CHIRAL SYMMETRY CONSTRAINTS ON THE K^+ INTERACTION WITH THE NUCLEAR PION CLOUD

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Abstract

The real part of the K^+ selfenergy for the interaction of the K^+ with the pion nuclear cloud is evaluated in lowest order of chiral perturbation theory and is found to be exactly zero in symmetric nuclear matter. This removes uncertainties in that quantity found in former phenomenological analyses and is supported by present experimental data on K^+ nucleus scattering.

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Systematic discrepancies between the impulse approximation (IA) and the data in K^+ nucleus scattering [1, 2] have stimulated work searching for the contribution of the K^+ interaction with the nuclear pion cloud [3, 4, 5] among others [6]. The relatively weak strength of the K^+N interaction led to the suggestion in [3] that the K^+ interaction with the virtual pion cloud in the nucleus could provide a correction of the size needed to explain the discrepancies. The idea was elaborated further in [4] and the contribution to the K^+ nucleus selfenergy, $\delta \Pi$, from the virtual pions was evaluated. Further work was done in [5] improving the work of [4] in two aspects: i) the static approximation of [4] was improved by using a covariant formalism and ii) further contributions to the imaginary part of the K^+ selfenergy were found which increased appreciably this magnitude. It was also found in [5] that the imaginary part of the K^+ selfenergy alone provided sufficient strength to account for the discrepancies of the IA with the data, account taken of extra nuclear corrections stemming from correlations and evaluated in [2]. However, both in [4] and [5], which used the same $K^+\pi$ amplitude, it was shown that the real part of the K^+ selfenergy was not precisely determined and the results depended strongly on the off-shell extrapolation of the $K^+\pi$ amplitude. These uncertainties in $\delta\Pi$ resulted in large uncertainties in the K^+ nucleus cross section at small K^+ energies, while at large energies $(T_{K^+} \sim 500 - 600$ MeV) these uncertainties were drastically reduced (see fig. 1). On the other hand, the imaginary part of the K^+ selfenergy produced sizeable corrections in the large energy region while, for obvious reasons of phase space, it provided no correction at small energies. Comparison with the data at low energies (see fig. 1) led to the conclusion in [5] that the K^+ nucleus data were best reproduced with the IA selfenergy and a contribution from the pion cloud such that $\operatorname{Re}(\delta \Pi) \simeq 0$.

In the present work we show that in the evaluation of $\operatorname{Re}(\delta\Pi)$ there are chiral terms, additional to the terms from the $K^+\pi$ interaction used in [4, 5], which cancel exactly these latter terms leading to $\operatorname{Re}(\delta\Pi) = 0$.

The K^+ selfenergy in [5], for the interaction of a K^+ with symmetric matter is given by

$$\delta \Pi(k) = i \int \frac{d^4 q}{(2\pi)^4} \, \delta D(q) \, \frac{3}{2} t^0(k,q;k,q) \,, \tag{1}$$

with

$$\delta D(q) = D(q) - D_0(q) - \rho \left(\frac{\partial D(q)}{\partial \rho}\right)_{\rho=0}$$
(2)

where k is the K^+ momentum, t^0 the isoscalar $K^+\pi$ amplitude and $D_0(q), D(q)$ the free pion propagator and the pion propagator in the nuclear medium, respectively. Note that in eq. (1) one evaluates the interaction of a K^+ with a pion in the medium and makes two subtractions: the one of the free pion, because it is already contained in the free K^+ mass, and the terms linear in ρ , the nuclear density, because they are contained in the IA K^+ selfenergy, $\Pi = t^0_{KN} \rho$. One thus picks up corrections of type $\rho^{\alpha} (\alpha > 1)$, essentially ρ^2 terms in eq. (1). They are depicted in figs. 2 a, d. The novelty introduced in this paper is the realization that in a systematic chiral perturbation expansion there are terms like those shown in figs. 2b, c, e, f which appear at the same order of the terms in figs. 2a, 2d and which cancel them exactly. This fact was already noticed in [8], studying the interaction of pions with the nuclear pion cloud, where a *partial* cancellation of the analogous terms to fig. 2, substituting the K^+ by a pion, was found. A different technical approach to that problem with a different interpretation of the results of [8] is given in [9].

In addition to the diagrams of fig. 2 one has the corresponding ones to a, b, c substituting one or the two ph excitations by a Δh excitation. However, there are no corrections of the type of diagrams d, e, f with Δh excitation since there is no Fermi sea of $\Delta's$. To clarify this point let us see the meaning of the arrows in the baryon lines. In the diagrams 2 a, b, c the meaning of the arrows is the standard one for particles and holes. In figs. 2d, e, f, one of the lines pointing down correspond to a hole line while the other one corresponds to the Pauli correction to the nucleon propagator, $2\pi i n(\vec{p})\delta(p^0 - \epsilon(\vec{p}))$, which appears when the terms linear in ρ are subtracted in the equivalent diagrams with the bubble representing a standard ph excitation (see appendix of ref. [7] for the appropriate many body details). In the absence of a Fermi sea of Δ 's, one has $n_{\Delta}(\vec{p}) = 0$ and the terms d, e, f with $\Delta's$ do not appear. We will omit other many-body details here since they will be unnecessary to prove that $\text{Re}(\delta\Pi) = 0$.

In order to evaluate the diagrams of fig. 2 we need the $KK\pi\pi$ vertex, the πNN vertex and the $\bar{N}NKK\pi$ contact term. All of them are directly obtained from the standard Chiral Perturbation Theory Lagrangian [10, 11, 12, 13], which contains the most general low-energy interactions of the pseudoscalar and baryon octets, consistent with the chiral symmetry properties of QCD. At lowest order in derivatives and quark masses it reads:

$$\mathcal{L}_{ChPT} = \mathcal{L}_2 + \mathcal{L}_1^{(B)} + \cdots$$
(3)

$$\mathcal{L}_2 = \frac{f^2}{4} \langle \partial_\mu U^+ \partial^\mu U + M(U+U^+) \rangle , \qquad (4)$$

$$\mathcal{L}_{1}^{(B)} = \langle \bar{B}i\gamma^{\mu}\nabla_{\mu}B\rangle - M_{B}\langle \bar{B}B\rangle + \frac{D+F}{2}\langle \bar{B}\gamma^{\mu}\gamma_{5}u_{\mu}B\rangle + \frac{D-F}{2}\langle \bar{B}\gamma^{\mu}\gamma_{5}Bu_{\mu}\rangle, \qquad (5)$$

where the symbol $\langle \rangle$ denotes the flavour trace of the SU(3) matrices and the dots in eq. (3) stand for interactions with more than two baryons and higher-order terms in the momentum expansion. The 3×3 unitary matrix

$$U(\phi) = u(\phi)^2 = \exp(i\sqrt{2}\Phi/f)$$
(6)

gives a very convenient parametrization of the pseudoscalar Goldstone fields

$$\Phi(x) \equiv \frac{\vec{\lambda}}{\sqrt{2}} \vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta_8 \end{pmatrix}.$$
 (7)

The baryon octet is collected in the 3×3 matrix B [11, 12, 13], which in our particular case, where only protons and neutrons are involved, simplifies to

$$B(x) = \begin{pmatrix} 0 & 0 & p \\ 0 & 0 & n \\ 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}(\lambda_4 + i\lambda_5)p + \frac{1}{2}(\lambda_6 + i\lambda_7)n.$$
(8)

The matrix M contains the explicit breaking of chiral symmetry generated by the non-zero quark masses. In the isospin limit $(m_u = m_d)$, it takes the simple form

$$M = \begin{pmatrix} m_{\pi}^2 & 0 & 0 \\ 0 & m_{\pi}^2 & 0 \\ 0 & 0 & 2m_K^2 - m_{\pi}^2 \end{pmatrix}.$$
 (9)

The baryon covariant derivative ∇_{μ} and u_{μ} are given by

$$\nabla_{\mu}B = \partial_{\mu}B + [\Gamma_{\mu}, B]; \qquad \Gamma_{\mu} = \frac{1}{2}(u^{+}\partial_{\mu}u + u\partial_{\mu}u^{+}); \qquad (10)$$

$$u_{\mu} = iu^{+}\partial_{\mu}Uu^{+} . \tag{11}$$

At lowest order, f equals the pion decay constant, $f = f_{\pi} = 92.4$ MeV, while D and F are the usual parameters of semileptonic hyperon decays, which satisfy $D + F = g_A = 1.257 \pm 0.003$.

Expanding $u(\phi)$ in a power series in Φ , we easily obtain the needed interactions:

$$\mathcal{L}_2 = \frac{1}{2} \langle \partial_\mu \Phi \partial^\mu \Phi - M \Phi^2 \rangle + \frac{1}{12f^2} \langle (\partial_\mu \Phi \Phi - \Phi \partial_\mu \Phi)^2 + M \Phi^4 \rangle + O(\Phi^6) , \qquad (12)$$

$$u_{\mu} = -\frac{\sqrt{2}}{f}\partial_{\mu}\Phi + \frac{\sqrt{2}}{12f^{3}}(\partial_{\mu}\Phi\Phi^{2} - 2\Phi\partial_{\mu}\Phi\Phi + \Phi^{2}\partial_{\mu}\Phi) + O(\Phi^{5}).$$
(13)

Only the $O(\Phi^4)$ interaction terms in \mathcal{L}_2 and the (D + F) and (D - F) couplings in $\mathcal{L}_1^{(B)}$ [i.e., the $O(\Phi)$ and $O(\Phi^3)$ terms in u_{μ}] contribute to the processes in fig. 2. The relevant baryon vertices are

$$\mathcal{L}_{1}^{(B)} = \frac{D+F}{2} \Big\{ \bar{p}\gamma^{\mu}\gamma_{5}u_{\mu}^{11}p + \bar{n}\gamma^{\mu}\gamma_{5}u_{\mu}^{22}n + \bar{n}\gamma^{\mu}\gamma_{5}u_{\mu}^{21}p + \bar{p}\gamma^{\mu}\gamma_{5}u_{\mu}^{12}n \Big\} + \frac{D-F}{2} \Big\{ \bar{p}\gamma^{\mu}\gamma_{5}u_{\mu}^{33}p + \bar{n}\gamma^{\mu}\gamma_{5}u_{\mu}^{33}n \Big\},$$
(14)

where u_{μ}^{ij} denote the (i, j) element of the matrix u_{μ} .

The terms in fig. 2 can be calculated in a systematic way from the Lagrangians of eqs. (12-14) and the cancellations appear in a subtle way in spin-isospin saturated nuclear matter. In order to show the cancellations it is useful to introduce the effective vertex of fig. 3 which allows then to rewrite the diagrams of the first row of fig. 2 as in fig. 4. We are only interested in the kinematics of the present case where

 $k^{\mu}=k'^{\mu}$, $k^2=k'^2=m_K^2,$ and the results in the following refer to this particular case.

In the effective vertex for a π^0 the pp and nn matrix elements can be cast into an isospin form given by $[N^T \equiv (p \ n)]$

$$-it_0 \equiv \frac{1}{12f^3} \bar{N} \gamma^{\mu} \gamma_5 q_{\mu} \left\{ -(D+F) \frac{1}{2} \tau_3 + (D+F) \frac{1}{2} (1+\tau_3) - (D-F) \right\} N, \quad (15)$$

where the first term in the bracket corresponds to the one half of the pion pole term in the effective vertex and the other two to the contact term. The terms with τ_3 cancel in eq. (15) and one obtains only the isoscalar contribution

$$-it_0 = \frac{3F - D}{2} \frac{1}{12f^3} \bar{N}\gamma^{\mu}\gamma_5 Nq_{\mu} \,. \tag{16}$$

It is then trivial to see the cancellation in fig. 4 for the neutral pions since the $\pi^0 \bar{N}N$ vertex below the effective vertex in the ph excitation has a +, - sign for p or n, and the sum over protons and neutrons cancels in symmetric nuclear matter.

For the case of charged pions it is easy to see that the sum of the matrix elements of the effective vertex for a π^+ and a π^- vanishes identically. In this case the effective vertices are

$$-it_{-} = \frac{1}{12\sqrt{2}f^{3}}\bar{N}\gamma^{\mu}\gamma_{5}\tau_{-}N\left\{-(D+F)\frac{q_{\mu}}{q^{2}-m_{\pi}^{2}}(q^{2}-m_{\pi}^{2}-6q\cdot k) + (D+F)(q_{\mu}-3k_{\mu})\right\}$$
(17)

$$-it_{+} = \frac{1}{12\sqrt{2}f^{3}}\bar{N}\gamma^{\mu}\gamma_{5}\tau_{+}N\left\{-(D+F)\frac{q_{\mu}}{q^{2}-m_{\pi}^{2}}(q^{2}-m_{\pi}^{2}+6q\cdot k)+(D+F)(q_{\mu}+3k_{\mu})\right\}$$
(18)

with q^{μ}, k^{μ} the pion and kaon momenta, respectively. Again the first term in the bracket corresponds to one half of the pion pole term and the second one to the contact term. Since the $\pi \bar{N}N$ vertex below the effective vertex has the same sign in p, n or n, p excitation ($\sqrt{2}$ isospin factor for charged pions), then the cancellation in fig. 4 holds exactly provided we have equal number of protons and neutrons in the system.

The isospin formalism used is particularly useful. The cancellation of the diagrams in figs. 2d, e, f follows the same arguments as before. On the other hand it is trivial to show the cancellation of the Δ terms. For this purpose we define the effective vertex at the quark level with quarks u, d. These vertices in all the isospin combinations have the same expression as in eqs. (16-18) up to some normalization factor. Then the sum of matrix elements of this effective vertex for π^+ and π^- vanishes identically while for a π^0 it is an isoscalar. If we take SU(4) wave functions for N and Δ then we recover the former matrix elements for the nucleons while the effective $\bar{N}\Delta KK\pi^0$ vertex is identically zero and the sum of these effective vertices with π^+ and π^- also vanishes. Since the lower vertex $\pi \bar{N} \Delta$ has the same isospin matrix element for all charged pion cases $(\sqrt{1/3} \text{ isospin coefficient})$, the cancellation for charged pions holds also exactly in symmetric nuclear matter. We have also checked these cancellations using the effective chiral Lagrangian incorporating explicitly the baryon decuplet [11, 15].

In summary, the approach followed here is different and more systematic than the phenomenological one followed in refs. [4, 5]. While in this latter work there were uncertainties in $\operatorname{Re}(\delta\Pi)$ tied to the off-shell extrapolation of the $K\pi$ amplitude, we have shown here that in lowest order in chiral perturbation theory there is an exact cancellation between different terms, for symmetric nuclear matter, and $\operatorname{Re}(\delta\Pi)$ is zero. Furthermore, the empiral analysis carried out in ref. [5] clearly favoured $\operatorname{Re}(\delta\Pi) = 0$. This is certainly a welcome feature. Obviously one may wonder what would happen if one goes to higher orders in the chiral expansion. The K^+ sector might be a privileged one for convergence of the chiral expansion, since one is far away from resonances in the $K^+\pi$ channel (the $K^*(892)$ considered in [4] has a negligible influence in the $K\pi$ amplitude in the range of energies considered in [4, 5]) and there are no resonances in the K^+N channel. In spite of that, a systematic expansion at higher orders, (as done in ref. [16] for the elastic KN scattering matrix) looking also for $\operatorname{Im}(\delta\Pi)$ as an alternative to the empirical evaluation done in refs. [4, 5], would be an interesting step forward.

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Figure Captions:

- Fig. 1: Comparison with the data for the total cross section of the impulse approximation (IA) and the (IA) plus the K^+ selfenergy form the interaction with the pion cloud, $\delta\Pi$. The results are from ref. [5] and the data form ref. [14]. The curve labelled IA shows the IA results. The curves labelled I, III, IV correspond to different values of Re($\delta\Pi$) coming from different $K^+\pi$ off-shell extrapolations and the same Im($\delta\Pi$) [5]. The curve labelled IV (solid line) corresponds to Re($\delta\Pi$) = 0.
- Fig. 2: Diagrams which enter the evaluation of eq. (1) (a, d) plus the extra terms (b, c, e, f) which appear in the chiral expansion at lowest order. Solid and dashed lines denote nucleons and mesons, respectively. In-coming and out-going mesons are kaons, whereas the exchanged mesons are pions.
- Fig. 3: Diagrammatic definition of the effective $\bar{N}NKK\pi$ vertex, depicted by the circlecross.
- Fig. 4: Detail of the diagrams of figs. 2a, b, c for the different isospin combinations in terms of the effective vertex, together with their cancellation.

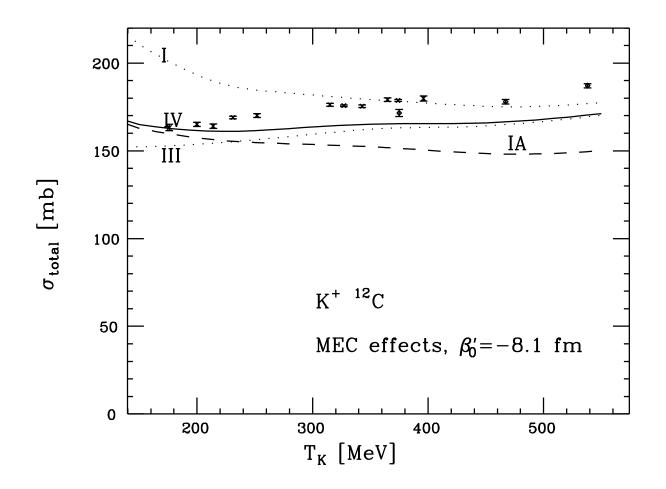
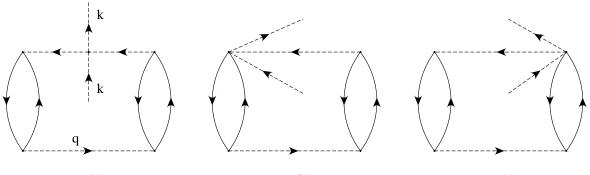


Figure 1









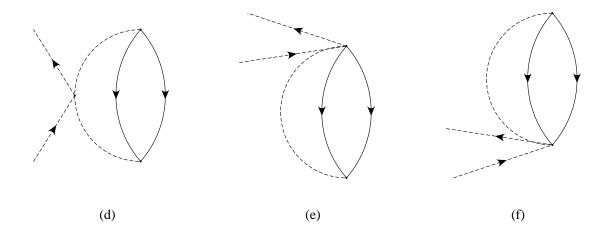


Figure 2

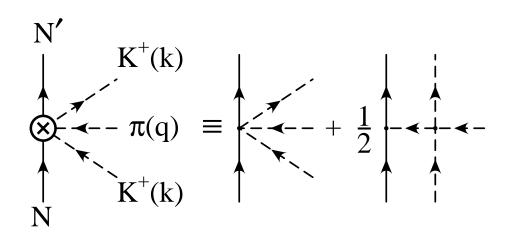


Figure 3

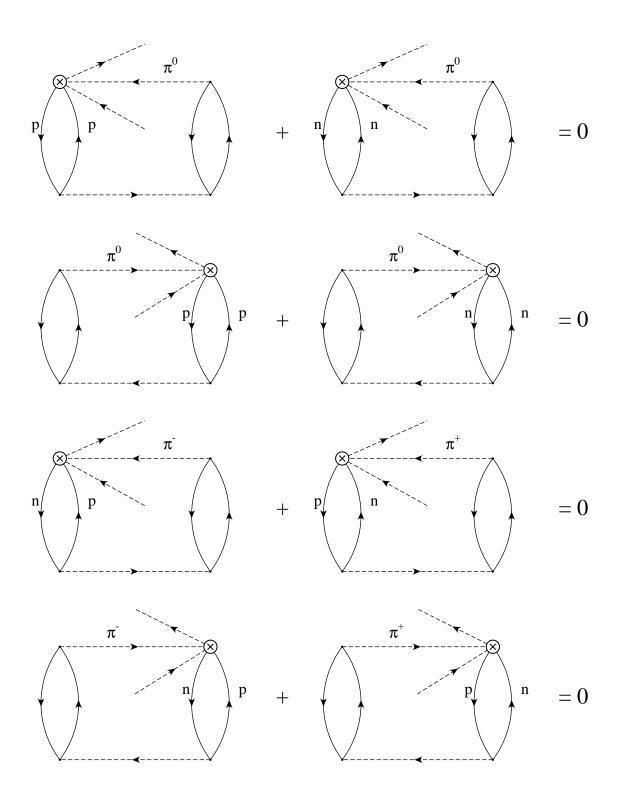


Figure 4