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Tau Polarization at the Z peak from the Acollinearity between both τ-Decay Products

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Abstract

We discuss the possibility of measuring the tau polarization distribution at the Z peak by studying the angular correlated distributions between the decay products of the tau pairs. We give explicit formulae for the energy-energy correlation and for the acollinearity distribution of the decay products of the τ-pairs, for both the cross section and the forward-backward asymmetry, as a function of the τ-polarization and the Z-polarization. These distributions provide an independent method of measuring the τ polarization, with a sensitivity comparable to the one that can be achieved by studying the energy spectrum of the produced pion in the decay $\tau \to \pi\nu_\tau$, and possibly better systematic errors. This new technique thus offers the prospect to achieve an increased precision in the determination of the electroweak mixing angle.

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1 Introduction

One of the main physics goals of the experiments at LEP is to perform high precision tests of the standard theory; this implies the careful determination of the theory parameters and the search for possible indications of departures from it. The LEP experiments have already achieved an impressive level of precision for the Z mass and width [1], while the precise determination of the mixing angle $\sin^2\theta_W$ remains as one of the major goals for the immediate future. The best determination of $\sin^2\theta_W$ is provided by several asymmetries to be determined on the Z peak [2,3]. These include the forward-backward asymmetry, the final state polarization asymmetry and the left-right cross-section asymmetry for a longitudinally polarized beam.

For the parity violating polarization asymmetry, the experimentally accessible process is the τ-pair production $e^-e^+ \to \tau^-\tau^+$. The use of the subsequent τ-decays as polarization analyzers of its spin state has been considered by several authors [4], following the principles discussed in the work of Tsai [5]. The polarization of the tau can be measured from the energy distribution of a given decay product; the sensitivity of the decay channel to the longitudinal τ-polarization depends on the spin of the product whose spectrum is measured. For the spin one ρ and a_1 channels, the loss of sensitivity with respect to the π channel can partially be recovered [6,7] by measuring the helicity of these systems from the subsequent $\rho \to \pi\pi$ and $a_1 \to \pi\pi\pi$ decays. Experimental results for the τ- polarization, analyzed along these lines, have been reported recently by the LEP Collaborations [8] and also at lower energies [9].

In this paper our main interest is to study the possibility of extracting the τ-polarization from angular correlations of the decay products of both taus produced at the Z-peak. For completeness we also study the energy-energy correlations, which have already been discussed in the literature [10]. The potentialities of the correlated distributions as analyzers of the τ properties cover an ample range, which includes not only information on the spin state of a single tau but also on the spin correlations between both parent taus [11,12] and on the parameters of the charged-current tau decay processes [10,13]. Here we shall show the sensitivity of these correlation observables to the polarization distribution of the τ. Namely, in terms of the acollinearity angle between the two decay products, we identify the acollinearity distribution of both the cross-section and the forward-backward asymmetry as the observables of interest. Assuming a standard theory description of the τ-decay, these measurable quantities depend only on P_τ and P_Z.

The interest of the method proposed here lies, first, in the possibility of reducing the overall error in the measurement of the τ polarization, since the angular correlations provide an *independent* measurement of this quantity which can, therefore, be combined with the value derived using the conventional method, and second, in the fact that the systematics associated with angular measurements in LEP-type detectors should be different than that associated with the measurement of the energy spectra and potentially smaller. It is interesting to notice that the present results for the τ-polarization based on the energy spectrum have already a systematic uncertainty comparable to the statistical error, even with the small number of available events.

The paper is organized as follows. In Section 2 we study the observables of the τ's produced at the Z-peak. The correlated distributions between both tau decay products are given in Section 3, where their information content on the parent τ-properties is shown. In Section 4 we discuss the general features of the experimental method to extract the τ polarization, and give sensitivities based on statistical errors, assuming universality $P_Z = P_\tau$. The possibility to perform a separate analysis of P_Z and P_τ is explored as well at the end of this Section. We summarize and present our conclusions in Section 5.

2 Observables for $e^-e^+ \to \tau^-\tau^+$

To lowest order in perturbation theory, the amplitude for the process

$$e^-(k_1) + e^+(k_2) \to \tau^-(p_1, s_1) + \tau^+(p_2, s_2)$$

is given in the standard theory by the sum of the contributions associated with γ- and Z-exchanges. The dominant electroweak radiative corrections around the Z resonance are included in the improved Born approximation [17]. This yields

$$\begin{aligned} M_{Born} &= i\,\frac{4\pi\alpha(M_Z^2)}{s}\bar{v}(k_2)\gamma_\mu u(k_1)\bar{u}(p_1)\gamma^\mu v(p_2) \\ &+ i\,\sqrt{2}GM_Z^2\rho\,\frac{\bar{v}(k_2)\gamma_\mu(v_e - a_e\gamma_5)u(k_1)\;\bar{u}(p_1)\gamma^\mu(v_\tau - a_\tau\gamma_5)v(p_2)}{s - M_Z^2 + i\frac{s}{M_Z^2}M_Z\Gamma_Z}\,, \end{aligned} \tag{1}$$

where α is the running fine structure constant, M_Z (Γ_Z) the physical mass (width) of the Z, G is the Fermi coupling constant, ρ includes the weak corrections due to a heavy top and v_f (a_f) is the vector (axial vector) coupling of the fermion f to the Z. Up to small non-universal vertex corrections, these neutral current couplings are given by

$$v_f = I_3^f - 2Q_f\bar{s}_W^2 \qquad a_f = I_3^f\,, \tag{2}$$

with $\bar{s}_W^2$ the effective mixing angle. From this amplitude, we obtain the differential cross section for the $\tau^-\tau^+$ production with polarization vectors s_1 and s_2, respectively. Using in the LAB frame the coordinate system where the z-axis is along the τ-direction, shown in Fig. 1, we get

$$\begin{aligned} \frac{d\sigma}{d\Omega}(s_1, s_2) &= \frac{1}{16s}\{(1 + s_{1z}^* s_{2z}^*)[F_0(s)(1+\cos^2\theta) + F_1(s)2\cos\theta] \\ &- (s_{1z}^* + s_{2z}^*)[G_0(s)(1+\cos^2\theta) + G_1(s)2\cos\theta] \\ &+ [(s_{1y}^* s_{2y}^* - s_{1x}^* s_{2x}^*)F_2(s) + (s_{1y}^* s_{2x}^* + s_{1x}^* s_{2y}^*)G_2(s)]\sin^2\theta\}\,, \end{aligned} \tag{3}$$

where s^* designes the polarization vector in the rest frame of the τ. The functions $F_i(s)$ ($i = 0,1,2$) are associated with parity conserving terms, whereas $G_i(s)$ ($i = 0,1,2$) correspond to parity violating observables. They are given in Table 1, where the contributions from γ-exchange, Z-exchange and their interference are shown separately. We have introduced the following notation:

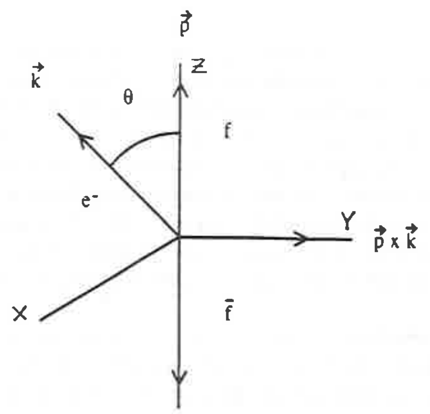

Figure 1: Coordinate system in the LAB frame.

$$P(s) = \frac{\sqrt{2}GM_Z^2\rho}{4\pi}\frac{s}{s - M_Z^2 + i\frac{s}{M_Z^2}M_Z\Gamma_Z} \qquad (4)$$

and

$$\begin{aligned} C_0 &= (|v_e|^2+|a_e|^2)(|v_\tau|^2+|a_\tau|^2) \\ C_1 &= 4\,\mathrm{Re}(v_e a_e^*)\ \mathrm{Re}(v_\tau a_\tau^*) \\ C_2 &= (|v_e|^2+|a_e|^2)(|a_\tau|^2-|v_\tau|^2) \\ \\ D_0 &= (|v_e|^2+|a_e|^2)\,2\,\mathrm{Re}(v_\tau a_\tau^*) \\ D_1 &= 2\,\mathrm{Re}(v_e a_e^*)(|v_\tau|^2+|a_\tau|^2) \\ D_2 &= -(|v_e|^2+|a_e|^2)\,2\,\mathrm{Im}(v_\tau a_\tau^*) \end{aligned} \qquad (5)$$

All observables but $G_2(s)$ have an interference term contribution proporcional to $\mathrm{Re}\,P(s)$, which vanishes on top of the Z. As a consequence, the Z-exchange term dominates over all the other contributions. The behaviour of $G_2(s)$ is different: it is not only a parity violating piece, but also a T-odd term, so that it gets contributions from the absorptive part of the Z-propagator (in the γ-Z term) and from the absorptive part of the vertex (in the Z-Z term) at comparable levels [14].

From the main result shown in eq. (3), we identify the different asymmetries and polarization observables. The forward-backward asymmetry of the event distribution, when the final polarizations are not observed, is given by:

$$A_{FB} = \frac{3}{4}\frac{F_1(s)}{F_0(s)} \qquad (6)$$

The longitudinal polarization of the τ lepton is a parity-violating observable, whose

	γ	γZ	Z
$F_0(s)$	$\alpha^2(M_Z^2)$	$2\alpha(M_Z^2)v_e v_\tau\,\mathrm{Re}\,P(s)$	$C_0\|P(s)\|^2$
$F_1(s)$		$2\alpha(M_Z^2)a_e a_\tau\,\mathrm{Re}\,P(s)$	$C_1\|P(s)\|^2$
$F_2(s)$	$-\alpha^2(M_Z^2)$	$-2\alpha(M_Z^2)v_e v_\tau\,\mathrm{Re}\,P(s)$	$C_2\|P(s)\|^2$
$G_0(s)$		$2\alpha(M_Z^2)v_e a_\tau\,\mathrm{Re}\,P(s)$	$D_0\|P(s)\|^2$
$G_1(s)$		$2\alpha(M_Z^2)a_e v_\tau\,\mathrm{Re}\,P(s)$	$D_1\|P(s)\|^2$
$G_2(s)$		$2\alpha(M_Z^2)v_e a_\tau\,\mathrm{Im}\,P(s)$	$D_2\|P(s)\|^2$

Table 1: *Contributions to the functions appearing in the differential cross section, eq. (3), from γ-exchange, Z-exchange and their interference.*

angular distribution is read from eq. (3)

$$P_\tau(\cos\theta) = -\frac{G_0(s)(1+\cos^2\theta) + G_1(s)\,2\cos\theta}{F_0(s)(1+\cos^2\theta) + F_1(s)\,2\cos\theta} \qquad (7)$$

As a consequence of the assumed helicity-conserving vertex, we see that $P_{\tau^+} = P_{\tau^-}$, and the longitudinal spin correlation is equal to one.

The angular asymmetry in the distribution of the longitudinal polarization is seen in eq. (7) to be controlled by $G_1(s)/F_0(s)$. On top of the Z resonance, this becomes the polarization of the Z,

$$P_Z = -\frac{D_1}{C_0} = -\frac{2\,\mathrm{Re}(v_e a_e^*)}{(|v_e|^2+|a_e|^2)}\,, \qquad (8)$$

disregarding the γ-exchange term. The Z-polarization (8) for unpolarized beams coincides with the left-right asymmetry of the cross section for a longitudinally polarized beam. With only the Z-exchange term, the distribution (7) can be written as:

$$P_\tau(\cos\theta) = \frac{P_\tau + P_Z\ 2\cos\theta/(1+\cos^2\theta)}{1 + P_\tau P_Z\ 2\cos\theta/(1+\cos^2\theta)}\,, \qquad (9)$$

where P_τ is the polarization averaged over all τ production angles

$$P_\tau = -\frac{D_0}{C_0} = -\frac{2\,\mathrm{Re}(v_\tau a_\tau^*)}{(|v_\tau|^2+|a_\tau|^2)} \qquad (10)$$

As we see, the forward-backward asymmetry is then given by $A_{FB} = \frac{3}{4}P_\tau P_Z$.

The analysis of the transverse spin correlations observed in eq. (3) has been performed in ref. [11], where it was shown that they lead to aplanarities of the decay products. Here we concentrate on the distributions of the decay products of both taus in order to extract P_τ (and P_Z). It is well known that this quantity is highly sensitive to the electroweak mixing angle in the standard theory.

3 Correlated Decay Distributions

Let us consider the 1-prong decay process

$$\tau^{\pm}(s^*) \to x_i^{\pm} + \ldots$$

The decay distribution in the τ rest frame (variables with $*$) is

$$d\Gamma^{\pm}_{(s^*)} = \Gamma(\tau \to \nu_\tau + x_i + \ldots)\, [A_1^{(i)} \mp \alpha_i\, \vec{q}_i^{\,*} \cdot \vec{s}^{\,*}\, A_2^{(i)}]\, d^3 q_i^* \tag{11}$$

For the purely leptonic processes, $\alpha_i = -\frac{1}{3}$ $(i = e, \mu)$ and the corresponding distribution functions may be written as [15]

$$A_1^{(i)} = \frac{a_1^{(i)}}{4\pi\lambda_i E_i^*} \qquad A_2^{(i)} = \frac{a_2^{(i)}}{4\pi\lambda_i E_i^*}\,, \tag{12}$$

which in the standard theory for the charged current weak interactions are:

$$a_1^{(i)} = -\frac{m_i^2}{6} + \frac{W_i}{2}E_i^* - \frac{1}{3}E_i^{*2} \tag{13}$$

$$a_2^{(i)} = -W_i + \frac{P_i}{2} + E_i^*\,, \tag{14}$$

where W_i (P_i) is the maximal energy (momentum) of the leptons in the τ rest frame

$$W_i = \frac{m_\tau^2 + m_i^2}{2m_\tau} \qquad P_i = \frac{m_\tau^2 - m_i^2}{2m_\tau} \tag{15}$$

The normalization factor λ_i turns out to be in the standard theory

$$\begin{aligned}\lambda_i &= \int_0^{P_i} \frac{a_1^{(i)}}{E_i^*}\, q_i^{*2} dq_i^* \\ &= \frac{1}{12}P_i^3 W_i - \frac{1}{8}m_i^2 P_i W_i + \frac{1}{8}m_i^4 \ln\frac{m_\tau}{m_i}\end{aligned} \tag{16}$$

For the semileptonic processes, $(x_i = \pi, \rho, a_1)$, we have a two-body decay with polarization analyzers

$$\alpha_\pi = 1 \qquad \alpha_i = \frac{m_\tau^2 - 2m_i^2}{m_\tau^2 + 2m_i^2} \quad (i = \rho, a_1)\,, \tag{17}$$

and momentum distribution [15]

$$A_1^{(i)} = P_i\, A_2^{(i)} = \frac{1}{4\pi}\frac{\delta(q_i^* - P_i)}{P_i^2} \tag{18}$$

The correlated decay distribution for the sequential process

$$e^- + e^+ \to \tau^- + \tau^+ \to x_1^- + x_2^+ \ldots$$

has been discussed for the low energy factories [16,15], where the production mechanism is due only to γ exchange. Including both γ and Z exchanges, we obtain from eqs. (3) and (11)

$$\begin{aligned}\frac{d^8\sigma}{d\Omega\, d^3q_1^* d^3q_2^*} = & K(s)\Big\{[A_1^{(1)}A_1^{(2)} + \alpha_1\alpha_2 A_2^{(1)}A_2^{(2)} q_1^* q_2^* \cos\theta_1^* \cos\theta_2^*] \\ & [F_0(s)(1+\cos^2\theta) + F_1(s) 2\cos\theta] \\ - & [\alpha_1 A_2^{(1)} A_1^{(2)} q_1^* \cos\theta_1^* + \alpha_2 A_1^{(1)} A_2^{(2)} q_2^* \cos\theta_2^*] \\ & [G_0(s)(1+\cos^2\theta) + G_1(s) 2\cos\theta] \\ + & \alpha_1\alpha_2 A_2^{(1)} A_2^{(2)} q_1^* q_2^* \sin\theta_1^* \sin\theta_2^* \\ & \sin^2\theta[F_2(s)\cos(\phi_2^* - \phi_1^*) + G_2(s)\sin(\phi_2^* - \phi_1^*)]\Big\}\end{aligned} \tag{19}$$

with

$$K(s) = \frac{1}{4s} Br(\tau \to x_1) Br(\tau \to x_2) \tag{20}$$

This distribution depends on the τ-direction θ in the LAB, as well as on the momenta $\vec{q}_1^{\,*}$, $\vec{q}_2^{\,*}$ of the particles x_1, x_2 in the $\tau^{\mp}$ rest frames, respectively. Without a sufficiently precise reconstruction of the τ-direction, one cannot measure these variables experimentally. Instead, one can rewrite the cross section in the LAB frame and use variables independent of the τ-direction. These transformations are performed below.

In the LAB frame, the momentum of x_i (q_i, θ_i, ϕ_i) $(i = 1,2)$ is referred to the direction of the $\tau^{\mp}$, $\vec{p}_i$ $(i = 1,2)$, respectively. The boost connecting the LAB and the $\tau^{\mp}$ rest frames is defined by the parameters $\gamma \simeq M_Z/(2m_\tau)$, $\beta = \sqrt{1-\gamma^{-2}}$. The variables in the two systems are related by

$$\begin{aligned}E_i^* &= \gamma(E_i - \beta q_i \cos\theta_i) \\ q_i^* \cos\theta_i^* &= \gamma(q_i \cos\theta_i - \beta E_i) \\ q_i^* \sin\theta_i^* &= q_i \sin\theta_i \\ \phi_i^* &= \phi_i\,,\end{aligned} \tag{21}$$

with the Jacobian

$$\frac{\partial(q_i^*, \cos\theta_i^*)}{\partial(q_i, \cos\theta_i)} = \frac{E_i^*}{q_i^{*2}}\frac{q_i^2}{E_i} \tag{22}$$

The range of the angular variables depends on the decay channel. For mesons, the two-body decay implies that the energy and the angle are not independent variables, so that

$$\theta_i = \zeta_i \equiv \arccos\left(\frac{\gamma E_i - W_i}{\beta\gamma q_i}\right) \tag{23}$$

For leptons, which appear in three-body decays, one has however $0 \le \theta_i \le \zeta_i$.

The variables θ_1, ϕ_1; θ_2, ϕ_2, which refer to the x_1 and x_2 directions with respect to their parent $\tau^{\mp}$, are not directly observable. The measurable quantities are instead the opening angle θ_{12} between x_1 and x_2 and the orientation of the x_1 x_2 momentum plane with respect to the electron beam. To analyze the correlated distribution in the LAB frame, it is highly convenient to define the **acollinearity angle** ϵ between the two decay products: $\epsilon = \pi - \theta_{12}$. The acollinearity angle ϵ is related to the azimuthal angle

ϕ between the $\tau^- x_1^-$ momentum plane and $\tau^+ x_2^+$ momentum plane in the LAB frame ($\phi = \phi_1 + \phi_2$) in the following way:

$$\cos\epsilon = \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2\cos\phi \tag{24}$$

The transverse and normal spin correlations between both taus only appear in eq. (19) associated with the azimuthal variable $\phi' = \phi_1 - \phi_2$, related to the orientation of the $x_1 x_2$ momentum plane. These correlations then give rise to aplanarity observables, considered in [11]. In this paper ϕ' is integrated out and we restrict our analysis to the acollinearity distribution

$$\begin{aligned}\frac{d^6\sigma}{d\Omega dE_1 dE_2 d\cos\epsilon} &= 4\pi K(s) q_1\ q_2 \int \frac{E_1^* E_2^*}{Q} d\cos\theta_1 d\cos\theta_2 \\ &\quad \Big\{[A_1^{(1)}A_1^{(2)} + \alpha_1\alpha_2 A_2^{(1)}A_2^{(2)} q_1^* \cos\theta_1^* q_2^* \cos\theta_2^*] \\ &\quad [F_0(s)(1+\cos^2\theta) + F_1(s) 2\cos\theta] \\ &- [\alpha_1 A_2^{(1)} A_1^{(2)} q_1^* \cos\theta_1^* + \alpha_2 A_1^{(1)} A_2^{(2)} q_2^* \cos\theta_2^*] \\ &\quad [G_0(s)(1+\cos^2\theta) + G_1(s) 2\cos\theta]\Big\}\end{aligned} \tag{25}$$

where E_i^* and $q_i^* \cos\theta_i^*$, defined in the $\tau^\pm$ rest frames, are given in eq. (21) in terms of the LAB variables E_i, q_i, $\cos\theta_i$, and Q^{-1} is the Jacobian $\frac{\partial\phi}{\partial(\cos\epsilon)}$, with

$$Q = \sqrt{\sin^2\theta_1 \sin^2\theta_2 - (\cos\theta_1\cos\theta_2 - \cos\epsilon)^2} \tag{26}$$

The non-observable angles θ_1 and θ_2 are integrated out in eq. (25). This result still depends on the production angle θ of the τ with respect to the beam. However, we can expand eq. (25) to the dominant order in the dilation factor γ, taking into account that $\theta_1, \theta_2, \epsilon \sim \gamma^{-1}$. As the angular distribution of the τ is explicitly dependent on $\cos\theta$, we can substitute $\cos\theta = \cos\theta_- + \mathcal{O}\left(\frac{1}{\gamma}\right)$, where θ_- is the polar angle between the x^- decay product and the e^- beam. This will be the physical meaning of $\cos\theta$ from now on.

3.1 Energy-Energy Correlation

The general distribution $\frac{d^4\sigma}{d\Omega dE_1 dE_2}$ can be obtained by integrating eq. (25) over ϵ:

$$\frac{d^4\sigma}{d\Omega dE_1 dE_2} = \int_{\cos(\theta_1-\theta_2)}^{\cos(\theta_1+\theta_2)} \frac{d^6\sigma}{d\Omega dE_1 dE_2 d\cos\epsilon} d\cos\epsilon \tag{27}$$

$$\begin{aligned} &= 4\pi^2 K(s)\ q_1\ q_2 \int E_1^* E_2^* d\cos\theta_1 d\cos\theta_2 \\ &\quad \Big\{[A_1^{(1)}A_1^{(2)} + \alpha_1\alpha_2 A_2^{(1)}A_2^{(2)} q_1^* \cos\theta_1^* q_2^* \cos\theta_2^*] \\ &\quad [F_0(s)(1+\cos^2\theta) + F_1(s) 2\cos\theta] \\ &- [\alpha_1 A_2^{(1)} A_1^{(2)} q_1^* \cos\theta_1^* + \alpha_2 A_1^{(1)} A_2^{(2)} q_2^* \cos\theta_2^*] \\ &\quad [G_0(s)(1+\cos^2\theta) + G_1(s) 2\cos\theta]\Big\}\end{aligned} \tag{28}$$

The energy-energy correlated distributions are considerably simpler than the angular distribution, due to the fact that the integration over $\cos\theta_1$ is independent of the integration over $\cos\theta_2$. Thus for leptons

$$\cos\zeta_i \leq \cos\theta_i \leq 1\ ,$$

and for mesons

$$\cos\theta_i = \cos\zeta_i\ ,$$

with ζ_i as given in eq. (23).

After integrating over the τ-direction, the distribution function around the Z resonance for any combination of leptons and mesons in the τ-decays can therefore be written as:

$$\begin{aligned}\frac{1}{\sigma}\frac{d^2\sigma}{dE_1 dE_2} &= \Big\{H_0^{(1)}(E_1)H_0^{(2)}(E_2) + \alpha_1\alpha_2 H_1^{(1)}(E_1)H_1^{(2)}(E_2) \\ &\quad + P_\tau[\alpha_1 H_1^{(1)}(E_1)H_0^{(2)}(E_2) + \alpha_2 H_0^{(1)}(E_1)H_1^{(2)}(E_2)\Big\}\end{aligned} \tag{29}$$

For leptons one gets [15]

$$\begin{aligned} H_0^{(i)}(E_i) &= \frac{q_i}{2\lambda_i}\int_{\cos\zeta_i}^1 a_1^{(i)} d\cos\theta_i \\ &= \frac{q_i}{2\lambda_i}\sum_{n=0}^{2} R_n^{(i)}(E_i)\frac{1}{n+1}(1-\cos^{n+1}\zeta_i)\end{aligned} \tag{30}$$

$$\begin{aligned} H_1^{(i)}(E_i) &= \frac{q_i}{2\lambda_i}\int_{\cos\zeta_i}^1 a_2^{(i)}(q_i^*\cos\theta_i^*) d\cos\theta_i \\ &= \frac{q_i}{2\lambda_i}\sum_{n=0}^{2} S_n^{(i)}(E_i)\frac{1}{n+1}(1-\cos^{n+1}\zeta_i)\ ,\end{aligned} \tag{31}$$

where the coefficients $R_n^{(i)}(E_i)$ and $S_n^{(i)}(E_i)$ are defined by writting the functions $a_1^{(i)}$ and $a_2^{(i)}(q_i^*\cos\theta_i^*)$ in the LAB frame

$$a_1^{(i)} = \sum_{n=0}^{2} R_n^{(i)}(E_i)\cos^n\theta_i \qquad a_2^{(i)}(q_i^*\cos\theta_i^*) = \sum_{n=0}^{2} S_n^{(i)}(E_i)\cos^n\theta_i\ , \tag{32}$$

whereas for mesons

$$H_0^{(i)}(E_i) = \frac{1}{2\gamma\beta P_i} \tag{33}$$

$$H_1^{(i)}(E_i) = \frac{1}{2\gamma\beta P_i}\frac{E_i - \gamma W_i}{\gamma\beta P_i} \tag{34}$$

Around the Z-resonance, we have found that the s-dependence of the energy correlation given by eq. (29) is quite smooth. When this observable is integrated over E_2, we reproduce the well known single τ-decay spectrum [18].

3.2 Angular Correlations

The angular distribution $\frac{d^2\sigma}{d\cos\theta d\epsilon}$ is obtained from eq. (25) by integrating out E_1 and E_2, as well as the azimuthal angle of the τ. This yields

$$\frac{d^2\sigma}{d\cos\theta d\epsilon} = 2\pi K(s)\left\{[Q_{11}(\epsilon)+\alpha_1\alpha_2 Q_{22}(\epsilon)][F_0(s)(1+\cos^2\theta)+F_1(s)2\cos\theta] - [\alpha_1 Q_{21}(\epsilon)+\alpha_2 Q_{12}(\epsilon)][G_0(s)(1+\cos^2\theta)+G_1(s)2\cos\theta]\right\} , \quad (35)$$

where the functions $Q_{ij}(\epsilon)$ are defined as

$$Q_{ij}(\epsilon) \equiv 4\pi\sin\epsilon\int\frac{E_1^* E_2^*}{Q}\, q_1\, q_2 dE_1 d\cos\theta_1 dE_2 d\cos\theta_2 \, A_i^{(1)}(q_1^*\cos\theta_1^*)^{i-1}A_j^{(2)}(q_2^*\cos\theta_2^*)^{j-1} \quad (36)$$

They satisfy

$$\int d\epsilon Q_{ij}(\epsilon) = \delta_{i1}\delta_{j1} , \quad (37)$$

showing that the information on the polarization distribution is lost when eq. (35) is integrated over ϵ.

As we can see from eq. (36), $Q_{12}(\epsilon) = Q_{21}(\epsilon)$ when the τ^+ and τ^- decay products have the same mass. The use of the acollinearity angle ϵ as an observable introduces a constraint between the (unobservable) quantities $\cos\theta_1, \cos\theta_2$ due to the relation (24). Specifying ϵ defines an allowed region in (θ_1, θ_2)-space, which is a rectangle whose boundaries are obtained for coplanar events ($\cos\phi = \pm 1$). This rectangle defines also an allowed region in (E_1, E_2)-space for a given ϵ and confines the region of integration over $\cos\theta_1$ and $\cos\theta_2$ to its interior. As a result, the integration over $\cos\theta_1$ depends on $\cos\theta_2$ and one cannot factorize the functions $Q_{ij}(\epsilon)$ in a similar way to the energy-energy correlation (see eq. (29)).

Although the functions $Q_{ij}(\epsilon)$ depend on the final states x_1 and x_2, the general shape is common for all the 1-prong decays considered *. In Fig. 2 we show those corresponding to the $\tau^- \to \pi^-\nu_\tau$, $\tau^+ \to \pi^+\bar{\nu}_\tau$ channel, referred to from now on as the $\pi^-\pi^+$ channel, at $s = M_Z^2$.

From eq. (35), we can define two independent observables which at the peak of the Z resonance depend on P_τ and P_Z only. Once we have integrated eq. (35) over $\cos\theta$, the normalized acollinearity distribution is given by

$$\frac{1}{\sigma}\frac{d\sigma}{d\epsilon} = Q_{11}(\epsilon) + \alpha_1\alpha_2 Q_{22}(\epsilon) + P_\tau\{\alpha_1 Q_{21}(\epsilon) + \alpha_2 Q_{12}(\epsilon)\} \quad (38)$$

which is an observable linear in P_τ. The s-dependence of the acollinearity distribution appears to be very smooth around the Z-resonance.

We can also define the acollinearity distribution of the forward-backward asymmetry

$$A_{FB}(\epsilon) = \frac{\int_{\cos\theta>0} d\sigma - \int_{\cos\theta<0} d\sigma}{\int_{\cos\theta>0} d\sigma + \int_{\cos\theta<0} d\sigma} , \quad (39)$$

*Indeed, in the limit $m_i = 0$ these functions become universal for all the leptonic and semileptonic decays [11].

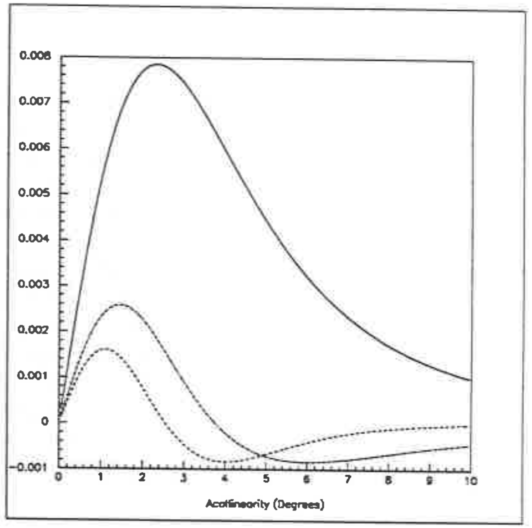

Figure 2: *Functions $Q_{ij}(\epsilon)$ for the $\pi^-\pi^+$ decay channel. Continuous line: Q_{11}; dashed line: Q_{22}; dashed-dotted line: $Q_{21} = Q_{12}$*

which on top of the Z has the following expression:

$$A_{FB}(\epsilon) = \frac{3}{4}\frac{P_Z\{P_\tau[Q_{11}(\epsilon)+\alpha_1\alpha_2 Q_{22}(\epsilon)] + \alpha_1 Q_{21}(\epsilon) + \alpha_2 Q_{12}(\epsilon)\}}{Q_{11}(\epsilon)+\alpha_1\alpha_2 Q_{22}(\epsilon) + P_\tau[\alpha_1 Q_{21}(\epsilon)+\alpha_2 Q_{12}(\epsilon)]} \quad (40)$$

Combined with eq. (38), this can separate the observables P_Z and P_τ. Alternatively, if universality is assumed, eqs. (38) and (40) provide independent methods to determine P_τ. We have found that the s-dependence of the acollinearity distribution of the forward-backward asymmetry around the Z-resonance is particularly strong, so we expect an appreciable sensitivity to the initial state radiation.

4 Measurement of P using correlated distributions

In the previous Section we have written explicit expressions for the correlated distributions as a function of P_Z and P_τ assuming the Standard Model for the τ-decays. In the next two subsections 4.1 and 4.2 we will also assume the universality hypothesis and write $P = P_\tau = P_Z$, while in subsection 4.3 we will explore the sensitivity of this method to separate out P_τ and P_Z.

The main problem to extract a measurement of P from the correlated distributions discussed in Section 3 is the relatively low efficiency for identifying τ final states (typically around 60-70 % for $\tau \to e\nu_e\nu_\tau$ and $\tau \to \mu\nu_\mu\nu_\tau$ and 40-50 % for $\tau \to \pi\nu_\tau$). Thus, the identification of the final state of the *two* taus would reduce the statistical significance of the measurement and a method whose "intrinsic" sensitivity to P is *a priori* excellent could become unpractical when low identification efficiencies and branching ratios are taken into account.

The "intrinsic" sensitivity of a given method to P can be quantified defining the "Analyzing power"of the method,

$$1/A_x = \frac{\delta_x}{\delta_\pi}|_{(N_x=N_\pi)} , \qquad (41)$$

where for the method X the inverse of the analyzing power A_x is defined as the error of the method in the determination of P relative to the error of the "reference" method (taken by convention to be the determination of P from the energy spectrum of the pion in the case of the decay $\tau \to \pi\nu_\tau$) when the number of events in the distribution used to measure P, N_x, is the same than the number of events in the reference method distribution N_π.

On the other hand, even if a method has a high analyzing power, its practical use can be restricted by low detection efficiencies and branching ratios. Indeed, the error on P measured from both the ALEPH and DELPHI collaborations [8] is lower for the channel $\tau \to \rho\nu_\tau$, in spite of the fact that its analyzing power is smaller than the one of the channel $\tau \to \pi\nu_\tau$ but with a branching ratio roughly two times larger and similar identification efficiencies. The "real" sensitivity of a method to P is quantified defining the "Sensitivity" of the method,

$$S_x = A_x\sqrt{\frac{N_x}{N_\pi}}|_{(N_{\tau\tau})} , \qquad (42)$$

where the analyzing power of the method is weighted with the statistical error due to the number of events that one has in the distribution used to measure P when the number of initial tau pairs $N_{\tau\tau}$ is fixed; N_x, N_π are obtained multiplying $N_{\tau\tau}$ by the appropiate efficiencies and branching ratios.

Measured channel		$\tau \to e\nu_e\nu_\tau$	$\tau \to \mu\nu_\mu\nu_\tau$	$\tau \to \pi\nu_\tau$	$\tau \to \rho\nu_\tau$	$\tau \to a_1\nu_\tau$
$d\sigma/dE$	A	0.31	0.31	1.00	0.93	0.40
$d\sigma/dE$	S	0.45	0.45	1.00	1.27	0.32

Table 2: *Summary of the different sensitivities for the standard method*

In Table 2 we show the analyzing power (A) and the sensitivity (S) of the "standard method" of measuring P, based in the study of the energy distributions [†] of single tau

[†] In the case of the spin one mesons, ρ and a_1, P is measured by separating the two helicity states of the meson, see [8].

decays. To obtain the analyzing power of the different channels we have fitted the energy spectra of a large number of events ($2 \cdot 10^5$) generated by the KORALZ Monte Carlo [20].

To compute the sensitivities of the different methods we assume "typical" detection efficiencies for the different tau decay channels, i.e. 50 % for $\tau \to \pi\nu_\tau, \tau \to \rho\nu_\tau, \tau \to a_1\nu_\tau$ and 75 % for $\tau \to e\nu_e\nu_\tau, \tau \to \mu\nu_\mu\nu_\tau$. We take all the exclusive and inclusive tau decay branching ratios from the Particle Data Group [19]. Notice that, by definition, the sensitivity and the analyzing power of the channel $\tau \to \pi\nu_\tau$ is one. All the other channels have analyzing powers smaller than one, while the sensitivity of the $\tau \to \rho\nu_\tau$ channel is bigger than one due to its large branching ratio. It is clear that, for a new method to be competitive, sensitivities of the order of one or larger are desirable.

The sensitivity of a method based on the explicit identification of the decay of the two taus is not competitive with the sensitivity of the standard method due to the double suppression associated with low identification efficiencies and branching ratios. In order to be able to achieve a competitive measurement of P it is necessary to avoid the identification of the two exclusive final states. The obvious solution is, therefore, to identify one of the final states and average over the final states of the decay of the second tau. Indeed, we classify the events where each tau decays to a single charged track (plus any number of neutrals) in three categories; (i) $\pi - X$, or *inclusive pion* where an identified pion recoils versus a charged track, (ii) $l - X$ or *inclusive lepton* where a lepton is identified in one side and the track on the other side is required *not to be a pion* to avoid double counting, and (iii) $h - h$ or *inclusive hadron* where we require each one of the two tracks of the event *to be* neither an electron nor a pion. This third category corresponds to states where a $\rho(a_1)$ recoils versus either ρ or a_1. We refer to the method of measuring P from correlated distributions using the combination of these three categories as the "inclusive method". We show below that this inclusive method using angular correlated distributions provides a sensitivity competitive with the standard method.

The calculation of the correlated distributions in Section 3 has been performed in the framework of the improved Born approximation, without including initial and final state radiation. In order to be able to study their effect on the observable quantities we have used the KORALZ Monte Carlo [20], which does include weak and QED radiative corrections as well as corrections to the decay of the tau pairs. To estimate the sensitivity of the correlated distributions to the τ-polarization we have generated a large number of events ($2 \cdot 10^5$) for different exclusive final states (i.e, $\pi - \pi, e - \mu, \pi - \mu$) as well as for the inclusive final states described above ($\pi - X, l - X, h - h$). The analyzing power is determined by fitting the corresponding distributions. The sensitivity of each method is finally obtained by weigthing with the statistical errors, taking into account the identification efficiencies and branching ratios for each exclusive final state, as described above.

4.1 Measurement of P using energy correlations

As we have already mentioned, the correlated energy distributions can offer [10,13] a method to determine the τ-decay parameters when the Standard Model is not assumed. We have shown in eq. (29) that they also provide an alternative method to measure

P. However these correlated distributions are not independent of the single tau energy distributions normally used and therefore they do not constitute an independent method of measuring P. Also, the systematic errors should be very similar to the standard method.

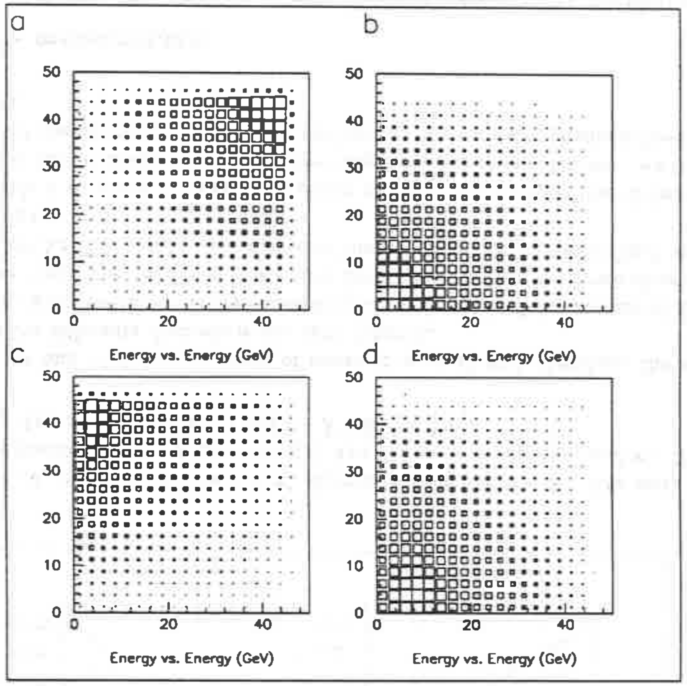

Figure 3: *Energy-energy correlation distribution for $\pi-\pi$ and $\pi-X$ channels. (a) $\pi-\pi$, P =1; (b) $\pi-\pi$, P =-1; (c) $\pi-X$, P =1; (d) $\pi-X$, P =-1.*

From equation (29) in Section 3 we can see that the energy-energy correlated distributions are linear in P. The best analyzing power corresponds obviously to the final state $\pi-\pi$ (both taus decaying via $\tau \to \pi\nu_\tau$) while the best sensitivity is achieved by the inclusive pion $\pi-X$. In Fig. 3 (a,b) we show the energy-energy distributions in the $\pi-\pi$ channel for $P=\pm1$, respectively; whereas Fig. 3 (c,d) show the corresponding distributions for the $\pi-X$ channel for $P=\pm1$. Table 3 shows the analyzing power and the sensitivity that can be achieved using energy-energy correlations for different final states. Notice that the best sensitivity is achieved with the inclusive final states $\pi-X, l-X$ and $h-h$.

Measured channel		$e-\mu$	$\pi-\pi$	$\pi-l$	$l-X$	$\pi-X$	$h-h$
$d^2\sigma/dE_1dE_2$	A	0.54	1.30	1.05	0.43	1.03	0.45
$d^2\sigma/dE_1dE_2$	S	0.34	0.29	0.39	0.64	0.92	0.40

Table 3: *Summary of the different sensitivities for energy correlation distributions*

4.2 Measurement of P using angular correlations

Unlike the correlated energy distributions, the angular spin correlation distributions constitute a method to measure the polarization P independent of the standard method. In addition this method has different systematic errors, which may be smaller than those of the standard method. In Section 3 we defined two independent observables: the "acollinearity distribution of the cross section", and the "acollinearity distribution of the forward-backward asymmetry". The first observable is linear on P

$$\frac{1}{\sigma}\frac{d\sigma}{d\epsilon} = Q_{11}(\epsilon) + \alpha_1\alpha_2 Q_{22}(\epsilon) + P\{\alpha_1 Q_{21}(\epsilon) + \alpha_2 Q_{12}(\epsilon)\} \tag{43}$$

Again, the $\pi-\pi$ the channel has the best analyzing power while the best sensitivity is achieved by the $\pi-X$ channel. In Fig. 4 we show the $d\sigma/d\epsilon$ distribution for the $\pi-\pi$ channel (a) and the $\pi-X$ channel (b) for $P=+1$ (solid lines) and $P=-1$ (dashed lines).

From eq. (29), setting $P = P_\tau = P_Z$ we have

Measured channel		$e-\mu$	$\pi-\pi$	$\pi-l$	$l-X$	$\pi-X$	$h-h$
$d\sigma/d\epsilon$	A	0.33	0.98	0.36	0.23	0.49	0.26
$d\sigma/d\epsilon$	S	0.21	0.22	0.13	0.22	0.44	0.25
$A_{FB}(\epsilon)$	A	0.52	0.66	0.42	0.50	0.55	0.30
$A_{FB}(\epsilon)$	S	0.33	0.15	0.16	0.75	0.50	0.29

Table 4: *Summary of the different sensitivities for angular correlated distributions*

$$A_{FB}(\epsilon) = \frac{3}{4}\frac{P^2[Q_{11}(\epsilon) + \alpha_1\alpha_2 Q_{22}(\epsilon)] + P[\alpha_1 Q_{21}(\epsilon) + \alpha_2 Q_{12}(\epsilon)]}{Q_{11}(\epsilon) + \alpha_1\alpha_2 Q_{22}(\epsilon) + P[\alpha_1 Q_{21}(\epsilon) + \alpha_2 Q_{12}(\epsilon)]} \tag{44}$$

Here the dependence on P is non-linear. However, the cross section term in eq. (43) $(Q_{11}(\epsilon) + \alpha_1\alpha_2 Q_{22}(\epsilon))$, which is independent of P, becomes suppressed by the factor P^2 in the numerator of eq. (44), thus increasing the sensitivity of the term proportional to $P\ (\alpha_1 Q_{21}(\epsilon) + \alpha_2 Q_{12}(\epsilon))$ in the $A_{FB}(\epsilon)$-observable. As a consequence, $A_{FB}(\epsilon)$ becomes linear for small P. The slope of $A_{FB}(\epsilon)$ with P depends on the final exclusive channel

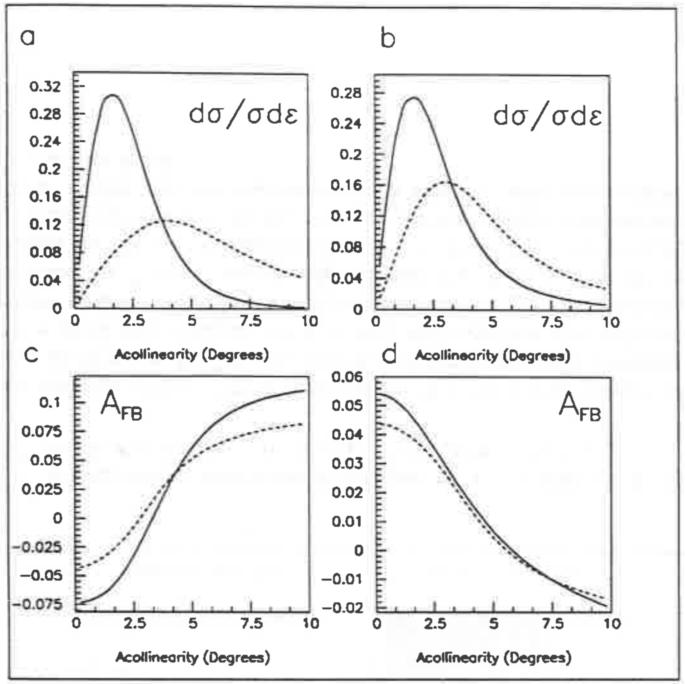

Figure 4: *(a) cross section for $\pi - \pi$; (b)cross section for $\pi - X$. The solid (dashed) line corresponds to $P = +1$ ($P = -1$). (c) A_{FB} for $\pi - \pi$ (solid line) and $\pi - X$ (dashed line); (d) A_{FB} $e - \mu$ (solid line) and $l - X$ (dashed line).*

$\pi - \pi, l - \pi$ and $l - l$, but its range of variation is small and, therefore, the analyzing power of the differents channels is not very different.

In Fig. 4 (c) we show the A_{FB} distribution for $P = -0.16$ in the case of the $\pi - \pi$ final state (solid line) and $\pi - X$ (dashed line) while in Fig. 4 (d) we show the A_{FB} distribution for $P = -0.16$ in the case of the $e - \mu$ final state (solid line) and $l - X$ (dashed line).

In Table 4 we show the analyzing power and the sensitivity of the different exclusive and inclusive channels using the two angular correlations defined. As it was the case for the energy-energy correlated distributions the inclusive channels have the best sensitivity.

4.3 P_Z measurement

As it was shown in Section 3 the expression for the acollinearity distribution of the forward-backward asymmetry contains P_Z and P_τ. Therefore it is possible to make a separate analysis of both quantities. To estimate the sensitivity on P_Z that can be

achieved using this method, we simply fix ‡ the value of P_τ in eq. (40) and perform the fit of the $A_{FB}(\epsilon)$ distribution in terms of P_Z. The sensitivities for the different channels are given in Table 5.

Measured channel		$e - \mu$	$\pi - \pi$	$\pi - l$	$l - X$	$\pi - X$	$h - h$	incl. Method
$A_{FB}(\epsilon)$	A	0.30	0.60	0.30	0.32	0.51	0.24	
$A_{FB}(\epsilon)$	S	0.19	0.10	0.11	0.48	0.46	0.21	0.70

Table 5: *Summary of sensitivities for P_Z using the acollinearity distribution of the forward-backward asymmetry*

Notice that the acollinearity distribution of the forward-backward asymmetry offers a promising way of measuring P_Z with good precision if a statistically significant sample of τ-pairs is obtained at LEP.

5 Conclusions

	P						P_Z
	Standard Method	Standard Method (without ρ)	$d^2\sigma/dE_1dE_2$	$A_{FB}(\epsilon)$	$d\sigma/d\epsilon$	$d\sigma/d\epsilon \oplus A_{FB}(\epsilon)$	$A_{FB}(\epsilon)$
S	1.76	1.22	1.20	0.95	0.59	1.12	0.70

Table 6: *Summary of the overall sensitivities for the different methods*

In this paper we have discussed the use of the correlated distributions of the decay products of τ-pairs produced at the Z peak to measure the polarization distribution of the tau. These are the energy-energy correlation, the acollinearity distribution of the cross section and the acollinearity distribution of the forward-backward asymmetry.

We have given explicit formulae for these measurable correlated distributions in terms of the average τ-polarization and the Z-polarization. Assuming a standard theory description for the τ-decay, the corresponding polarization analyzer functions (in energy-energy and in the acollinearity angle) have been obtained for the different decay channels of both taus.

We have compared the sensitivities of the measurements using correlated distributions to those of the standard method based on the energy and angular distributions

‡We assume the value of P_τ to be measured by any other method, i.e. the standard method or the acollinearity distribution of the cross section. We have checked that, allowing P_τ to vary inside the 1-σ error, the sensitivities shown in Table 5 remain similar.

of the decay products of a single τ. The sensitivity of the diferent methods discussed is shown in Table 6. Notice that the standard method including all the channels has still the highest sensitivity. However, we notice that its high sensitivity is mainly due to the final state $\tau \to \rho\nu_\tau$, whose large systematic error (already at the level of the present statistical errors of the LEP experiments [8]) will make its use very difficult when larger samples of τ pairs become available. Excluding this channel, the sensitivity of the standard method becomes comparable to those obtained by using correlated distributions.

On the other hand, the dominant source of systematic error in the standard method is the dependence of the acceptance with the energy spectrum. The use of angular variables such as the acollinearity angle, which are very well measured by the LEP detectors, may reduce substantially these systematic uncertanties. It seems, therefore, that the use of this technique can allow the improvement in the measurement of the tau polarization in the immediate future.

We also suggest a method to measure P_Z which seems promising for moderately high statistical τ-samples using the acollinearity distribution of the forward-backward asymmetry.

We conclude that the study of the correlated angular distributions of the decay products of τ-pairs can offer a complementary source of information on the polarization distribution of the τ, leading to better constraints on the Standard Model.

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