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Anomalous Non-Leptonic Kaon Decays *

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Abstract

We derive an effective $\Delta S = 1$ Lagrangian incorporating the chiral anomaly for non-leptonic K decays to $O(p^4)$ in chiral perturbation theory. A complete analysis to $O(p^4)$ of the decays $K^+ \to \pi^+\pi^0\gamma$ and $K_L \to \pi^+\pi^-\gamma$ is presented including the dominant effects at $O(p^6)$.

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1. The chiral anomaly [1] is a basic property of quantum field theories with chiral fermions and thus of the standard model (SM). Its fundamental status as an intrinsically quantum mechanical violation of a classical symmetry is matched by its unique realization at the hadronic level. Unlike for most other predictions of the SM, transferring the chiral anomaly from the underlying level of quarks and gluons to the composite level of hadrons is unambiguously determined by the Wess-Zumino-Witten (WZW) functional [2] in terms of pseudoscalar meson and external gauge fields.

Experimental tests of the chiral anomaly are therefore crucial for the theoretical basis of particle physics. Although the anomaly can be interpreted as a short-distance effect, it manifests itself most directly at low energies. In the usual chiral counting [3], the anomaly is an effect of $O(p^4)$. The classical and still most precise test is the decay $\pi^0 \to 2\gamma$. There are a number of other processes, all involving either electromagnetic fields (real or virtual photons) or virtual W fields (semi-leptonic decays), where the anomaly has been checked with varying precision [3,4].

The purpose of this letter is to present the first systematic and complete discussion of the chiral anomaly for non-leptonic K decays, i.e. for the $\Delta S = 1$ non-leptonic weak interactions. In this case, the anomaly appears in two different ways. The first, non-local manifestation is in a certain sense trivial: the non-leptonic weak interaction is accompanied by a WZW vertex such as in $K_S \to \pi^0\pi^0 \to \pi^0\gamma\gamma$ [5], $K_L \to 3\pi^0 \to \pi^0\pi^0\gamma\gamma$ [6], but also $K \to 2\pi \to 3\pi\gamma$. Although fundamentally of the same origin, the second manifestation of the chiral anomaly takes the closed form of a local $\Delta S = 1$ non-leptonic Lagrangian of $O(p^4)$ (octet only):

$$\mathcal{L}_{an}^{\Delta S=1} = \frac{-ieG_8}{8\pi^2 f}\widetilde{F}^{\mu\nu}\partial_\mu\pi^0 K^+ \overleftrightarrow{D}_\nu \pi^- + \frac{\alpha G_8}{6\pi f}\widetilde{F}^{\mu\nu}F_{\mu\nu}(K^+\pi^-\pi^0 - \frac{1}{\sqrt{2}}K^0\pi^+\pi^-) + h.c. \tag{1}$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor, $\widetilde{F}_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ its dual, and $D_\nu\varphi^\pm$ denotes the covariant derivative $(\partial_\nu \mp ieA_\nu)\varphi^\pm$; $f \simeq f_\pi = 93$ MeV to lowest order in chiral perturbation theory (CHPT) and G_8 is the only coupling constant of the $O(p^2)$ $\Delta S = 1$ effective Lagrangian [cf. Eq. (4); $|G_8| = 9 \times 10^{-6}$ GeV^{-2} from $K \to 2\pi$ decays at tree level with big corrections at $O(p^4)$ [7]].

We observe that the genuine non-leptonic manifestation of the chiral anomaly is restricted to the decays (with real or virtual photons)

$$\begin{aligned} &K^+ \to \pi^+\pi^0\gamma,\ \pi^+\pi^0\gamma\gamma \\ &K_L = K_2^0 \to \pi^+\pi^-\gamma\gamma \end{aligned} \tag{2}$$

in the limit of CP conservation. The remarkable feature of these decays is that the normally dominant bremsstrahlung amplitude is strongly suppressed, making the experimental verification of the anomalous amplitude substantially easier. This suppression has different origins: $K^+ \to \pi^+\pi^0$ proceeds through the small 27-plet part of the non-leptonic weak interactions, whereas $K_L \to \pi^+\pi^-$ is CP-violating.

2. We shall restrict the discussion here to the one-photon decays $K^+ \to \pi^+\pi^0\gamma$ and $K_L \to \pi^+\pi^-\gamma$, deferring all details of these and of the two-photon channels in (2) to a separate publication [8]. We include $K_L \to \pi^+\pi^-\gamma$ in our analysis even though the anomaly appears only at $O(p^6)$ for the same reason as in $K_L \to \gamma\gamma$: the π^0 and η contributions cancel to $O(p^4)$.

The amplitude for $K(P) \to \pi_1(p_1) + \pi_2(p_2) + \gamma(q)$ is written in the usual form

$$A = \frac{\varepsilon^\mu(q)}{M_K^3}[E(x_i)(p_1 q p_{2\mu} - p_2 q p_{1\mu}) + M(x_i)\varepsilon_{\mu\nu\rho\sigma}p_1^\nu p_2^\rho q^\sigma], \tag{3}$$

$$x_i = \frac{Pp_i}{M_K^2} \quad (i=1,2), \qquad x_3 = \frac{Pq}{M_K^2}, \qquad x_1 + x_2 + x_3 = 1$$

with invariant dimensionless amplitudes $E(x_i)$ (electric type) and $M(x_i)$ (magnetic type). To detect an interference between the E and M amplitudes requires measuring the photon helicity.

The decays $K^+ \to \pi^+\pi^0\gamma$ and $K_L \to \pi^+\pi^-\gamma$ have been treated in the literature since the early days of kaon physics (see Ref. [9] for more recent work), especially also in the context of CP violation. We present here the first systematic and complete analysis to $O(p^4)$ in CHPT, including what we expect to be the dominant effects at $O(p^6)$.

3. To lowest $O(p^2)$ in the chiral expansion, the amplitudes are pure bremsstrahlung. The relevant coupling constants G_8, $G_{27}^{(3/2)}$ are defined by the lowest-order effective chiral Lagrangian for $\Delta S = 1$ non-leptonic weak interactions[1] [10]

$$\mathcal{L}_2^{\Delta S=1} = G_8(L_\mu L^\mu)_{23} + G_{27}^{(3/2)}\{L_{\mu,21}L_{13}^\mu + L_{\mu,23}(L_{11}^\mu - L_{22}^\mu)\} + h.c. \tag{4}$$

in terms of the 3×3 matrix $L_\mu = if^2 U^\dagger D_\mu U$. The matrix field U transforms as

$$U \to g_R U g_L^\dagger \tag{5}$$

under chiral transformations $g_{L,R} \in SU(3)_L \times SU(3)_R$. With only electromagnetic gauge fields, the covariant derivative is

$$D_\mu U = \partial_\mu U - ieA_\mu[Q,U] \tag{6}$$

with the quark charge matrix $Q = \mathrm{diag}\,(2/3, -1/3, -1/3)$.

The derivative couplings in $\mathcal{L}_2^{\Delta S=1}$ seem to suggest the presence of direct emission in addition to the usual bremsstrahlung diagrams. However, gauge invariance (Low's theorem [11]) and the minimal number of two derivatives conspire to produce the same overall amplitude as in the case of non-derivative vertices. This can be formulated for any number of photons as a

Theorem [8]: Consider a general Lagrangian $\mathcal{L}_2(\varphi_i, D_\mu\varphi_i)$ $(i = 0,+,-)$ of $O(p^2)$ for three scalar fields φ_i with charges $0, 1, -1$. In addition to the gauge-invariant kinetic terms, $\mathcal{L}_2$ contains only cubic interactions with at most two derivatives. Then the amplitude for φ_0, φ_+, φ_- and any number n of photons in the initial or final states has the form

$$A(\varphi_0\varphi_+\varphi_-\gamma_1\ldots\gamma_n) = A_B(\varepsilon_a, q_a, p_i)A(\varphi_0\varphi_+\varphi_-) \qquad a = 1,\ldots,n; \quad i = 0,+,- \tag{7}$$

where $A(\varphi_0\varphi_+\varphi_-)$ is the on-shell amplitude for the decay of either scalar into the other two, and $A_B(\varepsilon_a, q_a, p_i)$ is the general bremsstrahlung amplitude (ε_a: photon polarization vectors, q_a: photon momenta, p_i: scalar momenta) independent of the structure of $\mathcal{L}_2$.

Although relatively trivial for $n = 1$, the theorem becomes quite useful for $n \geq 2$. It expresses the fact that the chiral structure of vertices is irrelevant to lowest order p^2 [12].

The bremsstrahlung amplitudes for the transitions under consideration are therefore

$$A(K^+ \to \pi^+\pi^0\gamma)_B \;=\; A(K^+ \to \pi^+\pi^0)\, e\, \varepsilon^\mu(q)\, \frac{p_+ q p_{0\mu} - p_0 q p_{+\mu}}{(Pq)(p_+q)} \tag{8a}$$

$$A(K_L \to \pi^+\pi^-\gamma)_B \;=\; \varepsilon\, A(K_1^0 \to \pi^+\pi^-)\, e\, \varepsilon^\mu(q)\, \frac{p_+ q p_{-\mu} - p_- q p_{+\mu}}{(p_+q)(p_-q)} \tag{8b}$$

[1] We neglect the very small $\Delta I = 1/2$ part of the 27-plet.

or, in the notation of Eq. (3),

$$E(x_i)_B = \frac{eA(K^+ \to \pi^+\pi^0)}{M_K x_3(1/2 - x_0)} \tag{9a}$$

$$E(x_i)_B = \frac{\varepsilon e A(K_1^0 \to \pi^+\pi^-)}{M_K(1/2 - x_+)(1/2 - x_-)} \tag{9b}$$

respectively. ε is the standard CP-violation parameter in $K \to \pi\pi$ decays and we have neglected ε'. The on-shell decay amplitudes are given by

$$\begin{aligned} A(K^+ \to \pi^+\pi^0) &= 3ifG_{27}^{(3/2)}(M_\pi^2 - M_K^2) \\ A(K_1^0 \to \pi^+\pi^-) &= 2if(G_8 + G_{27}^{(3/2)})(M_\pi^2 - M_K^2). \end{aligned} \tag{10}$$

Modulo radiative and higher-order chiral corrections [12], the ratio

$$\frac{G_{27}^{(3/2)}}{G_8} = \frac{1}{32} \tag{11}$$

is small (and positive), expressing the $\Delta I = 1/2$ rule in $K \to 2\pi$ decays.

For all amplitudes of $O(p^4)$ and higher we shall assume CP conservation and $G_{27}^{(3/2)} = 0$ (octet dominance).

4. The chiral anomaly enters the magnetic amplitude M at $O(p^4)$. The WZW functional with electromagnetic external fields takes the form of an anomalous action [2]

$$S_{an} = -N_c\Gamma[U, A] \tag{12}$$

$$\Gamma[U, A] = \Gamma_0[U] + \Gamma_1[U, A] + \Gamma_2[U, A]$$

with $N_c = 3$ and

$$\Gamma_1[U, A] = \frac{e\varepsilon_{\mu\nu\rho\sigma}}{48\pi^2}\int d^4x A^\mu\langle Q(\partial^\nu U^\dagger U)(\partial^\rho U^\dagger U)(\partial^\sigma U^\dagger U) + Q(U\partial^\nu U^\dagger)(U\partial^\rho U^\dagger)(U\partial^\sigma U^\dagger)\rangle \tag{13a}$$

$$\Gamma_2[U, A] = \frac{ie^2\varepsilon_{\mu\nu\rho\sigma}}{24\pi^2}\int d^4x \partial^\mu A^\nu A^\rho\langle Q^2(U\partial^\sigma U^\dagger + \partial^\sigma U^\dagger U) + \frac{1}{2}Q(UQ\partial^\sigma U^\dagger - U^\dagger Q\partial^\sigma U)\rangle \tag{13b}$$

$$\langle A\rangle := \mathrm{tr}\ A.$$

The functional $\Gamma_0[U]$, independent of A^μ, contains monomials with at least five meson fields. Since there can be at most three pions in the final state of K decays, $\Gamma_0[U]$ and all terms in $\Gamma_1[U, A]$, $\Gamma_2[U, A]$ with more than four meson fields are irrelevant for our purposes. As a matter of fact, due to parity at most three meson fields appear.

The non-leptonic weak Lagrangian $\mathcal{L}_2^{\Delta S=1}$ in (4), written out in terms of meson fields, contains in particular bilinear terms. It is convenient to eliminate those bilinear vertices by a simultaneous diagonalization of the quadratic terms both in $\mathcal{L}_2^{\Delta S=1}$ and in the corresponding strong Lagrangian $\mathcal{L}_2$. This "weak rotation" [13] induces weak vertices when applied to the effective chiral Lagrangian of strong interactions in the presence of external gauge fields. Applying it to the WZW anomalous action in (12) and retaining only terms with at most four meson fields yields precisely the anomalous Lagrangian of Eq. (1).

Unlike for the strong interactions, there are however also odd-parity terms in the effective chiral Lagrangian $\mathcal{L}_4^{\Delta S=1}$ of $O(p^4)$ which have nothing to do with the chiral anomaly. In the notation of Ref. [14], the relevant terms can be written as

$$\mathcal{L}_4^{\Delta S=1} = a_1\langle[\Delta, \widetilde{f}_-^{\mu\nu}]u_\mu u_\nu\rangle + a_2\langle\widetilde{f}_-^{\mu\nu}u_\mu\rangle\langle\Delta u_\nu\rangle + a_3\langle\widetilde{f}_+^{\mu\nu}u_\mu\rangle\langle\Delta u_\nu\rangle + \ldots \tag{14}$$

$$\Delta = G_8 f^2 u\lambda_6 u^\dagger, \qquad u_\mu = iu^\dagger D_\mu U u^\dagger, \qquad u = \sqrt{U}$$

$$f_\pm^{\mu\nu} = eF^{\mu\nu}(uQu^\dagger \pm u^\dagger Q u), \qquad \widetilde{f}_\pm^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} f_{\pm,\rho\sigma}\,.$$

The Lagrangian $\mathcal{L}_4^{\Delta S=1}$ contributes to all decays $K^+ \to \pi^+\pi^0\gamma(\gamma)$ and $K_L \to \pi^+\pi^-\gamma(\gamma)$. However, as we shall argue in detail in Ref. [8], we expect the dimensionless couplings a_i to be very small. First of all, one cannot generate such terms by applying a weak rotation to the strong chiral Lagrangian $\mathcal{L}_4$, since $\mathcal{L}_4$ clearly has no odd- parity terms. Our main justification to set

$$a_i = 0 \qquad (i = 1, 2, 3) \tag{15}$$

in the sequel is, however, the observation [8] that one cannot generate the local couplings (14) at the quark level. The main reason is that the coupling constants a_i are dimensionless constants that can be determined in the chiral limit. However, only the anomaly, which is independent of quark masses, survives the chiral limit for odd-parity terms of $O(p^4)$ and these terms are completely covered by $\mathcal{L}_{an}^{\Delta S=1}$ in Eq. (1). Note that (15) is a scale-independent statement, since there are no terms of the type (14) in the one-loop divergences for $\mathcal{L}_2^{\Delta S=1}$ [15].

To a good approximation, the magnetic amplitudes to $O(p^4)$ are therefore

$$M_4 = -\frac{eG_8 M_K^3}{2\pi^2 f} \qquad [K^+ \to \pi^+\pi^0\gamma] \tag{16a}$$

$$M_4 = 0 \qquad [K_L \to \pi^+\pi^-\gamma], \tag{16b}$$

determined entirely by the chiral anomaly.

5. The electric amplitudes of $O(p^4)$ are particularly important because they can interfere with the bremsstrahlung amplitudes (8). In general, there are two sources contributing to E_4. The first is due to the chiral Lagrangian of $O(p^4)$

$$\mathcal{L}_4^{\Delta S=1} = -\frac{i}{4}\{w_1\langle f_+^{\mu\nu}\{\Delta, u_\mu u_\nu\}\rangle + 2w_2\langle f_+^{\mu\nu}u_\mu\Delta u_\nu\rangle + w_1'\langle f_-^{\mu\nu}\{\Delta, u_\mu u_\nu\}\rangle + 2w_2'\langle f_-^{\mu\nu}u_\mu\Delta u_\nu\rangle\} + \ldots \tag{17}$$

restricted again to terms relevant for $K \to \pi\pi\gamma$. The notation is chosen so as to conform to the one used previously: w_1, w_2 also appear in radiative K decays with a single pion in the final state [16].

In the limit of CP conservation, $\mathcal{L}_4^{\Delta S=1}$ does not contribute to $K_2^0 = K_L \to \pi^+\pi^-\gamma$. As often in CHPT, the reason is again a clash between the chiral structure $[O(p^4)]$ and an additional symmetry (CP). CP demands an amplitude $E(x_i)$ antisymmetric with respect to $x_+ \leftrightarrow x_-$ ($\pi^+ \leftrightarrow \pi^-$). This can only be fulfilled by a vanishing amplitude to $O(p^4)$:

$$E_4^{\mathrm{local}}(x_i) = 0 \qquad [K_L \to \pi^+\pi^-\gamma]. \tag{18}$$

The situation is different for $K^+ \to \pi^+\pi^0\gamma$. The Lagrangian $\mathcal{L}_4^{\Delta S=1}$ in (17) gives rise to

$$E_4^{\mathrm{local}} = -\frac{ieG_8 M_K^3}{2f}(w_1 + 2w_2 - w_1' + 2w_2'). \tag{19}$$

Comparison with the divergent part of the one-loop functional for $\mathcal{L}_2^{\Delta S=1}$ [15] shows that $w_1+2w_2-w_1'$ and w_2' are separately scale-independent and therefore so is $E_4^{\rm local}$. This is an important message for the chiral practitioner: it implies that the one-loop diagrams for both $K_2^0 \to \pi^+\pi^-\gamma$ [because of Eq. (18)] and $K^+ \to \pi^+\pi^0\gamma$ [scale independence of Eq. (19)] are necessarily finite.

Before actually turning to the loop amplitudes, we take note of the fact that the combination $w_1 + 2w_2$ is already known from $K^+ \to \pi^+\ell^+\ell^-$ decays. More precisely, since $w_1 + 2w_2$ is scale-dependent, we can extract the renormalized coupling constants at an arbitrary scale μ. A recent experiment measuring $K^+ \to \pi^+e^+e^-$ [17] has resolved the original two-fold ambiguity [16] in favour of the solution

$$w_1^r(\mu = M_\rho) + 2w_2^r(\mu = M_\rho) \simeq 0.08. \tag{20}$$

Even accounting for the normalization of $\mathcal{L}_4^{\Delta S=1}$ in (17), this is a large coupling compared with naive power counting [18]. Although we do not know the coupling constants w_1', w_2' entering Eq.(19), we could expect a relatively big amplitude $E_4^{\rm local}$ interfering constructively with the bremsstrahlung amplitude (9a):

$$\begin{aligned} \frac{E_4^{\rm local}}{E_B} &= \frac{G_8 M_K^4}{12 G_{27}^{(3/2)} f^2 (M_K^2 - M_\pi^2)} x_3(1-2x_0)(w_1+2w_2-w_1'+2w_2') \\ &\simeq 6x_3(1-2x_0)(w_1+2w_2-w_1'+2w_2')/8\times 10^{-2}. \end{aligned} \tag{21}$$

Except for small photon momenta ($x_3 \to 0$, $x_0 \to 1/2$) where bremsstrahlung is bound to win, the amplitude $E_4^{\rm local}$ cannot be neglected. Measuring the energy spectrum of the photon should allow to isolate this amplitude and determine the coupling constant combination $w_1 + 2w_2 - w_1' + 2w_2'$. Although the experiment of Abrams et al. [19] is consistent with constructive interference, it has not been possible up to now to separate the amplitudes $E - E_B$ and M experimentally [19].

The only remaining pieces to $O(p^4)$ are the loop amplitudes which must be finite. It turns out that they are also quite small for both our transitions. The complete loop amplitudes will be given in Ref. [8]. Here, we confine ourselves to a qualitative discussion. The possible loop diagrams are shown in Fig. 1 for $K_2^0 \to \pi^+\pi^-\gamma$, but the structure is exactly the same for $K^+ \to \pi^+\pi^0\gamma$. The photon must be appended in all possible places: on the strong and weak vertices and on all charged lines.

We only discuss $K_2^0 \to \pi^+\pi^-\gamma$ here where CP invariance is the important ingredient. Writing the invariant amplitude $E_4^{\rm loop}(x_i)$ as a dispersion relation we observe that it must be unsubtracted in view of (18). For the diagram of Fig. 1a all absorptive parts vanish in the CP limit. Consider, e.g., the $\pi^+\pi^-$ intermediate state. There are no absorptive parts for the diagram of Fig. 1a because both $A(K_2^0 \to \pi^+\pi^-)$ and $A(K_L \to \pi^+\pi^-\gamma)_B$ of (8b) vanish when CP is conserved.

On the other hand, there are non-zero absorptive parts[2] in the diagrams of Fig. 1b, both for the $K_1^0\pi^\pm$ and the $K^\pm\eta$ intermediate states (the $K^\pm\pi^0$ loop only contributes for $G_{27}^{(3/2)} \neq 0$). The dominant contribution is due to the $K_1^0\pi^\pm$ intermediate states leading to

$$E_4^{\rm loop}(K_1^0\pi^\pm) = \frac{ieG_8 M_K(M_K^2 - M_\pi^2)}{8\pi^2 f}[g(x_-) - g(x_+)]. \tag{22}$$

As required by CP, the amplitude is antisymmetric in x_+, x_-. The exact result for $g(x)$ [8] simplifies considerably in the chiral $SU(2)$ limit ($M_\pi = 0$):

$$g(x)|_{M_\pi=0} = \frac{1}{2}\int_1^\infty \frac{dy(y-1)^2}{y^3(y-1+2x)} \simeq \frac{1}{12}(3-2x) \quad \text{for } 0 \le x \le 1/2, \tag{23}$$

[2] Albeit in the unphysical region: the dispersion integral is purely real.

where the linear approximation is entirely sufficient for our present purposes. Comparison with the bremsstrahlung amplitude (9b) gives

$$\left|\frac{E_4^{\rm loop}(K_1^0\pi^\pm)}{E(x_i)_B}\right| = \frac{M_K^2}{6|\varepsilon|(4\pi f)^2}|x_+-x_-|(\frac{1}{2}-x_+)(\frac{1}{2}-x_-) \simeq 13|x_+-x_-|(\frac{1}{2}-x_+)(\frac{1}{2}-x_-) < 4\times 10^{-2} \quad (24)$$

where the last limit holds over the whole Dalitz plot ($M_\pi \neq 0$ for the upper bound). Despite the rather big factor $M_K^2/(96\pi^2|\varepsilon|f^2)$, the loop amplitude is at most 4% of the bremsstrahlung amplitude. Of course, this small upper bound is due to CP invariance, which is responsible for the antisymmetry in x_+, x_- of $E_4^{\rm loop}$, forbidding in particular an electric dipole amplitude. Note also that because of $\arg\varepsilon \simeq \pi/4$ there is only partial interference between the two amplitudes. Only with very high statistics would one be able to detect the loop amplitude.

The analysis for the loop contributions to $K^+ \to \pi^+\pi^0\gamma$ is deferred to Ref. [8]. Once again, they are rather small because the usually dominant $\pi\pi$ loop is absent in the octet limit. Compared with $K_2^0 \to \pi^+\pi^-\gamma$, there are two opposite effects: there is no antisymmetry suppressing the amplitude, but $G_{27}^{(3/2)}/G_8$ is bigger than $|\varepsilon|$. Altogether, the dominant $K^0\pi^-$ loop amplitude is at most 6% of the bremsstrahlung amplitude (9a).

For most purposes, we can therefore neglect the loop contributions for both $K_L \to \pi^+\pi^-\gamma$ and $K^+ \to \pi^+\pi^0\gamma$. For $K_L \to \pi^+\pi^-\gamma$ it is then perfectly justified to subtract the bremsstrahlung contribution (9b) directly in the rate, whereas for $K^+ \to \pi^+\pi^0\gamma$ the interference of the amplitudes (9a) and (19) must be taken into account.

To $O(p^4)$, there is no other sizeable amplitude for $K_L \to \pi^+\pi^-\gamma$ in addition to bremsstrahlung, in striking disagreement with experiment [20], claiming a substantial direct emission rate. In order to understand the experimental observation, we must try to estimate the dominant contributions of $O(p^6)$.

6. In general, vector meson exchange plays an important rôle both for strong and non-leptonic weak transitions. In the usual terminology, the VMD contribution to a non-leptonic weak amplitude is obtained by performing a weak transition on each external leg of the corresponding strong amplitude. The more economical method consists in applying a weak rotation [13] to the corresponding strong Lagrangian.

Because of the odd number of mesons involved in the transitions under consideration, VMD only contributes to the magnetic amplitude M. If we limit ourselves to the chiral $SU(2)$ limit ($M_\pi = 0$), the strong VMD amplitude is unique. Of the list of chiral invariant couplings which are linear in the vector meson resonance field $\widehat{V}_\mu$ [21], only the following two matter :

$$\mathcal{L}_V = -\frac{ig_V}{2\sqrt{2}}\langle \widehat{V}_{\mu\nu}[u^\mu, u^\nu]\rangle + h_V\langle \widehat{V}^\mu\{u^\nu, \widetilde{f}_{+\mu\nu}\}\rangle + \ldots \quad (25)$$

$$\widehat{V}_{\mu\nu} = \nabla_\mu \widehat{V}_\nu - \nabla_\nu \widehat{V}_\mu.$$

Both coupling constants g_V, h_V are known rather well [18,21]; the precise form of the covariant derivative ∇_μ on chiral coset space will not concern us here. Contracting the vector meson fields (only the octet contributes) yields the VMD Lagrangian of $O(p^6)$

$$\mathcal{L}_6^{\rm VMD} = -\frac{ig_V h_V}{\sqrt{2}\,M_V^2}\langle\{u_\lambda, \widetilde{f}_+^{\nu\lambda}\}\nabla_\mu[u^\mu, u_\nu]\rangle, \quad (26)$$

$$M_V \simeq M_\rho.$$

Applying a weak rotation to $\mathcal{L}_6^{\rm VMD}$ produces the desired VMD non-leptonic weak Lagrangian of $O(p^6)$. Instead of writing down the complete, rather lengthy expression [8], we exhibit immediately the corresponding amplitudes ($M_\pi = 0$):

$$K^+ \to \pi^+\pi^0\gamma: \qquad M(x_i)_6^{\rm VMD} = -C_V \tag{27a}$$

$$K_L \to \pi^+\pi^-\gamma: \qquad M(x_i)_6^{\rm VMD} = 2C_V(1-3x_3) \tag{27b}$$

$$C_V = \frac{16\sqrt{2}\,G_8 e g_V h_V M_K^5}{3M_V^2 f} = 3.5\times 10^{-8}.$$

It is by now well established [18,22] that vector meson exchange contributes in still another way (direct transitions) to non-leptonic weak amplitudes. Although such direct terms cannot be neglected in general, there is at present no unambiguous method of calculation. We follow here an approach proposed recently (weak deformation model [18]) which has fared rather well so far in comparison with experiment [23].

With details deferred once more to Ref. [8], the weak deformation model produces the following amplitudes in addition to (27):

$$K^+ \to \pi^+\pi^0\gamma: \qquad M(x_i)_6^{\rm direct} = C_V \tag{28a}$$

$$K_L \to \pi^+\pi^-\gamma: \qquad M(x_i)_6^{\rm direct} = 2C_V x_3. \tag{28b}$$

As usual, the direct terms are comparable in size to the VMD amplitudes. Our best estimate for the effects of vector meson exchange, with due reference to the strong model dependence of the direct terms, is thus

$$K^+ \to \pi^+\pi^0\gamma: \qquad M(x_i)_6^{\rm VMD+direct} = 0 \tag{29a}$$

$$K_L \to \pi^+\pi^-\gamma: \qquad M(x_i)_6^{\rm VMD+direct} = 2C_V(1-2x_3). \tag{29b}$$

The VMD and direct terms cancel for $K^+ \to \pi^+\pi^0\gamma$. The amplitude for $K_L \to \pi^+\pi^-\gamma$ is symmetric in x_+, x_- (recall $x_3 = 1 - x_+ - x_-$) due to CP.

For $K^+ \to \pi^+\pi^0\gamma$ we can now present the total amplitude which is complete to $O(p^4)$ (except for the small loop amplitude) and the best we can do to $O(p^6)$:

$$E(x_i) = \frac{2eA(K^+\to\pi^+\pi^0)}{M_K x_3(1-2x_0)} - \frac{iG_8 e M_K^3}{2f}(w_1 + 2w_2 - w_1' + 2w_2') \tag{30a}$$

$$M(x_i) = -\frac{G_8 e M_K^3}{2\pi^2 f}. \tag{30b}$$

A quantitative analysis taking the amplitude E of Eq. (30a) into account must be left to the experimentalists. The magnetic amplitude M is completely determined by the anomaly even after including vector meson exchange to $O(p^6)$. Integrated over the whole Dalitz plot, the amplitude (30b) gives rise to

$$B(K^+ \to \pi^+\pi^0\gamma)_M = 8\times 10^{-6}\,[G_8/9\times 10^{-6}\ \mathrm{GeV}^{-2}]^2. \tag{31}$$

As expected, this partial branching ratio is smaller than the total direct emission branching ratio $B(K^+ \to \pi^+\pi^0\gamma)_{DE} = (1.8\pm 0.4)\times 10^{-5}$ with a cut on the energy of the charged pion and neglecting interference in the amplitude (30a) [19,24]. Omitting the direct term (28a) and keeping only the VMD amplitude (27a) together with the anomaly would raise the branching ratio to 1.1×10^{-5}.

7. For $K_L \to \pi^+\pi^-\gamma$, vector meson exchange cannot be the dominant mechanism for the magnetic amplitude. In fact, (29b) yields a branching ratio

$$B(K_L \to \pi^+\pi^-\gamma) = 2 \times 10^{-6}[G_8/9 \times 10^{-6} \text{ GeV}^{-2}]^2, \tag{32}$$

an order of magnitude smaller than the experimental direct emission branching ratio [20]

$$B(K_L \to \pi^+\pi^-\gamma) = (2.89 \pm 0.28) \times 10^{-5}. \tag{33}$$

Omitting the direct term (28b) and keeping only the VMD amplitude (27b) would decrease the branching ratio (32) to 8×10^{-7}.

As observed by many authors [9], the dominant part of the magnetic amplitude is again given by the anomaly even though it is an effect of $O(p^6)$ for $K_L \to \pi^+\pi^-\gamma$. The cancellation between the π^0 and η poles no longer holds at $O(p^6)$ where the η' and $\eta - \eta'$ mixing enter. Although there are certainly other contributions of $O(p^6)$ such as (29b), the main part is expected to be due to the anomalous amplitude

$$M_{an} = -\frac{eG_8 M_K^3}{2\pi^2 f} F_1 \tag{34}$$

$$F_1 = \frac{1}{1-r_\pi^2} - \frac{(c-\sqrt{2}s)(c+2\sqrt{2}\rho s)}{3(r_\eta^2 - 1)} + \frac{(\sqrt{2}c+s)(2\sqrt{2}\rho c - s)}{3(r_{\eta'}^2 - 1)}$$

$$r_i = M_i/M_K, c = \cos\theta, s = \sin\theta$$

where θ is the $\eta - \eta'$ mixing angle. We have assumed nonet couplings for the WZW vertices, which is supported by the observed $P \to \gamma\gamma$ widths. The constant ρ takes into account possible deviations from nonet symmetry for the non-leptonic weak vertices ($\rho = 1$ corresponds to nonet symmetry). The factor F_1 vanishes at $O(p^4)$ (the chiral counting applies to the amplitude) because the π^0 and η poles cancel for $\theta = 0$ due to the Gell–Mann–Okubo mass formula. Beyond $O(p^4)$, F_1 depends rather strongly on θ and ρ mainly because of the factor

$$c + 2\sqrt{2}\rho s \tag{35}$$

modulating the potentially large η-pole contribution. This can be demonstrated by observing that the expression (35) vanishes for $\rho = 1$ (nonet symmetry) and $s = -1/3$ corresponding to the phenomenologically favoured value $\theta \simeq -20°$. Moreover, for $0 \le \rho < 1$ and $\theta \simeq -20°$ the η and η' contributions have opposite signs.

Although we may therefore expect an amplitude (34) which is bigger than suggested by naive chiral counting, we cannot really predict the amplitude with any certainty. However, a similar factor

$$F_2 = \frac{1}{1-r_\pi^2} - \frac{(c-2\sqrt{2}s)(c+2\sqrt{2}\rho s)}{3(r_\eta^2 - 1)} + \frac{(2\sqrt{2}c+s)(2\sqrt{2}\rho c - s)}{3(r_{\eta'}^2 - 1)} \tag{36}$$

appears in the amplitude for $K_L \to \gamma\gamma$. Although F_2 is not equal to F_1, the ratio F_1/F_2 is less strongly dependent on θ and ρ than each factor separately. There could also be additional contributions, of $O(p^6)$ and higher, which cancel in F_1/F_2. To illustrate the plausibility of this conjecture, we use the experimental rate for $K_L \to \gamma\gamma$ [24] to obtain the following prediction for the direct emission branching ratio

$$B(K_L \to \pi^+\pi^-\gamma)_{DE} = 2.5 \times 10^{-5} \mid F_1/F_2 \mid^2 \tag{37}$$

comparing well with the experimental value (33) for $F_1 \simeq F_2$. As a final remark, we note that the experimental rate for $K_L \to \gamma\gamma$ corresponds to $F_2 \simeq 0.9$, suggesting that both $K_L \to \gamma\gamma$ and $K_L \to \pi^+\pi^-\gamma$ are dominated by the pion pole contribution in (34) and (36).

8. In conclusion, the magnetic amplitudes for both $K_L \to \pi^+\pi^-\gamma$ and $K^+ \to \pi^+\pi^0\gamma$ are determined mainly by the chiral anomaly. For $K_L \to \pi^+\pi^-\gamma$, the direct emission rate is a direct measure of the anomaly. For $K^+ \to \pi^+\pi^0\gamma$, there is in addition a potentially sizeable electric amplitude interfering with bremsstrahlung. This interference must be taken into account in the experimental analysis to extract the contribution of the anomaly to the rate.

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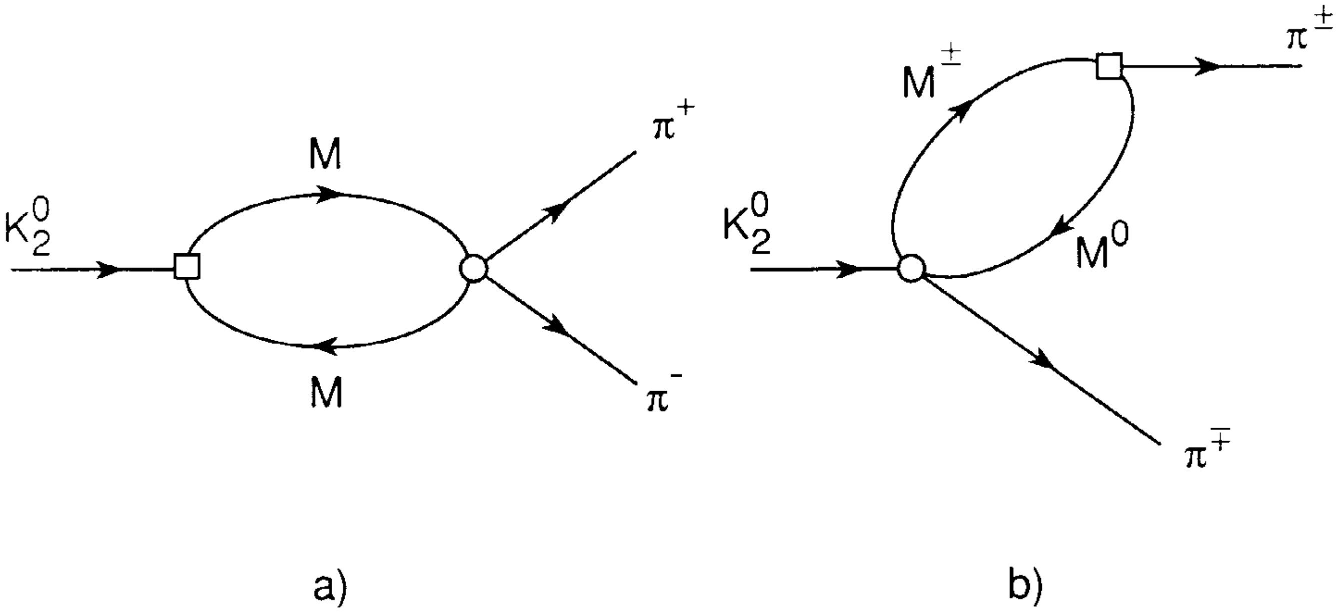

Figure 1: One-loop diagrams for $K_2^0 \to \pi^+\pi^-\gamma$. The photon must be appended on all charged lines and on both the non-leptonic weak vertex (square) and the strong one (circle).