

# Updating the Unitarity Triangle: Top Quark Mass Versus Nonperturbative Uncertainties

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## Abstract

We summarize the present knowledge on the non-perturbative hadronic inputs needed in the analysis of  $B^0$ - $\bar{B}^0$  mixing and the CP-violating parameter  $\varepsilon$  of the  $K^0$ - $\bar{K}^0$  system. Using this information, together with the recently determined value of the top-quark mass, we update the phenomenological constraints on the unitarity triangle.

# 1 Introduction

In the Standard Model [1], the GIM suppression of flavour-changing neutral-current processes [2] and the necessarily presence of three quark families to generate CP-violation effects make the top quark a key ingredient to analyze these phenomena. In both cases, the unitarity of the Cabibbo–Kobayashi–Maskawa (CKM) matrix implies vanishing effects in the limit of degenerate quark masses. Those processes, occurring through one-loop diagrams, are then very sensitive to the masses of the three equal-charge quarks running along the internal lines. Due to its large mass, the top-quark gives a very important (often dominant) contribution; thus, the unknown value of  $m_t$  has been up to now a crucial uncertainty in the phenomenological analyses.

The recent announcement of evidence for the top quark [3], with a “pole” mass

$$m_t = 174 \pm 10_{-12}^{+13} \text{ GeV} \quad (\text{CDF}), \quad (1)$$

should allow to improve the present determinations of the CKM parameters. This value of the top mass is in excellent agreement with the range obtained from the Standard Model electroweak fits at the  $Z$  peak [4],

$$m_t = 177_{-11}^{+11+18}_{-19} \text{ GeV} \quad (\text{Electroweak Fits}), \quad (2)$$

which gives further support to the CDF analysis.

One of the crucial tests of the Standard Model mechanism of CP violation involves the unitarity condition

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0. \quad (3)$$

This relation can be visualized as a triangle in the complex plane, the so-called “unitarity triangle” (UT). In the absence of CP violation, the triangle would degenerate into a segment along the real axis. It has become conventional to scale the triangle, dividing its sides by  $|V_{cb}^* V_{cd}|$ . In the Wolfenstein parametrization [5] of the CKM matrix,

$$\mathbf{V} = \begin{bmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} + O(\lambda^4), \quad (4)$$

$|V_{cb}^* V_{cd}|$  is real to an excellent accuracy [ $O(\lambda^7)$ ]. Therefore, the scaling aligns one side of the triangle along the real axis and makes its length equal to 1; the coordinates of the 3 vertices are then  $(0,0)$ ,  $(1,0)$  and  $(\bar{\rho}, \bar{\eta})$ , with [6]

$$\bar{\rho} \equiv \rho \left(1 - \frac{\lambda^2}{2}\right), \quad \bar{\eta} \equiv \eta \left(1 - \frac{\lambda^2}{2}\right). \quad (5)$$

The triangle is shown in Fig. 1.

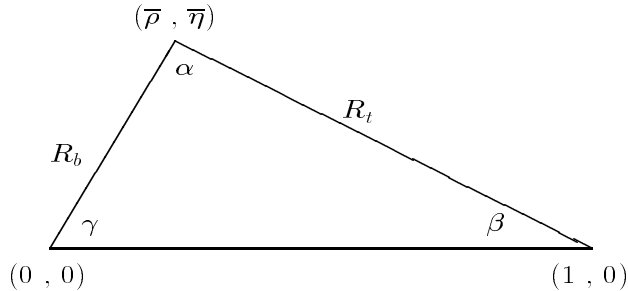


Figure 1: Unitarity triangle

The sides of this triangle,

$$R_b \equiv \left| \frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \right| = \left( 1 - \frac{\lambda^2}{2} \right) \left| \frac{V_{ub}}{\lambda V_{cb}} \right| = \sqrt{\bar{\rho}^2 + \bar{\eta}^2}, \quad (6)$$

$$R_t \equiv \left| \frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} \right| = \left| \frac{V_{td}}{\lambda V_{cb}} \right| = \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2}, \quad (7)$$

can be determined through CP-conserving measurements: the ratio  $\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c)$  fixes  $R_b$ , while  $R_t$  can be extracted from the observed  $B_d^0$ - $\bar{B}_d^0$  mixing. A third constraint is obtained from the measured value of the CP-violation parameter  $\varepsilon$ .

A precise test of the unitarity relation (3) is obviously required. Moreover, the angles  $\alpha$ ,  $\beta$  and  $\gamma$  of the UT regulate many interesting CP-violating asymmetries in the  $B$  system (see for instance [7]); thus, it is important to determine the triangle, in order to know the expected size of the CP-signals at future  $B$  factories.

The theoretical analysis of the UT is quite straightforward and has been performed many times in the past. Nevertheless, since the constraints obtained from  $\varepsilon$  and from  $B_d^0$ - $\bar{B}_d^0$  mixing strongly depend on the value of the top mass, the determination of the UT needs to be updated in view of the new information provided by CDF. A first study [6], “guessing” the CDF value before its public announcement, has already been done, and a second one [8] has just appeared, immediately after the CDF publication. Many more are probably going to show up soon.

Unfortunately, the knowledge of the top-quark mass is not enough to precisely fix the UT. Our limited ability to handle the long-distance effects of the strong interactions, translates into unavoidable hadronic uncertainties, which enter in the determination of the CKM parameters. Although a big theoretical effort has been made during the last decade to study the relevant hadronic matrix elements, the present status is certainly not satisfactory: theoretical errors are still large, and there is no universal agreement on the values of some non-perturbative parameters.

The resulting uncertainties are not always properly reflected in the usual UT analyses. Quite often, the non-perturbative parameters are fixed in a rather ad-hoc way, or following the last fashion (i.e. taking similar values to the latest published analyses);

thus, the mere repetition of a parameter set, rather than the scientific quality, is what finally makes a given choice of non-perturbative inputs the one “considered more reliable”. A very instructive example is provided by the  $B^0$ - $\bar{B}^0$  mixing parameter  $\xi_B \equiv f_B \sqrt{B_B}$ : while in 1988 it had been already established\* that  $\xi_B > f_\pi$  [9], many UT analyses done from 1988 to 1991 were still using  $\xi_B < f_\pi$  and getting, therefore, meaningless results.

In the following, we want to critically summarize the present status of those hadronic matrix elements which are relevant for the UT determination, and work out their phenomenological implications. We will try to put forward the arguments supporting our final choice of parameters and their associated error-bars. Using the measured value of the top-quark mass [3, 4], together with the most recent experimental information on the  $B$  system [11–15], we will finally analyze the present constraints on the UT.

## 2 Master formulae

The short-distance analysis of  $K^0$ - $\bar{K}^0$  and  $B^0$ - $\bar{B}^0$  mixing is well known. An excellent review has been given by Buras and Harlander [16]. Here, we only list the final formulae relevant for our discussion, referring to Refs. [6, 16] for further details.

The experimental value of  $\varepsilon$  specifies a hyperbola in the  $(\bar{\rho}, \bar{\eta})$  plane:

$$\bar{\eta} \left[ (1 - \bar{\rho}) A^2 \hat{\eta}_2 S(r_t) + P_0 \right] A^2 \hat{B}_K = \frac{3\sqrt{2}\pi^2 \Delta M_K |\varepsilon|}{G_F^2 M_W^2 f_K^2 M_K \lambda^{10}} \equiv C_\varepsilon, \quad (8)$$

where

$$S(r_t) = \frac{r_t}{4} \left[ 1 + \frac{9}{1 - r_t} - \frac{6}{(1 - r_t)^2} - \frac{6r_t^2 \ln r_t}{(1 - r_t)^3} \right] \quad (9)$$

contains the dominant top contribution, and the corrections coming from the  $cc$  and  $tc$  box diagrams are given by

$$P_0 = \frac{1}{\lambda^4} [\hat{\eta}_3 S(r_c, r_t) - \hat{\eta}_1 r_c], \quad (10)$$

$$S(r_c, r_t) = r_c \left[ \ln \left( \frac{r_t}{r_c} \right) - \frac{3r_t}{4(1 - r_t)} \left( 1 + \frac{r_t \ln r_t}{1 - r_t} \right) \right]. \quad (11)$$

Here,  $r_q \equiv m_q^2/M_W^2$  and the renormalization-scale-invariant factors

$$\hat{\eta}_1 = 1.10, \quad \hat{\eta}_2 = 0.57, \quad \hat{\eta}_3 = 0.36, \quad (12)$$

take into account the computed short-distance QCD corrections.

The main theoretical uncertainty stems from the so-called  $\hat{B}_K$  factor, parametrizing the hadronic matrix element of the  $\Delta S = 2$  four-quark operator:

$$\langle \bar{K}^0 | (\bar{s}\gamma^\mu(1 - \gamma_5)d) (\bar{s}\gamma_\mu(1 - \gamma_5)d) | K^0 \rangle \equiv \frac{8}{3} \left( \sqrt{2} f_K M_K \right)^2 B_K(\mu^2), \quad (13)$$

$$\hat{B}_K \equiv \alpha_s(\mu^2)^{-2/9} B_K(\mu^2). \quad (14)$$

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\* A summary of existing calculations of  $f_B$ ,  $B_B$  and  $\xi_B$  was given in Ref. [10].

The hadronic matrix element (and thus  $B_K$ ) depends on the chosen renormalization scale; this dependence is exactly cancelled by the short-distance renormalization-group factor  $\alpha_s(\mu^2)^{-2/9}$ , so that the combination  $\hat{B}_K$  appearing in (8) is renormalization-scale independent.

In the neutral  $B$  meson system, the mixing is completely dominated by the top contribution:

$$x_d \equiv \frac{\Delta M_{B_d}}{\Gamma_{B_d}} = \tau_{B_d} |V_{td}|^2 M_{B_d} \frac{G_F^2 M_W^2}{6\pi^2} \hat{\eta}_B S(r_t) (\sqrt{2} f_B)^2 \hat{B}_B. \quad (15)$$

The short-distance QCD-correction is collected in the the renormalization-scale-invariant factor  $\hat{\eta}_B$ , which has been computed to be

$$\hat{\eta}_B = 0.55, \quad (16)$$

and the long-distance  $\Delta B = 2$  hadronic matrix element is parametrized in terms of

$$\hat{\xi}_B \equiv f_B \sqrt{\hat{B}_B} \equiv \alpha_s(\mu^2)^{-3/23} \xi_B(\mu^2), \quad (17)$$

where the renormalization-scale-independent factor  $\hat{B}_B$  is defined by

$$\langle \bar{B}^0 | (\bar{b}\gamma^\mu(1-\gamma_5)d) (\bar{b}\gamma_\mu(1-\gamma_5)d) | B^0 \rangle \equiv \frac{8}{3} (\sqrt{2} f_B M_B)^2 B_B(\mu^2), \quad (18)$$

$$\hat{B}_B \equiv \alpha_s(\mu^2)^{-6/23} B_B(\mu^2), \quad (19)$$

in complete analogy to Eqs. (13) and (14). The different power of  $\alpha_s(\mu^2)$  in Eqs. (14) and (19) is due to the different number of “light” quark flavours (3 and 5, respectively) in the  $K$  and  $B$  systems.

The QCD parameters  $\hat{\eta}_1$ ,  $\hat{\eta}_2$  and  $\hat{\eta}_B$  have been computed at the next-to-leading-logarithm order [6]. At this level of accuracy, one needs to state how  $m_t$  is defined. The numerical values quoted in Eqs. (12) and (16) correspond to the running top quark mass in the  $\overline{MS}$  scheme evaluated at  $m_t$ , i.e. in Eqs. (8) to (19)  $m_t$  stands for  $\overline{m}_t(m_t)$  [6]. The relation with the “pole” mass, defined as the pole of the renormalized propagator, is given by [17]

$$m_t^{\text{pole}} = \overline{m}_t(m_t^{\text{pole}}) \left\{ 1 + \frac{4}{3} \frac{\alpha_s(m_t^{\text{pole}})}{\pi} + \left[ 16.11 - 1.04 \sum_{i=u,d,s,c,b} \left( 1 - \frac{m_i^{\text{pole}}}{m_t^{\text{pole}}} \right) \right] \left( \frac{\alpha_s(m_t^{\text{pole}})}{\pi} \right)^2 \right\}. \quad (20)$$

Thus,  $\overline{m}_t(m_t)$  is about 9 GeV lower than  $m_t^{\text{pole}}$ . The measured values (1) and (2) should be identified with  $m_t^{\text{pole}}$ .

### 3 Experimental inputs

The physical quantities defining the parameter  $C_\varepsilon$  are rather well measured [18]:

$$\begin{aligned}
 G_F &= 1.16639(2) \times 10^{-5} \text{ GeV}^{-2}, & M_W &= 80.22 \pm 0.16 \text{ GeV}, \\
 \Delta M_K &= (3.522 \pm 0.016) \times 10^{-12} \text{ MeV}, & M_{K^0} &= 497.671 \pm 0.031 \text{ MeV}, \\
 f_K &= 1.22 f_\pi = 113 \pm 1 \text{ MeV}, & \lambda &= 0.2205 \pm 0.0018, \\
 |\varepsilon| &= (2.26 \pm 0.02) \times 10^{-3}.
 \end{aligned}
 \tag{21}$$

Therefore,

$$C_\varepsilon = 0.220 \pm 0.019. \tag{22}$$

Owing to its large power ( $\lambda^{-10}$ ), the dominant uncertainty comes from the value of  $\lambda$ .

Except for the  $B^0$  meson mass [18],

$$M_{B^0} = 5.279 \pm 0.002 \text{ GeV}, \tag{23}$$

the experimental error-bars are somewhat larger in the  $B$  system. The averaged value of the  $b$ -lifetime has been continuously increasing as function of time; the 1990 value [19]  $\langle\tau_b\rangle = 1.18 \pm 0.11$  ps moved up to  $\langle\tau_b\rangle = 1.29 \pm 0.05$  ps in 1992 [18], and has been further increased by the recent LEP data. The present world average [11, 12] is:

$$\langle\tau_b\rangle = 1.49 \pm 0.04 \text{ ps}. \tag{24}$$

This value is in good agreement with the measured “exclusive” lifetime of the  $B^0$  meson [11, 12],

$$\langle\tau(B^0)\rangle = 1.5 \pm 0.1 \text{ ps}. \tag{25}$$

The world averaged value of the  $B^0$ - $\bar{B}^0$  mixing parameter is [11]:

$$x_d = 0.71 \pm 0.07. \tag{26}$$

The experimental determination of the CKM matrix elements  $|V_{cb}|$  and  $|V_{ub}|$  requires theoretical input and, therefore, suffers from systematic uncertainties related to the model-dependence involved in the analysis of semileptonic  $b$  ( $B$ ) decay. The cleanest determination of  $|V_{cb}|$  uses the decay  $B \rightarrow D^* l \bar{\nu}_l$  [20], where the relevant hadronic form factor ( $h_{A_1}$ ) can be controlled at the level of a few per cent, close to the zero-recoil region. In the infinite  $B$ -mass limit, the normalization of this form factor at zero recoil is fixed to be one, and the leading  $1/M$  corrections vanish [21] due to heavy-quark symmetry; thus, the theoretical uncertainty is of order  $1/M^2$  and therefore in principle small. The calculated short-distance QCD corrections [22] and the present estimates of the  $1/M^2$  contributions [23] result in  $h_{A_1}(1) = 0.97 \pm 0.04$  [24], implying [15]

$$|V_{cb}| \left( \frac{\tau(B_d^0)}{1.5 \text{ ps}} \right)^{1/2} = 0.037 \pm 0.004. \tag{27}$$

An independent determination can be done using the inclusive semi-leptonic decay spectra. The theoretical uncertainty is however bigger, because the total rate scales as  $m_b^5$  and, therefore, is very sensitive to the not so-well-known value of the  $b$ -quark mass. A compilation of experimental analyses [14] finds  $|V_{cb}| (\tau(B_d^0)/1.5 \text{ ps})^{1/2} = 0.040 \pm 0.005$ , in good agreement with (27); however, the quoted error does not take into account the uncertainty associated with  $m_b$ . A recent study of the  $b$ -quark mass definition within the heavy quark effective theory finds a slightly heavier  $b$  quark and consequently a slightly smaller value  $|V_{cb}| (\tau(B_d^0)/1.5 \text{ ps})^{1/2} = 0.036 \pm 0.005$  [25].

The value of  $|V_{cb}|$  quoted in Eq. (27) implies,

$$A = 0.76 \pm 0.08. \quad (28)$$

The present determination of  $|V_{ub}|$  is based on measurements of the lepton momentum spectrum in inclusive  $B \rightarrow X_q l \bar{\nu}_l$  decays, where  $X_q$  is any hadronic state containing a quark  $q = c$  or  $u$ . The method is very sensitive to the assumed theoretical spectrum near the kinematic limit for  $B \rightarrow D l \bar{\nu}_l$ . Using different models to estimate the systematic theoretical uncertainties, the analyses of the experimental data give [11, 13]:

$$\left| \frac{V_{ub}}{V_{cb}} \right| = 0.08 \pm 0.02. \quad (29)$$

## 4 $\widehat{B}_K$ factor

The matrix element (13) was first estimated [26] via the assumption of vacuum saturation, i.e. splitting the matrix element in a product of two currents and inserting the vacuum in all possible ways. The factor  $B_K$  parametrizes the deviation from this factorization estimate, so that

$$B_K(\mu^2) = 1 \quad (\text{Vacuum Saturation}) \quad (30)$$

corresponds to the vacuum saturation approximation. Clearly, this approximation can only be taken as an order-of-magnitude estimate, since it completely ignores the renormalization group factor  $\alpha_s(\mu^2)^{-2/9}$ . As it stands, Eq. (30) is meaningless because the value of  $\mu$  is not specified.

An improved factorization estimate of the renormalization-scale-independent factor  $\widehat{B}_K$  can be trivially performed in the large- $N_c$  limit, where  $N_c$  is the number of colours. In this limit, the anomalous dimension of the  $\Delta S = 2$  operator vanishes and factorization is then exact [27]:

$$\widehat{B}_K = B_K = \frac{3}{4} \quad (N_c \rightarrow \infty). \quad (31)$$

Another approach allowing a rigorous calculation of  $\widehat{B}_K$ , within a well-defined approximation, is Chiral Perturbation Theory (CHPT). The  $\Delta S = 2$  operator has the same chiral transformation properties than the four-quark operator mediating  $\Delta S = 1$ ,  $\Delta I = 3/2$  transitions. Both operators belong to the same  $(27_L, 1_R)$  representation of the

chiral  $SU(3)_L \otimes SU(3)_R$  group and, therefore, their matrix elements are trivially related by a Clebsch-Gordan coefficient. One can then directly measure the value of  $\hat{B}_K$ , using the  $K^+ \rightarrow \pi^+\pi^0$  decay rate [28]. One gets in this way

$$\hat{B}_K = 0.37 \quad (\text{CHPT at } O(p^2)). \quad (32)$$

At lowest-order in Chiral Perturbation Theory,  $O(p^2)$ , the only possible corrections to this result are  $SU(2)$  violations, induced by electromagnetism or proportional to  $m_d - m_u$  [29].  $SU(3)$ -breaking effects, spoiling the relation with the  $K^+ \rightarrow \pi^+\pi^0$  rate, appear first at the next order in the momentum expansion, i.e. at  $O(p^4)$ .

The discrepancy between the two determinations of  $\hat{B}_K$  in Eqs. (31) and (32) shows that higher-order corrections should be sizeable. A first estimate of the  $1/N_c$  corrections to (31) was performed by Bardeen, Buras and Gérard [30], with the result:

$$\hat{B}_K = 0.70 \pm 0.10 \quad (\text{Next-to-Leading } 1/N_c \text{ Estimate}). \quad (33)$$

The fact that the next-to-leading corrections in the  $1/N_c$  expansion are negative was further demonstrated in Ref. [31], using functional integration techniques. The numerical result of this analysis

$$\hat{B}_K = 0.4 \pm 0.2 \quad (\text{Next-to-Leading } 1/N_c \text{ Estimate, } O(p^2)), \quad (34)$$

has a rather large error, but the negative sign of the  $1/N_c$  corrections was clearly established [31]. At  $O(p^2)$ , this sign can in fact be proven to be negative in a model-independent way [32], because the  $1/N_c$  correction to  $\hat{B}_K$  is anticorrelated with the one enhancing the  $\Delta I = 1/2$   $K \rightarrow \pi\pi$  amplitude.

Since  $f_K/f_\pi = 1.22$ ,  $SU(3)$  breaking corrections to the result (32) could be expected to be important. The  $SU(3)$  rotation of the  $K^+ \rightarrow \pi^+\pi^0$  amplitude determines the product  $\hat{\xi}_K^2 \equiv f_K^2 \hat{B}_K$ . The value in Eq. (32) is obtained fixing (by definition) the kaon decay constant to its physical value; however, at  $O(p^2)$  in the chiral expansion,  $f_K = f_\pi$ . Using instead the pion decay constant, the same value of  $\hat{\xi}_K$  results in a 40% bigger  $\hat{B}_K$  factor:  $\hat{B}_K = 0.37(f_K/f_\pi)^2 = 0.55$ . In fact, those higher-order chiral corrections which are factorizable will precisely change  $f_\pi$  to  $f_K$ , increasing the lowest-order result (32) by the factor  $(f_K/f_\pi)^2$  and making it closer to the leading  $1/N_c$  estimate (31). Of course, the relevant question now concerns the magnitude and sign of the non-factorizable  $O(p^4)$  chiral contributions.

The explicit calculation of the 1-loop chiral corrections shows [33] indeed that the  $\Delta S = 2$   $K^0\text{-}\bar{K}^0$  matrix element (i.e.  $\hat{\xi}_K$ ) receives a large (and positive) logarithmic correction; a big part of this 1-loop contribution is just the usual (factorizable) correction to  $f_K^2$ . To perform a complete  $O(p^4)$  CHPT calculation, one needs also the non-logarithmic contributions coming from next-to-leading terms in the chiral weak Lagrangian. A recent estimate [34], based on the  $1/N_c$  techniques of Ref. [31], finds that the non-factorizable chiral corrections are negative. Adding all contributions, the final result of Ref. [34] is:

$$\hat{B}_K = 0.42 \pm 0.06 \quad (O(p^4) \text{ CHPT} + 1/N_c \text{ Estimate}). \quad (35)$$

Two clear qualitative conclusions emerge from the previous analyses:



- The next-to-leading corrections in the  $1/N_c$  expansion (non-factorizable corrections) are negative and, therefore, decrease the leading-order result (31).
- The factorizable  $O(p^4)$  corrections increase the lowest-order CHPT result (32); however, the non-factorizable contributions appear to be negative.

Thus, quite independently of any particular numerical estimate, one can pin down  $\widehat{B}_K$  to be within the interval:

$$0.35 < \widehat{B}_K < 0.75. \quad (36)$$

$\widehat{B}_K$  can be also calculated, through dispersion relations, using a QCD-hadronic duality approach. Making a dual description of the  $\Delta S = 2$  operator (CHPT + hadronic resonances at long distances and the usual quark-gluon description at short distances) and analyzing the corresponding two-point function correlator, it is possible to extract the value of the  $K^0$ - $\bar{K}^0$  matrix element [35]. Updating all the inputs of this analysis, one gets [36, 37]

$$\widehat{B}_K = 0.39 \pm 0.10 \quad (\text{QCD-Hadronic Duality}). \quad (37)$$

This result is in fact a calculation of the relevant  $O(p^2)$  chiral coupling; thus, one could probably expect a small increase, due to  $O(p^4)$  chiral corrections. The same approach provides a quite successful calculation of the  $K^+ \rightarrow \pi^+\pi^0$  decay rate [38], in good agreement with the experimental value. It overestimates this decay amplitude by less than 15% [39].

There have been several QCD sum rule calculations based on studies of three-point-function correlators [40–43]. After some initial disagreements, the final result seems to be [42] <sup>†</sup>:

$$B_K(\mu^2) = 0.5 \pm 0.1 \pm 0.2 \quad (\text{QCD Sum Rules -3 Point Functions}). \quad (38)$$

The theoretical uncertainty is rather large because the perturbative gluonic corrections have not been included yet, i.e.  $\widehat{B}_K \sim B_K(\mu^2)$  with an arbitrary renormalization-scale  $\mu$ .

Lattice calculations have given fluctuating results. The first (statistically) accurate determinations gave results compatible with vacuum saturation:  $\widehat{B}_K = 0.88 \pm 0.20$  [44],  $1.03 \pm 0.07$  [45],  $0.77 \pm 0.07$  [46]. It was realized later that finite-size effects were important and the extrapolation to the continuum limit could decrease the final numerical results; depending on the assumed extrapolation,  $\widehat{B}_K = 0.66 \pm 0.06$  or  $0.78 \pm 0.03$  (statistical errors only) was obtained [47]. An improved investigation of the lattice spacing errors, has recently given the more precise value [48]:

$$\widehat{B}_K = 0.825 \pm 0.035. \quad (\text{Lattice}). \quad (39)$$

This result has been obtained in the quenched approximation and with degenerate quarks with mass  $m_s/2$ . To assess the significance of these difficult calculations, one should keep

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<sup>†</sup> Larger values ( $B_K \sim 0.75$ -1) have been obtained in Refs. [43], where, following Ref. [40], only the non-factorizable piece is estimated. This type of analysis has been criticized in Refs. [41, 42].

$\widehat{B}_K$	Method	Reference
3/4	Leading $1/N_c$	[27]
0.37	Lowest-Order Chiral Perturbation Theory	[28]
$0.70 \pm 0.10$	Next-to-Leading $1/N_c$ Estimate	[30]
$0.4 \pm 0.2$	Next-to-Leading $1/N_c$ Estimate, $O(p^2)$	[31]
$0.42 \pm 0.06$	$\mathcal{O}(p^4)$ CHPT + $1/N_c$ Estimate	[34]
$0.39 \pm 0.10$	QCD-Hadronic Duality	[35, 36]
$0.5 \pm 0.1 \pm 0.2$	QCD Sum Rules (3-Point Functions)	[42]
$0.825 \pm 0.035$	Lattice (Quenched Approximation)	[48]

Table 1: Values of  $\widehat{B}_K$  obtained by various methods.

in mind the related  $K^+ \rightarrow \pi^+\pi^0$  decay amplitude. Present lattice calculations of  $\widehat{B}_K$  are still done in the  $SU(3)$  limit; therefore, they also provide a prediction for the  $K^+ \rightarrow \pi^+\pi^0$  decay rate. At present, the lattice calculation overestimates the  $K^+$  decay amplitude by a factor of 2.

Table 1 summarizes the different calculations. Except for the lattice value, which is somewhat bigger, all other results are in the range (36).

## 5 $B^0$ - $\bar{B}^0$ matrix element

The vacuum saturation approximation has been usually applied to estimate the  $\Delta B = 2$  matrix element in Eq. (18), i.e.  $B_B(\mu^2) = 1$  is generally assumed. However, in contrast to the kaon system, this assumption does not provide by itself an estimate of the hadronic parameter  $\xi_B$ , because the  $B^0$  decay constant has not been measured yet.

The theoretical determination of  $f_B$  has a quite confusing history, because many contradictory results have been published. To a large extent, the discrepancies among the different analyses stem from the different approximations assumed to be valid, and/or the different input values used in the final numerics.

Owing to the large mass of the  $b$  quark, non-relativistic potential models were supposed to provide a good estimate of  $f_B$  [49, 50]. However, relativistic and short-distance QCD corrections have been shown [50, 51] to be very significant. The meson decay constant is directly proportional to the meson wavefunction at the origin; thus, these calculations are very sensitive to the assumed short-distance behaviour of the potential, which explains the broad range of results obtained within this approach [49–51].

In the infinite quark-mass limit, the meson decay constant should scale as [52]

$$f_P \underset{M_Q \rightarrow \infty}{\sim} \frac{\alpha_s(M_Q)^{1/\beta_1}}{\sqrt{M_Q}}, \quad (40)$$

where  $\beta_1 = (2n_f - 33)/6$  is the first coefficient of the QCD  $\beta$  function, and  $n_f$  the number of unfrozen flavours. Assuming the charm-quark mass to be heavy enough, one should then expect  $f_B/f_D \sim 0.6$ . However, this is not supported by modern QCD sum rules [53–55] and lattice calculations [56] which rather prefer  $f_B \sim f_D$ , or even  $f_B \gtrsim f_D$ . The origin of this unexpected behaviour can be understood analyzing the leading corrections to the asymptotic result (40):

$$f_P = f_P^{\text{stat}} \left\{ 1 + \frac{c_P}{M_Q} + O(1/M_Q^2) \right\}. \quad (41)$$

The so-called static limit of the decay constant,  $f_P^{\text{stat}}$ , can be calculated using heavy-quark effective theory methods [22, 53–57]; moreover, several estimates of the  $1/M_Q$  correction have been performed [22, 54, 55, 57, 58]. These studies have shown that:

- The value of  $f_B^{\text{stat}}$  is quite large, typically  $f_B^{\text{stat}} \sim 2f_\pi$ . If short-distance QCD corrections are ignored, a much smaller value is obtained [53, 54, 59]; however, owing to the Coulombic interaction between the light and heavy quarks, there is a large<sup>‡</sup> perturbative gluonic correction of order 100% [60].
- The leading  $1/M_Q$  correction is negative and sizeable,  $c_P \sim -1$  GeV. It amounts to a 20% decrease of the decay constant at the  $b$ -quark scale, but it is of order 100% at the charm-mass scale, which makes a non-relativistic determination of  $f_D$  meaningless.

These large corrections allow us to understand the discrepancies among previous approximate calculations, but at the same time point out the difficulty of making a reliable determination of  $f_B$  within a heavy-quark or non-relativistic approach. Anyhow, it has been argued [57] that the large first-order gluonic correction gives already a very good description of the classical Coulomb interaction and, therefore, there is no reason to expect additional large contributions at higher-orders. Taking the pole  $b$ -quark mass to be  $m_b = 4.6$  (4.8) GeV, values around [22, 55, 57, 58]  $f_B \sim 1.4$  (1.1) $f_\pi$  have been estimated, using QCD sum rules in the heavy-quark effective theory.

The values of  $f_B$  obtained with QCD sum rules have a sizeable dependence on the input value of the heavy-quark mass. This effect induces a large uncertainty and is to a large extent responsible for the apparent discrepancy among different predictions [53, 54, 62–64]. Using the presently favoured range for the perturbative pole mass [63, 65–67],  $m_b = (4.6 \pm 0.1)$  GeV, one obtains [54]

$$f_B = (1.6 \pm 0.3) f_\pi \quad (\text{QCD Sum Rules}), \quad (42)$$

where larger values of the decay constant correspond to lower  $b$ -quark masses. Taking a larger mass  $m_b = 4.8$  GeV, one gets instead  $f_B \sim f_\pi$ , which shows the strong sensitivity to the value of  $m_b$ .

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<sup>‡</sup> The next-to-leading order renormalization group improvement of the currents in the heavy-quark effective theory [22, 55, 57, 60] shows that the strong coupling constant must be evaluated at a characteristic low-energy hadronic scale of order 1 GeV, rather than at the scale of the heavy quark. Thus, the relevant coupling  $\alpha_s(\mu)$  is much larger than the one used in previous analysis [61], which results in a sizeable increase of  $f_B^{\text{stat}}$ .

Lattice simulations with propagating heavy quarks face the problem of large systematic errors associated with the finite lattice spacing. One can only consider quark masses such that  $M_Q a \lesssim 1$  (for heavier masses, the associated Compton wavelengths are smaller than the lattice spacing  $a$ ), which implies that only the region around the charm-quark mass can be investigated with the presently available lattices. An extrapolation to the  $b$ -quark scale is then unavoidable. The currently quoted results [56] are in the range

$$f_B = (1.4 \pm 0.3) f_\pi \quad (\text{Lattice}). \quad (43)$$

Concerning the  $B_B$  factor, one would naively expect that the vacuum insertion approximation becomes more reliable with the increase of the quark mass. There are two published studies of this quantity, using QCD sum rules based on three-point-function correlators, which favour indeed a value of  $B_B$  around one:  $B_B(\mu^2) = 0.95 \pm 0.10$  [68]. However, perturbative  $\alpha_s$  corrections have not been considered and therefore the scale  $\mu$  is arbitrary. Moreover, what is actually computed is  $f_B^2 \xi_B = f_B^4 B_B$ ; thus, the value of  $f_B$  (or the sum rule used to fix the decay constant) is needed as input, which increases the theoretical uncertainty of the calculation.

So far, only one direct estimate of the relevant quantity  $\xi_B$  has been published [9]. This calculation is based on the two-point-function correlator associated with the  $\Delta B = 2$  four-quark operator. The usual QCD sum rule technology allows to fix the value of  $\xi_B^2$  directly, without any prior knowledge of  $f_B$ . Unfortunately, the result turns out to be very sensitive to the input value of the bottom quark mass. Taking the range  $m_b = (4.6 \pm 0.1)$  GeV, one gets [9]:

$$\xi_B(\mu^2) = (1.7 \pm 0.4) f_\pi \quad (\text{QCD Sum Rules}), \quad (44)$$

the larger  $\xi_B$  values corresponding to the lower masses. The comparison with Eq. (42) could give some support to the vacuum saturation approximation. However this comparison is not very meaningful because it is not specified at which scale it refers; again, perturbative gluonic contributions have not been included yet. One can argue that the relevant scale in the QCD sum rule is  $\mu = 2m_b$  [9]; the resummation of leading logarithms gives then rise to the renormalization-scale independent mixing parameter [see Eq. (17)]

$$\hat{\xi}_B = (2.0 \pm 0.5) f_\pi. \quad (45)$$

Taking instead  $\mu = m_b$  would not change the result within the quoted error-bar. Note that with  $\mu \sim m_b, 2m_b$ ,  $\hat{\xi}_B$  is a 20% larger than  $\xi_B(\mu^2)$ . To make a better estimate, a calculation of the next-to-leading-logarithm corrections is needed. Those corrections which are factorizable are already known from the analogous  $f_B$  calculation, and produce [39] a 18% increase with respect to  $\hat{\xi}_B$  in (45); the non-factorizable contributions remain, however, unknown<sup>§</sup>.

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<sup>§</sup> The non-factorizable contributions have been recently estimated to be smaller than 15% [69]. Unfortunately, this number refers to some special Fierz-symmetric renormalization-scheme. It is not clear how to translate it into the usual  $\overline{MS}$  scheme, which is needed for consistency with the rest of the theoretical analysis.

As in the  $f_B$  case, lattice simulations have estimated the parameter  $B_P$  in the charm region. Performing an extrapolation to the physical  $B$ -meson mass, the present lattice determination is [70]  $\hat{B}_B = 1.16 \pm 0.07$ . Using the  $f_B$  value obtained in the same lattice,  $f_B = (1.54 \pm 0.30) f_\pi$ , the quoted result for the renormalization-scale-independent parameter  $\hat{\xi}_B$  is [70]:

$$\hat{\xi}_B = (1.7 \pm 0.3) f_\pi \quad (\text{Lattice}), \quad (46)$$

in agreement with the QCD sum rule value (45).

## 6 Numerical analysis

In order to perform a numerical analysis of the UT constraints, we take  $\Lambda_{\overline{MS}}^{(5)} = 240 \pm 90$  MeV [71], the charm quark pole mass  $m_c = 1.47 \pm 0.05$  GeV [65],  $\lambda = 0.2205 \pm 0.0018$  and the inputs shown in Table 2. In view of the uncertainties discussed before, we have taken two different sets of parameters: the first corresponds to our best estimate, while the second one represents a somewhat more conservative choice.

Parameter	Best estimate	Conservative choice
$x_d$	$0.71 \pm 0.07$	$0.70 \pm 0.10$
$m_t^{\text{pole}}$	$174 \pm 16$ GeV	$175 \pm 25$ GeV
$\tau(B_d^0)$	$1.49 \pm 0.04$ ps	$1.50 \pm 0.10$ ps
$\tau(B_d^0)  V_{cb} ^2$	$(3.1 \pm 0.7) \times 10^9 \text{ GeV}^{-1}$	$(3.1 \pm 1.0) \times 10^9 \text{ GeV}^{-1}$
$ V_{ub} / V_{cb} $	$0.08 \pm 0.02$	$0.08 \pm 0.03$
$\hat{B}_K$	$0.50 \pm 0.15$	$0.55 \pm 0.25$
$\hat{\xi}_B/f_\pi$	$2.0 \pm 0.5$	$1.9 \pm 0.7$

Table 2: Input values for the UT analysis.

Figures 2 and 3 show the resulting UT constraints for the *best* and the *conservative*, respectively, choices of input parameters. The circles centered at  $(0, 0)$  show the present determination of  $R_b$ , defined in Eq. (6). The measured  $B^0$ - $\bar{B}^0$  mixing parameter  $x_d$ , constrains the value of  $R_t$ , defined in Eq. (7), forcing the vertex  $(\bar{\rho}, \bar{\eta})$  to be in the region between the two circles centered at  $(1, 0)$ ; the bigger circle corresponds to the smaller allowed values of  $\hat{\xi}_B$  and  $|V_{cb}|$ . The hyperbolae show the constraints from the  $K^0$ - $\bar{K}^0$  CP-violating parameter  $|\varepsilon|$ , which follow from Eq. (8); smaller values of  $\hat{B}_K$  and  $|V_{cb}|$  correspond to larger values of  $\bar{\eta}$ . The final allowed range of values for  $(\bar{\rho}, \bar{\eta})$  is given by the area which is common to the regions in between the hyperbolae, the circles centered at  $(0, 0)$  and the circles centered at  $(1, 0)$ . In Tables 3 and 4 we give the numerical results for  $\sin(2\alpha)$ ,  $\sin(2\beta)$ ,  $\sin(2\gamma)$ ,  $R_b$  and  $R_t$  from our analysis of the UT, using the two sets of input parameters introduced in Table 2.

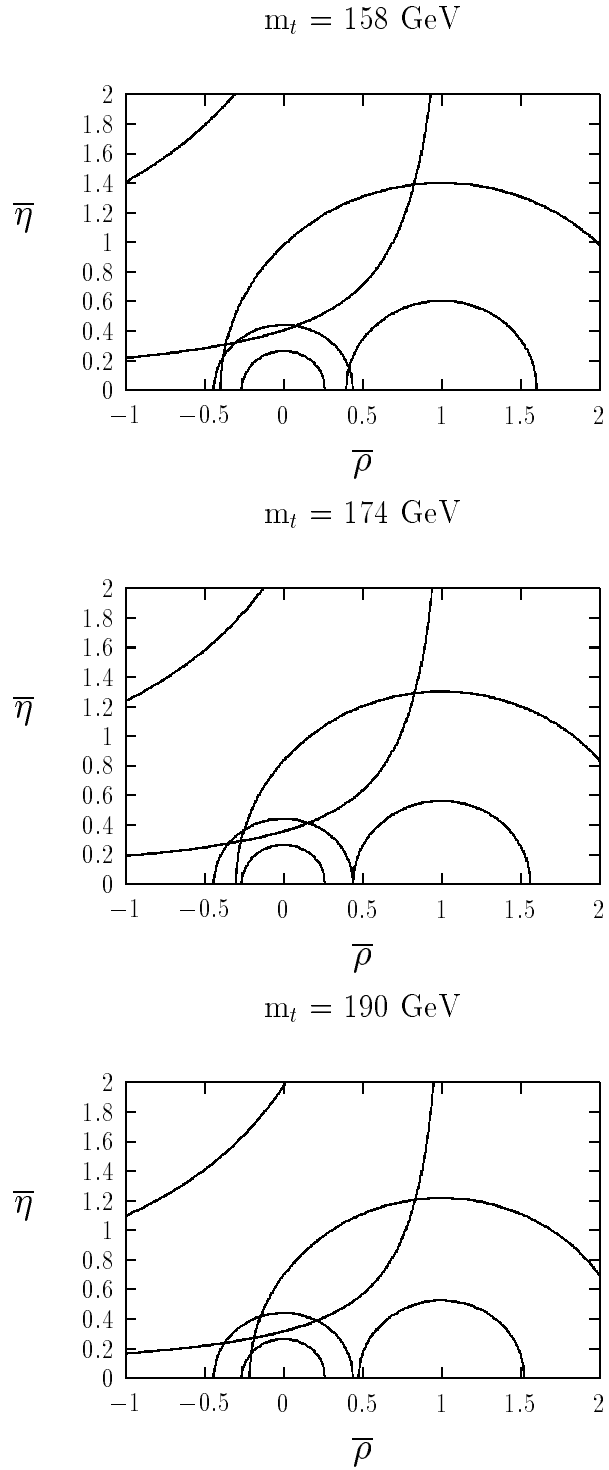


Figure 2: Constraints on the UT for the *best* estimate set of parameters in Table 2.

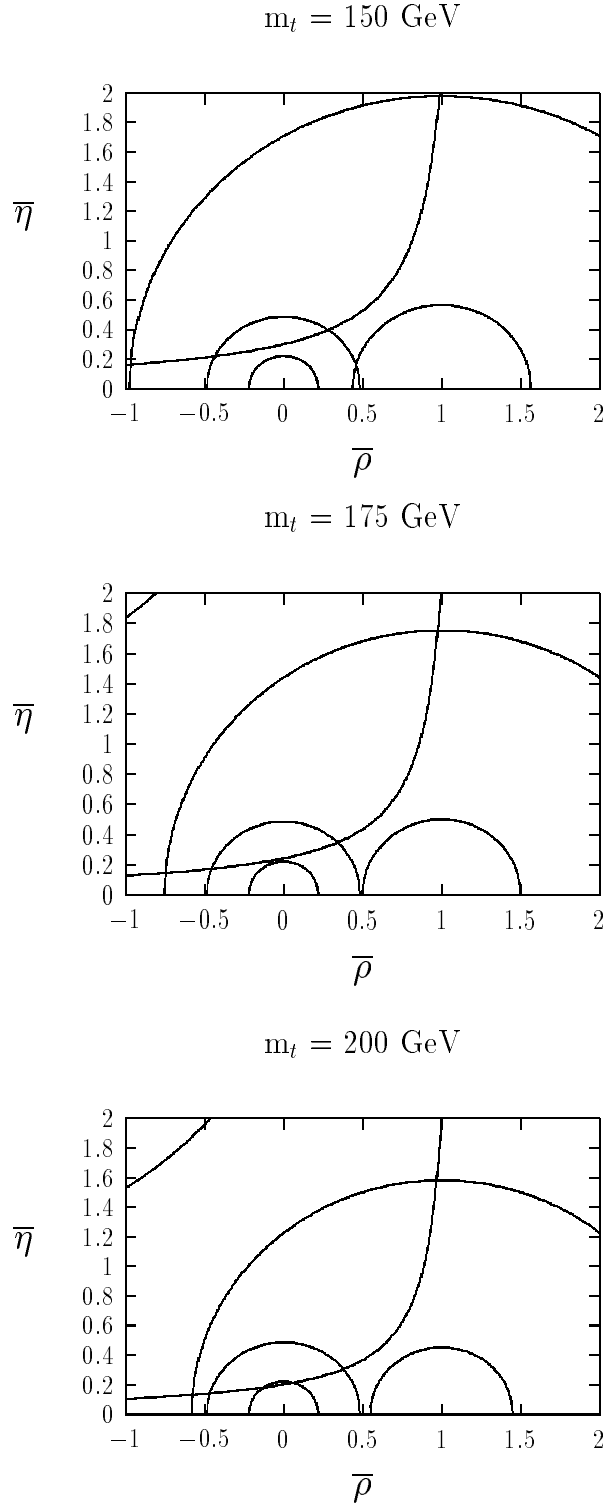


Figure 3: Constraints on the UT for the *conservative* choice of parameters in Table 2.

Parameter	$m_t^{\text{pole}} = 158 \text{ GeV}$	$m_t^{\text{pole}} = 174 \text{ GeV}$	$m_t^{\text{pole}} = 190 \text{ GeV}$
$\sin(2\alpha)_{\text{max}}$	0.96	0.97	0.98
$\sin(2\alpha)_{\text{min}}$	0.43	0.16	-0.10
$\sin(2\beta)_{\text{max}}$	0.76	0.81	0.81
$\sin(2\beta)_{\text{min}}$	0.43	0.44	0.43
$\sin(2\gamma)_{\text{max}}$	0.41	0.68	0.83
$\sin(2\gamma)_{\text{min}}$	-1.0	-1.0	-1.0
$R_b$	$0.42^{+0.03}_{-0.04}$	$0.39 \pm 0.07$	$0.38 \pm 0.07$
$R_t$	$1.17 \pm 0.17$	$1.12 \pm 0.18$	$1.06 \pm 0.18$

Table 3: Numerical results for the *best* estimate set of input parameters.

Parameter	$m_t^{\text{pole}} = 150 \text{ GeV}$	$m_t^{\text{pole}} = 175 \text{ GeV}$	$m_t^{\text{pole}} = 200 \text{ GeV}$
$\sin(2\alpha)_{\text{max}}$	0.95	0.95	0.95
$\sin(2\alpha)_{\text{min}}$	-0.25	-0.60	-0.80
$\sin(2\beta)_{\text{max}}$	0.86	0.87	0.87
$\sin(2\beta)_{\text{min}}$	0.29	0.25	0.18
$\sin(2\gamma)_{\text{max}}$	1.0	1.0	1.0
$\sin(2\gamma)_{\text{min}}$	-1.0	-1.0	-1.0
$R_b$	$0.40 \pm 0.10$	$0.36 \pm 0.14$	$0.36 \pm 0.14$
$R_t$	$1.12 \pm 0.30$	$1.09 \pm 0.36$	$1.07 \pm 0.39$

Table 4: Numerical results for the *conservative* choice of input parameters.

For the *best* estimate set of input parameters we get  $\sin(2\beta) > 0.43$  while for the *conservative* choice we have  $\sin(2\beta) > 0.29, 0.25$  and  $0.18$ , for  $m_t^{\text{pole}} = 150, 175$  and  $200$  GeV, respectively. In this last case, the constraints are mainly imposed by  $R_b$  and the CP-violating parameter  $\varepsilon$ , as can be seen from the figures. The difference between both estimates, namely, *best* versus *conservative*, gives a good idea of the present uncertainties.

Assuming the Standard Model mechanism of CP violation to be correct, one can use the UT analysis to pin down the values of the relevant input parameters. For instance, the measured low values of  $|V_{ub}|/|V_{cb}|$  imply that larger values of  $\widehat{B}_K$  and/or  $|V_{cb}|$  are preferred; alternatively, a low value of  $\widehat{B}_K$  could indicate that the theoretical uncertainties in the analyses of the decays  $B \rightarrow D^* l \bar{\nu}_l$  and  $B \rightarrow X_q l \bar{\nu}_l$  have been underestimated. Similarly, larger values of  $|V_{cb}|$  would favour smaller values of  $\widehat{\xi}_B$  (for a given top mass). However, this kind of analysis is not very satisfactory, because it misses the main motivation for studying the unitarity relation (3): to test the Standard Model mechanism of



CP violation. If more precise experimental data shows some discrepancy between different UT constraints, we would like to know if a violation of CKM universality has been established (i.e. new physics), or if the reason is just a wrong theoretical determination of some (less interesting) non-perturbative parameter. Clearly, a better theoretical understanding of long-distance effects is needed.

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