 + \frac{\alpha_s(M_W^2)}{\pi}B_\pm + 0(\frac{\alpha_s}{\pi})^2. \tag{3.12}$$

It has become conventional to lump together all the $\alpha_s(M_W^2)$ corrections and rewrite eq. (3.10) in the form

$$C_\pm(\mu^2) = \left(\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)}\right)^{-\gamma_\pm^{(1)}/\beta^{(1)}}\left\{1 + \frac{\alpha_s(\mu^2) - \alpha_s(M_W^2)}{\pi}R_\pm + 0(\frac{\alpha_s}{\pi})^2\right\}$$
$$\times\left\{1 + \frac{\alpha_s(\mu^2)}{\pi}B_\pm + 0(\frac{\alpha_s}{\pi})^2\right\}, \tag{3.13}$$

where now the coefficient

$$R_\pm \equiv K_\pm - B_\pm \tag{3.14}$$

is regularization and renormalization scheme independent, up to redefinitions of the effective coupling constant. $R_\pm$ was initially computed by Altarelli, Curci, Martinelli and Petrarca in ref. [19a], using dimensional reduction. The result has been recently confirmed by Buras and Weisz [19c], using three regularization schemes with different treatments of γ_5. With the $\overline{MS}$ prescription for α_s, they find

$$R_+ = -\frac{287}{648} \quad , \quad R_- = +\frac{413}{324}. \tag{3.15}$$

This additional α_s-correction, although moderate, further reinforces the enhancement (suppression) of the $\psi_{--}(\psi_{++})$ spectral functions. However, one still needs to include the contribution coming from the coefficient $B_\pm$, which depends on the renormalization scheme.

The two-point function results of eq. (3.7) have been obtained (in the $\overline{MS}$ scheme) using standard dimensional regularization with an anticommuting γ_5. The corresponding calculation of $B_\pm$, within the same scheme, has recently been done by Buras and Weisz in ref. [19c] with the result

$$B_+ = \frac{11}{12} \quad , \quad B_- = -\frac{11}{6}. \tag{3.16}$$

This correction is bigger than and of opposite sign to the one absorbed in the $R_\pm$ coefficients, making the total next-to-leading logarithm correction to the Wilson coefficients quite small. The final, μ^2-independent at next-to-leading order, result for the spectral functions is then

$$C_+(\mu^2)^2\frac{1}{\pi}Im\psi_{++}(t)\big|_{\text{no penguin}} = \frac{32}{15}\frac{t^4}{(16\pi^2)^3}\left(\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)}\right)^{4/9}$$
$$\times\left\{1 + \frac{\alpha_s(\mu^2)}{\pi}[ln(t/\mu^2) - \frac{1217}{810}] + \frac{\alpha_s(M_W^2)}{\pi}\frac{287}{324} + 0(\frac{\alpha_s}{\pi})^2\right\} \tag{3.17a}$$

$$C_-(\mu^2)^2\frac{1}{\pi}Im\psi_{--}(t)\big|_{\text{no penguin}} = \frac{16}{15}\frac{t^4}{(16\pi^2)^3}\left(\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)}\right)^{-8/9}$$
$$\times\left\{1 + \frac{\alpha_s(\mu^2)}{\pi}[-2ln(t/\mu^2) + \frac{6709}{810}] - \frac{\alpha_s(M_W^2)}{\pi}\frac{413}{162} + 0(\frac{\alpha_s}{\pi})^2\right\} \tag{3.17b}$$

3b. <u>The penguin two-point function in the large N_c limit.</u>

It would be interesting to see if other $\Delta I = 1/2$ correlators also get big α_s-corrections at next-to-leading order. Looking at the first-order anomalous dimension matrix (2.9), one immediately realizes that there is one operator which can be easily isolated : in the large N_c limit all entries are zero but for $\gamma_{66}^{(1)}$, i.e., in this limit there is no mixing among operators, and only Q_6 gets renormalized. Working at leading order in $1/N_c$, we can then try to compute the α_s-correction to the penguin two-point function $\psi_{66}(q^2)$, without worrying about the other operators.

We have seen before that the large N_c-limit is not a sensible approximation to calculate the $Q_\pm$ correlators, because their anomalous dimensions just vanish in this limit. This is however not the case for the penguin operator Q_6, since $\gamma_{66}^{(1)}$ is quite well approximated by the leading term in $1/N_c$. The large N_c-limit should hopefully provide a good estimate of $\frac{1}{\pi}Im\psi_{66}(t)$. In this approximation one gets

$$\frac{1}{\pi}Im\psi_{66}(t)|_{1/N_c}=\frac{12}{5}\frac{t^4}{(16\pi^2)^3}\left\{1+\frac{\alpha_s(\mu^2)}{\pi}[-\frac{9}{2}ln(t/\mu^2)+\frac{423}{20}]+0(\frac{\alpha_s}{\pi})^2\right\} \tag{3.18}$$

The coefficient of the logarithmic term is $\gamma_{66}^{(1)}|_{1/N_c}=-9/2$, as it should be, which provides a check of the calculation. The amazing thing is the very big positive factor, 423/20, one gets for the non-logarithmic correction, which is a factor 9/4 bigger than the one found for ψ_{--} in eq. (3.7b). Curiously enough 9/4 is just the ratio of the corresponding anomalous dimensions $\gamma_{66}^{(1)}|_{1/N_c}$ and $\gamma_{-}^{(1)}$; which means that both correlators have a common α_s-correction factor, once the corresponding anomalous dimension is factored out i.e.,

$$1+\frac{\alpha_s(\mu^2)}{\pi}\gamma^{(1)}[ln(t/\mu^2)-\frac{47}{10}]. \tag{3.19}$$

To leading order in $1/N_c$, the operator Q_6 factorizes as a product of $(\bar{s}_L q_R)$ and $(\bar{q}_R d_L)$ currents. As discussed in ref. [25], this implies that the corresponding Wilson coefficient, to the same leading order in $1/N_c$, scales as the square of the running quark mass i.e., $C_6(\mu^2)\sim m(\mu^2)^2$. From the known result of the two-loop anomalous dimension of the quark masses [26], we can then get $C_6(\mu^2)|_{1/N_c}$ at the two-loop level. With neglect of the small next-to-leading $\alpha_s(M_W^2)$ contributions we get

$$C_6(\mu^2)|_{1/N_c}\sim\left(\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)}\right)^{-9/11}[1+\frac{3027}{1936}\frac{\alpha_s(\mu^2)}{\pi}+0(\frac{\alpha_s}{\pi})^2] \quad . \tag{3.20}$$

The corresponding renormalization invariant spectral function to next-to-leading logarithmic approximation is then

$$\frac{1}{\pi}Im\widehat{\psi}_{66}(t)|_{1/N_c}=\frac{12}{5}\frac{t^4}{(16\pi^2)^3}\left(\frac{\alpha_s(M_W^2)}{\alpha_s(t)}\right)^{-18/11}\left\{1+\frac{117501}{4840}\frac{\alpha_s(t)}{\pi}+0(\frac{\alpha_s}{\pi})^2\right\} \quad . \tag{3.21}$$

Thanks to the factorization property of the quark currents in the large N_c-limit, the $O(\alpha_s^2)$ correction to $\frac{1}{\pi}Im\psi_{66}(t)$ can also be easily computed. To leading order in $1/N_c$, the ψ_{66}- correlator can be expressed as a convolution of simpler two-point functions already known to $0(\alpha_s^2)$:

$$\psi_{66}|_{1/N_c}=-16n_f \quad i\int\frac{d^4k}{(2\pi)^4}\phi(k^2)\phi((q-k)^2) \tag{3.22}$$

where

$$\phi(k^2)\equiv i\int d^4x e^{ikx} < 0|T((\bar{s}q)(x)(\bar{q}s)(0))|0 > . \tag{3.23}$$

The renormalization group invariant quantity $m^2\phi(k^2)$ was studied in ref. [27], at the two-loop level, where in fact a quite large α_s-correction was found. The calculation of the $O(\alpha_s^2)$ contribution was later reported in ref. [28] and there is a recent calculation which corrects for errors found in ref. [28]. In the large N_c-limit, the present result is

$$m(\mu^2)^2\frac{1}{\pi}Im\phi(s)|_{1/N_c}=-\frac{3}{8\pi^2}m(s)^2 s\left\{1+\frac{51}{8}\frac{\alpha_s(s)}{\pi}+41.53(\frac{\alpha_s(s)}{\pi})^2+0(\frac{\alpha_s}{\pi})^3\right\}. \tag{3.24}$$

With this information, together with the known values of the β-function and the anomalous dimension of the quark mass to three loops, it is not difficult to obtain the desired $0(\alpha_s^2)$ contribution

$$\frac{1}{\pi}Im\widehat{\psi}_{66}(t)|_{1/N_c}=\frac{12}{5}\frac{t^4}{(16\pi^2)^3}\left(\frac{\alpha_s(M_W^2)}{\alpha_s(t)}\right)^{-18/11} \times\left[1+\frac{117501}{4840}\frac{\alpha_s(t)}{\pi}+470.72\left(\frac{\alpha_s(t)}{\pi}\right)^2+0\left(\frac{\alpha_s}{\pi}\right)^3\right], \tag{3.25}$$

which, at least in this case, shows that the enhancement found in the $0(\alpha_s)$ correction also appears in the $0(\alpha_s^2)$ term. Here again, as in the case of the ψ_{--} spectral function, we find that the perturbative corrections in the ψ_{66} spectral function, at values of t of a few GeV^2, dominate by far the leading non-perturbative power corrections calculated in ref. [9].

3c. Conclusions from the short-distance behaviour of two-point functions.

From the previous analysis and the calculations reported in Appendix B we conclude that the short-distance behaviour of the $\Delta S=1$ correlators clearly shows a dynamical enhancement in the octet channel as a consequence of the interplay of gluonic corrections. The well-known enhancement of the C_- Wilson coefficient discussed in section 2, which already appears at the leading logarithmic approximation, is now modulated by a further enhancement from the $\Delta I=1/2$ spectral function. The latter enhancement appears first in the next-to-leading logarithmic approximation. The implications of this short-distance behaviour for long distance physics can be investigated within the framework of the so-called QCD-hadronic duality sum rules (refs. [7] to [11]). This however would take us far away from the mean purpose of this paper. Here we shall limit ourselves to a few remarks which we postpone until section 5, because first we need to recall some general properties of chiral symmetry in the non-leptonic weak sector.

4.— Chiral Symmetry Properties of Non-Leptonic Weak Amplitudes

In section 2 we have described the short-distance reduction from the Lagrangian of the Standard Model to a low energy four-quark Hamiltonian for $\Delta S = 1$ and $\Delta S = 2$ transitions. The evaluation of the hadronic matrix elements of these four-quark operators is now reduced to a problem of strong interactions i.e., to a problem of QCD in the sector of light quarks.

The QCD Lagrangian, in the limit where the masses of the u, d and s quarks are set to zero, has a global $SU(3)_L \times SU(3)_R$ invariance under rotations (V_L, V_R) of the (left, right) quark flavour triplets $\begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix}$ and $\begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix}$. At the level of the hadronic physical states, this symmetry is spontaneously broken down to $SU(3)_{V=L+R}$, the famous $SU(3)$-symmetry of the Eightfold Way. The pattern of spontaneously broken symmetry implies specific constraints on the dynamics of the strong interactions between the low-lying octet of pseudoscalar states (π, K, η) which are the Goldstone bosons associated to the broken axial symmetry generators. The purpose of this section is to spell out the dynamical constraints from chiral symmetry which are relevant to non-leptonic weak amplitudes.

The best way to formulate the chiral symmetry constraints is by means of an effective chiral Lagrangian involving the pseudoscalar degrees of freedom only [29], [30]. The meson fields $\vec{\varphi}(x)$ are collected in a unitary 3×3 matrix $U(UU^\dagger = 1)$ with det $U = 1$, which transforms as

$$U \to V_R U V_L^\dagger \tag{4.1}$$

under chiral $SU(3)_L \times SU(3)_R$ transformations. In practice, the representation (f_π = 93.3MeV)

$$U = exp(-i/f_\pi \vec{\lambda}.\vec{\varphi}(x)), \tag{4.2a}$$

with $\vec{\lambda}$ the 3×3 Gell-Mann matrices $(tr\lambda_a\lambda_b = 2\delta_{ab})$, is a convenient one. The octet of pseudoscalar fields is such that

$$\phi(x) \equiv \frac{\vec{\lambda}}{\sqrt{2}}.\vec{\varphi}(x) = \begin{pmatrix} \pi^o/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^o/\sqrt{2} + \eta/\sqrt{6} & K^o \\ K^- & \bar{K}^o & -2\eta/\sqrt{6} \end{pmatrix}. \tag{4.2b}$$

The form of the effective chiral Lagrangian is determined by symmetry requirements. It consists of a string of terms

$$\mathcal{L}_{\text{ext}} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \ldots \tag{4.3}$$

ordered in increasing number of derivatives, external vector v_μ and axial-vector a_μ fields, and powers of the quark mass matrix

$$\mathcal{M} = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}. \tag{4.4}$$

Terms with powers of $\mathcal{M}$ take care of the explicit chiral symmetry breaking effects due to the finite u, d and s quark mass terms in the QCD Lagrangian. We refer to $\mathcal{L}^{(2n)}$ in eq. (4.3) as the Lagrangian of order p^{2n} : the field U counts as a quantity of $O(p^o)$, the derivative ∂_μ as $O(p)$. The external fields v_μ and a_μ are Hermitian 3×3 matrices in flavour space with $trv_\mu = tra_\mu = 0$. They are booked as $O(p)$ and so is the covariant derivative

$$D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu). \tag{4.5}$$

In the presence of external v_μ and a_μ fields, the global chiral symmetry is promoted to a local gauge symmetry : the matrix U still transforms like in eq. (4.1) under local $(V_L(x), V_R(x))$ chiral $SU(3)_L \times SU(3)_R$ transformations; and the external v_μ, a_μ fields as non-Abelian gauge fields i.e.,

$$v_\mu - a_\mu \equiv l_\mu \to V_L l_\mu V_L^\dagger + iV_L \partial_\mu V_L^\dagger \tag{4.6a}$$

$$v_\mu + a_\mu \equiv r_\mu \to V_R r_\mu V_R^\dagger + iV_R \partial_\mu V_R^\dagger; \tag{4.6b}$$

hence the form of the covariant derivative in eq. (4.5). The quark mass matrix counts as $O(p^2)$ in accordance with the well-known current algebra Ward identity [31]

$$(m_u + m_d) < \bar{u}u + \bar{d}d >= -2f_\pi^2 m_\pi^2 (1 + O(\mathcal{M})). \tag{4.7}$$

Often in the literature, the quark mass matrix $\mathcal{M}$ is traded by a matrix χ, also of $O(p^2)$, such that

$$\frac{1}{4} f_\pi^2 \chi = v\mathcal{M}, \tag{4.8a}$$

$$v = \frac{f_\pi^2 m_\pi^2}{2(m_u + m_d)} = \frac{f_\pi^2 M_{K^+}^2}{2(m_u + m_s)} = \frac{f_\pi^2 M_{K^o}^2}{2(m_d + m_s)} = \frac{3 f_\pi^2 M_\eta^2}{2(m_u + m_d + 4m_s)}. \tag{4.8b}$$

The first term $\mathcal{L}^{(2)}$ in eq. (4.3) is the non-linear sigma model Lagrangian in the presence of external fields i.e.,

$$\mathcal{L}^{(2)} = \frac{1}{4} f_\pi^2 \Big[trD_\mu U D_\mu U^\dagger + tr(\chi U^\dagger + U^\dagger \chi) \Big]. \tag{4.9}$$

To next-to-leading order there are two types of contributions : one-loop graphs generated by $\mathcal{L}^{(2)}$ and tree graphs involving one vertex from $\mathcal{L}^{(4)}$. In fact $\mathcal{L}^{(4)}$ is the sum of the Wess-Zumino [32] effective Lagrangian $\mathcal{L}_{WZ}$ which accounts for the chiral anomaly [33] and a chiral invariant piece $\mathcal{L}^{(4)}$ which in full generality [30] involves ten dimensionless real constants $L_i, i = 1, 2, \ldots, 10$:

$$\mathcal{L}^{(4)} = \mathcal{L}_{WZ} + \mathcal{L}_4 \tag{4.10}$$

and

$$\begin{aligned}\mathcal{L}^{(4)}(x) \;&= L_1(trD_\mu U^\dagger D^\mu U)^2 + L_2 trD_\mu U^\dagger D_\nu U trD^\mu U^\dagger D^\nu U\\ &+L_3 trD_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U + L_4 trD_\mu U^\dagger D^\mu U tr(\chi^\dagger U + U^\dagger \chi)\\ &+L_5 trD_\mu U^\dagger D^\mu U(\chi^\dagger U + U^\dagger\chi) + L_6[tr(\chi^\dagger U + U^\dagger\chi)]^2\\ &+L_7[tr(\chi U - U^\dagger \chi)]^2 + L_8 tr(\chi^\dagger U\chi^\dagger U + \chi U^\dagger \chi U^\dagger)\\ &-iL_9 tr(F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U) + L_{10} trU^\dagger F_R^{\mu\nu} U F_{L\mu\nu}\\ &+\text{two contact terms in external fields.}\end{aligned} \tag{4.11}$$

The tensors $F_R^{\mu\nu}$ and $F_L^{\mu\nu}$ stand for the field strength associated to the non-Abelian external fields r_μ and l_μ. The phenomenological determination of the L_i's has been discussed in detail in refs. [30]. The way that resonances (in particular the low mass vector and axial-vector hadronic states) contribute to these constants has been recently analyzed in refs. [34]. In principle, the constants L_i should be calculable from the underlying QCD theory. We shall review in section 6 recent progress in this direction [1].

In QCD with three flavours, the Noether currents associated to the chiral $SU(3)_L \times SU(3)_R$ transformations are

$$(\bar{q}_j\gamma^\mu \frac{1-\gamma_5}{2} q_i) \quad \text{and} \quad (\bar{q}_j\gamma^\mu \frac{1+\gamma_5}{2} q_i), \tag{4.12a, b}$$

where i, j mean flavour indices ($u = q_1, d = q_2$ and $s = q_3$). The brackets in (4.12a,b) indicate an implicit summation over quark colour indices, as in eq. (1.2b). These current operators are the currents of the algebra of currents proposed by Gell-Mann in the 60's (prior to the development of QCD!). To lowest order in the chiral expansion the effective realization of these currents in terms of pseudoscalar fields is given by the $\mathcal{L}^{(2)}$ Lagrangian in eq. (4.9) i.e.,

$$(\bar{q}_{jL}\gamma^\mu q_{iL}) \doteq \frac{\delta\mathcal{L}^{(2)}}{\delta(l_\mu)_{ji}} = -\frac{if_\pi^2}{2}(U^\dagger D^\mu U)_{ij} + 0(p^3) \tag{4.13a}$$

and

$$(\bar{q}_{jR}\gamma^\mu q_{iR}) \doteq \frac{\delta\mathcal{L}^{(2)}}{\delta(r_\mu)_{ji}} = -\frac{if_\pi^2}{2}(UD^\mu U^\dagger)_{ij} + 0(p^3) \tag{4.13b}$$

where l_μ and r_μ are the left and right external fields of eqs. (4.6). Expanding U in (4.13) in powers of the ϕ matrix (see eqs. (4.2)), and in the absence of external fields, we find

$$(\bar{q}_{jL}\gamma_\mu q_{iL}) = -\frac{1}{\sqrt{2}} f_\pi \partial_\mu \phi_{ij} \quad - \quad \frac{i}{2}(\phi \overleftrightarrow{\partial}_\mu \phi)_{ij} + \dots \tag{4.14}$$

from which eqs. (1.3) and (2.25) follow as particular cases.

Here we are interested in the effective chiral realization of the four-quark Q_i-operators and $Q_{\Delta S=2}$-operator of section 2 in terms of pseudoscalar fields. The operators Q_-, Q_3, Q_5 and Q_6 which appear in $H_{\text{eff}}^{\Delta S=1}$ in eq. (2.10), they all transform like an $8_L \times 1_R$ operator under chiral $SU(3)_L \times SU(3)_R$; and they all have the $SU(3)_V$ flavour quantum numbers : $S = 1, I = 1/2, I_3 = -1/2$. To lowest order in the number of derivatives, there is only one possible Lorentz invariant bosonic operator with these requirements i.e.,

$$\mathcal{L}_{\underline{8}}^{(1/2)}(x) = \sum_i (\mathcal{L}_\mu)_{2i}(\mathcal{L}^\mu)_{i3} \tag{4.15}$$

where $\mathcal{L}_\mu(x)$ is shorthand for the 3×3 matrix

$$\mathcal{L}_\mu \equiv -\frac{if_\pi^2}{2} U^\dagger D_\mu U. \tag{4.16}$$

On the other hand, the operator $Q_+ = Q_2 + Q_1$ which also appears in eq. (2.10), is an admixture of the $(8_L \times 1_R)$ operator $-\frac{1}{5}(Q_1 - Q_2 + 2Q_3)$ and the operator $\frac{6}{5}(Q_1 + \frac{2}{3}Q_2 - \frac{1}{3}Q_3)$ which transforms as $(27_L \times 1_R)$. At lowest order in the number of derivatives, the only bosonic operator with the same $(27_L \times 1_R)$ quantum numbers is

$$\mathcal{L}_{\underline{27}} = \frac{2}{3}(\mathcal{L}_\mu)_{21}(\mathcal{L}^\mu)_{13} + (\mathcal{L}_\mu)_{23}(\mathcal{L}^\mu)_{11}. \tag{4.17}$$

$\mathcal{L}_{\underline{27}}$ induces both $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$ transitions via its components

$$\mathcal{L}_{\underline{27}} = \frac{1}{9}\mathcal{L}_{\underline{27}}^{(1/2)} + \frac{5}{9}\mathcal{L}_{\underline{27}}^{(3/2)} \tag{4.18}$$

where

$$\mathcal{L}_{\underline{27}}^{(1/2)} = (\mathcal{L}_\mu)_{21}(\mathcal{L}^\mu)_{13} + (\mathcal{L}_\mu)_{23}[4(\mathcal{L}^\mu)_{11} + 5(\mathcal{L}^\mu)_{22}] \tag{4.19a}$$

$$\mathcal{L}_{\underline{27}}^{(3/2)} = (\mathcal{L}_\mu)_{21}(\mathcal{L}^\mu)_{13} + (\mathcal{L}_\mu)_{23}[(\mathcal{L}^\mu)_{11} - (\mathcal{L}^\mu)_{22}]\,. \tag{4.19b}$$

We can now write down the most general form of the effective bosonic Lagrangian which, to lowest order in the number of derivatives, has the same $SU(3)_L \times SU(3)_R$ transformation properties as the four-quark Hamiltonian in eq. (2.10) :

$$\mathcal{L}_{\text{eff}}^{\Delta S=1}(x) = \frac{-G_F}{\sqrt{2}} V_{ud}V_{us}^* 4(g_{\underline{8}}^{(1/2)}\mathcal{L}_{\underline{8}}^{(1/2)} + g_{\underline{27}}^{(1/2)}\mathcal{L}_{\underline{27}}^{(1/2)} + g_{\underline{27}}^{(3/2)}\mathcal{L}_{\underline{27}}^{(3/2)}) + h.c. \tag{4.20}$$

where $g_{\underline{8}}^{(1/2)}, g_{\underline{27}}^{(1/2)}$ and $g_{\underline{27}}^{(3/2)}$ are dimensionless constants, real constants when CP--violation effects are neglected, which cannot be determined from symmetry arguments alone. In the $SU(3)_V$-limit $g_{\underline{27}}^{(3/2)} = 5g_{\underline{27}}^{(1/2)}$ as seen in eq. (4.18). In fact it is precisely the determination of these constants from QCD, albeit within a well-defined set of approximations, which will be discussed in the next section.

We have already mentioned in section 2 that in the strict large N_c limit, $H_{\text{eff}}^{\Delta S=1}$ in eq. (2.10) becomes the Hamiltonian of eq. (1.1) with the operator Q_2 factorized as a product of two currents. In this limit, the low energy bosonic Lagrangian in eq. (4.20) is simply given by the expression

$$\mathcal{L}_{\text{eff}}^{\Delta S=1}(x)|_{N_c\to\infty} = \frac{-G_F}{\sqrt{2}} V_{ud}V_{us}^{*} 4\mathcal{L}_\mu(x)_{21}\mathcal{L}^\mu(x)_{13} + h.c. \tag{4.21}$$

which corresponds to the values for the coupling constants

$$g_{\underline{8}}^{(1/2)}|_{N_c\to\infty} = \frac{3}{5},\quad g_{\underline{27}}^{(1/2)}|_{N_c\to\infty} = \frac{1}{15},\quad g_{\underline{27}}^{(3/2)}|_{N_c\to\infty} = \frac{1}{3}. \tag{4.22abc}$$

In full generality, the $K \to \pi\pi$ isospin amplitudes A_0 and A_2 introduced in eqs. (1.4), when calculated with the low-energy bosonic Lagrangian in eq. (4.18), are given by the expressions

$$A_0 = \frac{-G_F}{\sqrt{2}} V_{ud}V_{us}^{*}(g_{\underline{8}}^{(1/2)} + g_{\underline{27}}^{(1/2)})\sqrt{2} f_\pi (M_K^2 - m_\pi^2) \tag{4.23a}$$

$$A_2 = \frac{-G_F}{\sqrt{2}} V_{ud}V_{us}^{*} g_{\underline{27}}^{(3/2)} 2 f_\pi (M_K^2 - m_\pi^2). \tag{4.23b}$$

With this parametrization, and from its comparison to the experimental information on $K_S \to \pi^+\pi^-, K_S \to \pi^o\pi^o$ and $K^+ \to \pi^+\pi^o$ decays, one obtains

$$|g^{(1/2)}|_{\text{Exp}} = |g_{\underline{8}}^{(1/2)} + g_{\underline{27}}^{(1/2)}|_{\text{Exp}} = 5.1 \tag{4.24a}$$

and

$$|g_{\underline{27}}^{(3/2)}|_{\text{Exp}} = 0.16. \tag{4.24b}$$

We finally turn our attention to the $Q_{\Delta S=2}$-operator in eq. (2.17). It transforms like a $27_L \times 1_R$ operator under chiral $SU(3)_L \times SU(3)_R$ and carries $SU(3)_V$ flavour quantum numbers $S = 2, I = 1, I_3 = -1$. To lowest order in the number of derivatives, the only Lorentz invariant bosonic operator with these requirements is

$$\mathcal{L}_{\underline{27}}^{\Delta S=2}(x) = \mathcal{L}_\mu(x)_{23}\mathcal{L}^\mu(x)_{23} \tag{4.25}$$

To this approximation, the most general realization of an effective bosonic Lagrangian with the same symmetry properties as the $H_{eff}^{\Delta S=2}$ four-quark Hamiltonian in eq. (2.19) is

$$\mathcal{L}_{eff}^{\Delta S=2}(x) =$$

$$-\frac{G_F^2 M_W^2}{4\pi^2}\left[\lambda_c^2 F_1\left(\frac{m_i^2}{M_W^2}\right) + \lambda_t^2 F_2\left(\frac{m_i^2}{M_W^2}\right) + 2\lambda_c\lambda_t F_3\left(\frac{m_i^2}{M_W^2}\right)\right]\frac{4}{3}\frac{f_K^2}{f_\pi^2}\widehat{B}\mathcal{L}_{\underline{27}}^{\Delta S=2}(x), \tag{4.26}$$

where $\widehat{B}$ is a dimensionless coupling constant, μ^2-scale independent, and which is related to the conventional B-factor introduced in eq. (2.23) as follows

$$\widehat{B} = \alpha_s(\mu^2)^{-a} B(\mu^2). \tag{4.27}$$

In the strict large N_c-limit the operator $Q_{\Delta S=2}$ factorizes into a product of two left currents : $(\bar{s}_L\gamma_\mu d_L)$ times $(\bar{s}_L\gamma^\mu d_L)$. Each of these currents, to lowest order in the number of derivatives, has a bosonic realization as indicated in eq. (4.13a). This approximation results in a value $\widehat{B} = f_\pi^2/f_K^2 3/4$. There are however chiral corrections to this determination from the $O(p^4)$ Lagrangian in eq. (4.11) which are still leading in the large N_c-limit i.e.,

$$(\bar{s}_L\gamma_\mu d_L) \to -i\frac{f_\pi^2}{2}(U^\dagger D_\mu U)_{23} + \frac{\delta\mathcal{L}^{(4)}(x)}{\delta(l_\mu)_{32}}. \tag{4.28}$$

Keeping only the contribution from the L_5-term, which is the only one which can eventually modify the $\widehat{B}$ coupling in the large N_c-limit, one finds

$$(\bar{s}_L\gamma_\mu d_L) \to \frac{-1}{\sqrt{2}} f_\pi (1 + \frac{8M_{K^o}^2}{f_\pi^2}L_5)\left(1 - \frac{4M_{K^o}^2}{f_\pi^2}L_5\right)\partial_\mu K^o + \ldots, \tag{4.29}$$

where the second factor $\left(1 - \frac{4M_{K^o}^2}{f_\pi^2}L_5\right)$ comes from wave function renormalization. The net result is a renormalization of f_π. To the extent that this "renormalized" value of f_π can be identified with the physical f_K-constant (*) determined from the $K \to \mu\nu$ decay ($f_K = 114MeV$), the corresponding corrected value of $\widehat{B}$ then becomes [35b]

$$\widehat{B}|_{N_c\to\infty} = 3/4. \tag{4.30}$$

(*) The full calculation of the ratio f_K/f_π to $O(p^4)$ in ChPT can be found in the second reference in [30].

5.— Two-Point Functions of Four-Quark Operators at Long Distances

To leading order in chiral perturbation theory and in the chiral limit, where pseudoscalar masses vanish, the spectral functions of four-quark operators at small t-values behave like t^2 and receive contributions from two-pseudoscalar intermediate states only. This property follows from the structure of the effective bosonic Lagrangians in eqs. (4.20) and (4.26) which, to lowest order in the number of derivatives, realize the four-quark Hamiltonians of eqs. (2.10) and (2.19) respectively.

Let us first consider the spectral function associated to the $\Delta S = 2$ operator in (2.17). From our discussion in section 3; in particular eqs. (3.5) and (3.17a), we have learnt that at short distances (large t) the behaviour predicted by perturbative QCD is

$$\alpha_s(t)^{4/9}\frac{1}{\pi}Im\psi_{\Delta S=2}(t)|_{\text{t-large}} = \frac{2}{15}\frac{t^4}{(16\pi^2)^3}\left\{1 - \frac{1217}{810}\frac{\alpha_s(t)}{\pi} + 0\left(\frac{\alpha_s}{\pi}\right)^2\right\} \tag{5.1}$$

The corresponding behaviour at long distances (small t) for the same spectral function which follows from lowest order chiral perturbation theory is [7]

$$\alpha_s(t)^{4/9}\frac{1}{\pi}Im\psi_{\Delta S=2}(t)|_{\text{t-small}} = |\widehat{B}|^2\frac{2}{9}f_K^4\frac{t^2}{16\pi^2}\left(1 + 0\left(\frac{t}{16\pi^2 f_\pi^2}\right)\right) \quad . \tag{5.2}$$

We wish to plot the ratio

$$\frac{9}{2}\frac{16\pi^2}{t^2}\frac{1}{f_K^4}\alpha_s(t)^{4/9}\frac{1}{\pi}Im\psi_{\Delta S=2}(t) \equiv \rho_{\Delta S=2}(t) \tag{5.3}$$

versus t. At t very small, $\rho_{\Delta S=2} \to |\widehat{B}|^2$; and at t sufficiently large, where the asymptotic freedom behaviour sets ,

$$\rho_{\Delta S=2} \to \frac{3}{5}\frac{t^2}{(16\pi^2 f_K^2)^2}\left(1 - \frac{1217}{810}\frac{\alpha_s(t)}{\pi}\right) \quad . \tag{5.4}$$

At $t = 4\text{GeV}^2$, the α_s-correction for $\Lambda_{\overline{MS}} = 200\text{MeV}$ is 15%. At these t-values, the leading non-perturbative power correction is negative and quite small [7]. We therefore consider that the t-behaviour from $t = 4\text{GeV}^2$ is well represented by the QCD-asymptotic expression in eq. (5.4). This corresponds to the continuous line in the plot shown in Fig. 7. If the physical value of $\widehat{B}$ lies between

$$1/4 \le \widehat{B} \le 1 \quad ,$$

we see from Fig. 7 that there is in principle no difficulty in matching the two behaviours, large t (the continuous line) and small t (the big dots at small t), with contributions from hadronic intermediate states i.e., multipseudoscalars and resonances simulated in Fig. 7 by the dashed line. Of course, to make a firm prediction for $\widehat{B}$ requires a detailed analysis, as discussed in ref. [7], where the value

$$|\widehat{B}| = 0.33 \pm 0.09 \tag{5.5}$$

was found. Recently, a reanalysis of this determination has been made [36], with a result compatible with (5.5).

Next, we consider the spectral function associated with the octet part of the four-quark Hamiltonian extracted from eq. (2.10), which we call $\frac{1}{\pi}Im\psi_{\underline{8}}(t)$. The large t behaviour can be obtained from the short-distance analyses of sections 2 and 3. The small t-behaviour follows from the long-distance chiral behaviour discussed in section 4, with the result [10]

$$\frac{1}{\pi}Im\psi_{\underline{8}}(t) = |g_{\underline{8}}|^2\frac{5}{3}f_\pi^4\frac{t^2}{16\pi^2}\left(1 + 0\left(\frac{t}{16\pi^2 f_\pi^2}\right)\right) . \tag{5.6}$$

Here again, we propose to plot the ratio

$$\frac{3}{5}\frac{16\pi^2}{t^2}\frac{1}{f_\pi^4}\frac{1}{\pi}Im\psi_{\underline{8}}(t) \equiv \rho_{\underline{8}}(t) \tag{5.7}$$

versus t. At very small t values $\rho_{\underline{8}}(t) \to |g_{\underline{8}}|^2$; at large t-values $\rho_{\underline{8}}(t) \to t^2$. The precise expression for t large can be read off from the appropriate formula for the four-quark correlations in section 3 and Appendix B, and the Wilson coefficients discussed in section 2. The difference with the previous discussion of the $\Delta S = 2$ spectral function is that now, in the case of the $\Delta I = 1/2$ spectral function, the perturbative α_s corrections at $t = 4\text{GeV}^2$ are still too large. They are almost 100%. To make sure that we are in the asymptotic freedom regime we have to go to much higher t-values. At $t = 14\text{GeV}^2$, the α_s corrections, which give an overall positive increase, are still 50%. As seen in Fig. 8 it is possible to match the QCD asymptotic behaviour for $t \approx 14\text{GeV}^2$ or larger (the continuous line) with the large experimental value $|g_{\underline{8}}|^2 \approx 25$ (the big dots at small t-values). If by contrast, we take the onset of asymptotic freedom at $t \ge 4\text{GeV}^2$ (i.e., the same as for the $\Delta S = 2$ case) we find that the contribution from hadronic states in the intermediate region would require rather unusual destructive interferences to provide the matching.

An attempt to determine $|g_{\underline{8}}|$ via QCD hadronic sum rules was made in ref. [9] and failed to reproduce the experimental value by a large factor. The previous discussion, which in essence is a simplified version of our work in ref. [10], clearly shows why we failed : the α_s-corrections to the $\Delta I = 1/2$ spectral function were not known at that time; (and in fact, from the result of the normal size α_s term in the $\Delta S = 2$ correlator which was already known, there was no reason a priori to suspect such a large enhancement from that source!) It is possible to try again a determination of $|g_{\underline{8}}|$ along the lines discussed in ref. [9], in the presence of the calculated α_s-corrections. The drawback is that this requires knowledge about contributions from hadronic states in a very large range of t-values which are difficult to estimate.

The next sections offer a new, more direct, approach to this difficult problem.

6.— The QCD Effective Action at Long Distances

This section is a summary of recent work by Espriu, de Rafael and Taron [1] which is necessary to understand the calculational framework we shall be using in the next section. In what follows we shall refer to ref. [1] as ERT. The QCD Lagrangian with three light flavours $q \equiv \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ and in the presence of external fields $v_\mu(x), a_\mu(x), s(x)$ and $p(x)$ is defined by

$$\mathcal{L}_{\text{QCD}}(q,\bar{q},G;v,a,s,p) = \mathcal{L}^{o}_{\text{QCD}} + \bar{q}\gamma^\mu(v_\mu + \gamma_5 a_\mu)q - \bar{q}(s - i\gamma_5 p)q, \qquad (6.1a)$$

where

$$\mathcal{L}^{o}_{\text{QCD}} = -\frac{1}{4}\sum_{a=1}^{8} G^{(a)}_{\mu\nu}G^{(a)\mu\nu} + i\bar{q}\gamma^\mu(\partial_\mu + iG_\mu)q. \qquad (6.1b)$$

Here,

$$G_\mu \equiv g_s \sum_{a=1}^{8} \frac{\lambda^{(a)}}{2} G^{(a)}_\mu(x) \qquad (6.2)$$

is the gluon field matrix in the fundamental $SU(3)_c$ representation, and $G^{(a)}_{\mu\nu}$ the gluon field strength tensor

$$G^{(a)}_{\mu\nu} = \partial_\mu G^{(a)}_\nu - \partial_\nu G^{(a)}_\mu - g_s f_{abc} G^{(b)}_\mu G^{(c)}_\nu, \qquad (6.3)$$

with g_s the colour coupling constant ($\alpha_s = g_s^2/4\pi$). The external fields : vector $v_\mu(x)$, axial-vector $a_\mu(x)$, scalar $s(x)$ and pseudoscalar $p(x)$ are Hermitian 3×3 matrices with $trv = tra = 0$. The external v_μ and a_μ fields have already been introduced in section 4 (see eq. (4.5)). The field $s(x)$ contains in particular the quark mass matrix $\mathcal{M}$ of eq. (4.4).

The Lagrangian $L_{\text{QCD}}(x)$ is invariant under local chiral $SU(3)_L \times SU(3)_R$ transformations :

$$q_L \equiv \frac{1}{2}(1 - \gamma_5)q(x) \to V_L(x)q_L \qquad (6.4a)$$

$$q_R \equiv \frac{1}{2}(1 + \gamma_5)q(x) \to V_R(x)q_R \qquad (6.4b)$$

$$l_\mu \equiv v_\mu - a_\mu \to V_L l_\mu V_L^+ + iV_L\partial_\mu V_L^+ \qquad (6.4c)$$

$$r_\mu \equiv v_\mu + a_\mu \to V_R r_\mu V_R^+ + iV_R\partial_\mu V_R^+ \qquad (6.4d)$$

$$s + ip \to V_R(s + ip)V_L^+, \qquad (6.4e)$$

where V_L and V_R are unitary unimodular matrices of $SU(3)_L$ and $SU(3)_R$.

The generating functional for the Green's functions of vector, axial-vector, scalar and pseudoscalar quark currents will be denoted by $\Gamma(v,a,s,p)$. It is convenient to use a path integral representation for Γ

$$e^{i\Gamma(v,a,s,p)} = \frac{1}{Z}\int \mathcal{D}G_\mu \mathcal{D}\bar{q}\mathcal{D}q \exp\left(i\int d^4x \mathcal{L}_{\text{QCD}}(q,\bar{q},G;v,a,s,p)\right) \qquad (6.5a)$$

$$= \int \mathcal{D}G_\mu \exp\left(-i\int d^4x \frac{1}{4}G^{(a)}_{\mu\nu}G^{(a)\mu\nu}\right)\mathcal{D}\bar{q}\mathcal{D}q \exp\left(i\int d^4x \bar{q}iDq\right) \qquad (6.5b)$$

where D denotes the Dirac operator

$$D = \gamma^\mu(\partial_\mu + iG_\mu) - i\gamma^\mu(v_\mu + \gamma_5 a_\mu) + i(s - i\gamma_5 p). \qquad (6.6)$$

The normalization factor Z in eq. (6.5) is fixed so that $\Gamma(0,0,0,0) = 1$. The generating functional $\Gamma(v,a,s,p)$ is not invariant under local chiral transformations due to the existence of the axial anomaly at the one-loop level. The structure of the anomalous piece in Γ is known however from the work of Bardeen [33c] and Wess and Zumino [32].

Chiral symmetry implies that Γ admits a low energy representation

$$e^{i\Gamma(v,a,s,p)} = \int \mathcal{D}U \exp\left(i\int d^4x \mathcal{L}_{\text{eff}}(U,v,a,s,p)\right), \qquad (6.7)$$

where $\mathcal{L}_{\text{eff}}$ is the effective chiral Lagrangian discussed in the previous section [eqs. (4.3), (4.9), (4.10) and (4.11)] with U the unitary unimodular 3×3 matrix which collects the low lying pseudoscalar fields of the hadronic spectrum, and which transforms linearly (see eq. (4.1)) under chiral $SU(3)_L \times SU(3)_R$.
Once $\Gamma(v,a,s,p)$ is known, the functional dependence on U of the low energy effective action

$$W(U,v,a,s,p) = i\int d^4x \mathcal{L}_{\text{eff}}(U,v,a,s,p) \qquad (6.8)$$

can in principle be obtained from the chiral transformation properties of Γ. This is certainly the case for the Wess-Zumino term [32] associated to the chiral anomaly. However, as discussed in ERT, the explicit determination of the rest of the effective action depends, in general, on the details of the dynamics of spontaneous chiral symmetry breaking.

We propose to use the same phenomenological parametrization of spontaneous chiral symmetry breaking as the one introduced in ERT; i.e., to the QCD Lagrangian in (6.1) we add the following term

$$-M_Q(\bar{q}_R U q_L + \bar{q}_L U^\dagger q_R), \qquad (6.9)$$

which serves to introduce the U field in a way non-invariant under $U \to -U$, and a mass parameter M_Q which regulates the infra-red behaviour of the low energy effective action. In the presence of this term the operator $\bar{q}q$ acquires a vacuum expectation value

$$\langle \bar{q}q \rangle \equiv \langle \bar{q}q \rangle_{G_\mu} - \langle \bar{q}q \rangle_{G_\mu = 0}, \tag{6.10}$$

where the subtraction term means the vacuum condensate generated when the gluon fields are switched off. We assume M_Q to be sufficiently large so that an expansion in inverse powers of M_Q of the low energy effective action can be accepted. It then follows that

$$\langle \bar{q}q \rangle = -\frac{1}{12}\frac{\alpha_S}{\pi}\frac{\langle GG \rangle}{M_Q} - \frac{1}{360}\frac{\alpha_S}{\pi} g_s f_{abc} \frac{\langle G^a G^b G^c \rangle}{M_Q^3} + \ldots \tag{6.11}$$

where $\langle GG \rangle, f_{abc}\langle G^a G^b G^c \rangle, \ldots$ are the gluon condensates introduced by Shifman, Vainshtein and Zakharov [37]. The term in eq. (6.9) is therefore an effective way to generate the order parameter due to spontaneous chiral symmetry breaking. In the presence of this term it is then convenient to introduce new quark fields Q_L and Q_R which we call the "rotated basis" defined as follows

$$\begin{aligned} Q_L &= \xi q_L \qquad \bar{Q}_L = \bar{q}_L \xi^\dagger \\ Q_R &= \xi^\dagger q_R \qquad \bar{Q}_R = \bar{q}_R \xi \end{aligned} \tag{6.12}$$

with ξ chosen such that

$$U = \xi^2 \,. \tag{6.13a}$$

Under chiral $SU(3)_L \times SU(3)_R$ transformations (V_L, V_R)

$$\xi(x) \to V_R \xi(x) h^\dagger(x) = h(x)\xi(x) V_L^\dagger \tag{6.13b}$$

which defines the compensating $SU(3)_V$ transformation $h(\varphi(x))$, the wanted ingredient for a non-linear representation of the chiral group [38]. The quark fields of the rotated basis $Q_{L,R}$ transform like

$$Q_L \to h(x) Q_L \quad ; \quad Q_R \to h(x) Q_R \tag{6.14}$$

and the new term in eq. (6.9)

$$-M_Q \left(\bar{q}_R U q_L + \bar{q}_L U^\dagger q_R\right) = -M_Q \left(\bar{Q}_R Q_L + \bar{Q}_L Q_R\right) \tag{6.15}$$

is invariant. The quark fields $Q_L Q_R$ can be interpreted as "constituent chiral quarks" and M_Q as a "constituent quark mass".

The QCD Lagrangian in the rotated basis and in Euclidean space then has the following form

$$\mathcal{L}_{\rm QCD}^{(E)} = \frac{-1}{4} G_{\mu\nu}^{(a)} G_{\mu\nu}^{(a)} + \bar{Q} D_E Q, \tag{6.16}$$

with D_E the Euclidean Dirac operator (see appendix A for notations)

$$D_E = \tilde{\gamma}_\mu \nabla_\mu + \mathrm{M} = \tilde{\gamma}_\mu (\partial_\mu + \mathcal{A}_\mu) + \mathrm{M} \tag{6.17a}$$

where

$$\mathcal{A}_\mu = i G_\mu + \Gamma_\mu - \frac{i}{2}\gamma_5 \xi_\mu \quad \text{and} \quad \mathrm{M} = \frac{-1}{2}(\Sigma - \gamma_5 \Delta) - M_Q. \tag{6.17b, c}$$

The external vector v_μ and axial-vector a_μ fields appear now in Γ_μ and ξ_μ:

$$\Gamma_\mu = \frac{1}{2}\left[\xi^\dagger(\partial_\mu - i r_\mu)\xi + \xi(\partial_\mu - i l_\mu)\xi^\dagger\right] \tag{6.18a}$$

$$\xi_\mu = i\left[\xi^\dagger(\partial_\mu - i r_\mu)\xi - \xi(\partial_\mu - i l_\mu)\xi^\dagger\right]; \tag{6.18b}$$

while the external s and p fields have been frozen to the quark mass matrix $\mathcal{M}$ in eq. (4.4) i.e.,

$$\Sigma = \xi^\dagger \mathcal{M} \xi^\dagger + \xi \mathcal{M} \xi \quad \text{and} \quad \Delta = \xi^\dagger \mathcal{M} \xi^\dagger - \xi \mathcal{M} \xi. \tag{6.19a, b}$$

Notice that

$$\Gamma_\mu = -\Gamma_\mu^\dagger; \xi_\mu = \xi_\mu^\dagger; \Sigma = \Sigma^\dagger; \Delta = -\Delta^\dagger; \mathcal{M} = \mathcal{M}^\dagger \tag{6.20}$$

The Σ and Δ terms break explicitly the chiral symmetry.

To $\mathcal{L}_{\rm QCD}^{(E)}$ in eq. (6.16) we associate the Euclidean effective action $W_E(U, v, a, \mathcal{M}, M_Q)$ as follows

$$\exp W_E(U, v, a, \mathcal{M}, M_Q) = \frac{1}{Z}\int \mathcal{D}G_\mu \exp(-\int d^4z \frac{1}{4} G_{\mu\nu}^{(a)} G_{\mu\nu}^{(a)}) \exp\Gamma_E(\mathcal{A}, \mathrm{M}) \tag{6.21a}$$

with

$$\exp\Gamma_E(\mathcal{A}, \mathrm{M}) = \int \mathcal{D}\bar{Q}\mathcal{D}Q \exp \int d^4z \bar{Q} D_E Q = det D_E. \tag{6.21b}$$

We recall that here we are concerned with the non-anomalous part of the effective action. It is then sufficient to consider the modulus of the determinant ; i.e.,

$$\Gamma_E(\mathcal{A}, \mathrm{M}) = \frac{1}{2}\log \det D_E^\dagger D_E. \tag{6.22}$$

Since $D_E^\dagger D_E$ is a second-order elliptic operator, $det D_E^\dagger D_E$ can be defined using e.g. the ξ-function regularization method (for a review, see ref. [39] and refs. therein)

$$\det D^\dagger D = \exp\left(\frac{-d}{ds}\zeta_{D^\dagger D}(s)\big|_{s=o}\right) \tag{6.23a}$$

with

$$\zeta_{D^\dagger D}(s) = \frac{\nu^{2s}}{\Gamma(s)}\int_0^\infty d\tau \tau^{s-1} Tr \exp(-\tau D^\dagger D). \tag{6.23b}$$

where Tr stands for trace over Dirac γ-matrices, colour $SU(3)$ matrices, flavour $SU(3)$ matrices and Euclidean space ; ν is an arbitrary mass scale, so that $\zeta(s)$ remains dimensionless, which serves to regulate ultra-violet divergences. In what follows, we shall distinguish between this UV-regulator mass scale ν^2 and the perturbative QCD scale μ^2 which appears, via $\bar{M}S$-renormalization, in the Wilson coefficients of the effective weak Hamiltonian of section 2. Explicit calculations can now be made using the powerful technique of the heat kernel expansion [39], [40] applied to the operator $D_E^\dagger D_E - M_Q^2$ i.e.,

$$(x|e^{-\tau D^\dagger D}|y) = \frac{1}{16\pi^2\tau^2}e^{-(x-y)^2/4\tau}\sum_{n=0}^{\infty}H_n(x,y)\tau^n e^{-\tau M_Q^2} \tag{6.24}$$

The coefficient functions $H_n(x,x)$ are the so-called Seeley-DeWitt coefficients [40]. For the reader's convenience we have tabulated those that will be needed in our calculations in Appendix A.

The derivation of a low energy effective chiral Lagrangian within this framework has been extensively discussed in ERT. The results obtained for the L_i-couplings of the $O(p^4)$ Lagrangian are very satisfactory; in particular the couplings L_1, L_2, L_3, L_9 and L_{10} which exist in the chiral limit. This success has prompted us to compute, within the same framework, the effective action of four-quark operators which we discuss in the following sections.

7.— The Effective Action of Four-Quark Operators

7a. Introduction

We shall begin by giving a formal definition, within the functional formalism, of the bosonic realization of the current algebra quark currents that we have discussed in section 4, i.e., eqs. (4.13).

In QCD with external sources (see eq. (6.1)), the effective action of the bilinear quark operator $\bar{q}_{jL}(x)\gamma^\mu q_{iL}(x)$ is given by the functional average

$$\langle\bar{q}_{jL}\gamma^\mu q_{iL}\rangle = \frac{\int \mathcal{D}G_\mu\mathcal{D}\bar{q}\mathcal{D}q\ i\int d^4x(q_{jL}\gamma^\mu q_{iL}(x))\exp\left(i\int d^4z\mathcal{L}_{\mathrm{QCD}}(q,\bar{q},G;v,a,s,p)\right)}{\int \mathcal{D}G_\mu\mathcal{D}\bar{q}\mathcal{D}q\exp\left(i\int d^4zL_{\mathrm{QCD}}(z)\right)} \tag{7.1}$$

With $\mathcal{L}_{\mathrm{QCD}}^{(E)}$ defined in eq. (6.16), and using eq. (6.21), this corresponds to an effective bosonic current

$$\langle(\bar{q}_{jL}\tilde{\gamma}^\mu q_{iL})(x)\rangle = i\frac{\delta W_E}{\delta l_\mu(x)_{ji}}\ , \tag{7.2}$$

where $\tilde{\gamma}_\mu$ denote Euclidean γ-matrices. (See Appendix A).

$$\frac{\delta W_E}{\delta l_\mu(x)_{ji}} = \frac{\int\mathcal{D}G_\mu\exp(-\int d^4x\frac{1}{4}G_{\mu\nu}^{(a)}G_{\mu\nu}^{(a)})\frac{\delta\Gamma_E}{\delta(l_\mu)_{ji}}\exp\Gamma_E}{\int\mathcal{D}G_\mu\exp(-\int d^4x\frac{1}{4}G_{\mu\nu}^{(a)}G_{\mu\nu}^{(a)})\exp\Gamma_E}\ , \tag{7.3}$$

and

$$\frac{\delta\Gamma_E}{\delta l_\mu} = \frac{1}{2}Tr\left(\frac{\delta D_E}{\delta l_\mu}D_E^{-1} + \frac{\delta D_E^\dagger}{\delta l_\mu}(D_E^\dagger)^{-1}\right). \tag{7.4}$$

In deriving the last equation we have used eq. (6.22) and the identity

$$\delta(\det A) = \det A\,Tr(A^{-1}\delta A)\ . \tag{7.5}$$

Equation (7.4) provides an explicit recipe to construct the effective action, in an external gluonic field background, of the quark current operator associated to l_μ i.e.,

$$\langle(\bar{q}_{jL}\tilde{\gamma}^\mu q_{iL})(x)\rangle_{\mathcal{A},\mathrm{M}} = i\frac{\delta\Gamma_E(\mathcal{A},\mathrm{M})}{\delta(l_\mu)_{ji}}. \tag{7.6}$$

We have then two ways to compute the effective action of a quark current operator. Either we already know the effective action W_E, in which case we simply have to take the variation with respect to an external source ; or otherwise, in general, we will have to compute the r.h.s. in eq. (7.4) and integrate over gluons as indicated in eq. (7.3).

7b. Proper-time representation of the Dirac propagator

In order to use the heat kernel expansion technique to compute eq. (7.4), we need a proper-time representation of the Dirac propagator :

$$(x|D^{-1}|y) = (x|(D^\dagger D)^{-1}D^\dagger|y) =$$

$$\int_0^\infty d\tau (x|e^{-\tau D^\dagger D}D^\dagger|y) = \int_0^\infty d\tau (x|e^{-\tau D^\dagger D}|y)\overleftarrow{D}^\dagger_y. \tag{7.7}$$

For $x = y$, this representation needs a regularization. As for the determinant, we use the ζ-function regularization. Then

$$(x|D^{-1}|x) = \frac{d}{ds}\left(\frac{\nu^{2s}}{\Gamma(s)}\int_0^\infty d\tau\tau^s \frac{e^{-\tau M_Q^2}}{16\pi^2\tau^2}\sum_{n=0}^\infty \mathcal{H}_n(x,x)\tau^n\right)\Big|_{s=0} \tag{7.8}$$

and the problem is reduced to the calculation of the derivative coefficient functions [40]

$$\mathcal{H}_n(x,x) \equiv H_n(x,y)\overleftarrow{D}^\dagger_y|_{y=x}, \tag{7.9}$$

where $H_n(x,y)$ are the coefficient functions of the heat kernel expansion in eqs. (6.24).

With D_E given in eqs. (6.17) and the hermiticity properties (6.20),

$$D_E^\dagger = -\tilde{\gamma}_\mu \widehat{\nabla}_\mu + \widehat{\mathrm{M}}, \tag{7.10a}$$

where

$$\widehat{\nabla}_\mu = \partial_\mu + iG_\mu + \Gamma_\mu + \frac{i}{2}\gamma_5\xi_\mu \quad \text{and} \quad \widehat{\mathrm{M}} = \frac{-1}{2}(\Sigma + \gamma_5\Delta) - M_Q. \tag{7.10b, c}$$

We then have

$$\mathcal{H}_n(x,x) = H_n(x,x)\widehat{\mathrm{M}} - R_{n\mu}(x)\tilde{\gamma}_\mu, \tag{7.11}$$

where $H_n(x,x)$ are the usual Seeley-DeWitt coefficients, and the required new coefficient functions

$$R_{n\mu}(x) \equiv H_n(x,y)\overleftarrow{\nabla}_\mu\big|_{y=x} \tag{7.12}$$

can be calculated from the general heat kernel properties. Tables of the Seeley-DeWitt coefficients and the $R_{n\mu}(x)$ coefficient functions which are needed for the explicit calculations of the results presented in this paper can be found in Appendix A.

The integral over proper time in eq. (7.8) is a Laplace transform which can be done explicitly

$$I(n+1,s) \equiv \frac{\nu^{2s}}{\Gamma(s)}\int_0^\infty d\tau\tau^{s+n-2}e^{-\tau M_Q^2} =$$

$$= (M_Q^2)^{1-n}\frac{\Gamma(s+n-1)}{\Gamma(s)}\exp\left(-s\ln\frac{M_Q^2}{\nu^2}\right). \tag{7.13}$$

In practice we shall require the factors

$$F_n \equiv \frac{d}{ds}I(n+1,s)|_{s=0}, \tag{7.14}$$

for $n = 0, 1, 2,$ and 3 :

$$F_0 = -M_Q^2(1 + \ln\frac{\nu^2}{M_Q^2}) \tag{7.15a}$$

$$F_1 = \ln\frac{\nu^2}{M_Q^2} \tag{7.15b}$$

$$F_2 = \frac{1}{M_Q^2} \tag{7.15c}$$

$$F_3 = \left(\frac{1}{M_Q^2}\right)^2 \tag{7.15d}$$

In general, for the convergent integrals ($n > 1$)

$$F_n = \frac{(n-2)!}{(M_Q^2)^{n-1}} \quad \text{for} \quad n > 1 \tag{7.16}$$

7c. Four-quark operators

Let us consider as a specific example the $Q_{\Delta S=2}$-operator of eq. (2.17). We shall use the rotated basis of eq. (6.12). Therefore

$$Q_{\Delta S=2} \equiv (\bar{s}_L\gamma_\mu d_L)(\bar{s}_L\gamma^\mu d_L)$$

$$= (\bar{Q}_L\xi\lambda_{32}\xi^\dagger\gamma_\mu Q_L)(\bar{Q}_L\xi\lambda_{32}\xi^\dagger\gamma^\mu Q_L), \tag{7.17}$$

where we have introduced the flavour matrix notation

$$(\lambda_{ab})_{ij} = \delta_{ai}\delta_{bj}. \tag{7.18}$$

In Euclidean space

$$Q^{(E)}_{\Delta S=2} = (\bar{Q}_L\xi\lambda_{32}\xi^\dagger\tilde{\gamma}_\mu Q_L)(\bar{Q}_L\xi\lambda_{32}\xi^\dagger\tilde{\gamma}_\mu Q_L) \tag{7.19}$$

with $\tilde{\gamma}_\mu$ Euclidean γ-matrices. (See Appendix A).

In QCD, the effective action of the four-quark operator $Q_{\Delta S=2}(x)$ is formally defined by the average

$$\langle Q_{\Delta S=2}\rangle = \frac{\int \mathcal{D}G_\mu\mathcal{D}\bar{q}\mathcal{D}qi\int d^4x Q_{\Delta S=2}(x)\exp\left(i\int d^4z\mathcal{L}_{\mathrm{QCD}}(q,\bar{q},G,v,a,s,p)\right)}{\int \mathcal{D}G_\mu\mathcal{D}\bar{q}\mathcal{D}q\exp\left(i\int d^4z\mathcal{L}_{\mathrm{QCD}}(z)\right)} \tag{7.20}$$

With $\mathcal{L}^{(E)}_{\rm QCD}$ defined in eq. (6.16), this corresponds to an effective bosonic operator

$$\langle Q_{\Delta S=2}(x)\rangle = \frac{\int[\mathcal{D}G_\mu]\mathcal{D}\bar{Q}\mathcal{D}Q Q^{(E)}_{\Delta S=2}(x)\exp\left(\int d^4z\bar{Q}D_EQ\right)}{\int[\mathcal{D}G_\mu]\mathcal{D}\bar{Q}\mathcal{D}Q\exp\left(\int d^4z\bar{Q}D_EQ\right)} \qquad (7.21)$$

where we have introduced the shorthand notation

$$[\mathcal{D}G_\mu] \equiv \mathcal{D}G_\mu \exp\left(-\int d^4z \frac{1}{4}G^{(a)}_{\mu\nu}(z)G^{(a)}_{\mu\nu}(z)\right). \qquad (7.22)$$

Using the definition of the Euclidean effective action in eqs. (6.21), we can rewrite $\langle Q_{\Delta S=2}(x)\rangle$ in a way more accessible to explicit calculation :

$$\langle Q_{\Delta S=2}(x)\rangle = \frac{1}{Z\exp W_E}\int d^4y\int\frac{d^4k}{(2\pi)^4}e^{-ik.(x-y)}\int[\mathcal{D}G_\mu]\int\mathcal{D}\bar{Q}\mathcal{D}Q$$

$$(\bar{Q}_L(x)\xi(x)\lambda_{32}\xi^\dagger(x)\tilde{\gamma}_\mu Q_L(x))\,(\bar{Q}_L(y)\xi(y)\lambda_{32}\xi^\dagger(y)\tilde{\gamma}_\mu Q_L(y))\exp\left(\int d^4z\bar{Q}D_EQ\right). \qquad (7.23)$$

The integral over the four-vector k is a delta function which, upon integration over the y-variable, brings all the fields of the $Q^{(E)}_{\Delta S=2}$-operator to the same point. The technical advantage of writing $\langle Q_{\Delta S=2}(x)\rangle$ in such a form is that now we can introduce effective bosonic operators in an external gluonic background as variations of the Euclidean action $\Gamma_E(\mathcal{A},\mathrm{M})$ in eq. (6.22) with respect to appropriate external sources :

$$i\frac{\delta}{\delta l_\mu(x)}\exp\Gamma_E(\mathcal{A},\mathrm{M}) = \int\mathcal{D}\bar{Q}\mathcal{D}Q\,(\bar{Q}_L\xi\lambda\xi^\dagger\tilde{\gamma}_\mu Q_L)\,(x)\exp\left(\int d^4z\bar{Q}D_EQ\right), \qquad (7.24)$$

as in eq. (7.3) ; and also

$$ii\frac{\delta^2}{\delta l_\mu(x)\delta l_\nu(y)}\exp\Gamma_E(\mathcal{A},\mathrm{M}) =$$

$$\int\mathcal{D}\bar{Q}\mathcal{D}Q\,(\bar{Q}_L\xi\lambda\xi^\dagger\tilde{\gamma}_\mu Q_L)\,(x)\,(\bar{Q}_L\xi\lambda\xi^\dagger\tilde{\gamma}_\nu Q_L)\,(y)\exp\left(\int d^4z\bar{Q}D_EQ\right). \qquad (7.25)$$

From eqs. (7.23) and (7.25) it then follows that

$$\langle Q_{\Delta S=2}(x)\rangle = \int d^4y\int\frac{d^4k}{(2\pi)^4}e^{-ik.(x-y)}\frac{\int[\mathcal{D}G_\mu]\frac{(-1)\delta^2}{\delta l_\mu(x)_{32}\delta l_\mu(y)_{32}}\exp\Gamma_E(\mathcal{A},\mathrm{M})}{\int[\mathcal{D}G_\mu]\exp\Gamma_E(\mathcal{A},\mathrm{M})} =$$

$$= -\int d^4y\int\frac{d^4k}{(2\pi)^4}e^{-ik.(x-y)}\frac{\int[\mathcal{D}G_\mu]\left\{\frac{\delta\Gamma_E}{\delta l_\mu(x)_{32}}\frac{\delta\Gamma_E}{\delta l_\mu(y)_{32}}+\frac{\delta^2\Gamma_E}{\delta l_\mu(x)_{32}\delta l_\mu(y)_{32}}\right\}\exp\Gamma_E(\mathcal{A},\mathrm{M})}{\int[\mathcal{D}G_\mu]\exp\Gamma_E(\mathcal{A},\mathrm{M})}. \qquad (7.26)$$

The calculation of the term in brackets {} above can now be made using the fermionic determinant (recall eq. (7.4)) and the proper-time representation of the Dirac propagator :

$$\frac{\delta\Gamma_E(\mathcal{A},\mathrm{M})}{\delta l_\mu(x)} = Tr\left(D_E^{-1}\frac{\delta D_E}{\delta l_\mu(x)}\right), \qquad (7.27a)$$

$$\frac{\delta^2\Gamma_E(\mathcal{A},\mathrm{M})}{\delta l_\mu(x)\delta l_\nu(y)} = Tr\frac{\delta D_E^{-1}}{\delta l_\mu(x)}\frac{\delta D_E}{\delta l_\nu(y)} = -Tr\left(D_E^{-1}\frac{\delta D_E}{\delta l_\nu(y)}D_E^{-1}\frac{\delta D_E}{\delta l_\mu(x)}\right), \qquad (7.27b)$$

where in deriving the last equation we have used the fact that D_E only depends linearly on the external l_μ-source. To avoid lengthy expressions it will be understood from here onwards that the r.h.s. of eqs.(7.27) stand for an average over the contributions from D_E and $D_E^\dagger$ as explicitly shown in eq.(7.4). The problem of calculating the effective bosonic realization of the four-quark operator $Q_{\Delta S=2}(x)$ has then been reduced to a calculation first of the quantity

$$\langle Q_{\Delta S=2}(x)\rangle_{\mathcal{A},\mathrm{M}} = -TrD_E^{-1}\frac{\delta D_E}{\delta l_\mu(x)_{32}}TrD_E^{-1}\frac{\delta D_E}{\delta l_\mu(x)_{32}}$$
$$+\int d^4y\int\frac{d^4k}{(2\pi)^4}Tre^{-ik.(x-y)}D_E^{-1}\frac{\delta D_E}{\delta l_\mu(x)_{32}}D_E^{-1}\frac{\delta D_E}{\delta l_\mu(y)_{32}}, \qquad (7.28)$$

and then of the gluonic average

$$\langle Q_{\Delta S=2}(x)\rangle = \frac{\int[\mathcal{D}G_\mu]\langle Q_{\Delta S=2}(x)\rangle_{\mathcal{A},\mathrm{M}}\exp\Gamma_E(\mathcal{A},\mathrm{M})}{\int[\mathcal{D}G_\mu]\exp\Gamma_E(\mathcal{A},\mathrm{M})}. \qquad (7.29)$$

The two terms in eq. (7.28) correspond to the factorized and unfactorized patterns shown in Fig. 9. As we shall see, eq. (7.28) is particularly useful for extracting the contribution of gluon condensates to the effective action of four-quark operators.

The functional framework we have presented is particularly well adapted to our purposes ; however, it is not indispensable. We can reproduce our results using a more conventional perturbative framework, in Minkowski space, where

$$\langle Q_{\Delta S=2}(x)\rangle = \langle 0|T(\bar{Q}_L(x)\xi(x)\lambda_{32}\xi^\dagger(x)\gamma_\mu Q_L(x))(\bar{Q}_L(x)\xi(x)\lambda_{32}\xi^\dagger(x)\gamma^\mu Q_L(x))|0\rangle =$$

$$= \int d^4y \int \frac{d^4k}{(2\pi)^4} e^{-ik.(x-y)} \langle 0|T(\bar{Q}_L\xi\lambda_{32}\xi^\dagger\gamma_\mu Q_L)^{(0)}_{in}(x)(\bar{Q}_L\xi\lambda_{32}\xi^\dagger\gamma^\mu Q_L)^{(0)}_{in}(y)$$

$$\times \; \exp(i\int d^4z\mathcal{L}^{(0)}_{\rm INT}(z))|0\rangle_{\rm connected}. \qquad (7.30)$$

The index (0) means free fields, and $\mathcal{L}^{(0)}_{\rm INT}(z)$ denotes the interaction term between free fields of the QCD-Lagrangian with external sources $\Gamma_\mu(z), \xi_\mu(z), \Sigma(z)$ and $\Delta(z)$ defined in eqs. (6.18) and (6.19). The representation in eq. (7.30) is particularly useful for the calculation of the gluonic perturbative contributions to the effective action of four-quark operators. Like in the previous functional framework, the mass parameter M_Q acts as an infra-red regulator. It is convenient to regularize the UV-divergences using the $\overline{MS}$-version of dimensional regularization so as to make the matching with the μ^2-dependence of the Wilson coefficients, calculated in the $\overline{MS}$-scheme as well, more evident.

8.— Calculation of $\langle Q_{\Delta S=2}(x)\rangle$ to Order α_s

We shall first consider the factorized term; i.e., the first term on the r.h.s. of eq. (7.28). To the extent that gluons communicating the two fermionic traces are neglected; i.e., gluons like in fig. 10, the contribution from the factorized pattern becomes simply a local product of two currents

$$\langle Q^{(E)}_{\Delta S=2}(x)\rangle|_{FP} = \int \frac{d^4k}{(2\pi)^4} \langle(\bar{s}_L\gamma_\mu d_L)(x)\rangle\langle(\bar{s}_L\gamma_\mu d_L)(y)\rangle e^{-ik.(x-y)}. \qquad (8.1)$$

Because of colour conservation ($tr\lambda^a = 0$) two gluons at least are needed to communicate the factorized fermionic loops. They give $0(\alpha_s^2)$ contributions at least which we neglect, since here we are going to limit ourselves to $0(\alpha_s)$ contributions only. From the point of view of the large N_c-limit, and as already mentioned in the introduction, the rule of approximation we propose is to keep the leading $0(N_c^2)$ terms and next-to-leading $0(N_c)$ and $0[(\alpha_s N_c)N_c]$ terms, with neglect of higher power $0[(\alpha_s N_c)^p N_c]$ terms for $p \geq 2$.

To lowest order in the chiral expansion, the bosonic effective action associated to the factorized pattern of eq. (7.28) is then, in Minkowski space,

$$\langle Q_{\Delta S=2}\rangle_{FP} = i\int d^4x\mathcal{L}_\mu(x)_{23}\mathcal{L}^\mu(x)_{23}, \qquad (8.2)$$

where $\mathcal{L}_\mu(x)$ has been defined in eq. (4.16),

$$\mathcal{L}_\mu(x) = -i\frac{f_\pi^2}{2}U^\dagger(x)D_\mu U(x).$$

Had we carried through the details of the calculation of $TrD_E^{-1}\frac{\delta D_E}{\delta(l_\mu)_{32}}$ with the proper-time representation for D_E^{-1} given in eq. (7.8) we would have found the same result of course, but with an explicit expression for f_π^2, the same as the one found in ERT i.e.,

$$f_\pi^2 = \frac{N_c}{16\pi^2}4M_Q^2\Big[\log\frac{\nu^2}{M_Q^2} + \frac{\pi^2}{6N_c}\frac{\langle\frac{\alpha_s}{\pi}GG\rangle}{M_Q^4} + \frac{1}{360N_c}\frac{\langle g^3GGG\rangle}{M_Q^6} + \ldots\Big]. \qquad (8.3)$$

If, furthermore, we relax the chiral limit assumption and allow for chiral corrections, then there will also be a contribution from the L_5 coupling in the $0(p^4)$ effective chiral Lagrangian (see eq. (4.11)) with a net effect which will consist of an overall change of f_π^2 to "some approximate" f_K^2 in the final effective action ; the same change as already discussed at the end of section 4. It is important that we are able to identify unambiguously the various sources of contributions which, within our approach, go to f_π and f_K. Even if we cannot compute f_π (or f_K) because of the UV cut-off dependence in eq. (8.3), we can find out which contributions can be reabsorbed in f_π^2 or f_K^2, which in practice we shall take from experiment.

We shall next consider the unfactorized term in eq. (7.28) i.e.,

$$(\langle Q_{\Delta S=2}(x)\rangle_{\mathcal{A},\rm M})_{UP} \equiv \int d^4y\int\frac{d^4k}{(2\pi)^4}Tr e^{-ik.(x-y)}D_E^{-1}\frac{\delta D_E}{\delta l_\mu(x)_{32}}D_E^{-1}\frac{\delta D_E}{\delta l_\mu(y)_{32}}. \qquad (8.4)$$

This requires explicit calculation. From D_E in eqs. (6.17) and (6.18) it follows that

$$\frac{\delta D_E}{\delta l_\mu(x)_{ji}} = \tilde{\gamma}_\mu \left(\frac{1-\gamma_5}{2}\right)(-i)\xi(x)\lambda_{ji}\xi^\dagger(x), \tag{8.5}$$

with λ_{ji} the flavour matrix defined in eq. (7.18). We then have to compute (Sp denotes trace of γ-matrices; tr_c over colour indices; tr_F over flavour)

$$(\langle Q_{\Delta S=2}(x)\rangle_{\mathcal{A},\mathrm{M}})_{UP} = -\int d^4y \int \frac{d^4k}{(2\pi)^4} e^{-ik.(x-y)} Sptr_c tr_F \tilde{\gamma}_\mu \left(\frac{1-\gamma_5}{2}\right)\xi(x)\lambda_{32}\xi^\dagger(x).$$

$$(x|D_E^{-1}|y)\tilde{\gamma}_\mu \left(\frac{1-\gamma_5}{2}\right)\xi(y)\lambda_{32}\xi^\dagger(y)(y|D_E^{-1}|x) =$$

$$-Sptr_c tr_F \tilde{\gamma}_\mu \left(\frac{1-\gamma_5}{2}\right)\xi(x)\lambda_{32}\xi^\dagger(x)(x|D_E^{-1}|x)\tilde{\gamma}_\mu \left(\frac{1-\gamma_5}{2}\right)\xi(x)\lambda_{32}\xi^\dagger(x)(x|D_E^{-1}|x). \tag{8.6}$$

From here onwards, the use of the heat kernel expansion in eq. (7.8), reduces the calculation to purely algebraic operations. We shall only describe the origin of the various contributions which appear to $0(\alpha_s)$, and the corresponding results. For this purpose we shall use the notation $\left(\langle Q_{\Delta S=2}(x)\rangle_{\mathcal{A},\mathrm{M}}^{(m,n)}\right)_{UP}$ to indicate contributions coming from the term $\mathcal{H}_m(x,x)$ in the proper-time expansion of the first Dirac propagator and the term $\mathcal{H}_n(x,x)$ in the second Dirac propagator in the last line of eq. (8.6).

The first contribution comes from $m=1$ and $n=1$; more precisely, from the term

$$iM_Q^2\xi_\alpha\tilde{\gamma}_\alpha\gamma_5 \quad \text{in} \quad \mathcal{H}_1(x,x). \tag{8.7}$$

From this we get

$$\left(\langle Q_{\Delta S=2}^{(E)}(x)\rangle_{\mathcal{A},\mathrm{M}}^{(1,1)}\right)_{UP} = \left(\frac{N_c}{16\pi^2}M_Q^2 ln\frac{\nu^2}{M_Q^2}\right)^2 \frac{1}{N_c} tr_F S_p$$

$$\left[\tilde{\gamma}_\mu \left(\frac{1-\gamma_5}{2}\right)\xi\lambda_{32}\xi^\dagger\xi_\alpha\tilde{\gamma}_\alpha\gamma_5\tilde{\gamma}_\mu \left(\frac{1-\gamma_5}{2}\right)\xi\lambda_{32}\xi^\dagger\xi_\beta\tilde{\gamma}_\beta\gamma_5\right]. \tag{8.8}$$

The overall factor comes from the proper-time integrals (see eq. (7.15b)). The term $\frac{N_c}{16\pi^2}M_Q^2 ln\,(\nu^2/M_Q^2)$ gives the leading contribution to $f_\pi^2/4$, as seen in eq. (8.3). With the remaining traces performed, the final result in Minkowski space is then

$$\left(\langle Q_{\Delta S=2}(x)\rangle_{\mathcal{A},\mathrm{M}}^{(1,1)}\right)_{UP} =$$

$$= \frac{16}{N_c}\frac{N_c}{16\pi^2}M_Q^2 ln\frac{\nu^2}{M_Q^2}\left(\frac{-i}{2}U^\dagger D_\mu U\right)_{23}\frac{N_c}{16\pi^2}M_Q^2 ln\frac{\nu^2}{M_Q^2}\left(\frac{-i}{2}U^\dagger D^\mu U\right)_{23} \tag{8.9}$$

This contribution is represented by the diagram of Fig. 11a.

The lowest gluonic correction to this result from the terms $m=3, n=1$ and $m=1, n=3$ are shown in the diagram of Fig. 11b,c . They come from the term

$$\frac{+i}{12}M_Q^2\xi_\rho G_{\mu\nu}G_{\alpha\beta}\Big\{\frac{1}{2}[\sigma_{\mu\nu}\sigma_{\alpha\beta}\tilde{\gamma}_\rho + \sigma_{\mu\nu}\tilde{\gamma}_\rho\sigma_{\alpha\beta} + \tilde{\gamma}_\rho\sigma_{\mu\nu}\sigma_{\alpha\beta}]\gamma_5$$

$$-\delta_{\mu\alpha}\delta_{\nu\beta}\tilde{\gamma}_\rho\gamma_5\Big\} \quad \text{in} \quad \mathcal{H}_3(x,x). \tag{8.10}$$

Since eventually, as indicated in eq. (7.26), we shall be taking an average over the gluonic fields, we can make the identification

$$G_{\mu\nu}G_{\alpha\beta} = g_s^2\frac{\lambda^a}{2}\frac{\lambda^b}{2}G_{\mu\nu}^{(a)}G_{\alpha\beta}^{(b)} \Rightarrow$$

$$4\pi\alpha_s\frac{\lambda^a}{2}\frac{\lambda^b}{2}\frac{1}{12}(\delta_{\mu\alpha}\delta_{\nu\beta}-\delta_{\mu\beta}\delta_{\nu\alpha})\frac{1}{8}\delta_{ab}\langle\sum_a G_{\rho\sigma}^{(a)}G_{\rho\sigma}^{(a)}\rangle \tag{8.11}$$

in the previous expression. The trace over colour then gives a factor

$$tr_c G_{\mu\nu}G_{\alpha\beta} \Rightarrow \frac{\pi^2}{6}\langle\frac{\alpha_s}{\pi}GG\rangle(\delta_{\mu\alpha}\delta_{\nu\beta}-\delta_{\mu\beta}\delta_{\nu\alpha}); \tag{8.12}$$

and using the identities

$$\sigma_{\mu\nu}\sigma_{\mu\nu} = 12\mathbb{1} \quad \text{and} \quad \sigma_{\mu\nu}\tilde{\gamma}_\rho\sigma_{\mu\nu} = 0,$$

we have

$$tr_c\langle\mathcal{H}_3(x,x)\rangle \Rightarrow iM_Q^2\xi_\rho\tilde{\gamma}_\rho\gamma_5\frac{\pi^2}{6}\langle\frac{\alpha_s}{\pi}GG\rangle. \tag{8.13}$$

If we now compare this result with the one obtained from the $\mathcal{H}_1(x,x)$ term in eq. (8.7) and take into account the corresponding proper-time integrals in eqs. (7.15b) and (7.15c), we see that the net effect of $\mathcal{H}_3$ is to modify the previous contribution from $\mathcal{H}_1$ as follows

$$F_1\mathcal{H}_1 \doteq iM_Q^2\xi_\mu\tilde{\gamma}_\mu\gamma_5 ln\frac{\nu^2}{M_Q^2} \quad \to \quad F_1\mathcal{H}_1 + F_3\mathcal{H}_3 \doteq$$

$$iM_Q^2\xi_\mu\tilde{\gamma}_\mu\gamma_5\left(ln\frac{\nu^2}{M_Q^2}+\frac{\pi^2}{6N_c}\frac{\langle\frac{\alpha_s}{\pi}GG\rangle}{M_Q^4}\right). \tag{8.14}$$

From the comparison of this result with the expression of f_π^2 in eq. (8.3) we conclude that the contribution from H_3 is reabsorbed in the physical f_π^2 value. Therefore

$$\langle Q_{\Delta S=2}\rangle_{UP}^{(1,1)} + \langle Q_{\Delta S=2}\rangle_{UP}^{(1,3)} + \langle Q_{\Delta S=2}\rangle_{UP}^{(3,1)} =$$

$$\frac{1}{N_c} i \int d^4x \mathcal{L}_\mu(x)_{23} \mathcal{L}^\mu(x)_{23}. \tag{8.15}$$

We observe that the sum of the contribution from the factorized pattern in eq. (8.2) with the contribution from the unfactorized pattern in eq. (8.15) reproduces the result of the vacuum saturation approximation (VSA) often used in the literature.

The first $0(\alpha_s)$ contribution, not included in the VSA, and by definition neglected in the leading large N_c-limit approximation, comes from the $m = 2$ and $n = 2$ terms in the unfactorized pattern i.e., the term $\left(\langle Q^{(E)}_{\Delta S=2}(x)\rangle^{(2,2)}_{\mathcal{A},\mathrm{M}}\right)_{UP}$ which corresponds to the diagram shown in Fig. 11d . More specifically, it comes from the term

$$-\frac{i}{4} M_Q^2 G_{\mu\nu} \left\{\sigma_{\mu\nu}, \tilde{\gamma}_\alpha \gamma_5\right\}_+ \xi_\alpha \quad \text{in each} \quad \mathcal{H}_2(x,x) \quad \text{factor} \tag{8.16}$$

Each integral over proper time gives a F_2-factor eq. (7.15c), and we have

$$\left(\langle Q^{(E)}_{\Delta S=2}(x)\rangle^{(2,2)}_{\mathcal{A},\mathrm{M}}\right)_{UP} = \frac{-1}{(16\pi^2)^2} \frac{1}{M_Q^4} \left(\frac{-i}{4} M_Q^2\right)^2 Sptr_F tr_c \Big[G_{\mu\nu} \left\{\sigma_{\mu\nu}, \tilde{\gamma}_\rho \gamma_5\right\}_+ \xi_\rho$$

$$\tilde{\gamma}_\delta \frac{1-\gamma_5}{2} \xi \lambda_{32} \xi^\dagger G_{\alpha\beta} \left\{\sigma_{\alpha\beta}, \tilde{\gamma}_\sigma \gamma_5\right\}_+ \xi_\sigma \tilde{\gamma}_\delta \frac{1-\gamma_5}{2} \xi \lambda_{32} \xi^\dagger \Big]. \tag{8.17}$$

Performing the gluonic average like in (8.11) and doing first the colour trace, eq. (8.12), renders the rest of the calculation rather straightforward. The result we find in Minkowski space is

$$\langle Q_{\Delta S=2}\rangle^{(2,2)}_{UP} = -\frac{1}{2} \frac{\langle \frac{\alpha_s}{\pi} GG\rangle}{16\pi^2 f_\pi^4} i \int d^4x \mathcal{L}_\mu(x)_{23} \mathcal{L}^\mu(x)_{23} \tag{8.18}$$

Our final result to $0(\alpha_s)$ from gluonic condensates is then

$$\langle Q_{\Delta S=2}\rangle = \left\{1 + \frac{1}{N_c}\left(1 - \frac{N_c}{2} \frac{\langle \frac{\alpha_s}{\pi} GG\rangle}{16\pi^2 f_\pi^4} + 0(\alpha_s N_c)^2\right)\right\} i \int d^4x \mathcal{L}_\mu(x)_{23} \mathcal{L}^\mu(x)_{23}. \tag{8.19}$$

Notice that in the large N_c-limit counting rules

$$\langle \frac{\alpha_s}{\pi} GG\rangle \text{ is } 0(N_c) \quad \text{and} \quad f_\pi \text{ is } O(\sqrt{N_c})$$

Therefore

$$\frac{N_c}{2} \frac{\langle \frac{\alpha_s}{\pi} GG\rangle}{16\pi^2 f_\pi^4} \text{ is } 0(1) \; .$$

The uncalculated terms $0(\alpha_s N_c)^2$ are however $0(1)$ as well ; hopefully smaller since they involve condensates of higher dimension which therefore require higher powers of the normalization scale. The only way to check this is of course to calculate these higher order

terms, as has been done for f_π^2 in eq. (8.3) and for $\langle\bar{\psi}\psi\rangle$ in eq. (6.11). This is however beyond the scope of this first paper.

The next question we wish to discuss is the dependence on the renormalization scale μ^2 in the effective action and its eventual cancellation with the μ^2-dependence in the corresponding Wilson coefficient. For this discussion, the calculational framework explained in eq. (7.30) turns out to be more adequate. To order α_s, there is a logarithmic perturbative contribution to the effective action of the $Q_{\Delta S=2}(x)$ operator, with the result (in the $\overline{MS}$-scheme)

$$\langle Q_{\Delta S=2}\rangle = -\frac{3}{4}\left(1 - \frac{1}{N_c}\right) \frac{\alpha_s}{\pi} ln \frac{\mu^2}{M_Q^2} i \int d^4x \mathcal{L}_\mu(x)_{23} \mathcal{L}^\mu(x)_{23}; \tag{8.20}$$

i.e., the effective action at order α_s has the same anomalous dimension as the $Q_{\Delta S=2}$ four-quark operator [see eqs. (2.20) and (2.21)]. The arbitrariness in the μ^2-dependence then disappears and the net result for the scale invariant $\widehat{B}$-factor in eq. (4.27) is then

$$\widehat{B} = \alpha_s(M_Q^2)^{-2/9} \frac{3}{4} \left\{1 + \frac{1}{N_c}\left(1 - \frac{N_c}{2} \frac{\langle \frac{\alpha_s}{\pi} GG\rangle}{16\pi^2 f_\pi^4} + 0(\alpha_s N_c)^2\right)\right\} \tag{8.21}$$

We postpone the discussion of numerical estimates of this result to section 10.

9.— Calculation of $\langle Q_{\pm}(x)\rangle$ to Order α_s

The $\Delta S = 1$ four-quark operators

$$Q_+ = Q_2 + Q_1 \qquad \text{and} \qquad Q_- = Q_2 - Q_1 \tag{9.1a,b}$$

have been introduced in section 2. We recall that

$$Q_2 = 4(\bar{s}_L\gamma_\mu u_L)(\bar{u}_L\gamma^\mu d_L) = 4(\bar{Q}_L\xi\lambda_{31}\xi^\dagger\gamma_\mu Q_L)(\bar{Q}_L\xi\lambda_{12}\xi^\dagger\gamma^\mu Q_L) \tag{9.1}$$

and

$$Q_1 = 4(\bar{s}_L\gamma_\mu d_L)(\bar{u}_L\gamma^\mu u_L) = 4(\bar{Q}_L\xi\lambda_{32}\xi^\dagger\gamma_\mu Q_L)(\bar{Q}_L\xi\lambda_{11}\xi^\dagger\gamma^\mu Q_L) \tag{9.2}$$

where the expressions on the r.h.s. correspond to those of the rotated basis of eqs. (6.12).

From the point of view of flavour dynamics there is a crucial difference between the operators $Q_{\Delta S=2}$ [eq. (7.17)] and Q_1 on the one hand, and the operator Q_2 on the other. It is the simple fact that the flavour matrix products

$$\lambda_{32}\lambda_{32} = 0 \qquad \text{and} \qquad \lambda_{32}\lambda_{11} = 0 \tag{9.3a,b}$$

while

$$\lambda_{31}\lambda_{12} = \lambda_{32} \neq 0. \tag{9.4}$$

As we shall see, this simple property implies the existence of extra configurations in the calculation of $\langle Q_2\rangle$ as compared to the common configurations which contribute to $\langle Q_{\Delta S=2}\rangle\langle Q_2\rangle$ and $\langle Q_1\rangle$.

9a. Common configurations

We start with a discussion of the contribution to the effective action of the operators Q_2 and Q_1 from the so-called common configurations. These are the equivalent of the factorized and unfactorized patterns in Fig. 9 already calculated for $\langle Q_{\Delta S=2}\rangle$ in the previous section. To obtain the corresponding results for $\langle Q_2\rangle$ and $\langle Q_1\rangle$ it is enough to replace

$$\lambda_{32}\otimes\lambda_{32} \quad \text{in} \quad Q_{\Delta S=2} \quad \text{by} \quad 4\lambda_{31}\otimes\lambda_{12} \quad \text{in} \quad Q_2$$

and

$$\lambda_{32}\otimes\lambda_{32} \quad \text{in} \quad Q_{\Delta S=2} \quad \text{by} \quad 4\lambda_{32}\otimes\lambda_{11} \quad \text{in} \quad Q_1$$

From these common configurations we obtain the results

$$\langle Q_2\rangle = i\int d^4x 4\Big[\mathcal{L}_\mu(x)_{21}\mathcal{L}^\mu(x)_{13} + \frac{1}{N_c}\left(1 - \frac{N_c}{2}\frac{\langle\frac{\alpha_s}{\pi}GG\rangle}{16\pi^2 f_\pi^4}\right)\mathcal{L}_\mu(x)_{23}\mathcal{L}^\mu(x)_{11}\Big] \tag{9.5}$$

and

$$\langle Q_1\rangle = i\int d^4x 4\Big[\mathcal{L}_\mu(x)_{23}\mathcal{L}^\mu(x)_{11} + \frac{1}{N_c}\left(1 - \frac{N_c}{2}\frac{\langle\frac{\alpha_s}{\pi}GG\rangle}{16\pi^2 f_\pi^4}\right)\mathcal{L}_\mu(x)_{21}\mathcal{L}^\mu(x)_{13}\Big]; \tag{9.6}$$

which in terms of Q_- and Q_+ implies

$$\langle Q_-\rangle = i\int d^4x\Big[1 - \frac{1}{N_c}\left(1 - \frac{N_c}{2}\frac{\langle\frac{\alpha_s}{\pi}GG\rangle}{16\pi^2 f_\pi^4}\right)4[\mathcal{L}_\mu(x)_{21}\mathcal{L}^\mu(x)_{13} - \mathcal{L}_\mu(x)_{23}\mathcal{L}^\mu(x)_{11}], \tag{9.7}$$

and

$$\langle Q_+\rangle = i\int d^4x\Big[1 + \frac{1}{N_c}\left(1 - \frac{N_c}{2}\frac{\langle\frac{\alpha_s}{\pi}GG\rangle}{16\pi^2 f_\pi^4}\right)4[\mathcal{L}_\mu(x)_{21}\mathcal{L}^\mu(x)_{13} + \mathcal{L}_\mu(x)_{23}\mathcal{L}^\mu(x)_{11}]. \tag{9.8}$$

The result is that the same gluon condensate correction which decreases the VSA estimate of the B-factor now increases the VSA estimate of $\langle Q_-\rangle$ and decreases that of $\langle Q_+\rangle$. This is certainly a correction which, for $\Delta S = 1$ transitions, goes in the right direction of the observed phenomenological features ! But, as already announced, it is not yet the full contribution to $\langle Q_-\rangle$ at the order of approximation at which we are working.

9b. Singlet-like configurations

Let us again consider the contribution to the effective bosonic realization of the Q_2-operator from the unfactorized pattern i.e., the analogue of the term in eq. (8.6)

$$(\langle Q_2(x)\rangle_{\mathcal{A},\mathbf{M}})_{UP} = -4S_p tr_c tr_F \tilde{\gamma}_\mu \frac{1-\gamma_5}{2}\xi\lambda_{31}\xi^\dagger(x|D_E^{-1}|x)\tilde{\gamma}_\mu\frac{1-\gamma_5}{2}\xi\lambda_{12}\xi^\dagger(x|D_E^{-1}|x). \tag{9.9}$$

Because of the flavour property stated in eq. (9.4) this trace admits possible contributions from flavour singlet terms in the first Dirac propagator (flavour singlet contributions from the second Dirac propagator give an overall null result because $\lambda_{12}\lambda_{31} = 0$). In the proper time representation of eq. (7.8), the lowest order flavour singlet contribution to $(x|D_E^{-1}|x)$ with an odd number of γ-matrices comes from the terms

$$\frac{i}{6}(\nabla_\nu G_{\nu\mu})\tilde{\gamma}_\mu - \frac{1}{4}\sigma_{\alpha\beta}(\nabla_\mu G_{\alpha\beta})\tilde{\gamma}_\mu \qquad \text{in} \qquad \mathcal{H}_1(x,x). \tag{9.10}$$

These terms, when inserted in the r.h.s. of eq. (9.9), and taking into account the F_1 factor in eq. (7.15b) from the proper-time integral, lead to the following expression

$$(\langle Q_2(x)\rangle_{\mathcal{A},\mathbf{M}})_{UP}^{\text{singlet}} =$$

$$\frac{1}{4\pi^2}ln\frac{\nu^2}{M_Q^2}Sptr_c tr_F\Big[\xi^\dagger(x|D_E^{-1}|x)\xi\lambda_{32}\tilde{\gamma}_\rho\left(\frac{i}{3}\nabla_\sigma G_{\sigma\rho} + \frac{1}{2}\sigma_{\alpha\beta}\nabla_\rho G_{\alpha\beta}\right)\frac{1-\gamma_5}{2}\Big]$$

$$= -\frac{i}{4\pi^2} ln \frac{\nu^2}{M_Q^2}(1-\frac{1}{3}) Sptr_c tr_F \Big[\xi^\dagger(x|D_E^{-1}|x)\xi\lambda_{32}(\nabla_\alpha G_{\alpha\beta})\tilde{\gamma}_\beta \frac{1-\gamma_5}{2}\Big], \qquad (9.11)$$

where, in deriving the last line, we have used the Bianchi identity

$$\nabla_\rho \tilde{G}_{\rho\mu} = 0 \quad ; \quad \tilde{G}_{\rho\mu} \equiv \frac{1}{2}\epsilon_{\rho\mu\alpha\beta} G_{\alpha\beta} \,. \qquad (9.12)$$

We can now use the equations of motion of the gluon field, to replace the covariant derivative of the gluon strength tensor by its fermionic content

$$\nabla_\mu G^{(a)}_{\mu\nu}(x) = -ig_s Sptr_c tr_F \tilde{\gamma}_\nu \frac{\lambda^a}{2}(x|D_E^{-1}|x), \qquad (9.13)$$

with the result, which we represent diagramatically in Fig. 12,

$$(\langle Q_2(x)\rangle_{\mathcal{A},\mathrm{M}})^{\mathrm{singlet}}_{UP} = \frac{-2}{3}\frac{\alpha_s}{\pi} ln \frac{\nu^2}{M_Q^2} \sum_a Sptr_c tr_F \left(\tilde{\gamma}_\mu \frac{\lambda^a}{2} D_E^{-1}\right) Sptr_c tr_F$$

$$\left(\xi^\dagger D_E^{-1} \xi \lambda_{32} \frac{\lambda^a}{2}\tilde{\gamma}_\mu \frac{1-\gamma_5}{2}\right). \qquad (9.14)$$

Up to an overall factor $\frac{-2}{3}\frac{\alpha_s}{\pi} ln \frac{\nu^2}{M_Q^2}$, the result on the r.h.s. is precisely the one corresponding to the bosonic realization of the factorized pattern of the four-quark operator

$$\sum_a \left(\bar{Q}_L \tilde{\gamma}_\mu \xi \lambda_{32} \xi^\dagger \frac{\lambda^a}{2} Q_L\right)\left(\bar{Q}\tilde{\gamma}_\mu \frac{\lambda^a}{2} Q\right) \qquad (9.15a)$$

in an external background of fields $\mathcal{A}$ and $\mathcal{M}$. In the bases of four quark operators Q_i introduced in eqs. (2.3) to (2.6), this operator corresponds to the combination

$$\frac{1}{8}\Big[Q_4 + Q_6 - \frac{1}{N_c}(Q_3 + Q_5)\Big]. \qquad (9.15b)$$

The result in eq. (9.14) can then also be written in the form

$$(\langle Q_2(x)\rangle_{\mathcal{A},\mathrm{M}})^{\mathrm{singlet}}_{UP} = -\frac{1}{6}\frac{\alpha_s}{\pi}\frac{1}{2} ln \frac{\nu^2}{M_Q^2} < Q_4(x) + Q_6(x) - \frac{1}{N_c}[Q_3(x) + Q_5(x)] >_{FP} \quad (9.16)$$

If now we take the average over gluonic fields, we find the relation, valid to $0(\alpha_s)$:

$$< Q_2 >^{\mathrm{singlet}}_{UP} = -\frac{1}{6}\frac{\alpha_s}{\pi}\frac{1}{2} ln \frac{\nu^2}{M_Q^2} < Q_4 + Q_6 - \frac{1}{N_c}[Q_3 + Q_5] >_{FP} \qquad (9.17)$$

This result, when compared to the anomalous dimension matrix equation in (2.8), shows that the same gluonic mixing also appears between the effective action of the Q_2-operator and the effective action of the four-quark penguin operators. From eq. (9.16) we read the anomalous dimensions

$$\gamma_{24} = \gamma_{26} = -N_c\gamma_{23} = -N_c\gamma_{25} = +\frac{1}{6}, \qquad (9.18)$$

in agreement with eq. (2.9b).

10.— The Coupling Constants $g_{\underline{8}}^{(1/2)}$ and $g_{\underline{27}}^{(3/2)}$ to Order α_s

We shall now proceed to the determination of the coupling constants of the lowest order chiral Lagrangian in eq. (4.20). Let us first discuss the coupling $g_{\underline{27}}^{(3/2)}$ responsible for the $\Delta I = 3/2$ transition $K^+ \to \pi^+\pi^0$. (See eqs. (1.4c) and (4.23b).) Only the operator Q_+ contributes to the transition. The bosonic realization of the operator which we have found is given in eq. (9.8). This contains an $(8_L \otimes 1_R)$ piece and a $(27_L \otimes 1_R)$ term. From the expressions in eqs. (4.15) and (4.19), it follows that the $\Delta I = 3/2$ content is given by

$$\langle Q_+\rangle|_{\Delta I=3/2} = \frac{2}{3}\left[1 + \frac{1}{N_c}\left(1 - \frac{N_c}{2}\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{16\pi^2 f_\pi^4} + 0(\alpha_s N_c)^2\right)\right] 4i\int d^4x \mathcal{L}_{\underline{27}}^{(3/2)}(x) \qquad (10.1)$$

To extract $g_{\underline{27}}^{(3/2)}$ we have to modulate this result with the Wilson coefficient $C_+(M_Q^2)$ scaled at $\mu^2 = M_Q^2$. As discussed in the (analogous) case of the $\widehat{B}$-parameter, this is the scaling which reabsorbs the perturbative logarithmic term in $\langle Q_+\rangle_{\Delta I=3/2}$. We then find

$$g_{\underline{27}}^{(3/2)} = \frac{1}{2}C_+(M_Q^2)\frac{2}{3}\left[1 + \frac{1}{N_c}\left(1 - \frac{N_c}{2}\frac{\langle\frac{\alpha_s}{\pi}GG\rangle}{16\pi^2 f_\pi^4} + 0(\alpha_s N_c)^2\right)\right] \qquad (10.2)$$

From the decomposition in eqs (4.19) it also follows that the result for the $g_{\underline{27}}^{(1/2)}$ coupling is

$$g_{\underline{27}}^{(1/2)} = \frac{1}{2}C_+(M_Q^2)\frac{2}{15}\left[1 + \frac{1}{N_c}\left(1 - \frac{N_c}{2}\frac{\langle\frac{\alpha_s}{\pi}GG\rangle}{16\pi^2 f_\pi^4} + 0(\alpha_s N_c)^2\right)\right] \qquad (10.3)$$

The extraction of $g_{\underline{8}}^{(1/2)}$ is a little bit more involved. As already mentioned, if we do an overall $0(\alpha_s)$ calculation, only $\langle Q_-\rangle$ and the $(8_L \otimes 1_R)$ piece of $\langle Q_+\rangle$ are required to be known at that order. The other operators are modulated by $0(\alpha_s)$ Wilson coefficients and therefore we shall only need their effective bosonic realization to leading order in the $N_c \to \infty$ limit. In this limit

$$\langle Q_3\rangle \;=\; \langle Q_5\rangle \;\to\; 0\,. \qquad (10.4)$$

We are left with $\langle Q_6\rangle$ which we need in the $N_c \to \infty$ limit. Historically, this has been the subject of a few controversies. The tricky point is that the lowest order chiral realization of this operator, which naively could be $0(1)$, gives nothing because $U^\dagger U = 1$. To find the expected $0(p^2)$ realization requires knowledge of $0(p^4)$ terms in the chiral strong interaction Lagrangian of eq. (4.11). In fact, only the L_5 coupling contributes in this case, with the result [41], [25]

$$\langle Q_6\rangle = -16L_5\left(\frac{\langle\bar\psi\psi\rangle}{f_\pi^3}\right)^2 4i\int d^4x \mathcal{L}_{\underline{8}}^{(1/2)}(x)\,. \qquad (10.5)$$

Several remarks concerning this result are in order :

i) In the $N_c \to \infty$ limit L_5 is $0(N_c)$; $\langle\bar\psi\psi\rangle$ is $0(N_c)$ and f_π^2 is $0(N_c)$. Therefore $\langle Q_6\rangle$ is $0(1)$ and finite.

ii) In chiral perturbation theory, the constant L_5 is scale dependent. L_5 is one of the constants needed to renormalize the UV-behaviour of the lowest order chiral loop. However, in the $N_c \to \infty$ limit, chiral loops are suppressed by $\frac{1}{N_c}$ powers and there is no way to keep track of the scale dependence of quantities like L_5. In practice, the question becomes : which numerical value do we take for $L_5|_{N_c\to\infty}$? We claim - without offering a justification which would take us a long way from the topic of this paper - that

$$L_5|_{N_c\to\infty} \simeq L_5(\nu^2 = 4M_Q^2) \qquad (10.6a)$$

At $M_Q = 320$MeV, this corresponds to a value

$$L_5(4M_Q^2) \simeq 1.8\times 10^{-3} \qquad (10.6b)$$

iii) A similar question of renormalization scale dependence appears with $\langle\bar\psi\psi\rangle$. As emphasized in ref. [25], the renormalization scale dependence of $\langle\bar\psi\psi\rangle$ cancels with the renormalization scale dependence of the $C_6(\mu^2)$ Wilson coefficient. Since the latter will eventually be scaled at $\mu^2 = M_Q^2$, the value we must take for $\langle\bar\psi\psi\rangle$ is the running value at $\mu^2 = M_Q^2$. After these considerations we finally give the expression we find for $g_{\underline{8}}^{(1/2)}$:

$$g_{\underline{8}}^{(1/2)} = \frac{1}{2}C_-(M_Q^2)\left[1 - \frac{1}{N_c}\left(1 - \frac{N_c}{2}\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{16\pi^2 f_\pi^4} + 0(\alpha_s N_c)^2\right)\right]$$
$$+\frac{1}{2}C_+(M_Q^2)\frac{1}{5}\left[1 + \frac{1}{N_c}\left(1 - \frac{N_c}{2}\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{16\pi^2 f_\pi^4} + 0(\alpha_s N_c)^2\right)\right]$$
$$+ReC_4(M_Q^2) - ReC_6(M_Q^2)16L_5\left(\frac{\langle\bar\psi\psi\rangle}{f_\pi^3}\right)^2 + 0(\alpha_s N_c)\,. \qquad (10.7)$$

This, together with eqs. (10.2) and (10.3), and the expression for the $\widehat{B}$-parameter in eq. (8.21), are the main new results of this paper.

10a. Numerical evaluation

Our results depend crucially on the value of the gluon condensate $\langle\frac{\alpha_s}{\pi}G^2\rangle$. Unfortunately this is a parameter which is poorly known. The latest phenomenological determination from $e^+e^- \to$ hadrons data made in ref. [42] finds

$$\langle\frac{\alpha_s}{\pi}GG\rangle = [(410\pm 80)\text{MeV}]^4\ ; \qquad (10.8)$$

while the value proposed by Shifman, Vainshtein and Zakharov in their early work on QCD sum rules is

$$\langle\frac{\alpha_s}{\pi}GG\rangle_{\rm SVZ} = (331\text{MeV})^4\,. \qquad (10.9)$$

The values found in lattice simulations are rather large [43]

$$\langle\frac{\alpha_s}{\pi}GG\rangle_{\rm Lattice} \simeq (500\text{MeV})^4\ ; \qquad (10.10)$$

but large uncertainties are expected from lattice artifacts on this determination.

Another drawback for a precise numerical analysis is that, in our approach, the value of the "constituent" chiral quark mass M_Q is correlated to the gluon condensate via eq. (6.11). Fortunately, the error in the value of $\langle \bar{q}q \rangle$ extracted from QCD sum rules is smaller than the one of the gluon condensate. The latest result from the analysis of the two-point function associated to the divergence of the axial current is [44]

$$\langle \widehat{\bar{q}q} \rangle = -\left[(194 \pm 8)\mathrm{MeV}\right]^3 , \qquad (10.11a)$$

where $\langle \widehat{\bar{q}q} \rangle$ is a scale invariant quantity which, at the one-loop level, is defined by

$$\langle \widehat{\bar{q}q} \rangle = \left(\frac{1}{2}\ln M_Q^2/\Lambda_{MS}^2\right)^{-4/9} \langle \bar{\psi}\psi \rangle_{M_Q^2} ; \qquad (10.11b)$$

i.e., at $M_Q = e\bar{\Lambda}_{MS}$, the invariant quark condensate coincides with the running quark condensate at $\nu^2 = M_Q^2$. With the central values of eqs. (10.8) and (10.11a) and from the first term only in eq. (6.11) one gets

$$M_Q \simeq 320\mathrm{MeV} \qquad (10.12)$$

i.e., the typical value of a constituent quark mass in phenomenological models [45]. For this value of M_Q, the influence of the three-gluon condensate in eq. (6.11) estimated with the dilute gas instant model of ref. [37], and using again the central value in eq. (10.8), is 25%.

In our opinion, at the present stage of the theoretical development which has provided the results of $g_{\underline{8}}^{(1/2)}$, $g_{\underline{27}}^{(3/2)}$ and $\widehat{B}$, and in view of the errors in the parameters $\langle \frac{\alpha_s}{\pi} G^2 \rangle$, $\langle \widehat{\bar{q}q} \rangle$ and $\Lambda_{\bar{M}S}$, it does not seem necessary to go through an elaborate numerical analysis. It seems worthwhile however to see what values one gets for the couplings of the chiral Lagrangian we have calculated for a "reasonable" input of parameter values within the errors we have discussed. For that purpose we suggest fixing M_Q to the phenomenological value $M_Q = 320$MeV, leaving $\Lambda_{\bar{M}S}$ to vary within the range 100MeV to 200MeV, and seeing what the output is for values of the condensate within the intermediate range of values

$$(360\mathrm{MeV})^4 \le \langle \frac{\alpha_s}{\pi} GG \rangle \le (390\mathrm{MeV})^4 . \qquad (10.13)$$

Smaller values of the gluon condensate than $(360\mathrm{MeV})^4$ would necessarily imply a value for M_Q too small if $\bar{\Lambda}_{MS}$ is kept within the range mentioned above. Bigger values than $(390\mathrm{MeV})^4$ could be perfectly well accommodated provided we increase M_Q accordingly. Table 1 is a compilation of the results we find for the various Wilson coefficients needed for the determination of the $g_{\underline{8}}^{(1/2)}$, $g_{\underline{27}}^{(3/2)}$ and $\widehat{B}$ parameters. For the determination of C_6 we use the values $m_u \simeq 0$ and $m_c = 1.4$GeV, and neglect the running effects in these quark masses.

Table 2 is a compilation of our final numerical results for various values of the input parameters. We have also incorporated in this table the experimental values $g^{(1/2)}$ and

$g_{\underline{27}}^{(3/2)}$ - without corrections due to final state interactions and/or electromagnetic corrections - and for the sake of comparison, we also give the leading $N_c \to \infty$ limit predictions. We find values for the $\widehat{B}$-parameter definitely below the $\widehat{B} = 3/4$ prediction. The values for $g_{\underline{27}}^{(3/2)}$ and $g^{(1/2)}$ we find are a clear improvement with respect to the leading $N_c \to \infty$ predictions. In view of the fact that finite quark mass corrections have not been taken into account, neither electromagnetic corrections, our results look very promising.

TABLE 1

Numerical values of the Wilson coefficients C_+, C_-, ReC_4 and ReC_6 defined in eqs. (2.11) and (2.13) to (2.16) for three input values of $\Lambda_{\overline{MS}}$, and $\mu^2 = M_Q^2$.

$\Lambda_{\overline{MS}}$(MeV)	C_+	C_-	(ReC_4)	(ReC_6)
100	0.678	2.18	−0.0538	−0.0870
150	0.625	2.56	−0.0733	−0.135
200	0.568	3.10	−0.101	−0.218

TABLE 2

Values of the coupling constants $g^{(1/2)} = g_8^{(1/2)} + g_{\underline{27}}^{(1/2)}$, $g_{\underline{27}}^{(3/2)}$ and the $\widehat{B}$-parameter for different input values of the gluon condensate and $\Lambda_{\overline{MS}}$.

INPUT		$g^{(1/2)}$	$g_{\underline{27}}^{(3/2)}$	$\widehat{B}$
"Experiment"		5.1	0.16	?
Leading $N_c \to \infty$ limit		2/3	1/3	3/4
$\langle \frac{\alpha_s}{\pi} GG \rangle$	$\Lambda_{\overline{MS}}$			
$(390\text{MeV})^4$	200MeV	3.0	0.069	0.25
	150MeV	2.4	0.076	0.28
	100MeV	2.0	0.083	0.31
$(380\text{MeV})^4$	200MeV	2.9	0.087	0.32
	150MeV	2.3	0.096	0.35
	100MeV	1.9	0.10	0.39
$(370\text{MeV})^4$	200MeV	2.7	0.10	0.38
	150MeV	2.2	0.11	0.42
	100MeV	1.8	0.12	0.46
$(360\text{MeV})^4$	200MeV	2.6	0.12	0.43
	150MeV	2.1	0.13	0.48
	100MeV	1.7	0.14	0.53

11.— Comparison with other Approaches, Conclusions and Outlook

The idea of "factorization" explained in the introduction goes back to early work by Feynman [46]. It is a beautifully simple idea but fails to explain non-leptonic weak interactions. There are many models in the literature which advocate the factorization idea, but only applied to the four-quark operators matrix elements; the short-distance effects from the Wilson coefficients are maintained. The question is then to know at which renormalization scale this is a good approximation. Our approach clearly shows that this phenomenological procedure does not make sense in QCD. We find that gluonic effects in the matrix elements of the four-quark operators are very important.

The importance of non-factorizable contributions was already pointed out, several years ago, by Preparata and collaborators [47]. Their approach proceeds by analogy to the neutron-proton mass difference analysis using the Cottingham dispersive approach [48]. However, in order to be implemented convincingly, this approach requires very detailed knowledge of pseudoscalar structure functions in the full Q^2, ν plane. Also, the direct relation to the underlying QCD theory is lost.

An interesting semiphenomenological framework has been suggested by Stech and collaborators [49]. Their crucial point is that the Q_--operator can also be viewed as a product of "diquark" operators. If one adopts the point of view that diquark states play an important rôle in the intermediate regime of bosonization, it is possible to show, following the work of these authors, that one obtains a very good quantitative picture for a description of both K-decays and hyperon decays.

The analysis closest in spirit to ours is the $1/N_c$-approach proposed by Bardeen, Buras and Gerard (BBG) [21], [35b], [50], [51], [52]; and in fact, as already mentioned, our short-distance analysis follows entirely their work in [21]. Both approaches attempt to extract from QCD the next-to-leading terms in a $1/N_c$-expansion. The BBG method extracts this information from the unfactorizable loops in a chiral perturbation theory approach where the starting chiral Lagrangian is the one in the $N_c \to \infty$ limit. It is interesting to compare their results, in the chiral limit, with ours. For the $\widehat{B}$-factor, the BBG-approach finds

$$\widehat{B} = \alpha_s(\mu^2)^{-2/9} \frac{3}{4} \left(1 - 2\frac{M^2}{16\pi^2 f_\pi^2}\right) ,$$

where M is a quadratic cut-off appearing in their chiral loop integrals. They propose, without justification, to match $\mu^2 = M^2$ and in order to make numerical predictions they use an optimization procedure which suggests the range $M_K^2 \leq M^2 \leq 0.64\text{GeV}^2$ as a reasonable one. (Their numerical analysis however also incorporates explicit breaking effects from pseudoscalar mass terms, which ours, so far, does not.) It is interesting that both approaches give an overall next-to-leading correction of the same sign ! Both negative. In fact, it is tempting to identify the two expressions and see how they compare numerically. If one sets

$$2\frac{M^2}{16\pi^2 f_\pi^2} = \frac{1}{2}\frac{\langle \frac{\alpha_s}{\pi} GG \rangle}{16\pi^2 f_\pi^4} - \frac{1}{N_c} ,$$

it is quite remarkable that a choice of the central value in eq. (10.8) for the gluon condensate corresponds to $M = 763$ MeV ; a very reasonable value for the BBG cut-off.

The comparison between the BBG-approach and ours for the $g_{\underline{27}}$ couplings, in the chiral limit, is of course analogous to the one of the $\widehat{B}$-factor. For the $g^{(1/2)}$ coupling however it is slightly different. The contribution from the $\langle Q_-\rangle$ matrix element to $g_{\underline{8}}$ in the BBG approach is $1+3\frac{M^2}{16\pi^2 f_\pi^2}$, to be compared to our result : $1+\frac{1}{2}\frac{\langle\frac{\alpha_s}{\pi}GG\rangle}{16\pi^2 f_\pi^4}-\frac{1}{N_c}$. We think the difference is due to extra contributions in the chiral loops, which in our approach do not appear at the level of approximations we have retained.

There are however differences between the two approaches. In our approach there is no explicit quadratic cut-off to be fixed ; and therefore we don't see a justification for identifying μ^2 and M^2. However, these differences may perhaps become less striking if both approaches go beyond their respective approximations.

A lot of effort is being spent at the moment on lattice calculations of weak matrix elements. So far, however, only the calculations of the $\widehat{B}$-parameter have reached the level of clear numerical "predictions". To implement $K \to \pi\pi$ decays in the lattice is an extremely complicated problem; however, a lot of progress has been made in identifying the technical difficulties to overcome. Several groups have reported numerical values for the $\widehat{B}$-parameter [53]. At the moment, the most precise determination comes from the group working with staggered fermions [53b]. Using $\beta = 6.0$ and 31 configurations in a $16^3 \times 40$ volume they find for the B-parameter, at a scale of 2GeV,

$$B(\mu = 2\text{GeV})\big|_{\text{lattice}} = 0.70 \pm 0.01 \,.$$

There is no evidence from the present simulations in the lattice that in the approximation in which they are working the running in μ^2 is the one provided by the anomalous dimension of the continuum QCD. If one assumes it is, then the value above corresponds to a rather large $\widehat{B} \sim 0.94$, compared to the values we find.

We wish to conclude with an outlook of possible improvements we expect to bring to our present calculations in the near future.

i) Order $0(\alpha_s N_c)^2$ corrections in the chiral limit.

As already mentioned, the three-gluon condensate effects in f_π^2 and $\langle\bar\psi\psi\rangle$ are relatively small. It would be nice to check their influence, albeit within the dilute instanton gas model, in $g_{\underline{8}}^{(1/2)}$ and $\widehat{B}$.

ii) Explicit chiral breaking effects.

Here the prospects of obtaining an estimate are rather good. There has been progress in the evaluation of the one-loop chiral corrections [54], [55]; and there exists a classification of all possible local terms which can contribute to the non-leptonic weak interaction chiral Lagrangian to $0(p^4)$. Our approach can provide at least an educated estimate of the $0(p^4)$ couplings ; and therefore an evaluation of the $g_{\underline{8}}^{(1/2)}$, $g_{\underline{27}}^{(3/2)}$, $g_{\underline{27}}^{(1/2)}$ coupling constants and the $\widehat{B}$-parameter to $0(p^4)$. Related to that is the question of the actual size of final state interactions [56] which contribute to these constants.

iii) Our approach can also give estimates for the chiral coupling constants relevant to CP-violation phenomenology (electroweak-penguins) and rare K-decays. This, and hopefully the previous issues too, we plan to discuss in forthcoming publications.

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Appendix A : Notation and useful formulae

1. To go to Euclidean space, we adopt the usual prescription

$$x^\mu \equiv (x^0, x^i) = (-i\bar{x}^0, \bar{x}^i) \tag{A.1}$$

$$\partial_\mu \equiv (\partial_0, \partial_i) = (+i\bar\partial_0, \bar\partial_i) \tag{A.2}$$

For arbitrary 4-vectors a_μ, b_μ, one has then

$$a^\mu b_\mu = -\bar{a}_\mu \bar{b}_\mu . \tag{A.3}$$

It is convenient to work with Hermitian gamma matrices with positive metric. We take

$$\tilde\gamma_\mu \equiv -i\bar\gamma_\mu , \tag{A.4}$$

i.e., $\tilde\gamma_0 = -\gamma_o$ and $\tilde\gamma_i = -i\gamma_i$, which have the required properties

$$\tilde\gamma_\mu^+ = \tilde\gamma_\mu \quad ; \quad \{\tilde\gamma_\mu, \tilde\gamma_\mu\} = 2\delta_{\mu\nu} . \tag{A.5a, b}$$

Other useful relations are

$$\gamma_5 \equiv -i\gamma_0\gamma_1\gamma_2\gamma_3 = \bar\gamma_0\bar\gamma_1\bar\gamma_2\bar\gamma_3 = \tilde\gamma_0\tilde\gamma_1\tilde\gamma_2\tilde\gamma_3 = \gamma_5^\dagger \tag{A.6}$$

$$\bar\sigma_{\mu\nu} = \frac{i}{2}[\bar\gamma_\mu, \bar\gamma_\nu] = -\frac{i}{2}[\tilde\gamma_\mu, \tilde\gamma_\nu] . \tag{A.7}$$

In the remainder of this appendix we will omit the bars in the Euclidean quantities.

2. We shall next recall the Seeley-Dewitt coefficients of the heat kernel expansion of the operator

$$D_E^\dagger D_E - M_Q^2 \equiv -\nabla_\mu \nabla_\mu + E, \tag{A.8}$$

with D_E the Euclidean Dirac operator defined in eqs. (6.17) to (6.18) ; ∇_μ the full covariant derivative in eq. (6.17), and E the quantity [1] :

$$E = iM_Q\tilde\gamma_\mu\gamma_5\xi_\mu - \frac{i}{2}\sigma_{\mu\nu}R_{\mu\nu}$$

$$+\frac{i}{4}\gamma_\mu\{\gamma_5\xi_\mu, \Sigma - \gamma_5\Delta\}_+ + \frac{1}{2}\tilde\gamma_\mu d_\mu(\Sigma - \gamma_5\Delta)$$

$$+M_Q\Sigma + W + \gamma_5 W_5. \tag{A.9}$$

In this expression, $R_{\mu\nu}$ is the full strength tensor i.e.,

$$R_{\mu\nu} = [\nabla_\mu, \nabla_\nu] = iG_{\mu\nu} + \Gamma_{\mu\nu} - \frac{1}{4}[\xi_\mu, \xi_\nu] - \frac{i}{2}\gamma_5\xi_{\mu\nu} =$$

$$= iG_{\mu\nu} - i\left(\frac{1+\gamma_5}{2}\xi^\dagger F_{R_{\mu\nu}}\xi + \frac{1-\gamma_5}{2}\xi F_{L_{\mu\nu}}\xi^\dagger\right), \tag{A.10}$$

with

$$G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + i[G_\mu, G_\nu] \tag{A.11a}$$

$$\Gamma_{\mu\nu} = \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu + i[\Gamma_\mu, \Gamma_\nu] \tag{A.11b}$$

$$\xi_{\mu\nu} = d_\mu \xi_\nu - d_\nu \xi_\mu, \tag{A.11c}$$

and d_μ the covariant derivative with respect to the Γ_μ-connection i.e.,

$$d_\mu A \equiv \partial_\mu A + [\Gamma_\mu, A] \tag{A.12}$$

The expressions Σ and Δ are constructed with the quark mass matrix $\mathcal{M}$ as indicated in eqs. (6.19) ; and W and W_5 denote the $0(\mathcal{M}\mathcal{M})$ expressions [1]

$$W = \frac{1}{2}\left(\xi^\dagger \mathcal{M}\mathcal{M}^\dagger \xi + \xi \mathcal{M}^\dagger \mathcal{M} \xi^\dagger\right) \tag{A.13a}$$

$$W_5 = \frac{1}{2}\left(\xi^\dagger \mathcal{M}\mathcal{M}^\dagger \xi - \xi \mathcal{M}^\dagger \mathcal{M} \xi^\dagger\right). \tag{A.13b}$$

In the chiral limit $\mathcal{M} \to 0$, only the first line on the r.h.s. of E is relevant.

The heat kernel expansion of the operator $D_E^\dagger D_E - M_Q^2$ in eq. (6.24) defines the Seeley-Dewitt coefficients $H_n(x,x)$ we are interested in. (For an excellent recent review of the subject see ref. [39]). In our calculations we need these coefficients up to $n = 3$:

$$H_0(x,x) = 1 \tag{A.14a}$$

$$H_1(x,x) = -E \tag{A.14b}$$

$$H_2(x,x) = \frac{1}{2}E^2 - \frac{1}{6}\nabla_\mu\nabla_\mu E + \frac{1}{12}R_{\mu\nu}R_{\mu\nu} \tag{A.14c}$$

$$\begin{aligned} H_3(x,x) = & -\frac{1}{6}E^3 + \frac{1}{12}[E\nabla_\mu\nabla_\mu E + (\nabla_\mu\nabla_\mu E)E + (\nabla_\mu E)(\nabla_\mu E)] \\ & - \frac{1}{60}\nabla_\mu\nabla_\mu\nabla_\nu\nabla_\nu E \\ & - \frac{1}{30}(ER_{\mu\nu}R_{\mu\nu} + \frac{1}{2}R_{\mu\nu}ER_{\mu\nu} + R_{\mu\nu}R_{\mu\nu}E) \\ & + \frac{1}{60}[(\nabla_\mu E)\nabla_\nu R_{\nu\mu} - (\nabla_\nu R_{\nu\mu})\nabla_\mu E] \\ & + \frac{1}{15}[\frac{1}{3}(\nabla_\mu R_{\nu\rho})\nabla_\mu R_{\nu\rho} + \frac{1}{12}(\nabla_\mu R_{\nu\mu})\nabla_\rho R_{\nu\rho} \\ & + \frac{1}{4}(\nabla_\mu\nabla_\mu R_{\nu\rho})R_{\nu\rho} + \frac{1}{4}R_{\nu\rho}(\nabla_\mu\nabla_\mu R_{\nu\rho}) \\ & - \frac{1}{2}R_{\mu\nu}R_{\nu\rho}R_{\rho\mu}] \end{aligned} \tag{A.14d}$$

As discussed in section 7b, the proper-time representation of the Dirac propagator requires also knowledge of the coefficient $R_{n\mu}(x)$ defined in eq. (7.12). To the approximation to which we are working in this paper, only the first two coefficients are needed :

$$R_{0\mu}(x) = 0 \tag{A.15a}$$

$$R_{1\mu}(x) = -\frac{1}{2}\nabla_\mu E(x) + \frac{1}{6}\nabla_\alpha R_{\alpha\mu}(x). \tag{A.15b}$$

Appendix B : Exact results of the two-point function calculations

We want to study two-point functions of the type

$$\psi_{ij}(q^2) \equiv i\int d^4x e^{iqx} < 0|T(Q_i(x)Q_j(0)^+)|0 >, \tag{B.1}$$

with Q_i one of the $\Delta S = 1$ operators listed in eqs. (2.1) to (2.6), or the $Q_{\Delta S=2}$ operator of eq. (2.17).

Let us discuss first the simplest case of $Q_{\Delta S=2}$, which is a multiplicatively renormalizable operator. The explicit calculation of the associated $\Delta S = 2$ correlator will result in an expression of the form (we use dimensional regularization with $D = 4 + 2\epsilon$ and an anticommuting γ_5)

$$\begin{aligned} \psi_{\Delta S=2}(q^2,\epsilon) = & \frac{(-q^2)^4}{(16\pi^2)^3}(\mu^2)^{3\epsilon}a \\ & \times\left\{\Gamma(-3\epsilon)\left(\frac{-q^2}{4\pi\mu^2}\right)^{3\epsilon}[1 + b\epsilon + \cdots]\right. \\ & \left. + \frac{\Gamma(-4\epsilon)}{\epsilon}\left(\frac{-q^2}{4\pi\mu^2}\right)^{4\epsilon}\frac{\alpha_s}{\pi}c[1 + d\epsilon + \cdots] + 0\left(\frac{\alpha_s}{\pi}\right)^2\right\}. \end{aligned} \tag{B.2}$$

The lowest order contribution is easily obtained from the evaluation of the three-loop diagrams of figure 5. One gets

$$a = \frac{2}{15} \quad ; \quad b = -\frac{319}{30}. \tag{B.3}$$

The calculation of the $0(\alpha_s)$ gluonic corrections, from the diagrams in figure 6 (note that diagram 6.c is not present in this case, due to the flavour structure of $Q_{\Delta S=2}$) is more involved. We will omit the details of this four-loop calculation ; nevertheless, we give in appendix C the result of a non-trivial integral, which is necessary in order to make the otherwise straightforward computation. We get

$$c = 1 \quad ; \quad d = -\frac{157}{12}. \tag{B.4}$$

The composite four-quark operator needs to be renormalized

$$Q^R_{\Delta S=2}(\mu) = (1 - \frac{\alpha_s}{2\pi}\frac{\gamma^{(1)}}{\epsilon} + \cdots)Q_{\Delta S=2}. \tag{B.5}$$

Taking that into account, the renormalized two-point function is, in the $\overline{MS}$ scheme,

$$\psi_{\Delta S=2}(q^2,\mu^2) = -\frac{(-q^2)^4}{(16\pi^2)^3}a\Big\{ln(-q^2/\mu^2)$$

$$+\frac{\alpha_s(\mu^2)}{\pi}[(c-\gamma^{(1)})\frac{1}{\epsilon}ln(-q^2/\mu^2) + (2c - \frac{3}{2}\gamma^{(1)})ln^2(-q^2/\mu^2)$$

$$+(cd - \gamma^{(1)}b)ln(-q^2/\mu^2) + \cdots]$$

$$+0(\frac{\alpha_s}{\pi})^2\Big\}, \tag{B.6}$$

where we have only kept the interesting logarithmic terms. The absence of logarithmic divergences fixes the anomalous dimension factor $\gamma^{(1)} = c = 1$, in agreement with the known value given in eq. (2.21). The spectral function we are looking for is then

$$\frac{1}{\pi}Im\psi_{\Delta S=2}(t) = \theta(t)\frac{t^4}{(16\pi^2)^3}a\Big\{1 + \frac{\alpha_s(\mu^2)}{\pi}\gamma^{(1)}[ln(t/\mu^2)$$

$$+(d-b)] + 0(\frac{\alpha_s}{\pi})^2\Big\}, \tag{B.7}$$

i.e.,

$$\frac{1}{\pi}Im\psi_{\Delta S=2}(t) = \theta(t)\frac{t^4}{(16\pi^2)^3}\frac{2}{15}\Big\{1 + \frac{\alpha_s(\mu^2)}{\pi}[ln(t/\mu^2)$$

$$-\frac{49}{20}] + 0(\frac{\alpha_s}{\pi})^2\Big\} \tag{B.8}$$

The α_s-correction turns out to be identical to the one given in eq. (3.7a) for ψ_{++}. The reason is that both correlators are (when penguin diagrams are neglected) in the same $(27_L, 1_R)$ representation of the flavour chiral group.

As explained in section 3, the result (B.8) is still unphysical in the sense that one still needs to include the Wilson coefficient, computed to the same next-to-leading order, in order to get a renormalization-scale independent result. Since QCD is flavour blind, the μ^2-dependence of $C_{\Delta S=2}(\mu^2)$ and $C_+(\mu^2)$ (without penguins) will be the same; thus, we can use the known $C_+(\mu^2)$ expression, given in eqs. (3.13) to (3.16). The final μ^2-independent result is then

$$C_{\Delta S=2}(\mu^2)^2\frac{1}{\pi}Im\psi_{\Delta S=2}(t) \sim$$

$$\sim \theta(t)\frac{2}{15}\frac{t^4}{(16\pi^2)^3}\alpha_s(\mu^2)^{-4/9}\Big\{1 + \frac{\alpha_s(\mu^2)}{\pi}[ln(t/\mu^2) - \frac{1217}{810}]$$

$$+0(\frac{\alpha_s}{\pi})^2\Big\}. \tag{B.9}$$

The calculation of the two-point function correlators $\psi_{ij}(q^2)$, associated with the $\Delta S = 1$ operators $Q_i(i = 1,2,3,5,6)$, is technically more difficult, because these operators mix when gluonic corrections are taken into account. The result can be written in the form

$$\psi_{ij}(q^2,\epsilon) = \frac{(-q^2)^4}{(16\pi^2)^3}(\mu^2)^{3\epsilon}\Big\{\Gamma(-3\epsilon)\left(\frac{-q^2}{4\pi\mu^2}\right)^{3\epsilon}[A_{ij} + B_{ij}\epsilon + \cdots]$$

$$+\frac{\Gamma(-4\epsilon)}{\epsilon}\left(\frac{-q^2}{4\pi\mu^2}\right)^{4\epsilon}\frac{\alpha_s}{\pi}[C_{ij} + D_{ij}\epsilon + \cdots] + 0(\frac{\alpha_s}{\pi})^2\Big\} \tag{B.10}$$

where A_{ij}, B_{ij}, C_{ij} and $D_{ij}(i,j = 1,2,3,5,6)$ are the following 5×5 matrices ($\xi \equiv -653/60$)

$$A = \frac{4}{5}\begin{bmatrix} 1 & 1/3 & 1 & 0 & 0 \\ 1/3 & 1 & 1/3 & 0 & 0 \\ 1 & 1/3 & 11/3 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix} \tag{B.11a}$$

$$B = \frac{4}{5}\begin{bmatrix} \xi & \frac{\xi+1}{3} & \xi & 0 & 0 \\ \frac{\xi+1}{3} & \xi & \frac{\xi+1}{3} & 0 & 0 \\ \xi & \frac{\xi+1}{3} & \frac{11\xi+2}{3} & 0 & 0 \\ 0 & 0 & 0 & 3\xi & \xi \\ 0 & 0 & 0 & \xi & 3(\xi-\frac{1}{2}) \end{bmatrix} \tag{B.11b}$$

$$C = \begin{bmatrix} 0 & \frac{16}{15} & 0 & 0 & 0 \\ \frac{16}{15} & \frac{16}{135} & \frac{176}{135} & 0 & \frac{16}{45} \\ 0 & \frac{176}{135} & \frac{352}{135} & 0 & \frac{32}{45} \\ 0 & 0 & 0 & 0 & -\frac{16}{5} \\ 0 & \frac{16}{45} & \frac{32}{45} & -\frac{16}{5} & -\frac{128}{15} \end{bmatrix} \tag{B.11c}$$

$$D = \begin{bmatrix} \frac{8}{5} & -\frac{140}{9} & \frac{8}{5} & 0 & 0 \\ -\frac{140}{9} & \frac{28}{405} & -\frac{1508}{81} & 0 & -\frac{668}{135} \\ \frac{8}{5} & -\frac{1508}{81} & -\frac{13136}{405} & 0 & -\frac{1336}{135} \\ 0 & 0 & 0 & \frac{24}{5} & \frac{140}{3} \\ 0 & -\frac{668}{135} & -\frac{1336}{135} & \frac{140}{3} & \frac{6232}{45} \end{bmatrix} \qquad (B.11d)$$

At the leading logarithmic approximation, the mixing of the set of operators $Q_i (i = 1, 2, 3, 5, 6)$ under renormalization, is governed by the 5×5 anomalous dimension matrix

$$\gamma^{(1)} = \begin{bmatrix} -1/2 & 3/2 & 0 & 0 & 0 \\ 4/3 & -1/3 & 1/9 & -1/18 & 1/6 \\ -11/6 & 11/6 & 11/9 & -1/9 & 1/3 \\ 0 & 0 & 0 & 1/2 & -3/2 \\ -1/2 & 1/2 & 1/3 & -1/6 & -7/2 \end{bmatrix} \qquad (B.12)$$

As pointed out by Buras and Slominski [15g], it is convenient to choose a basis of diagonal operators $\tilde{Q}_i$, which are multiplicatively renormalizable

$$\begin{pmatrix} \tilde{Q}_1 \\ \tilde{Q}_2 \\ \tilde{Q}_3 \\ \tilde{Q}_4 \\ \tilde{Q}_5 \end{pmatrix} = W \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{pmatrix} \qquad (B.13)$$

Diagonalizing the matrix $\gamma^{(1)}$, one finds

$$W = \begin{bmatrix} 0.21524 & -0.21524 & -0.064018 & 0.035902 & 1 \\ -1 & 1 & -0.043324 & 0.026217 & 0.069604 \\ -0.052403 & 0.052403 & 0.16479 & 1 & -0.35570 \\ 1 & 2/3 & -1/3 & 0 & 0 \\ -0.60176 & 0.60176 & 1 & -0.20233 & 0.15230 \end{bmatrix} \qquad (B.14)$$

The corresponding eigenvalues of the anomalous dimension matrix $\gamma^{(1)}$ are

$$\tilde{\gamma}^{(1)} = \mathrm{diag}(-3.6110, -1.8779, 0.53806, 1, 1.3398) \qquad (B.15)$$

The operators $Q_2 - Q_1, Q_3, Q_5$ and Q_6, and therefore $\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3$ and $\tilde{Q}_5$, transform as $(8_L, 1_R)$ under $SU(3)_L \otimes SU(3)_R$ flavour transformations. On the other hand, $\tilde{Q}_4 = Q_1 + \frac{2}{3}Q_2 - 1/3Q_3$ transforms as $(27_L, 1_R)$ under the chiral group. Note that $\tilde{Q}_4$ has the same anomalous dimension $\tilde{\gamma}_4^{(1)} \equiv \tilde{\gamma}_{44}^{(1)} = 1$ as $Q_{\Delta S=2}$, because both operators are related by a chiral flavour transformation.

We define now new two-point functions, associated with the diagonal operators $\tilde{Q}_i$

$$\tilde{\psi}_{ij}(q^2) \equiv i \int d^4x e^{iqx} < 0|T(\tilde{Q}_i(x), \tilde{Q}_j(0)^\dagger)|0 > . \qquad (B.16a)$$

$$\tilde{\psi}_{ij}(q^2) = \sum_{k,l} W_{ik} \quad W_{jl} \quad \psi_{kl}(q^2). \qquad (B.16b)$$

These correlators can be written in the same form given in eq. (B.10), but with new coefficient matrices

$$\left[\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}\right] \equiv W \left[A, B, C, D\right] W^T. \qquad (B.17)$$

The renormalization procedure is now straightforward. Keeping only the logarithmic terms, one gets in the $\overline{MS}$-scheme ($\tilde{\gamma}_i^{(1)} \equiv \tilde{\gamma}_{ii}^{(1)}$)

$$\begin{aligned} \tilde{\psi}_{ij}(q^2, \mu^2) = -\frac{(-q^2)^4}{(16\pi^2)^3} \Big\{ & \tilde{A}_{ij} ln(-q^2/\mu^2) \\ & + \frac{\alpha_s}{\pi} \Big[\left(\tilde{C}_{ij} - \tilde{A}_{ij} \frac{\tilde{\gamma}_i^{(1)} + \tilde{\gamma}_j^{(1)}}{2} \right) \frac{1}{\epsilon} ln(-q^2/\mu^2) \\ & + \left(2\tilde{C}_{ij} - \frac{3}{2}\tilde{A}_{ij} \frac{\tilde{\gamma}_i^{(1)} + \tilde{\gamma}_j^{(1)}}{2} \right) ln^2(-q^2/\mu^2) \\ & + \left(\tilde{D}_{ij} - \tilde{B}_{ij} \frac{\tilde{\gamma}_i^{(1)} + \tilde{\gamma}_j^{(1)}}{2} \right) ln(-q^2/\mu^2) + \cdots \Big] + 0 \left(\frac{\alpha_s}{\pi} \right)^2 \Big\}. \end{aligned} \qquad (B.18)$$

One can easily check that the consistency condition

$$\tilde{C}_{ij} - \tilde{A}_{ij} \frac{\tilde{\gamma}_i^{(1)} + \tilde{\gamma}_j^{(1)}}{2} = 0 \qquad (B.19)$$

is satisfied, as required in order to have no logarithmic divergences. The diagonal spectral functions of the multiplicatively renormalizable operators, are finally ($\tilde{A}_i \equiv \tilde{A}_{ii}$)

$$\begin{aligned} \frac{1}{\pi} Im\tilde{\psi}_{ii}(t) = \theta(t) \frac{t^4}{(16\pi^2)^3} \tilde{A}_i \Big[1 + & \frac{\alpha_s(\mu^2)}{\pi} \left(\tilde{\gamma}_i^{(1)} ln(t/\mu^2) + \tilde{\Delta}_i \right) \\ & + 0 \left(\frac{\alpha_s}{\pi} \right)^2 \Big]. \end{aligned} \qquad (B.20)$$

The coefficients $\tilde{A}_i$ and $\tilde{\Delta}_i$ are given in table B-1

Table B-1

i	$\tilde{A}_i$	$\tilde{\Delta}_i$
1	2.507	17.21
2	1.135	9.21
3	2.208	1.06
4	32/27	-49/20
5	2.782	-3.08

Although the spectral functions $\tilde{\psi}_{ij}(q^2)$ are diagonal at the leading logarithmic approximation, off-diagonal corrections are also generated at the next-to-leading order

$$\frac{1}{\pi} Im\tilde{\psi}_{ij}(t) = \theta(t)\frac{t^4}{(16\pi^2)^3}\left[K_{ij}\frac{\alpha_s(\mu^2)}{\pi} + 0\left(\frac{\alpha_s}{\pi}\right)^2\right], (i \neq j). \qquad (B.21)$$

The parameters $K_{ij} = K_{ji}(i \neq j)$, are given in table B-2. The result $K_{i4} = 0(i \neq 4)$ is obvious, since the $(27_L, 1_R)\tilde{Q}_4$ operator doesn't mix with the octet ones.

Table B-2

K_{12}	K_{13}	K_{15}	K_{23}	K_{25}	K_{35}	K_{i4}
0.5929	-4.1140	1.9712	-0.3226	0.8022	-1.2575	0

The results (B.20) and (B.21) should now be multiplied by the appropriate Wilson coefficient factors. These coefficients are, however, only known at the leading logarithmic approximation

$$\tilde{C}_i(\mu^2) \sim \alpha_s(\mu^2)^{\tilde{\gamma}_i^{(1)}/\beta^{(1)}}. \qquad (B.22)$$

In order to get complete μ^2-independent results at next-to-leading order, it would be necessary to know the anomalous dimension matrix at $0(\alpha_s^2)$, which has not been computed to date.

Let's compare now the results of our complete calculation of the spectral functions, with the approximate formulae of section 3. The result given in eq. (3.7a) for the ψ_{++} correlator was already an exact (to order α_s) calculation of the $(27_L, 1_R)$ spectral function (our approximation there was just to neglect the octet contamination originating from diagram 6.c) ; therefore the α_s-correction is identical to the one obtained for $\tilde{\psi}_{44}$ and $\psi_{\Delta S=2}$. It is more interesting to look at the results in the octet sector, where the mixing complications appear. From eq. (B.14), we see that

$$\tilde{Q}_1 \simeq Q_6 - 0.22Q_- \\ \tilde{Q}_2 \simeq Q_- + 0.07Q_6 \qquad (B.23)$$

up to small ($< 7\%$) mixing with Q_3 and Q_5. We can then compare the results obtained for $\tilde{\psi}_{11}$ and $\tilde{\psi}_{22}$, with the approximate expressions for ψ_{66} and ψ_{--} in eqs. (3.18) and (3.7b). The values $\gamma_6^{(1)}\Big|_{1/N_c} = -9/2$ and $\gamma_-^{(1)} = -2$, used in section 3, are in fact not very different from the eigenvalues $\tilde{\gamma}_1^{(1)} = -3.6$ and $\tilde{\gamma}_2^{(1)} = -1.9$ respectively. Moreover, $\tilde{A}_2 = 1.135$ and $\tilde{\Delta}_2 = 9.21$ are very similar to the values $16/15 \simeq 1.07$ and $47/5 \sim 9.4$ in eq. (3.7b), while $\tilde{A}_1 = 2.507$ and $\tilde{\Delta}_1 = 17.21$ are also in good agreement with the results $12/5 \simeq 2.4$ and $423/20 \simeq 21.2$ in eq. (3.18). The discrepancies between the exact and approximate results are never bigger than $20 - 25\%$, which is already at the level of the Q_--contamination present in $\tilde{Q}_1$. We can then conclude that the simplified expressions discussed in section 3 are very good approximations to the true results.

Appendix C : A useful integral

The calculation of the two-point functions $\psi_{ij}(q^2)$ at $0(\alpha_s)$ requires the knowledge of the integral

$$T^{\alpha\beta\mu\nu}(k) \equiv \int \frac{d^Dk_1}{(2\pi)^D}\frac{d^Dk_2}{(2\pi)^D}\frac{k_1^\alpha k_2^\beta (k-k_1)^\mu (k-k_2)^\nu}{k_1^2k_2^2(k-k_1)^2(k-k_2)^2(k_1-k_2)^2}. \qquad (C.1)$$

It is easy to see that the following symmetry relations are satisfied

$$T^{\alpha\beta\mu\nu}(k) = T^{\beta\alpha\nu\mu}(k) = T^{\mu\nu\alpha\beta}(k) = T^{\nu\mu\beta\alpha}(k). \qquad (C.2)$$

Therefore, the integral has the Lorentz decomposition

$$\begin{aligned} T^{\alpha\beta\mu\nu}(k) = k^2[&a_1 g^{\alpha\beta}g^{\mu\nu} + a_2 g^{\alpha\mu}g^{\beta\nu} + a_3 g^{\alpha\nu}g^{\beta\mu}] \\ &+ b_1[g^{\alpha\beta}k^\mu k^\nu + g^{\mu\nu}k^\alpha k^\beta] + b_2[g^{\alpha\mu}k^\beta k^\nu + g^{\beta\nu}k^\alpha k^\mu] \\ &+ b_3[g^{\alpha\nu}k^\beta k^\mu + g^{\beta\mu}k^\alpha k^\nu] + c \quad k^\alpha k^\beta k^\mu k^\nu / k^2. \end{aligned} \qquad (C.3)$$

A straightforward but lengthy calculation gives the following values for the scalar coefficients ($D = 4 + 2\epsilon$)

$$\begin{aligned} a_1 \simeq a_3 &\simeq \left\{-\frac{3}{8}\epsilon + 0(\epsilon^2)\right\}\phi \\ a_2 &\simeq \left\{1 - \frac{51}{8}\epsilon + 0(\epsilon^2)\right\}\phi \\ b_1 &\simeq \left\{\frac{7}{4}\epsilon + 0(\epsilon^2)\right\}\phi \\ b_2 &\simeq \left\{1 - \frac{21}{4}\epsilon + 0(\epsilon^2)\right\}\phi \\ b_3 &\simeq \left\{-\frac{5}{4}\epsilon + 0(\epsilon^2)\right\}\phi \\ c &\simeq 0(\epsilon^2), \end{aligned} \qquad (C.4)$$

where

$$\phi \equiv -\frac{1}{24}\frac{\Gamma(-2\epsilon)}{\epsilon}\frac{(-k^2)^{2\epsilon}}{(16\pi^2)^{2+\epsilon}}. \qquad (C.5)$$

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Figure Captions

Fig.1 Tree level contribution to $\Delta S = 1$ transitions in the electroweak model.

Fig.2 Gluonic contribution giving rise to mixing with the new Q_1-operator in eq.(2.1).

Fig.3 Gluonic penguin-like diagrams giving rise to the operators Q_3 to Q_6 in eqs.(2.3) to (2.6).

Fig.4 Box diagrams in the electroweak model contributing to $\Delta S = 2$ transitions.

Fig.5 Lowest order contributions to the spectral functions in eq.(3.2). The four quark lines at each vertex emerge from the same space-time point. The drawing shows the leading $O(N_C^2)$ configuration in diagram (a) and the next-to-leading $O(N_C)$ configuration in diagram (b).

Fig.6 α_s-corrections to the lowest order diagrams of Fig. 5.

Fig.7 Plot of the function $\rho_{\Delta S=2}(t)$ defined by equation (5.3) versus t. The continuous line corresponds to the behaviour of $\rho_{\Delta S=2}$ at large t predicted by perturbative QCD, with α_s-corrections included. The dashed line simulates the effect of multipseudoscalars and resonances which interpolate the large t QCD behaviour with the constant behaviour at small t-values corresponding to $|\underset{\sim}{B}|^2$.

Fig.8 Plot of the function $\rho_{\underline{8}}(t)$ defined by equation (5.7) versus t. The solid line corresponds to the behaviour predicted by perturbative QCD with α_s-corrections included. The thin dashed line simulates the effect of hadronic contributions which interpolate the large t QCD behaviour with the constant behaviour at small t-values corresponding to $|g_8|^2 \approx 25$.

Fig.9 In the presence of external flavour sources $\times \times \times \ldots \times$ the propagator of the quark lines of the four-quark operator $Q_{\Delta S=2}$ can close following two patterns : the factorized pattern (FP) which is leading in the $N_C \to \infty$ limit ; and the unfactorized pattern (UP) which is next to leading.

Fig.10 Two gluon exchange at least is required to communicate the factorized quark loops of Fig. 9.

Fig.11 Gluonic corrections (b),(c) and (d) to the unfactorized pattern (UP) of diagram (a). Notice that the diagram (a) is the same as the UP-diagram in Fig. 9. The corrections in (b) and (c) are reabsorbed in the physical value of f_π^2, as explained in the text. The correction in (d) is a genuinely new correction not included in the vacuum saturation approximation.

Fig.12 Diagrammatic representation of the singlet-like configuration contributing to the unfactorized pattern of the Q_2-operator to order α_s.

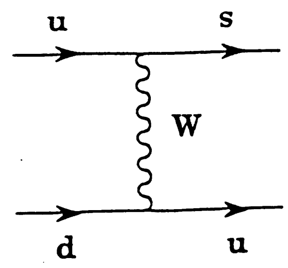


Fig.1

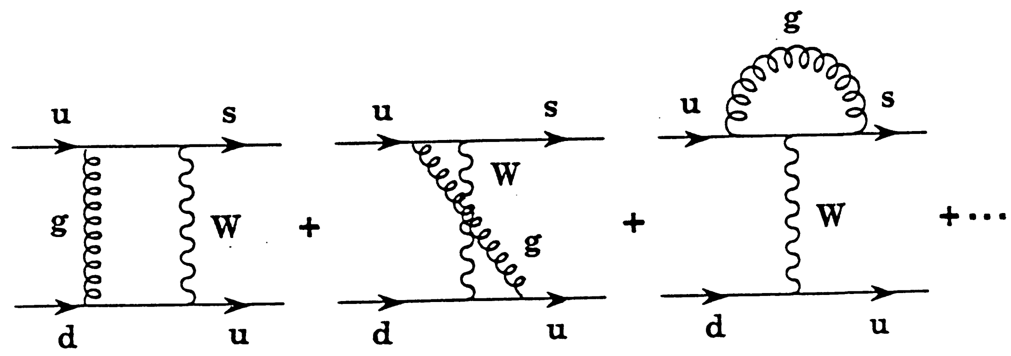


Fig.2

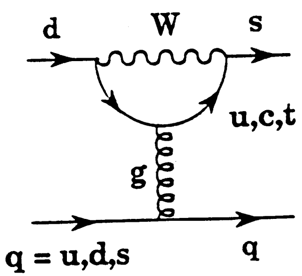


Fig . 3

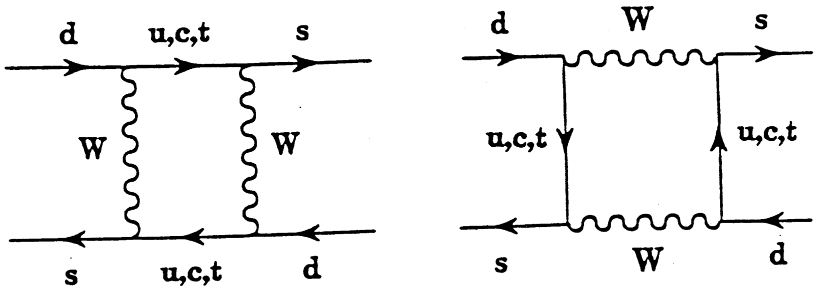


Fig .4

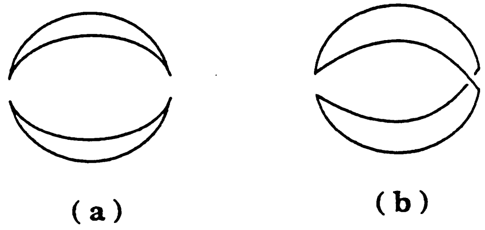


Fig . 5

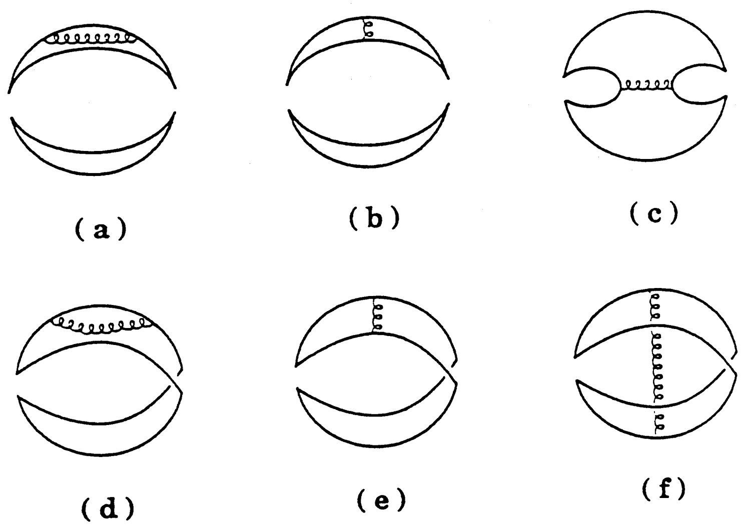
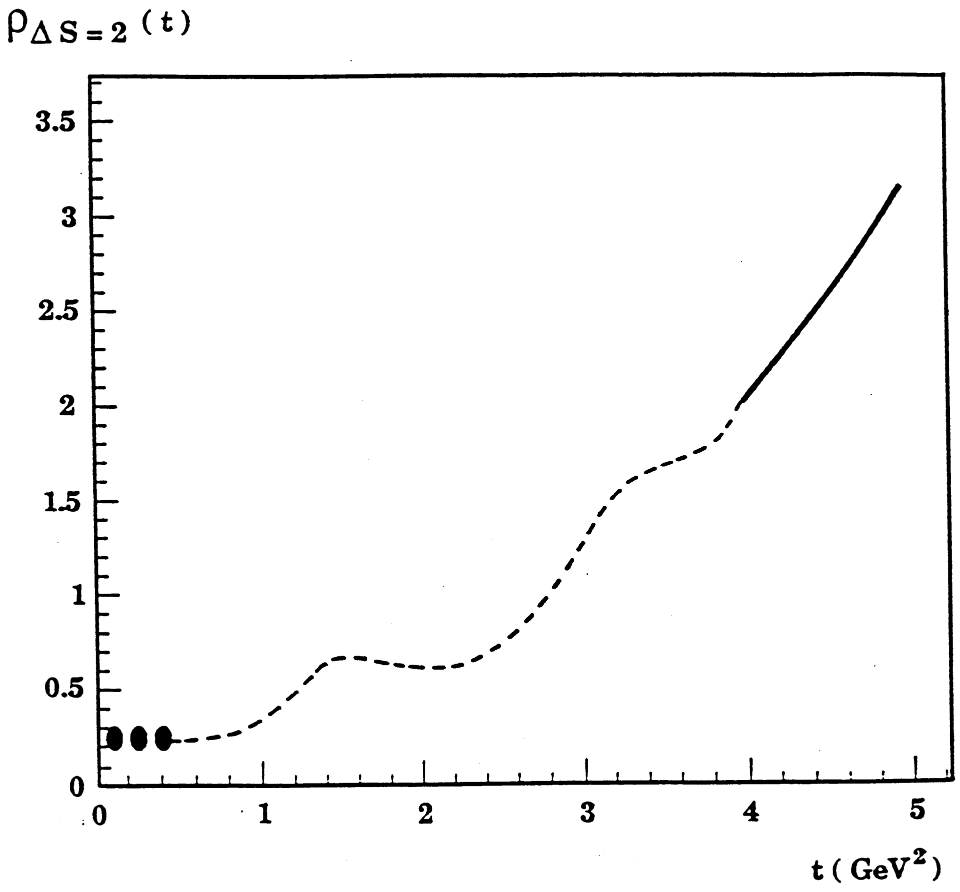
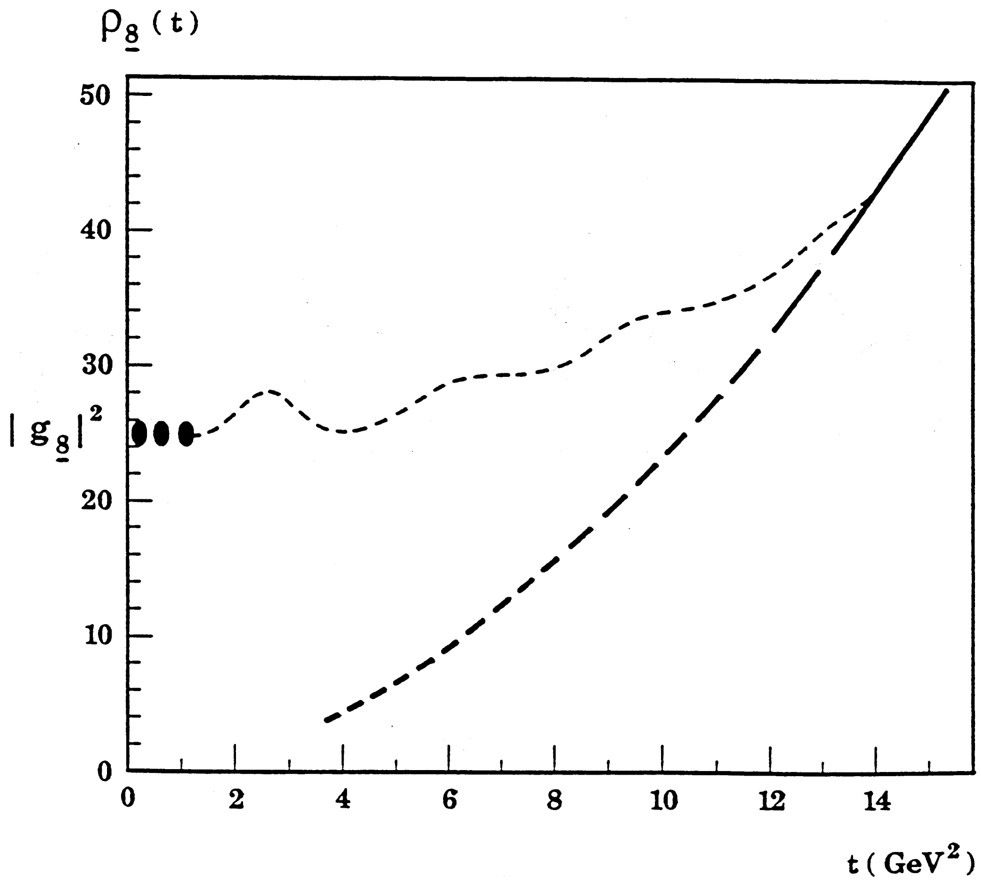


Fig . 6



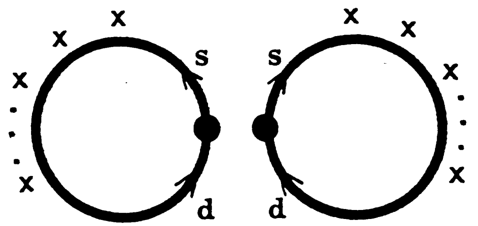
$\rho_{\Delta S=2}(t)$
3.5
3
2.5
2
1.5
1
0.5
0
0
1
2
3
4
5
$t\,(GeV^2)$

Fig. 7

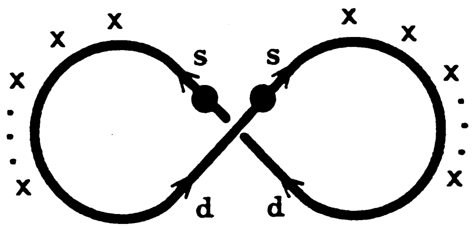


$\rho_{\underline{8}}(t)$
50
40
30
$|g_{\underline{8}}|^2$
20
10
0
0
2
4
6
8
10
12
14
$t\,(GeV^2)$

Fig. 8



(FP)



(UP)

Fig.9

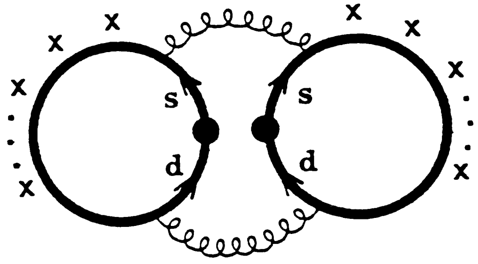
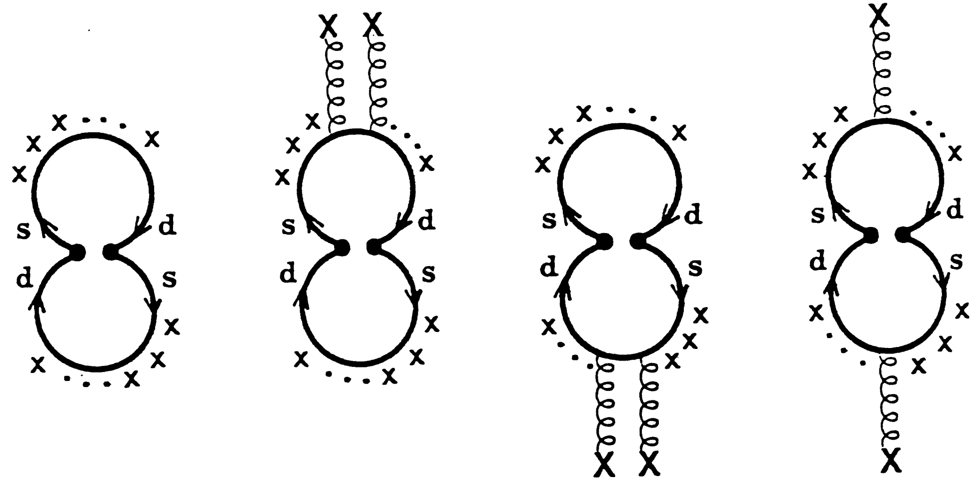


Fig.10



a) b) c) d)

Fig.11

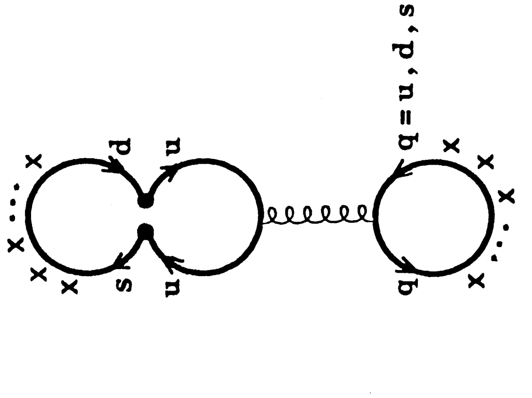


Fig. 12